JOURNAL OF MODERN OPTICS, 2000, VOL. 47, NO. 12, 2165–2180

Subdynamics as a mechanism for objective description

LUDOVICO LANZ†, OLAF MELSHEIMER‡ and BASSANO VACCHINI†‡

† Dipartimento di Fisica dell'Università di Milano and INFN, Sezione di Milano, Via Celoria 16, I-20133, Milano, Italy

‡ Fachbereich Physik, Philipps–Universität, Renthof 7, D-35032, Marburg, Germany

(*Received 25 November 1999*)

Abstract. The relationship between microsystems and macrosystems is considered in the context of quantum field formulation of statistical mechanics: it is argued that problems on foundations of quantum mechanics can be solved relying on this relationship. This discussion requires some improvement of non-equilibrium statistical mechanics which is briefly presented.

1. Evolution of the atomistic idea: the new relationship micro-macrosystems

Classical physics was fully compatible with a deeply rooted, venerable philosophy: the wealth of objects, called *macrosystems*, we become aware of in our experience at a certain space–time scale, arises by different structures of very many, very elementary, possibly of very few kinds, unavoidably elusive objects, *microsystems* at the previous scale; already in ancient times Lucretio called them *primordia rerum*. For a physicist the fundamental challenge is to learn about microsystems and discover the basic links; what remains is to face complexity: correlations of simple properties of single microsystems provide all describable variety. One expects that this scheme can be reproduced at smaller time scales, to leave open the adventure of discerning more remote links. Since generally a human being feels himself as not completely fitting inside the aforementioned scheme, he might be happy to cut himself apart from the world of objects, assuming the role of an *observer* and, quantum mechanics neatly seemed to support this dream. Belief in all this, although popular, has however a drawback: it makes typical quantum mechanical features like entanglement between microsystems paradoxical and puzzling. Therefore much effort has been done to experimentally check these typical features, which are deeply rooted in the basic mathematical structure of quantum mechanics: as a result quantum mechanics has passed these tests in a brilliant way. Now a more subtle philosophy should be assumed, which, as always happens in physics, does not mean complete dismission of the previous one, but a reappraisal of it: something of it remains true, e.g. when some classical limit holds; the basic ideas however should change. As is well known, quantum mechanics is equivalent to a quantum field theory based on quantization of classical interacting and self-interacting Schrödinger fields (one for each type of particle); in the case of

Journal of Modern Optics ISSN 0950–0340 print/ISSN 1362–3044 online
http://www.tandf.co.uk/journals

macrosystems, as will be shown in section 3, the field formulation becomes very important for the construction of the densities of conserved quantities and the related currents; finally the fact that the field approach becomes mandatory if relativity is taken into account is decisive. The atomistic model essentially transforms into a picture of dynamics for any system, of varying complexity, which is driven by local, simple, universal interactions between fields. Quantization on the other hand implies that mass and electric charges and other conserved quantities have a discrete structure, thus pointing to the existence of *particles* that acquire a leading role in any irreducible *event* produced by field dynamics. We would like to call attention to a crucial point, that is taken into due consideration in quantum optics but is generally overlooked, probably due to the old philosophy, in quantum field theory: the macroscopic setting which characterizes any concrete system implies boundary conditions on the fields by which *normal modes* arise. Mass, charge and energy of the system are then distributed through quantum population of this huge set of normal modes which are in turn involved if these populations are changed due to local interactions. In this way an objective macroscopic property of the physical system, precisely its *shape*, influences in an important way what the microsystems leading the quantum dynamics are, at complete variance with the old philosophy. Take for example massive matter inside a container, with perfectly reflecting walls, enclosing a space region ω; normal modes are then a set of stationary waves, complete and orthonormal in $L^2(\omega)\otimes \mathbf{C}^{2s+1}$, $u_r(\mathbf{x},\sigma)\times \exp[-(\mathrm{i}W_r t]/h)$, σ denoting the spin variable, such that

$$-\frac{\hbar^2}{2m}\Delta_2 u_r(\mathbf{x},\sigma) = W_r u_r(\mathbf{x},\sigma), \quad u_r(\mathbf{x},\sigma) = 0, \mathbf{x}\in\partial\omega.$$

The Schrödinger operator is given by

$$\hat{\psi}(\mathbf{x},\sigma) = \sum_r \hat{a}_r u_r(\mathbf{x},\sigma),$$

with $[\hat{a}_r, \hat{a}^\dagger_{r'}]_\pm = \delta_{r,r'}$, the basic quantity is mass $\hat{M} = \int_\omega \mathrm{d}^3\mathbf{x}\hat{\rho}(\mathbf{x}) = m\sum_r \hat{a}^\dagger_r \hat{a}_r$, where the density is $\hat{\rho}(\mathbf{x}) = \sum_\sigma m\hat{\psi}^\dagger(\mathbf{x},\sigma)\hat{\psi}(\mathbf{x},\sigma)$. More refined representations of $\hat{M}$ in terms of a phase-space density occur in kinetic theory. The Hamiltonian is

$$\hat{H} = \sum_\sigma \int_\omega \mathrm{d}^3\mathbf{x}\hat{e}(\mathbf{x},\sigma)$$

with

$$\hat{e}(\mathbf{x},\sigma) = \frac{\hbar^2}{2m}\nabla\hat{\psi}^\dagger(\mathbf{x},\sigma)\cdot\nabla\hat{\psi}(\mathbf{x},\sigma) + \hat{\psi}^\dagger(\mathbf{x},\sigma)U(\mathbf{x},t)\hat{\psi}(\mathbf{x},\sigma),$$
$$+\tfrac{1}{2}\sum_{\sigma'}\int_\omega \mathrm{d}^3\mathbf{y}\hat{\psi}^\dagger(\mathbf{x},\sigma)\hat{\psi}(\mathbf{y},\sigma')V(|\mathbf{x}-\mathbf{y}|)\hat{\psi}(\mathbf{y},\sigma')\hat{\psi}(\mathbf{x},\sigma), \qquad (1)$$

where for simplicity only one massive self-interacting field has been considered and $V(|\mathbf{x}-\mathbf{y}|)$ describes the short range (mass) density–density interaction, while $U(\mathbf{x},t)$ describes an external field affecting all systems constituted by the field $\hat{\psi}(\mathbf{x},\sigma)$ (e.g. a gravitational field). Also momentum is an important quantity

$$\hat{\mathbf{p}} = \tfrac{1}{2}\sum_\sigma\int_\omega \mathrm{d}^2\mathbf{x}\{[\mathrm{i}\hbar\nabla\hat{\psi}^\dagger(\mathbf{x},\sigma)]\hat{\psi}(\mathbf{x},\sigma) + h.c.\}.$$

