A Probabilistic View on Decoherence Theory

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Abstract. The phenomenon of decoherence appears when the quantum behaviour of a microsystem is investigated. If the shielding of the considered microsystem from the macroscopic environment is not perfect, typical quantum features, such as the capability to exhibit quantum fringes, are suppressed as an unavoidable result of the interaction with the macroscopic background. Focusing on the centre of mass degrees of freedom of a massive test particle decoherence can typically be described in terms of random momentum transfers with the environment, thus naturally calling for a probabilistic standpoint. In this framework a general connection between the characteristic function of a Lévy process and loss of coherence of a massive quantum system interacting through momentum transfer events with an environment will be put into evidence, relying on Holevo's characterization of quantum dynamical semigroups in the presence of translational invariance. The relationship with microphysical models and recent experiments on decoherence will also be considered.

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In recent times the word decoherence[1, 2, 3, 4] has become quite fashionable in order to describe a range of utterly different physical situations, which however all exhibit a common qualitative feature: a quantum system, due to its unavoidably imperfect isolation from the surrounding environment, shows in its dynamical evolution the suppression of typical quantum coherence properties, such as interference capability.

The basic ideas are actually very old and as it was recently stressed in[5] can be essentially traced back to the first studies on the measurement problem in quantum mechanics in the 50's (for a reference see[6, 7]). These studies, in which the main concepts related to decoherence already appeared, has led by now to relevant improvements in the formulation of quantum mechanics, going beyond Dirac's presentation and leading to the new concept of effect, positive operator valued measure, operation and instrument; these more advanced and flexible tools in the description of quantum systems are by now extensively used in quantum information and communication theory[8, 9], as well as open system theory[2]. Most importantly these lines of research have disclosed most fruitful and interesting connections with the theory of stochastic processes; it appears indeed that many useful clues in studying the theory of quantum systems can be obtained from classical probability theory. In this contribution we will focus on recent work[10] showing how the characteristic function of a classical Lévy process, i.e. the Fourier transform of its probability density, is the natural quantity in order to describe loss of coherence in interferometry experiments with massive particles, thus providing an example of the relevance of concepts and ideas taken from classical probability theory for the description of a typical quantum dynamics. A probabilistic approach to decoherence has been also pursued, from a different perspective, in[11, 12], where the reduction of

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a quantum-dynamical semigroup to a classical Markov semigroup on an Abelian subalgebra of observables has been studied, accounting for the suppression of off-diagonal matrix elements, so-called coherences, of the statistical operator in the long time limit.

The aforementioned studies on the foundations of quantum mechanics also give a natural setting for the understanding of decoherence. Following Ludwig's approach to quantum mechanics[6] microsystems only appear anchored to an objectively given macroscopic reality. The very concept of microsystem described inside one-particle quantum mechanics than turns out to be a highly schematized description of a typical non equilibrium system, where a "source part" affects in a stochastic way a "detecting part". In this spirit a field theoretical description of non equilibrium systems, due to Zubarev[13], where dynamics is given through classical state parameters, has been reformulated[14] in order to provide an objective background for microsystems inside quantum mechanics. It has been suggested[15] that the breakdown of a deterministic dynamics of state parameters can be linked with a stochastic behaviour giving evidence of a microsystem. Just in the relativistic case, which strictly speaking does not allow the existence of a one-particle dynamics in presence of external fields, this apparently cumbersome approach might actually be unavoidable. The macroscopic background in which these microsystems emerge indicates that decoherence might naturally occur and methods of open system theory come in the foreground. Such a description should apply to all situations in which the almost perfect shielding of this microscopic interaction channel necessary for the full appearance of quantum mechanical features cannot be obtained.

