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Nobel Preis inspired notes on

Bell inequalities and entanglement

Bassano Vacchini @ FG3

Overview

- Press release
- Awardees
- Scientific background
- The heart of the matter
- Detailed achievements
- Further achievements
- Entanglement generation strategies
- Potential for applications

Press release

PRESS RELEASE

4 October 2022

The Nobel Prize in Physics 2022

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2022 to

Alain Aspect
Institut d'Optique Graduate School – Université Paris-Saclay and École Polytechnique, Palaiseau, France

John F. Clauser
J.F. Clauser & Assoc., Walnut Creek, CA, USA

Anton Zeilinger
University of Vienna, Austria

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

that is …

PRESSMEDDELANDE

4 oktober 2022

Nobelpriset i fysik 2022

Kungl. Vetenskapsakademien har beslutat utdela Nobelpriset i fysik 2022 till

Alain Aspect
Institut d'Optique Graduate School – Université Paris-Saclay och École Polytechnique, Palaiseau, Frankrike

John F. Clauser
J.F. Clauser & Assoc., Walnut Creek, CA, USA

Anton Zeilinger
Universität Wien, Österrike

"för experiment med sammanflätade fotoner som påvisat brott mot Bell-olikheter och banat väg för kvantinformationsvetenskap"

Awardees

Alain Aspect
born 1946
in Agen, France

A. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982)

John Francis Clauser
born 1942
in Pasadena, CA, USA

S.J. Freedman and J.F. Clauser, Phys. Rev. Lett. **28**, 938 (1972)

Anton Zeilinger
born 1945
in Ried im Innkreis, Austria

J.-W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter and A. Zeilinger, Nature 4**00**, 515 (1998)

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Scientific background

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There are infinite alternative representations of quantum states

$\hat{A}$-representation

$$\psi = \sum_n \langle u_n^A | \psi \rangle u_n^A$$

$\hat{B}$ -representation

$$\psi = \sum_m \langle v_m^B | \psi \rangle v_m^B$$

with

$$A u_n^A = a_n u_n^A \qquad B v_m^B = b_m v_m^B$$

and generally $[\hat{A}, \hat{B}] \neq 0$

Scientific background

That a pure quantum state is entangled means that it is not separable; for the simplest case of two distinct spinless particles moving on a line, being separable means that the wave function can be written as

$$\psi(x, y) = \psi_1(x)\psi_2(y)$$

while the general form of the wave function is

$$\psi(x, y) = \sum_i c_i \psi_i(x) \psi_i(y)$$

Where the c_i are complex numbers. This basic definition can be generalized not only to states with many particles, and many quantum numbers (such as spin or charge), but also to what are called mixed states that describe classical statistical mixtures of pure states, typically thermal states.[6]

$$\psi = \frac{1}{\sqrt{2}}(\phi_+^{\mathbf{n}} \otimes \phi_-^{\mathbf{n}} - \phi_-^{\mathbf{n}} \otimes \phi_+^{\mathbf{n}}) \qquad \mathbf{n} = \mathbf{x}, \mathbf{y}, \mathbf{z}$$

$$[\hat{S}_i, \hat{S}_i] = i\hbar \sum_{k=1}^{3} \varepsilon_{ijk} \hat{S}_k$$

[6]Also in this respect the the Norwegian Nobel Committee highly recommends the ***Dispense del corso di Fisica Generale 3*** used by students at the Math Department of UNIMI

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The heart of the matter

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Understanding quantum mechanics especially the relationship between the whole and the parts

DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS

BY E. SCHRÖDINGER

[Communicated by Mr M. BORN]

[*Received* 14 August, *read* 28 October 1935]

1. When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. To disentangle them we must

Attention has recently* been called to the obvious but very disconcerting fact that even though we restrict the disentangling measurements to *one* system, the representative obtained for the *other* system is by no means independent of the particular choice of observations which we select for that purpose and which by

* A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.* 47 (1935), 777.

