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## Mathematical characterization and physical examples of translation-covariant Markovian master equations

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The notion of covariance under a given symmetry group has proved to be very powerful in characterizing different mathematical structures relevant for quantum mechanics and especially for quantum information and computation, such as positive operator-valued measures and channels. In the same spirit, more recent work has been devoted to exploit the notion of covariance in order to point out interesting structures of generators of quantum-dynamical semigroups, which describe the Markovian dynamics of an open system.

In particular Holevo has given a full characterization of possible generators of quantum-dynamical semigroups for the case of covariance under translations, relying on a non-commutative generalization of the Lévy-Khintchine formula [1]. These results provide a natural setting to look in a unified way at different master-equations used for the description of decoherence of the center of mass degrees of freedom. Since one generally has to deal also with unbounded operators, the general strategy has been to characterize the so-called form-generators, which essentially amounts to provide a formal operator expression and an invariant domain. The generator may be expressed in the Heisenberg picture as

$$\mathcal{L}[\hat{X}] = i[H(\hat{\mathbf{p}}), \hat{X}] + \mathcal{L}_G[\hat{X}] + \mathcal{L}_P[\hat{X}]$$

putting into evidence a Gaussian and a Poisson component

$$\mathcal{L}_G[\hat{X}] = i\left[\hat{\mathbf{y}}_0 + \frac{1}{2i}\sum_{k=1}^{3}(\hat{\mathbf{y}}_k L_k(\hat{\mathbf{p}}) - L_k^\dagger(\hat{\mathbf{p}})\hat{\mathbf{y}}_k), \hat{X}\right]$$
$$+\sum_{k=1}^{3}\left[(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}}))^\dagger \hat{X}(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}})) - \frac{1}{2}\left\{(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}}))^\dagger(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}}))\,, \hat{X}\right\}\right]$$

$$
\begin{aligned}
\mathcal{L}_P[\hat{X}] =& \int \sum_{j=1}^{\infty} \left[ L_j^\dagger(\mathbf{q},\hat{\mathbf{p}}) \hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) L_j(\mathbf{q},\hat{\mathbf{p}}) - \frac{1}{2} \left\{ L_j^\dagger(\mathbf{q},\hat{\mathbf{p}}) L_j(\mathbf{q},\hat{\mathbf{p}}), \hat{X} \right\} \right] d\mu(\mathbf{q}) \\
&+ \int \sum_{j=1}^{\infty} \Big[ \omega_j(\mathbf{q}) L_j^\dagger(\mathbf{q},\hat{\mathbf{p}}) (\hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) - \hat{X}) \\
&\qquad + (\hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) - \hat{X}) L_j(\mathbf{q},\hat{\mathbf{p}}) \omega_j^*(\mathbf{q}) \Big] d\mu(\mathbf{q}) \\
&+ \int \sum_{j=1}^{\infty} \left[ \hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) - \hat{X} - i \frac{[\hat{X}, \mathbf{q}\cdot\hat{\mathbf{x}}]}{1+|\mathbf{q}|^2} \right] |\omega_j(\mathbf{q})|^2 d\mu(\mathbf{q})
\end{aligned}
$$

with $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$ position and momentum operators, $\hat{\mathbf{y}}_j = \sum_{i=1}^{3} a_{ji} \hat{\mathbf{x}}_i, a_{ji} \in \mathbf{R}$ $(j = 0, \ldots, 3)$, $\hat{U}(\mathbf{q}) = e^{i\mathbf{q}\cdot\hat{\mathbf{x}}}$, and the other functions depending on the system considered.

A general structure of master-equation for the description of both dissipation and decoherence of the center of mass degrees of freedom of a quantum system interacting through collisions with a homogeneous fluid has been obtained in [2] providing a physical realization of the Poisson component according to

$$
\begin{aligned}
&\mathcal{L}[\hat{\rho}] = -i \left[ H_0(\hat{\mathbf{p}}), \hat{\rho} \right] \\
&+ \frac{2\pi}{\hbar} (2\pi\hbar)^3 n \int d^3\mathbf{q}\, |\tilde{t}(q)|^2 \left[ \hat{U}(\mathbf{q}) \sqrt{S(\mathbf{q},\hat{\mathbf{p}})} \hat{\rho} \sqrt{S(\mathbf{q},\hat{\mathbf{p}})} \hat{U}^\dagger(\mathbf{q}) - \frac{1}{2} \left\{ S(\mathbf{q},\hat{\mathbf{p}}), \hat{\rho} \right\} \right]
\end{aligned}
$$

corresponding to the Schödinger picture, with $S(\mathbf{q},\mathbf{p})$ a two-point correlation function known as dynamic structure factor, here appearing operator-valued, $n$ the particle density in the fluid, $\tilde{t}(q)$ the Fourier transform of the interaction potential. Neglecting the dependence on the momentum operator, which is responsible for the dissipative effects, one recovers the typical structure of master-equation recently used in the quantitative experimental assessment of collisional decoherence [3]. Considering furthermore the limit of small momentum and energy transfer one obtains a quantum description of Brownian motion [4]

$$
\begin{aligned}
&\mathcal{L}[\hat{\rho}] = -i \left[ H_0(\hat{\mathbf{p}}), \hat{\rho} \right] \\
&\qquad - \eta \sum_{i=1}^{3} \left\{ \frac{i}{2\hbar} \left[ \hat{\mathbf{x}}_i, \{ \hat{\mathbf{p}}_i, \hat{\rho} \} \right] + \frac{\Delta p_{\text{th}}^2}{\hbar^2} \left[ \hat{\mathbf{x}}_i, [\hat{\mathbf{x}}_i, \hat{\rho}] \right] + \frac{\Delta x_{\text{th}}^2}{4\hbar^2} \left[ \hat{\mathbf{p}}_i, [\hat{\mathbf{p}}_i, \hat{\rho}] \right] \right\}
\end{aligned}
$$

with $\eta$ a microscopically determined friction coefficient, $\Delta p_{\text{th}}^2 = M/\beta$ and $\Delta x_{\text{th}}^2 = \beta\hbar^2/4M$, used in the recoilless approximation in order to estimate decoherence effects and giving a physical realization of the Gaussian component.

It thus appears that all these different master-equations used in the physical literature share the common feature of translation covariance, which strongly characterizes their structure.

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## Quantum Information as Private Information

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### 1. No measurement without disturbance

Quantum information theory deals with the kind of information carried by systems described by quantum theory. Among the characteristic differences between these systems and classical ones is their great sensitivity to perturbations, demonstrated, for example in Heisenberg's Uncertainty Relations.

Another sharp formulation of this fundamental fact is the Theorem summarized as "No measurement without disturbance". It refers to a general kind of measurement, by which some classical data are obtained from a quantum system, leaving the system in a typically changed state for further experimentation. The Theorem considers measurements introducing *no* disturbance, in the sense that all statistical experiments with the output particles (without selecting according to the measurement outcomes) give exactly the same expectations as the corresponding experiment on the input particles. The conclusion is that in this case the measured outcomes are independent of the input, i.e., the whole measurement can effectively be replaced by a classical random generator producing "outcomes" completely unrelated to the quantum system. In other words, such a device does not measure anything.

This guarantees privacy of transmitted information in a very strong way: if sender and receiver operating the channel $T$ can verify that their channel is ideal, then they can be sure that nothing whatsoever can be learned from observing the environment of the channel: no tapping of wires, and no "receive, read and resend" (or "man in the middle") attack has a chance. Of course, verifying that the channel is ideal is itself a statistical problem, so there is a subtle tradeoff