This is in fact a very primitive model: its concrete use requires many fields (each molecule or ion denotes a field!) and therefore many phenomenological entries. Too small space–time scales must be avoided since a simple hard sphere model is assumed for $V(r)$ at very small distances; still it copes with a large part of physics. A much more profound model would already be available: electric charges carrying fields interacting via an electromagnetic field, that is quantum electrodynamics (QED). It is used with strong schematizations in quantum optics and is a well established subject in particle physics, which is however generally biased by the old philosophy. When matter inside the container is suitably isolated, in due time an equilibrium state is prepared by spontaneous evolution of the system; equilibrium states are then very well described by statistical operators $\hat{\varrho}(\beta, \mu)$ labelled by two parameters, temperature $1/\beta$ and chemical potential μ. These parameters together with the geometry ω of the container provide an objective and exhaustive characterization of the physical system. The description given by the statistical operator is essentially statistical: often the expectations of the number of quanta in a normal mode r, i.e. $\mathrm{Tr}\left(\hat{a}_r^\dagger \hat{a}_r \hat{\varrho}(\beta, \mu)\right)$, is exceedingly small (typically it approaches zero in the limit of infinite volume of ω, an impressive exception is however ground state occupation in the case of Bose–Einstein condensation [1]), indicating a strongly erratic behaviour of the microsystems in occupation of the huge set of normal modes. If particles are introduced via *field theory–identical particle* correspondence, these particles are entangled (this entanglement being just a key point for the correspondence) leading to a variety of low-temperature phenomena: only for high temperatures can entanglement be forgotten and the picture of the system as composed by particles, each endowed with typical dynamical properties, becomes useful. By the quasi-local structure of the interaction term in (1) one is immediately led to the idea that settings in a space region $\omega_I \subset \omega$ can be prepared in which no interaction occurs, i.e., $\hat{\psi}(\mathbf{x}, \sigma)\hat{\psi}(\mathbf{y}, \sigma')\hat{\varrho} = 0, \ \mathbf{x}, \mathbf{y} \in \omega_I$, so that the dynamics of a single microsystem can be exhibited. Then its interaction with other suitable systems can be arranged, i.e. possibly interaction and entanglement with other microsystems. Such an investigation is indeed necessary to gain information on the *two-body* potential $V(|\mathbf{x} - \mathbf{y}|)$ in the energy density (1), which represents the basic phenomenological entry for the model we are considering: similar problems arise also in QED, where the electromagnetic structure of the microsystems is the basic input. From an experimental point of view all this has been achieved to an astonishing level of performance: a region ω_I means vacuum technique, shielding devices, noise control; systems feeding ω_I with possibly entangled microsystems are now available as sources; systems that can be macroscopically affected by microsystems prepared in ω_I are now available as high efficiency detectors. However, such a very satisfactory experimental situation has no theoretical representation inside the quantum field theoretical description of systems. In fact such description exists at present only for equilibrium systems, while the situation we have now described is a highly non-equilibrium situation: this is immediately clear if we think about the extremely dishomogeneous mass distribution between ω_I and the rest of ω. So there is at present a real gap inside theoretical physics on the general subject of non-equilibrium statistical mechanics. In section 3 some proposal is presented to fill this gap; one expects that inside any reasonable proposal a natural way should exist to characterize the highly non-equilibrium situation in which a *source part* and a *detecting part* of an isolated system can interact through a directed microphysical channel. However, in the absence of this not yet available theoretical setting, no problem has arisen in the practical use of quantum mechanics to describe what happens inside ω_I. Indeed there exists a large consistent theoretical frame including quantum dynamics of microsystems (e.g. scattering processes) and equilibrium statistical mechanics. As was declared by Bell, quantum mechanics is valid *for all practical purposes* [2]. But this is actually the point: quantum mechanics is just the providential short cut through the forementioned untilled ground. Clear statement of this role of quantum mechanics dissipates, in our opinion, all that is called the problem on foundations of quantum mechanics, as we will discuss further in the next sections.

2. The role of quantum mechanics

Let us assume that inside some region $\omega_I \subset \omega$ of the system, interaction between the fields representing its microphysical structure drops out, during a time interval $[t_0, t_1]$: one has

$$\hat{\psi}(\mathbf{x},\sigma)\hat{\psi}(\mathbf{y},\sigma)\hat{\varrho}_t = 0, \quad \mathbf{x},\mathbf{y} \in \omega_I, \quad t \in [t_0,t_1]. \tag{2}$$

This happens trivially if

$$\hat{\psi}(\mathbf{y},\sigma)\hat{\varrho}_t = 0, \quad \mathbf{y} \in \omega_I, \quad t \in [t_0,t_1]$$

and in this case we shall say that in region ω_I one has in this time interval a perfect vacuum. Equation (2) however also holds if $\hat{\varrho}_t$ has the following more complex structure:

$$\hat{\varrho}_t = \sum_{\sigma,\sigma'} \int_{\omega_I} d^3\mathbf{y} \int_{\omega_I} d^3\mathbf{y}' \Psi_t^{(1)}(\mathbf{y},\sigma)\hat{\psi}^\dagger(\mathbf{y},\sigma)\hat{\varrho}'_t\hat{\psi}(\mathbf{y}',\sigma')\Psi_t^{(1)*}(\mathbf{y}',\sigma') \tag{3}$$

with $\hat{\psi}(\mathbf{y},\sigma)\hat{\varrho}'_t = 0$, $\mathbf{y} \in \omega_I$, $t \in [t_0,t_1]$ and $\Psi_t^{(1)}(\mathbf{y},\sigma) = 0$, $\mathbf{y} \in \partial\omega_I$. One has:

$$\mathrm{Tr}\,\hat{\varrho}_t = \sum_{\sigma} \int_{\omega_I} d^3\mathbf{y} |\Psi_t^{(1)}(\mathbf{y},\sigma)|^2 = 1. \tag{4}$$

Using $[\hat{\psi}(\mathbf{y},\sigma),\ \hat{\psi}^\dagger(\mathbf{y}',\sigma')]_\pm = \delta^3(\mathbf{y}-\mathbf{y}')\delta_{\sigma,\sigma'}$ one can show that $\hat{\varrho}_t$ is a solution of the Liouville–von Neumann equation $d\hat{\varrho}_t/dt = -(i/\hbar)[\hat{H}_t, \hat{\varrho}_t]$ with Hamiltonian (1) whenever $\hat{\varrho}'_t$ is and if the function $\Psi_t^{(1)}(\mathbf{y},\sigma)$ satisfies a remarkable equation. This equation, if one considers points $\mathbf{y}$ at a distance from $\partial\omega_I$ larger than the range of $V(r)$ and neglects an *operator valued* surface term, takes a system independent form and is precisely the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi_t^{(1)}(\mathbf{y},\sigma) = -\frac{\hbar^2}{2m}\Delta_2\Psi_t^{(1)}(\mathbf{y},\sigma) + U(\mathbf{y},t)\Psi_t^{(1)}(\mathbf{y},\sigma) \tag{5}$$