We now focus on the issue of decoherence of the centre of mass degrees of freedom of massive test particles, an object of recent and very accurate quantitative experimental investigations[16, 17, 18, 19], showing how these different situations can be addressed within a unified theoretical approach which puts into evidence how the loss of coherence in the off-diagonal position matrix elements of the statistical operator is generally described by the characteristic function of a Lévy process. The common feature of the abovementioned experiments is the fact that, provided dissipative effects which take place on a much longer time scale are neglected, the interaction causing decoherence can be characterized through momentum transfer events. In the Markovian case a common description of such dynamics can be obtained referring to the general structure of translation-covariant quantum-dynamical semigroups obtained by Holevo[20], relying on a quantum generalization of the Lévy-Khintchine formula. Lévy processes are a class of processes, including Gaussian processes, which despite obeying the Chapman-Kolmogorov equation characterizing Markov processes not necessarily have finite variance, so that the central limit theorem does not always apply. Such processes were in fact found looking for generalizations of such theorem, and are both space and time homogeneous, thus naturally arising when considering space translation invariance. The general structure of the characteristic function of such processes is given by the famous Lévy-Khintchine formula (for a most compact presentation see[2] and references therein). The relevance of Lévy process in physics is growing[21, 22], since they allow to cope with situations not encompassed by the central limit theorem.

We first start by introducing in a way adapted to our purposes the results by Holevo[20]. If the dynamics causing decoherence is Markovian and described in terms of momentum transfers, so that in the absence of an external potential one has translation invariance, the generator of the quantum-dynamical semigroup generally has the structure

$$d\hat{\rho}/dt = \mathscr{L}_G[\hat{\rho}] + \mathscr{L}_P[\hat{\rho}] \tag{1}$$

with $\hat{\rho}$ the statistical operator of the test particle; $\mathscr{L}_G$ a so-called Gaussian component given by

$$\mathscr{L}_G[\hat{\rho}] = -ia[\hat{x},\hat{\rho}] - \frac{1}{2}D[\hat{x},[\hat{x},\hat{\rho}]], \tag{2}$$

written for simplicity in the one dimensional case, with $a \in \mathbb{R}$, $D > 0$ and $\hat{x}$ the position operator of the test particle; $\mathscr{L}_P$ the so-called Poisson component

$$\mathscr{L}_P[\hat{\rho}] = \int dq\, |\omega(q)|^2 \left[e^{\frac{i}{\hbar}q\hat{x}}\hat{\rho}e^{-\frac{i}{\hbar}q\hat{x}} - \hat{\rho} - \frac{i}{\hbar}\frac{q[\hat{x},\hat{\rho}]}{1+q^2/q_0^2} \right] \tag{3}$$

where $|\omega(q)|^2 dq$ is a positive measure, also called Lévy measure, with $|\omega(q)|^2$ possibly divergent in zero but such that the Lévy condition

$$\int dq\, |\omega(q)|^2 \frac{q^2}{1+q^2} < \infty$$

holds, the integration variable q has the dimension of momentum and the meaning of momentum transfer, the parameter q_0 only appearing for dimensional purposes in the regularizing factor. In stating the result we have neglected free evolution and dissipative effects which are relevant only on a much longer time scale, so that the momentum of the test particle has essentially been treated as a $\mathbb{C}$-number. Focusing on the position matrix elements the master-equation takes the form

$$\frac{d}{dt}\langle x|\hat{\rho}|y\rangle = -\Psi(x-y)\langle x|\hat{\rho}|y\rangle, \tag{4}$$

so that one immediately has the general solution

$$\langle x|\hat{\rho}_t|y\rangle = e^{-t\Psi(x-y)}\langle x|\hat{\rho}_0|y\rangle, \tag{5}$$

with

$$\Psi(x-y) = ia(x-y) + \frac{1}{2}D(x-y)^2 - \int dq\, |\omega(q)|^2 \left[e^{\frac{i}{\hbar}q(x-y)} - 1 - \frac{i}{\hbar}\frac{q(x-y)}{1+q^2/q_0^2} \right]. \tag{6}$$

It turns out[10, 20] that the function

$$\Phi(t,x-y) \equiv e^{-t\Psi(x-y)} \tag{7}$$

is the characteristic function of a Lévy process, $\Psi(x-y)$ being called its characteristic exponent, the quantity actually fully characterized by the Lévy-Khintchine formula. The fact that $\Phi(t,x-y)$ is a characteristic function automatically entails that its modulus is less than one and the value one for $x-y$ tending to zero, i.e. the natural properties in order to predict the reduction of the off-diagonal matrix elements in (5) due to decoherence. This suppression of coherence however happens with a variety of behaviours going far beyond the quadratic common lore corresponding to Gaussian statistics, depending on the process characterizing the physical interaction.