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The heart of the matter

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received 4 November 1964)

Bell was awarded many international honours, and had been nominated for a Nobel Prize, but died suddenly of a stroke in 1990

PROPOSED EXPERIMENT TO TEST LOCAL HIDDEN-VARIABLE THEORIES*

John F. Clauser†
Department of Physics, Columbia University, New York, New York 10027
and
Michael A. Horne
Department of Physics, Boston University, Boston, Massachusetts 02215
and
Abner Shimony
Departments of Philosophy and Physics, Boston University, Boston, Massachusetts 02215
and
Richard A. Holt
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 4 August 1969)

A theorem of Bell, proving that certain predictions of quantum mechanics are inconsistent with the entire family of local hidden-variable theories, is generalized so as to apply to realizable experiments. A proposed extension of the experiment of Kocher and Commins, on the polarization correlation of a pair of optical photons, will provide a decisive test between quantum mechanics and local hidden-variable theories.

$$\left|\mathbb{E}_{LHV}[(a_1+a_2)b_1+(a_2-a_1)b_2]\right| \leq 2$$

$$\left|\mathbb{E}_{QM}[(a_1+a_2)b_1+(a_2-a_1)b_2]\right| \leq 2\sqrt{2}$$

$$LHV \iff p(a_i, b_j \mid \lambda) = p(a_i \mid \lambda)\, p(b_j \mid \lambda) \qquad QM \iff FG3$$

Detailed achievements

In the early 1970s, **John Francis Clauser** succeeded in obtaining a first experimental confirmation that experimental data are compatible with quantum mechanics and violate the bound to correlations set by local hidden-variable theories.

Experimental Test of Local Hidden-Variable Theories*

Stuart J. Freedman and John F. Clauser
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 4 February 1972)

We have measured the linear polarization correlation of the photons emitted in an atomic cascade of calcium. It has been shown by a generalization of Bell's inequality that the existence of local hidden variables imposes restrictions on this correlation in conflict with the predictions of quantum mechanics. Our data, in agreement with quantum mechanics, violate these restrictions to high statistical accuracy, thus providing strong evidence against local hidden-variable theories.

Since quantum mechanics was first developed, there have been repeated suggestions that its statistical features possibly might be described by an underlying deterministic substructure. Such features, then, arise because a quantum state represents a statistical ensemble of "hidden-variable states." Proofs by von Neumann and others, demonstrating the impossibility of a hid-

Detailed achievements

In 1982, **Alen Aspect** highly improved the experimental setup introducing in particular acousto-optical switches to randomly set the polarizers direction, thus successfully addressing the locality loop-hole.

Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard,[(a)] and Gérard Roger
Institut d'Optique Théorique et Appliquée, F-91406 Orsay Cédex, France

(Received 27 September 1982)

Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.

PACS numbers: 03.65.Bz, 35.80.+s

Bell's inequalities apply to any correlated measurement on two correlated systems. For instance, in the optical version of the Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*,[1] a source emits pairs of photons (Fig. 1). Measurements of the correlations of linear polarizations are performed on two photons belonging to the same pair. For pairs emitted in suitable states, the correlations are strong. To account for these correlations, Bell[2] considered theories which invoke common properties of both members of the

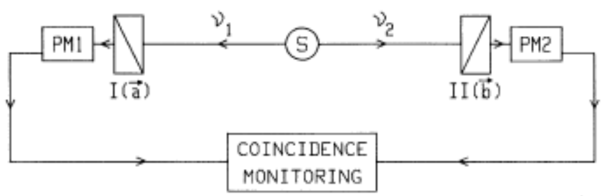

FIG. 1. Optical version of the Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*. The pair of photons ν_1 and ν_2 is analyzed by linear polarizers I and II (in orientations $\vec{a}$ and $\vec{b}$) and photomultipliers. The coincidence rate is monitored.

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Detailed achievements

Anton Zeilinger contributed to closure of the locality and detection loopholes, experimentally demonstrated entanglement swapping, quantum teleportation as well as exclusion of local-hidden variables via GHZ states. He further contributed to obtain efficient generation of entangled photon pairs and demonstration of interference with macroscopic systems.