(near to the boundary $\partial\omega_I$ an effective additional operator valued contribution to $U(\mathbf{y},t)$ appears). By (4) the role of the Hilbert space $\mathcal{H}^{(1)} = L^2(\omega_I) \otimes \mathbf{C}^{2s+1}$ immediately appears as the natural setting for (5) and from now on we shall take over formalism and notation developed by the use of $\mathcal{H}^{(1)}$ in quantum mechanics (bra, ket, observables). An important generalization of the situation indicated by (3) occurs if $\hat{\varrho}_t$ has the form:

$$\hat{\varrho}_t = \sum_{\sigma,\sigma'} \int_{\omega_I} d^3\mathbf{y} \int_{\omega_I} d^3\mathbf{y}' \hat{\psi}^\dagger(\mathbf{y},\sigma)\hat{\varrho}'_t\hat{\psi}(\mathbf{y}',\sigma')\langle\mathbf{y},\sigma|\varrho_t^{(1)}|\mathbf{y}',\sigma'\rangle, \tag{6}$$

where $\varrho_t^{(1)}$ is a statistical operator on $\mathcal{H}^{(1)}$, such that

$$\frac{\mathrm{d}\varrho_t^{(1)}}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}[H_t^{(1)}, \varrho_t^{(1)}], \tag{7}$$

with $H_t^{(1)}$ the Hamilton operator by which (5) is written as $\mathrm{i}\hbar(\mathrm{d}\Psi^{(1)}/\mathrm{d}t) = H_t^{(1)}\Psi^{(1)}$; still $\hat{\varrho}_t$ is a solution of the Liouville–von Neumann equation of the system if $\hat{\varrho}_t'$ is and if $\varrho_t^{(1)}$ satisfies (7). Representation (6) becomes (3) if $\varrho_t^{(1)}$ is a pure state:

$$\langle \mathbf{y},\sigma|\varrho_t^{(1)}|\mathbf{y}',\sigma'\rangle = \Psi_t^{(1)}(\mathbf{y},\sigma)\Psi_t^{(1)*}(\mathbf{y}',\sigma'). \tag{8}$$

Let us further assume that for $t = t_0$, $\Psi_t^{(1)}(\mathbf{y},\sigma)$ is practically different from zero only in a region inside ω_I which is very small at the space-scale of our macroscopic description: then $\hat{\varrho}_{t_0}$ can be described as the source of a microsystem localized in this region at time t_0. Let $\hat{A}$ be an observable of the system, typically a *relevant variable* of a measured quantity (see section 3) or the spectral measure $\hat{E}^A(M)$ (M being a Borel set), by which expectations of $\hat{A}$ can be related to probability distributions $p_t^A(M)$. One has:

$$\begin{aligned}\langle \hat{A}\rangle_t &= \mathrm{Tr}\,(\hat{A}\hat{\varrho}_t) \qquad (9)\\ &= \sum_{\sigma,\sigma'}\int_{\omega_I}\mathrm{d}^3\mathbf{y}\int_{\omega_I}\mathrm{d}^3\mathbf{y}'\Psi_t^{(1)}(\mathbf{y},\sigma)A_t^{(1)}(\mathbf{y}',\sigma',\mathbf{y},\sigma)\Psi_t^{(1)*}(\mathbf{y}',\sigma') = \langle\Psi_t^{(1)}|A_t^{(1)}|\Psi_t^{(1)}\rangle,\end{aligned}$$

$$p_t^A(M) = \mathrm{Tr}\,(\hat{E}^A(M)\hat{\varrho}_t) = \langle\Psi_t^{(1)}|F_t^{(1)A}(M)|\Psi_t^{(1)}\rangle, \tag{10}$$

where

$$A_t^{(1)}(\mathbf{y}',\sigma',\mathbf{y},\sigma) = \mathrm{Tr}\,(\hat{A}\hat{\psi}^\dagger(\mathbf{y},\sigma)\hat{\varrho}_t'\hat{\psi}(\mathbf{y}',\sigma')) \tag{11}$$

$$\langle\mathbf{y}',\sigma'|F_t^{(1)A}(M)|\mathbf{y},\sigma\rangle = \mathrm{Tr}\,(\hat{E}^A(M)\hat{\psi}^\dagger(\mathbf{y},\sigma)\hat{\varrho}_t'\hat{\psi}(\mathbf{y}',\sigma')). \tag{12}$$

More generally one has $\langle\hat{A}\rangle_t = \mathrm{Tr}_{\mathcal{H}^{(1)}}\,(A_t^{(1)}\varrho_t^{(1)})$, $p_t^A(M) = \mathrm{Tr}_{\mathcal{H}^{(1)}}\,(F_t^{(1)A}(M)\varrho_t^{(1)})$. Expectations $\langle\hat{A}\rangle_t$ and probability distributions $p_t^A(M)$ are related to symmetric operators $A_t^{(1)}$ in $\mathcal{H}^{(1)}$ and to their POV spectral measures $F_t^{(1)A}(M) : A_t^{(1)} = \int_{\mathbf{R}}\lambda\,\mathrm{d}F_t^{(1)A}$. Notice how naturally the well known Dirac's quantum mechanical formalism arises in this context, which initially had nothing to do with it. Let us now discuss the basic relations (9) and (10): they provide the dynamics of the (*macro*)-system prepared as given by (3) or (6). The dynamics consists of the time dependence of $\Psi_t^{(1)}(\varrho_t^{(1)})$ and of $A_t^{(1)}$ and $F_t^{(1)A}$. A basic difference now arises: $\Psi_t^{(1)}(\varrho_t^{(1)})$ are solutions of a *universal* (as far as the microphysical field theoretical model reaches) equation in which only fundamental aspects of quasi-local interactions enter. This is of course true in so far as the operator valued surface term neglected in (5) and given explicitly by

$$\frac{\hbar^2}{2m}\left[\sum_{\sigma,\sigma'}\int_{\partial\omega_I}\mathrm{d}^2\mathbf{a}\int_{\omega_I}\mathrm{d}^2\mathbf{y}'\mathbf{n}_I\cdot\nabla\Psi_t^{(1)}(\mathbf{y},\sigma)\hat{\psi}^\dagger(\mathbf{y},\sigma)\hat{\varrho}_t'(\mathbf{y}',\sigma')\Psi_t^{(1)*}(\mathbf{y}',\sigma')\right.$$

$$\left.-\sum_{\sigma,\sigma'}\int_{\omega_I}\mathrm{d}^3\mathbf{y}\int_{\partial\omega_I}\mathrm{d}^2\mathbf{a}'\mathbf{n}_I\cdot\Psi_t^{(1)}(\mathbf{y},\sigma)\hat{\psi}^\dagger(\mathbf{y},\sigma)\hat{\varrho}_t'\hat{\psi}(\mathbf{y}',\sigma')\nabla\Psi_t^{(1)*}(\mathbf{y}',\sigma')\right],$$