We now briefly present a microphysical model giving a specific realizations of (4). As it turns out Eq.(4) actually encompasses all known models of decoherence for the centre of mass degrees of freedom[1]. Let us consider the motion of a massive test particle interacting through collisions with a background gas, studied in detail in[23] and further developed in[24], where also dissipative effects have been taken into account, relying on a kinetic approach. Neglecting free motion and dissipation the result becomes

$$\frac{d}{dt}\langle \boldsymbol{x}|\hat{\rho}|\boldsymbol{y}\rangle = n(2\pi)^4\hbar^2 \int d^3\boldsymbol{q}\, |\tilde{t}(q)|^2 S(\boldsymbol{q},E) \left[e^{\frac{i}{\hbar}\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})} - 1 \right] \langle \boldsymbol{x}|\hat{\rho}|\boldsymbol{y}\rangle \tag{8}$$

where n is the gas density, $\tilde{t}(q)$ the Fourier transform of the interaction potential and S a two-point correlation function characterizing the gas known as dynamic structure factor depending on both momentum and energy transfer ($\boldsymbol{q}$ and E). For a finite macroscopic scattering cross-section $\sigma = (2\pi)^4\hbar^2(M/p_0)\int d^3\boldsymbol{q}\, |\tilde{t}(q)|^2 S(\boldsymbol{q},E)$, with M the mass of the test particle and p_0 its incoming momentum, one can introduce a scattering rate $\Lambda \equiv n(p_0/M)\sigma$ and a suitably normalized probability density

$$\mathscr{P}(\boldsymbol{q}) = \frac{n}{\Lambda}(2\pi)^4\hbar^2 |\tilde{t}(q)|^2 S(\boldsymbol{q},E), \tag{9}$$

so that (5) reads

$$\langle \boldsymbol{x}|\hat{\rho}_t|\boldsymbol{y}\rangle = e^{-\Lambda[1-\Phi_{\mathscr{P}}(\boldsymbol{x}-\boldsymbol{y})]t} \langle \boldsymbol{x}|\hat{\rho}_0|\boldsymbol{y}\rangle, \tag{10}$$

where we have introduced the characteristic function $\Phi_{\mathscr{P}}$ associated to the probability density $\mathscr{P}$, i.e. its Fourier transform. Here no confusion should arise: the exponential function in (10) is the characteristic function of a Lévy process which in this particular case can be expressed in terms of the characteristic function $\Phi_{\mathscr{P}}$ of the probability density $\mathscr{P}$. Eq.(10) is a particular realization of (5), corresponding to a compound Poisson process[25]. The physical picture behind it is the following: the dynamics is driven by collisions, the probability of having a definite number of collisions in a time t being given by a Poisson distribution with intensity Λ and mean Λt, each collision however is not characterized by a fixed, deterministic value of the transferred momentum $\boldsymbol{q}$, but rather by a certain probability density $\mathscr{P}(\boldsymbol{q})$ depending in the case under consideration on the two-body interaction potential and a suitable correlation function. The result (10) generally applies to a situation in which one has a collection of momentum transfer events each characterized by a certain probability density (to be obtained or introduced by means of some microscopic or phenomenological model) corresponding to a compound Poisson process. It has been used for the theoretical analysis of decoherence experiments with fullerenes, both in the case of collisional decoherence[17] and of decoherence due to thermal emission of radiation[18]. Both situations correspond to compound Poisson processes, where the relevant probability density $\mathscr{P}(\boldsymbol{q})$ is obtained in terms of the collisional cross-section and the spectral photon emission rate respectively. According to a detailed theoretical analysis the final visibility is obtained by an average of the characteristic exponent in (10) over the possible scattering positions in the interferometer[26, 27].

We have thus sketched a simple theoretical framework allowing for a unified analysis of various decoherence experiments. This approach should generally be valid in the presence of translational invariance, leading to the realization of different Lévy processes depending on the relevant environment and interactions considered.

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