PHYSICAL REVIEW LETTERS

VOLUME 80 | 4 MAY 1998 | NUMBER 18

Experimental Entanglement Swapping: Entangling Photons That Never Interacted

Jian-Wei Pan, Dik Bouwmeester, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 6 February 1998)

We experimentally entangle freely propagating particles that never physically interacted with one another or which have never been dynamically coupled by any other means. This demonstrates that quantum entanglement requires the entangled particles neither to come from a common source nor to have interacted in the past. In our experiment we take two pairs of polarization entangled photons and subject one photon from each pair to a Bell-state measurement. This results in projecting the other two outgoing photons into an entangled state. [S0031-9007(98)05913-4]

PACS numbers: 03.65.Bz, 03.67.–a, 42.50.Ar

PHYSICAL REVIEW LETTERS

VOLUME 80 | 9 FEBRUARY 1998 | NUMBER 6

Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

D. Boschi,[1] S. Branca,[1] F. De Martini,[1] L. Hardy,[1,2] and S. Popescu[3,4]
[1]*Dipartimento di Fisica, Istituto Nazionale di Fisica Nucleare, Istituto Nazionale di Fisica della Materia, Università "La Sapienza," Roma 00185, Italy*
[2]*Clarendon Laboratory, University of Oxford, Oxford OX1 3PU, United Kingdom*
[3]*Isaac Newton Institute, University of Cambridge, Cambridge CB3 0EH, United Kingdom*
[4]*BRIMS, Hewlett-Packard Laboratories, Bristol BS12 SQZ, United Kingdom*
(Received 28 July 1997)

articles

Experimental quantum teleportation

Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger

Institut für Experimentalphysik, Universität Innsbruck, Technikerstr. 25, A-6020 Innsbruck, Austria

Quantum teleportation—the transmission and reconstruction over arbitrary distances of the state of a quantum system—is demonstrated experimentally. During teleportation, an initial photon which carries the polarization that is to be transferred and one of a pair of entangled photons are subjected to a measurement such that the second photon of the entangled pair acquires the polarization of the initial photon. This latter photon can be arbitrarily far away from the initial one. Quantum teleportation will be a critical ingredient for quantum computation networks.

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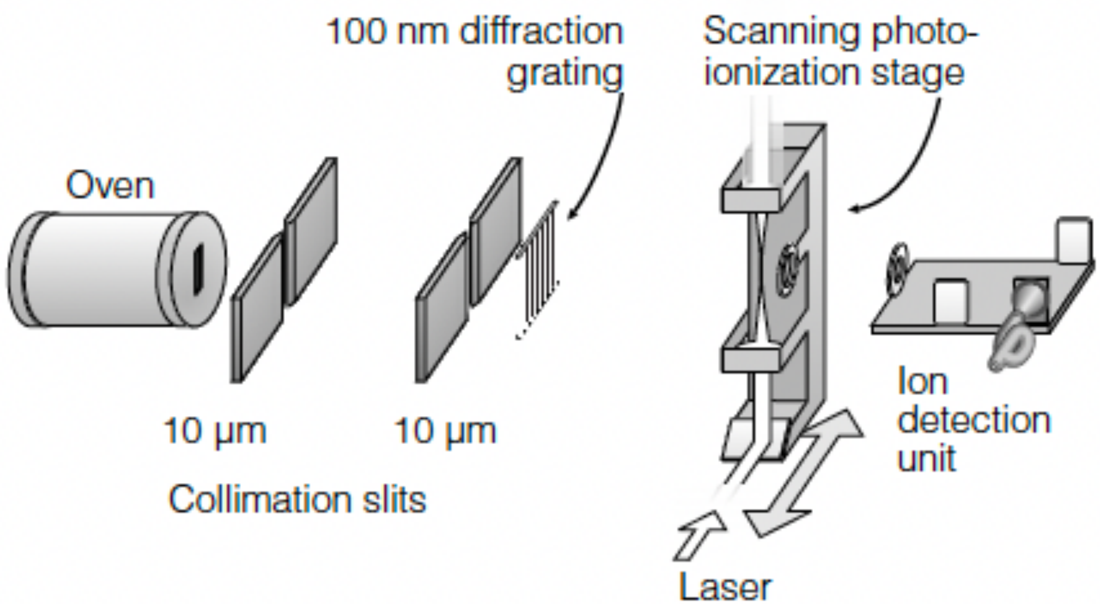
Further achievements