where $\mathbf{n}_I$ is the normal to the surface of ω_I, can actually be disregarded. The relevance of this contribution provides a control mechanism for the feasibility of the considered description. The (*macro*)-system only provides different initial conditions at time t_0, i.e. different sources can be described, but there is no other influence of $\hat{\varrho}_t$ on $\Psi_t^{(1)}(\varrho_t^{(1)})$. The time dependence of $A_t^{(1)}$ and $F_t^{(1)A}$ is instead related to the time evolution of $\hat{\varrho}_t'$, that is to say to the irreversible dynamics of the macroscopic background part of the system, when the microsystem we are dealing with has been cut out. Let us look closer at this time dependence. Taking $\hat{\psi}(\mathbf{y},\sigma)\hat{\varrho}_t' = 0$ into account one has by (11) and (12)

$$\langle \mathbf{y}',\sigma'|A_t^{(1)}|\mathbf{y},\sigma\rangle = \delta^3(\mathbf{y}-\mathbf{y}')\delta_{\sigma,\sigma'}\,\mathrm{Tr}\,(\hat{A}\hat{\varrho}_t') + \mathrm{Tr}\,(\hat{\mathcal{A}}(\mathbf{y}',\sigma',\mathbf{y},\sigma)\hat{\varrho}_t'], \tag{13}$$

where

$$\hat{\mathcal{A}}(\mathbf{y}',\sigma',\mathbf{y},\sigma) = \tfrac{1}{2}([\hat{\psi}(\mathbf{y}',\sigma'),\hat{A}]\hat{\psi}^\dagger(\mathbf{y},\sigma) + \hat{\psi}(\mathbf{y}',\sigma')[\hat{A},\hat{\psi}^\dagger(\mathbf{y},\sigma)]). \tag{14}$$

By the splitting (13), $A_t^{(1)} = I^{(1)}\,\mathrm{Tr}\,(\hat{A}\hat{\varrho}_t') + \mathcal{A}_t^{(1)}$ or also $\mathcal{A}_t^{(1)} = \int_{\mathbf{R}}(\lambda - \mathrm{Tr}\,(\hat{A}\hat{\varrho}_r')) \times \mathrm{d}F_t^{(1)A}$, a *microsystem independent* dynamics of the system is brought into evidence and can be considered separately. Conversely the operators $\mathcal{A}_t^{(1)}$ provide a contribution to the dynamics of the microsystem, arising from the background irreversible dynamics of the system, given by $\mathrm{Tr}\,(\hat{A}\hat{\varrho}_t)$, $\mathrm{Tr}\,(\hat{E}^A(M)\hat{\varrho}_t)$, provides an insight into and is adequately described by the dynamics of the microsystem, which is described by the *universal* element $\Psi_t^{(1)}(\varrho_t^{(1)})$. The additional time dependence induced by $\hat{\varrho}_t'$ in $A_t^{(1)}$ must be negligible during the time evolution of the microsystem: $A_t^{(1)} \approx A^{(1)}$ for $t \in [t_0,t_1]$. In this case the system is not only the source but also becomes a possible measuring device of the observable $A^{(1)}$ of the microsystem and the POV measure $F^{(1)A}$ provides its probability distribution. Of prominent importance and simplicity are the projection valued measures on $\mathcal{H}^{(1)}$: if $F^{(1)A}(M) = (F^{(1)A}(M))^2$, then orthonormal resolutions of the identity $I^{(1)}$ are associated to measurements and the physics related to the microsystem can be displayed by well-known quantum mechanics using geometry of $\mathcal{H}^{(1)}$. The main question is to characterize $\hat{\varrho}_t$ so that $A_t^{(1)} \approx A^{(1)}$ in correspondence to variables $\hat{A}$ of the system: is this possible? This question cannot be solved explicitly at the present stage; however just the experimental success of quantum mechanics provides a phenomenological affirmative answer. Not only, but typical basic features of quantum mechanics indicate limitations in this assertion, may be unexpected, at least if one still endorses the old philosophy. By the existence of non-compatible observables in $\mathcal{H}^{(1)}$, e.g. position and momentum, one must expect that for a given $\hat{\varrho}_t$ only for some subset variables $A_t^{(1)} \approx A^{(1)}$ can hold: different $\hat{\varrho}_t$ must be taken if $A^{(1)}$ is a sharp measurement of position or if $A^{(1)}$ is a sharp measurement of momentum; similarly if different components of spin are considered. To enlighten what a microsystem is, e.g. what its spin is, related to a field $\hat{\psi}(\mathbf{x},\sigma)$, the dynamics of a *suitably large set of different* (*macro*)-*systems*, structured in terms of $\hat{\psi}(\mathbf{x},\sigma)$ is necessary: thus this micro-macrosystem relationship is deeply different from the classical one. In our opinion quantum mechanics is and must be seen as puzzling and paradoxical when just this point is missed. Let us now return briefly to the simplest microsystem in our model: if source and potential $U(\mathbf{x},t)$ allow a classical limit the naive concept of a particle with mass m and magnetic moment related to the spin s can be finally recovered in well known ways. In the general case the term *one quanton microsystem* is more appropriate as suggested in a recent very pedagogical paper by Englert [3]. $\Psi_t^{(1)}(\mathbf{y},\sigma)$ represents the source and subsequent evolution, $A^{(1)}$ an observable, and all this can be embedded in the dynamics of some suitable system $\hat{\varrho}_t$. More complicated microsystems arise in obvious ways, taking for example

$$\hat{\varrho}_t = \sum_{\substack{\sigma_1,\sigma_2\\ \sigma_1',\sigma_2'}} \int_{\omega_I} \mathrm{d}^3\mathbf{y}_1 \int_{\omega_I} \mathrm{d}^3\mathbf{y}_2 \int_{\omega_I} \mathrm{d}^3\mathbf{y}_1' \int_{\omega_I} \mathrm{d}^3\mathbf{y}_2'$$

$$\times \frac{1}{2!}\Psi_t^{(2)}(\mathbf{y}_1,\sigma_1;\mathbf{y}_2,\sigma_2)\hat{\psi}^\dagger(\mathbf{y}_1,\sigma_1)\psi^\dagger(\mathbf{y}_2,\sigma_2)\hat{\varrho}_t'\hat{\psi}(\mathbf{y}_2',\sigma_2')\Psi(\mathbf{y}_1',\sigma_1')\Psi_t^{(2)*}(\mathbf{y}_1',\sigma_1';\mathbf{y}_2',\sigma_2').$$