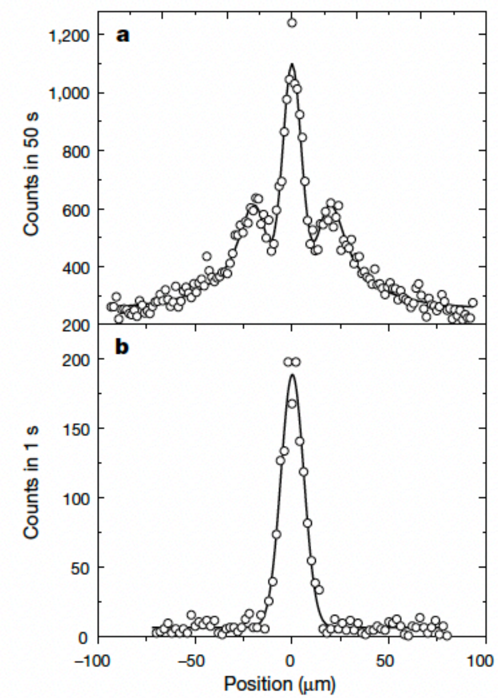
letters to nature

Wave–particle duality of C_{60} molecules

Markus Arndt, Olaf Nairz, Julian Vos-Andreae, Claudia Keller, Gerbrand van der Zouw & Anton Zeilinger

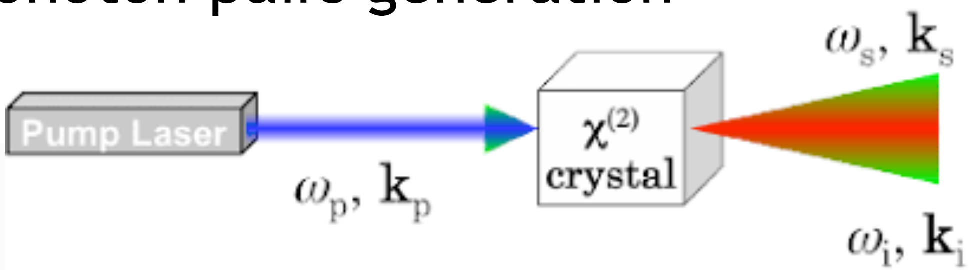
Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, A-1090 Wien, Austria