instead of equation (3), one has a two quanton microsystem described by the symmetric $[(\hat{\psi}(\mathbf{y}_1,\sigma_1),\hat{\psi}(\mathbf{y}_2,\sigma_2)]_- = 0)$ or the antisymmetric $([\hat{\psi}(\mathbf{y},\sigma_1),\hat{\psi}(\mathbf{y}_2,\sigma_2)]_+ = 0)$ wave function $\Psi_t^{(2)}(\mathbf{y}_1,\sigma_1;\mathbf{y}_2,\sigma_2)$: here entanglement between the two microsystems is mandatory. By the dynamics of this $\hat{\varrho}_t$ interaction, the two microsystems can be investigated and the basic entry $V(|\mathbf{x}-\mathbf{y}|)$ inside (1), which also enters the Schrödinger equation ruling $\Psi_t^{(2)}$, can be checked with high accuracy. Also to test the extremely simple structure of a total spin 0 system, more than one system $\hat{\varrho}_t$ is necessary. This is the root of the long debated famous EPR paradox: when the spin of only one of two spin 1/2 microsystems is measured, the simple statistical operator $\frac{1}{2}I_s^{(1)}$ in one particle spin space $\mathbf{C}^2$ arises. However measuring the spin of the second particle this statistical operator can be demixed in different ways, according to which component of the spin of the second particle is measured: these are however measurements involving different $\hat{\varrho}_t$. Demixtures, to be related to different choices of $\hat{\varrho}_t$, are called different *as if realities* or different *blends* in Englert's paper [3]. This is actually the most reasonable point of view if quantum mechanics is discussed as the *theory of microsystems*. A very deep, unavoidably abstract and ponderous, location of quantum mechanics as the description of a microphysical channel inside a phenomenological description of experimental settings was given by Ludwig starting in the 1960s and collected in [4]: in this approach all usual paradoxes and puzzles were already foreseen and reduced to reasonable problems and also modern tools in measurement theory like POV measures, operations and instruments were anticipated, even if called different names. Our proposal points to a realization of Ludwig's approach in the context of a quantum field theoretical description of isolated physical systems. The question $A_t^{(1)} \approx A^{(1)}$, $t \in [t_0,t_1]$ has another far reaching implication; one cannot expect that by the right choice of $\hat{\varrho}_t$ and of the observables a strict time independence arises, i.e. $\mathrm{d}A_t^{(1)}/\mathrm{d}t = 0$. One must instead expect that a sufficiently weak coupling to the irreversible dynamics of $\hat{\varrho}_t'$ exists, so that the universal behaviour of the microsystem is not obscured too much by the system, depending on the dynamics of $A^{(1)}$; the study of $\mathrm{Tr}\left(\hat{\mathcal{A}}(\mathbf{y}',\sigma',\mathbf{y},\sigma)\hat{\varrho}_t'\right)$ is a difficult many-body problem and since the variables $\mathbf{y}',\sigma'$ and $\mathbf{y},\sigma$ are jointly involved one can expect that just due to the weak coupling situation the dynamics of the microsystem receives non-Hamiltonian corrections and that a dynamical semigroup evolution for $\varrho^{(1)}$ arises; one takes care in this way of the fact that the universal behaviour alone is a too strict schematization, as already indicated by the so-called infrared problem of field theory. All this is related to the debated problem of decoherence: this concept arises now in a very natural way. Obviously in our scheme any modification of fundamental dynamics to add decoherence would be superfluous: it is automatically present if microsystems are described, while objectivity of the description of systems is granted by the structure of the theory, as will be shown in the next section. The problem of preserving coherence in the physics of microsystems, faced by experimentalists with modern high technology, should have, in our opinion, the theoretical counterpart in the forementioned study of $\mathrm{Tr}\,(\hat{\mathcal{A}}(\mathbf{y}',\sigma',\mathbf{y},\sigma)\hat{\varrho}_t')$ in quantum field theory of non-equilibrium systems.

3. Isolated non-equilibrium macrosystems

According to the point of view substantiated in sections 1 and 2, which establishes the foundations of quantum theory on a suitable theory of non-equilibrium quantum statistical mechanics, we will sketch a proposal pointing in this direction, based on some recent work [5], and will give a brief account of it. In order to tackle the problem it is quite natural to cope first with isolated systems, this attitude being taken not only for the sake of simplicity, but also in connection with the search for objective state parameters, as we shall see later on. The very concept of isolation is however far from trivial as it immediately appears, taking into account the existence of correlations and quantum mechanical entanglement, which prevent any real physical system to be utterly isolated from the rest of the world. This issue is strongly related to the problem of decoherence (for a recent review see [6]) and its technical and philosophical consequences. Despite these facts, the notion of isolation is still meaningful and useful, provided it is related to a specific time scale, so that only a suitable subset of all possible observables is to be considered, specifically those observables which are slowly varying on this time scale. Relying on this restriction a physically realizable preparation procedure should be conceivable, which actually implements this effective isolation. Conversely if variables sensible to any time or energy scale were considered, shielding from the influence of the environment would be unfeasible. The set of fundamental fields (e.g. associated to charged elementary particles as in QED, or to molecules in a neutral continuum) and the choice of relevant observables (e.g. hydrodynamic or kinetic description of a massive continuum) determine the level of description and therefore actually define the considered macrosystem and its relevant time scale. We are thus led to look for *subsets of relevant slow variables*, and a natural choice are the densities of conserved quantities. In fact, as stressed in non-equilibrium statistical mechanics (see for example [7]), such densities, averaged with suitable probe functions, do provide natural candidates for relevant, not too fast changing observables. The basic structure which characterizes a suitable description of an isolated system at a certain space–time scale is an adequate choice of a subset $\mathcal{M}$ of *relevant variables* $\hat{A}_j(\xi)$: for any realistic system $\mathcal{M}$ will not be invariant under time evolution generated by the Hamiltonian of the model; this can be seen as the root of the second principle of thermodynamics.