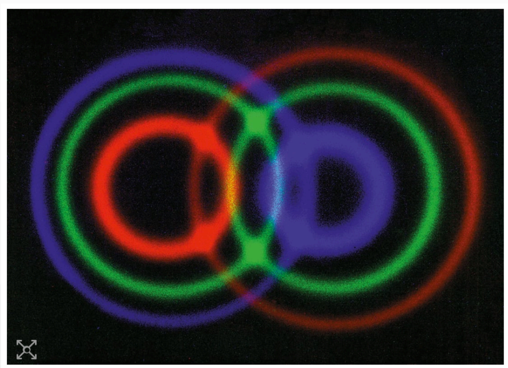


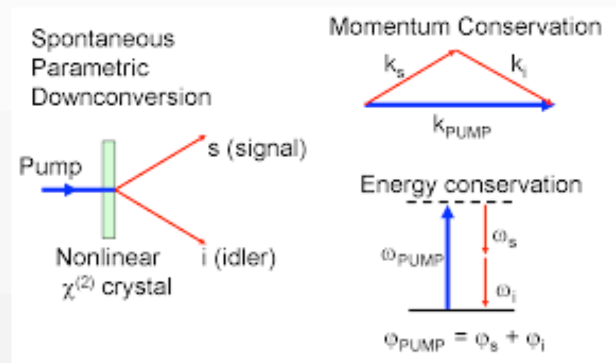


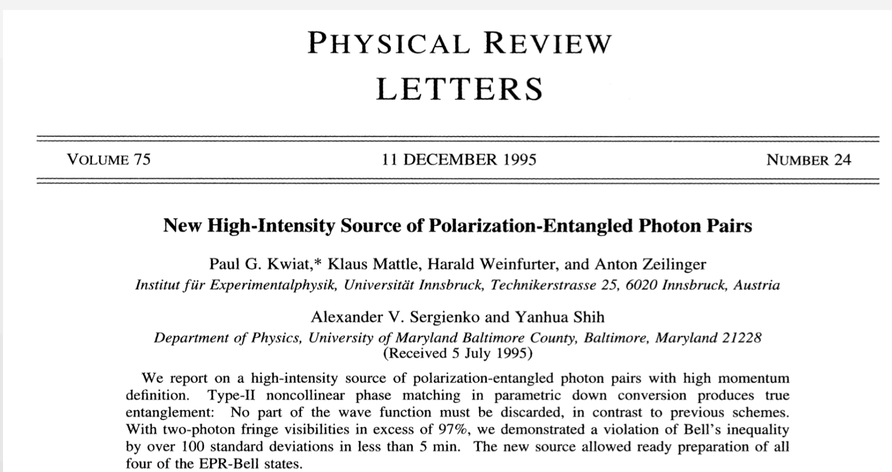
Entanglement generation strategies

Entangled photon pairs generation









PHYSICAL REVIEW LETTERS

VOLUME 75 | 11 DECEMBER 1995 | NUMBER 24

New High-Intensity Source of Polarization-Entangled Photon Pairs

Paul G. Kwiat,* Klaus Mattle, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, 6020 Innsbruck, Austria

Alexander V. Sergienko and Yanhua Shih
Department of Physics, University of Maryland Baltimore County, Baltimore, Maryland 21228
(Received 5 July 1995)

We report on a high-intensity source of polarization-entangled photon pairs with high momentum definition. Type-II noncollinear phase matching in parametric down conversion produces true entanglement: No part of the wave function must be discarded, in contrast to previous schemes. With two-photon fringe visibilities in excess of 97%, we demonstrated a violation of Bell's inequality by over 100 standard deviations in less than 5 min. The new source allowed ready preparation of all four of the EPR-Bell states.

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Entanglement generation strategies

nature Vol 461|24 September 2009|doi:10.1038/nature08363

LETTERS

Violation of Bell's inequality in Josephson phase qubits

Markus Ansmann[1], H. Wang[1], Radoslaw C. Bialczak[1], Max Hofheinz[1], Erik Lucero[1], M. Neeley[1], A. D. O'Connell[1], D. Sank[1], M. Weides[1], J. Wenner[1], A. N. Cleland[1] & John M. Martinis[1]

Observation of Bell inequality violation in B mesons

APOLLO GO (Belle Collaboration)
Department of Physics, National Central University Chung-Li, Taiwan; e-mail: apollo.go@cern.ch

(*Received 31 October 2003*)

Abstract. A pair of $B^0\bar{B}^0$ mesons from $\Upsilon(4S)$ decay exhibit EPR type non-local particle-antiparticle (flavour) correlation. It is possible to write down the Bell inequality (in the CHSH form: $S \leq 2$) to test the non-locality assumption of EPR. Using semileptonic B^0 decays of $\Upsilon(4S)$ at Belle experiment, a clear violation of the Bell inequality in particle–antiparticle correlation is observed:

$$S = 2.725 \pm 0.167_{\rm stat} \pm 0.092_{\rm syst}.$$

Experimental violation of a Bell's inequality with efficient detection

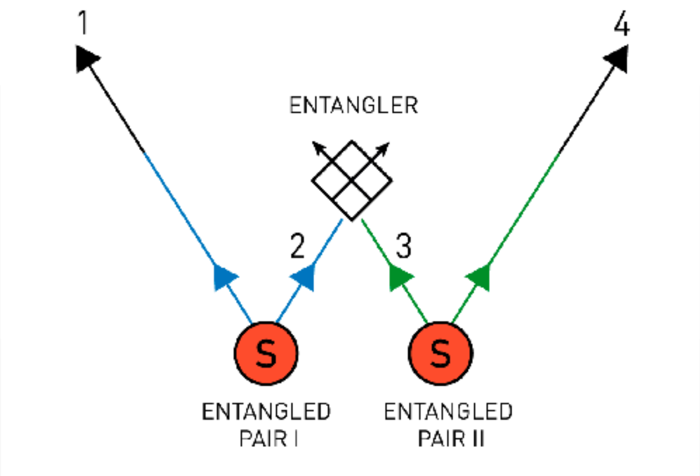
M. A. Rowe*, D. Kielpinski*, V. Meyer*, C. A. Sackett*, W. M. Itano*, C. Monroe† & D. J. Wineland*

* *Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80305, USA*
† *Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*

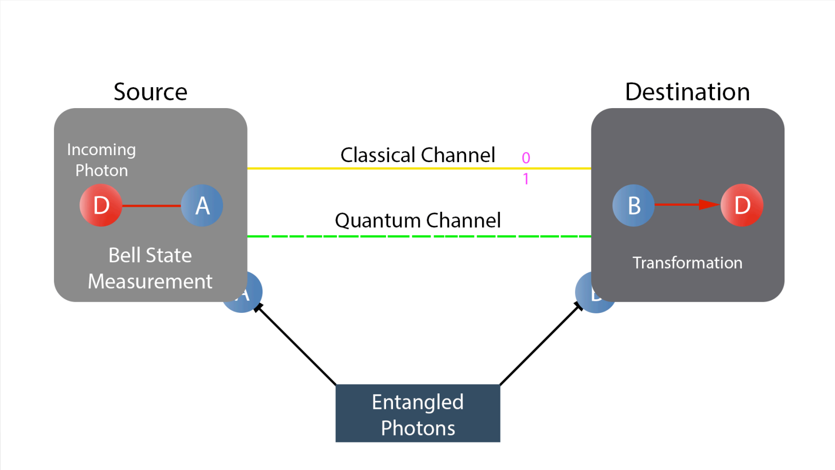
Local realism is the idea that objects have definite properties whether or not they are measured, and that measurements of these properties are not affected by events taking place sufficiently far away[1]. Einstein, Podolsky and Rosen[2] used these reasonable assumptions to conclude that quantum mechanics is incomplete. Starting in 1965, Bell and others constructed mathematical inequalities whereby experimental tests could distinguish between quantum mechanics and local realistic theories[1,3–5]. Many experiments[1,6–15] have since been done that are consistent with quantum mechanics and inconsistent with local realism. But these conclusions remain the subject of considerable interest and debate, and experiments are still being refined to overcome 'loopholes' that might allow a local realistic interpretation. Here we have measured correlations in the classical properties of massive entangled particles ($^9Be^+$ ions): these correlations violate a form of Bell's inequality. Our measured value of the appropriate Bell's 'signal' is 2.25 ± 0.03, whereas a value of 2 is the maximum allowed by local realistic theories of nature. In contrast to previous measurements with massive particles, this violation of Bell's inequality was obtained by use of a complete set of measurements. Moreover, the high detection efficiency of our apparatus eliminates the so-called 'detection' loophole.

Potential for applications

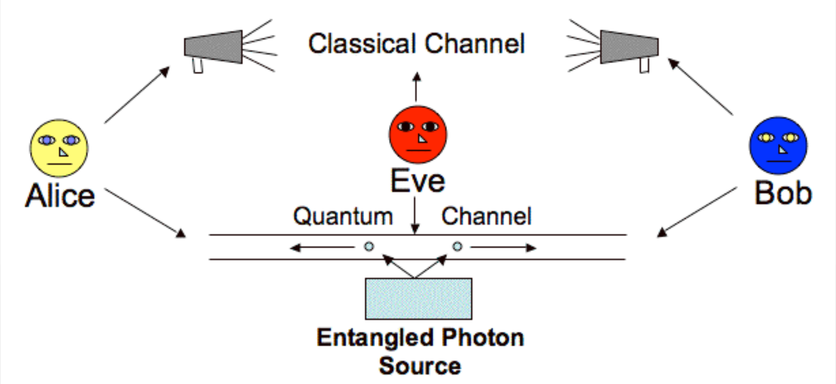
Quantum entanglement swapping



Quantum teleportation

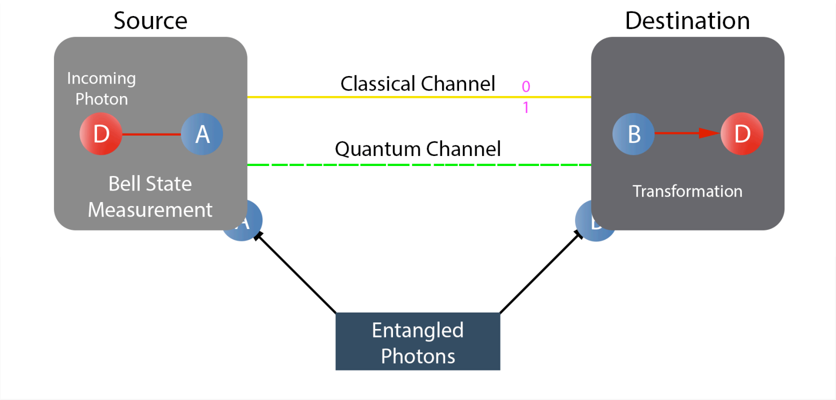


Quantum cryptography

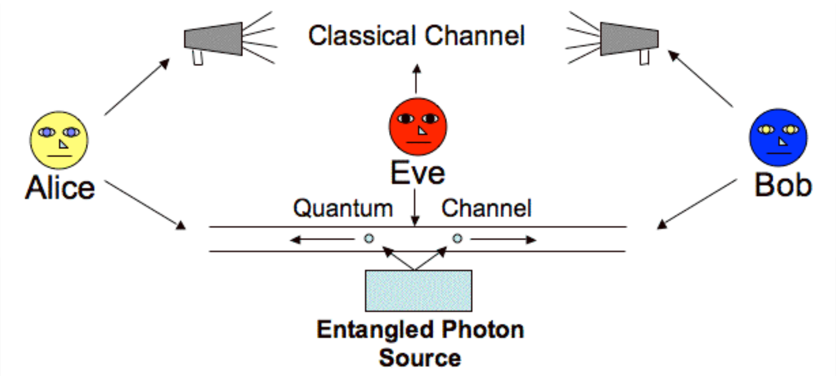


Potential for applications

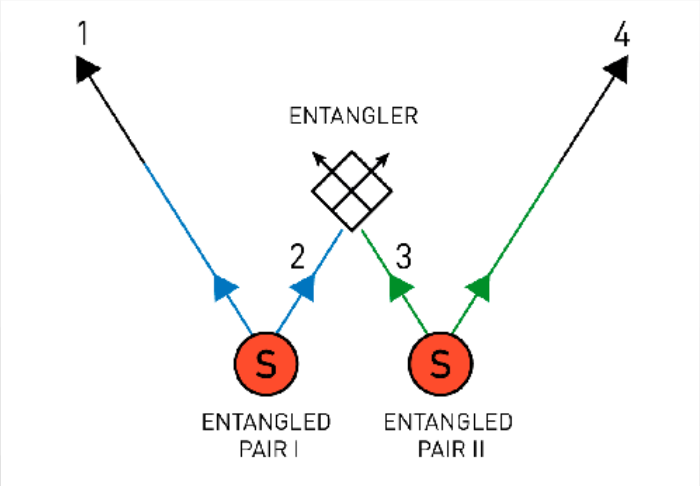
Quantum teleportation



Quantum cryptography



Quantum entanglement swapping



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Sources

- The Nobel Prize in Physics 2022
- Press release
- Scientific motivation
- DPG announcement
- Physik Journal article