The set $\mathcal{A}_t$ of expectation values of the relevant variables $\hat{A}_j(\xi)$ at any time t fixes a set $\mathcal{K}_{\mathcal{A}_t}$ of statistical operators, compatible with the given expectations. For a macrosystem one cannot generally expect that $\mathcal{A}_t$ uniquely determines a statistical operator, so that a further selection inside $\mathcal{K}_{\mathcal{A}_t}$ has to be carried out. This can be meaningfully accomplished by maximizing the von Neumann entropy (supposed finite for every element of $\mathcal{K}_{\mathcal{A}_t}$), so that one is generally led to consider generalized Gibbs states of the form

$$\hat{w}[\zeta(t)] = \frac{\exp\left\{\sum_j \int d\xi \zeta_j(\xi,t)\hat{A}_j(\xi)\right\}}{Z[\zeta(t)]} \equiv \exp\left\{-\zeta_0(t)\hat{\mathbf{1}} - \sum_j \int d\xi \zeta_j(\xi,t)\hat{A}_j(\xi)\right\}, \quad (15)$$

where $\zeta_0(t) = \log Z[\zeta(t)]$, $Z[\zeta(t)] = \mathrm{Tr}\,\exp\{-\sum_j \int d\xi \zeta_j(\xi,t)\hat{A}_j(\xi)\}$ being the partition function of the system at time t. The maximization of $-k\,\mathrm{Tr}\,\hat{\sigma}\log\hat{\sigma}$, $\hat{\sigma} \in \mathcal{K}_{\mathcal{A}_t}$, leads to the appearance of the Lagrange parameters $\zeta_j(\xi,t)$ linked to the mean values at time t of the relevant observables. These classical parameters represent a generalization of temperature and chemical potential that we meet in the usual applications of equilibrium statistical mechanics; they characterize the statistical collection and can be naturally taken as properties of each member of the statistical collection, thus assuming an objective meaning. These can be considered as the *beables*, so called in contrast to *observables* by Bell, who claimed physics should be founded on the former rather than on the latter. This is substantiated by phenomenological evidence, for example in the description of macrosystems inside mechanics of continua, where the velocity field $\mathbf{v}(\mathbf{x},t)$ (related to the reference frame in which the continuum is at rest) can be endowed with an objective meaning for each individual system, together with the fact that $\mathbf{v}(\mathbf{x},t)$ is given by the ratio between the mean values of momentum density and mass density in the statistical collection. This distinguished role of mean values in leading to objective features of the statistical collection, reflecting the fact that the measurement of these relevant observables does indeed provide a negligible influence on their expectation values, is confirmed in the framework of continuous measurement theory [8]. There one can check that in the limit in which the coupling to the measuring apparatus, responsible for the continuous observation and the removal of the system's isolation, vanishes, the trajectories for the mean values still remain meaningful, while second or higher order momenta of the probability distribution diverge. Entropy of the system is defined as a function of $\{\zeta(t)\}$ by

$$-k\,\mathrm{Tr}\,\hat{w}[\zeta(t)]\log\hat{w}[\zeta(t)].$$

In order to give the dynamics of the macrosystem for times $t > t_0$ (supposing the isolation effective with respect to the relevant variables begins at time t_0) we have to give a recipe for the determination of the statistical operator $\hat{\varrho}_t$ of the macrosystem, with which to calculate the mean values and therefore determine the state parameters. A straightforward and naive approach would be to take

$$\hat{\varrho}_{t_0} = \hat{w}[\zeta(t_0)] \quad (16)$$

with $\hat{w}[\zeta(t_0)]$ given by (15) and exploiting for later times the unitary evolution, we would have:

$$\hat{\varrho}_t = \hat{U}(t-t_0)\hat{w}[\zeta(t_0)]\hat{U}^\dagger(t-t_0) = \exp\left\{-\zeta_0(t_0)\hat{\mathbf{1}} - \sum_i \int d\,\xi\zeta_i(\xi,t_0)\hat{A}_i(\xi-(t-t_0))\right\}. \quad (17)$$

The choice (16), corresponding to the standpoint of *information thermodynamics*, amounts to considering the history up to the time point t_0 completely negligible, while this is no more true for later times, as can be seen from (17). A more realistic viewpoint consists in taking $\hat{\varrho}_t$ as the representative of the spontaneous time evolution the system has undergone from time t_0 to time t and of a suitable preparation procedure operated in the finite time interval $[T, t_0]$, controlling and measuring the system before enforcing isolation at time t_0. In this perspective the following quantities should be considered in assigning $\hat{\varrho}_{t_0}$ as a result of a concrete preparation procedure: the expectations of the relevant variables at sharp time points T and t_0; the previous history in the time interval $[T, t_0]$ controlled by measurements of variables $\int_T^{t_0} dt' \hat{A}_j(\boldsymbol{\xi}, t') h_\alpha(t')$ (with $h_\alpha(t')$ suitable test functions, e.g. $h_\alpha(t) = \cos \omega_\alpha t$). Together with the densities $\hat{A}_j(\boldsymbol{\xi}, t)$ one should also consider the corresponding current $\hat{\mathbf{J}}_j(\boldsymbol{\xi})$ related to them by conservation equations of the form $\dot{\hat{A}}_j(\boldsymbol{\xi}, t) = -\nabla \cdot \hat{\mathbf{J}}_j(\boldsymbol{\xi}, t)$, where time dependence is given in the Heisenberg picture $\hat{A}_j(\boldsymbol{\xi}, t) = \exp[+(i/\hbar)\hat{H}t]\hat{A}_j(\boldsymbol{\xi}) \exp[-(i/\hbar)\hat{H}t]$. In the end one obtains

$$\begin{aligned}\hat{\varrho}_{t_0} = \exp \Bigg\{ & -\sum_j \int d\boldsymbol{\xi} \gamma_j(\boldsymbol{\xi}, t_0) \hat{A}_j(\boldsymbol{\xi}) + \sum_{j\alpha} \int d\boldsymbol{\xi} \gamma_{j\alpha}(\boldsymbol{\xi}) \int_T^{t_0} dt' \hat{A}_j(\boldsymbol{\xi}, -(t_0 - t')) h_{j\alpha}(t') \\ & + \sum_{j\alpha} \int d\boldsymbol{\xi} \gamma_{j\alpha}(\boldsymbol{\xi}) \int_T^{t_0} dt' \hat{\mathbf{J}}_j(\boldsymbol{\xi}, -(t_0 - t')) h_{j\alpha}(t') \\ & - \sum_j \int d\boldsymbol{\xi} \gamma_j(\boldsymbol{\xi}, T) \hat{A}_j(\boldsymbol{\xi}, -(t_0 - T)) \Bigg\}. \end{aligned} \tag{18}$$

A crucial step is now to assume that

$$\gamma_j(\boldsymbol{\xi}, t_0) = \zeta_j(\boldsymbol{\xi}, t_0), \tag{19}$$

so as to stress the distinguished role of the expectations of the relevant observables $\hat{A}_j(\boldsymbol{\xi}, t)$ at time t_0, $\{\zeta(t_0)\}$ being the parameters characterizing the macroscopic state. Equation (19) will hold at least for some suitable preparation procedures. In this way a time arrow is introduced, because of the asymmetry between $\gamma_j(\boldsymbol{\xi}, t_0)$ and $\gamma_j(\boldsymbol{\xi}, T)$. One has, therefore, considering time evolution of the isolated system up to time t:

$$\begin{aligned}\hat{\varrho}_t &= \exp[-(i/\hbar)\hat{H}(t - t_0)] \hat{\varrho}_{t_0} \exp[+(i/\hbar)\hat{H}(t - t_0)] \\ &= \exp \Bigg\{ -\zeta_0(t)\hat{\mathbf{1}} - \sum_j \int d\boldsymbol{\xi} \zeta_j(\boldsymbol{\xi}, t_0) \hat{A}_j(\boldsymbol{\xi}, -(t - t_0)) \\ & \quad + \sum_{j\alpha} \int d\boldsymbol{\xi} \gamma_{j\alpha}(\boldsymbol{\xi}) \int_T^{t_0} dt' \hat{A}_j(\boldsymbol{\xi}, -(t - t')) h_{j\alpha}(t') \\ & \quad + \sum_{j\alpha} \int d\boldsymbol{\xi} \gamma_{j\alpha}(\boldsymbol{\xi}) \int_T^{t_0} dt' \hat{\mathbf{J}}_j(\boldsymbol{\xi}, -(t - t')) j_{j\alpha}(t') \\ & \quad - \sum_j \int d\boldsymbol{\xi} \gamma_j(\boldsymbol{\xi}, T) \hat{A}_j(\boldsymbol{\xi}, -(t - T)) \Bigg\}. \end{aligned} \tag{20}$$

Exploiting this expression one can determine the macrostate at time t, using the expectations

$$\langle \hat{A}_j(\boldsymbol{\xi}) \rangle_t = \mathrm{Tr}\,(\hat{A}_j(\boldsymbol{\xi}) \hat{\varrho}_t) \tag{21}$$

to determine the parameters $\{\zeta(t)\}$ in $\hat{w}[\zeta(t)]$. Equations (20) and (21) give the objective dynamics of the system. The existence of $\{\zeta(t)\}$ is granted by the mathematical structure. An immense complexity is hidden in the Heisenberg picture of the operators inside (20) for any realistic system, despite the apparently simple formulas. Using the function $\zeta_j(\boldsymbol{\xi},t)$ we can rewrite $\zeta_j(\boldsymbol{\xi},t_0)\hat{A}_j(\boldsymbol{\xi},-(t-t_0))$ in the following way:

$$\begin{aligned}
\zeta_j(\boldsymbol{\xi},t_0)\hat{A}_j(\boldsymbol{\xi},-(t-t_0)) &= \zeta_j(\boldsymbol{\xi},t)\hat{A}_j(\boldsymbol{\xi}) - \int_{t_0}^{t} \mathrm{d}t' \frac{\mathrm{d}}{\mathrm{d}t'}[\zeta_j(\boldsymbol{\xi},t')\hat{A}_j(\boldsymbol{\xi},-(t-t'))] \\
&= \zeta_j(\boldsymbol{\xi},t)\hat{A}_j(\boldsymbol{\xi}) - \int_{t_0}^{t} \mathrm{d}t'\dot{\zeta}_j(\boldsymbol{\xi},t')\hat{A}_j(\boldsymbol{\xi},-(t-t')) - \int_{t_0}^{t} \mathrm{d}t'\zeta_j(\boldsymbol{\xi},t')\dot{\hat{A}}_j(\boldsymbol{\xi},-(t-t')) \\
&= \zeta_j(\boldsymbol{\xi},t)\hat{A}_j(\boldsymbol{\xi}) - \int_{t_0}^{t} \mathrm{d}t'\dot{\zeta}_j(\boldsymbol{\xi},t')\hat{A}_j(\boldsymbol{\xi},-(t-t')) + \int_{t_0}^{t} \mathrm{d}t'\zeta_j(\boldsymbol{\xi},t')\nabla\cdot\hat{\mathbf{J}}_j(\boldsymbol{\xi},-(t-t')),
\end{aligned}$$

so that we can rewrite (20) in the form

$$\begin{aligned}
\hat{\varrho}_t = \exp\Big\{ &-\zeta_0(t)\hat{\mathbf{1}} - \sum_j \int \mathrm{d}\boldsymbol{\xi}\zeta_j(\boldsymbol{\xi},t)\hat{A}_j(\boldsymbol{\xi}) + \sum_j \int_{t_0}^{t} \mathrm{d}t' \int \mathrm{d}\boldsymbol{\xi}\dot{\zeta}_j(\boldsymbol{\xi},t')\hat{A}_j(\boldsymbol{\xi},-(t-t')) \\
&+ \sum_{j\alpha} \int_{T}^{t_0} \mathrm{d}t' \int \mathrm{d}\boldsymbol{\xi}\gamma_{j\alpha}(\boldsymbol{\xi})\hat{A}_j(\boldsymbol{\xi},-(t-t'))h_{j\alpha}(t') \\
&- \sum_j \int_{t_0}^{t} \mathrm{d}t' \int \mathrm{d}\boldsymbol{\xi}\zeta_j(\boldsymbol{\xi},t')\nabla\cdot\hat{\mathbf{J}}_j(\boldsymbol{\xi},-(t-t')) \\
&+ \sum_{j\alpha} \int_{T}^{t_0} \mathrm{d}t' \int \mathrm{d}\boldsymbol{\xi}\gamma_{j\alpha}(\boldsymbol{\xi})\cdot\hat{\mathbf{J}}_j(\boldsymbol{\xi},-(t-t'))h_{j\alpha}(t') \\
&- \sum_j \int \mathrm{d}\boldsymbol{\xi}\gamma_j(\boldsymbol{\xi},T)\hat{A}_j(\boldsymbol{\xi},-(t-T))\Big\}.
\end{aligned} \tag{22}$$

Comparing $\hat{\varrho}_t$ given by (22) with $\hat{\varrho}_{t_0}$ given by (18) one sees that the basic structure is preserved: $\hat{\varrho}_t$ accounts for a preparation procedure terminating at time t, replacing t_0, the initial parameters $\{\zeta(t_0)\}$ being replaced by $\{\zeta(t)\}$. The contribution referring to the past history extends now from T to t and a new part is displayed, related to the time interval $[t_0,t]$. In place of the parameters $\sum_\alpha \gamma_{j\alpha}(\boldsymbol{\xi})h_{j\alpha}(t')$ relative to the preparation procedure in the time interval $[T,t_0]$, now the parameters $\dot{\zeta}_j(\boldsymbol{\xi},t)$ appear, and in place of the term $\sum_{j\alpha}\gamma_{j\alpha}(\boldsymbol{\xi})\cdot\hat{\mathbf{J}}_j(\boldsymbol{\xi},-(t-t'))h_{j\alpha}(t')$ one deals with $-\sum_j \zeta_j(\boldsymbol{\xi},t')\nabla\cdot\hat{\mathbf{J}}_j(\boldsymbol{\xi},-(t-t'))$. This internal consistency gives the *justification for the assumptions* (18), (19) for the $\hat{\varrho}_{t_0}$ assigned to a suitable preparation procedure of a macrosystem. The structure of the obtained $\hat{\varrho}_t$ looks very similar to the *non-equilibrium statistical operator* proposed by Zubarev [7]. A main difference has however to be pointed out: the choice of $T \to -\infty$ in Zubarev's approach means that no preparation time interval is taken into account. The limit $T \to -\infty$ presupposes a thermodynamic limit, and therefore the idealization of infinite systems. This is a very useful technical procedure in order to get rid of boundary conditions, but should be avoided in a fundamental approach, finite-sized systems often being the realistic, experimentally testable realizations of interesting physical systems (e.g. the recent and outstanding example of Bose–Einstein condensation). Nevertheless the huge set of practical applications of Zubarev's formalism also fit in the present scheme.

Let us give the part $-\sum_j \int d\xi \zeta_j(\xi,t)\hat{A}_j(\xi)$ for the case of hydrodynamical description of a continuum in terms of the typical parameters $\beta(\mathbf{x},t)$, $\mu(\mathbf{x},t)$, $\mathbf{v}(\mathbf{x},t)$:

$$\int_\omega d^3\mathbf{x}\beta(\mathbf{x})\left[\hat{e}_0(\mathbf{x},\mathbf{v}(\mathbf{x},t)) + \frac{\mu(\mathbf{x})}{m}\hat{\rho}(\mathbf{x})\right]$$

with

$$\hat{e}(\mathbf{x}) = \hat{e}_0(\mathbf{x},\mathbf{v}(\mathbf{x},t)) + \mathbf{v}(\mathbf{x},t)\cdot\hat{\mathbf{p}}_0(\mathbf{x},\mathbf{v}(\mathbf{x},t)) + \tfrac{1}{2}\mathbf{v}^2(\mathbf{x},t)\hat{\rho}(\mathbf{x}),$$
$$\hat{\mathbf{p}}(\mathbf{x}) = \hat{\mathbf{p}}_0(\mathbf{x},\mathbf{v}(\mathbf{x},t)) + \mathbf{v}(\mathbf{x},t)\hat{\rho}(\mathbf{x}).$$

Notice that $\beta(\mathbf{x},t)$ is related to the energy density $\hat{e}_0(\mathbf{x})$ in the local rest frame, obtained by replacing $\mp i\hbar\nabla$ with $\mp i\hbar\nabla - m\mathbf{v}(\mathbf{x},t)$ in the expressions for energy and momentum density.

Let us indicate in a sketchy way how one obtains from the general equation the most simple form of quantum dynamics of a macroscopic system. We shall call this situation, which already copes with so-called linear non-equilibrium thermodynamics, *simple dynamics*. Looking at (22) it is quite natural to exploit a perturbative expansion around the first two contributions in the argument of the exponential, linked to normalization and the mean value of the relevant observables. Indeed by (19) the first term is already all that is needed for the dynamics of $\hat{A}_j(\xi) \in \mathcal{M}$. This can be neatly done in terms of the so-called *cumulant expansion* [9], which allows one to express the first-order perturbation in terms of two point *Kubo correlation functions*. Setting $\hat{W} = e^{\hat{A}}/\mathrm{Tr}\, e^{\hat{A}}$ one has

$$\frac{\mathrm{Tr}\,\hat{C}e^{\hat{A}+\hat{B}}}{\mathrm{Tr}\,e^{\hat{A}+\hat{B}}} = \mathrm{Tr}\,\hat{C}\hat{W} + \mathrm{Tr}\,\hat{C}\int_0^1 du\, e^{u\hat{A}}\hat{B}e^{-u\hat{A}}\hat{W} - \mathrm{Tr}\,\hat{C}\hat{W}\,\mathrm{Tr}\,\hat{B}\hat{W} + \cdots$$
$$\equiv \mathrm{Tr}\,\hat{C}\hat{W} + \langle\hat{C},\hat{B}\rangle_{\hat{W}} + \cdots, \tag{23}$$

where the Kubo correlation function with respect to the statistical operator $\hat{W}$ has been implicitly introduced. By the very definition of a macrostate one may write

$$\frac{d}{dt}\,\mathrm{Tr}\,(\hat{A}_j(\xi)\hat{\varrho}_t) = \frac{d}{dt}\,\mathrm{Tr}\,(\hat{A}_j(\xi)\hat{w}[\zeta(t)]) = -\sum_l \int d\xi' \langle\hat{A}_j(\xi),\langle\hat{A}_l(\xi')\rangle_{\hat{w}[\zeta(t)]}\dot{\zeta}_l(\xi',t), \tag{24}$$

with $\hat{w}[\zeta(t)] = \exp\{-\zeta_0(t)\hat{\mathbf{1}} - \sum_j \int d\xi\zeta_j(\xi,t)\hat{A}_j(\xi)\}$ as in (15), so that, exploiting the Liouville–von Neumann equation and the cumulant expansion (23) one comes to an evolution equation for the parameters $\{\zeta(t)\}$ in the form:

$$-\sum_l \int d\xi' \langle\hat{A}_j(\xi),\hat{A}_l(\xi')\rangle_{\hat{w}[\zeta(t)]}\dot{\zeta}_l(\xi',t)$$
$$= \mathrm{Tr}\left(\frac{i}{\hbar}[\hat{H},\hat{A}_j(\xi)]\hat{w}_{[\zeta(t)]}\right) + \int_T^t dt' \left\langle \frac{i}{\hbar}[\hat{H},\hat{A}_j(\xi)],\hat{\mathcal{S}}(t')\right\rangle_{\hat{w}[\zeta(t)]}$$
$$-\sum_l \int d\xi' \left\langle \frac{i}{\hbar}[\hat{H},\hat{A}_j(\xi)],\hat{A}_l(\xi',-(t-T))\right\rangle_{\hat{w}[\zeta(t)]}\gamma_e(\xi',T) + \cdots .$$

The term $\hat{\mathcal{S}}(t')$, keeping track of the previous history together with $\gamma_l(\boldsymbol{\xi}',T)$, has a different expression for the two different time intervals $[T,t_0]$ and $[t_0,t]$ relative to the preparation time and the spontaneous time evolution respectively. The precise structure of $\hat{\mathcal{S}}(t')$ can be read from (3.8) and is given by

$$\begin{cases} \sum_{j\alpha} \int d\boldsymbol{\xi}' [\gamma_{j\alpha}(\boldsymbol{\xi}') \hat{A}_j(\boldsymbol{\xi}',-(t-t')) + \gamma_{j\alpha}(\boldsymbol{\xi}') \cdot \hat{\mathbf{J}}_j(\boldsymbol{\xi}',-(t-t'))] h_{j\alpha}(t'), & T \leqslant t' \leqslant t_0, \\ \sum_{j} \int d\boldsymbol{\xi}' [\dot{\zeta}_j(\boldsymbol{\xi}',t') \hat{A}_j(\boldsymbol{\xi}',-(t-t')) - \zeta_j(\boldsymbol{\xi}',t') \nabla \cdot \hat{\mathbf{J}}_j(\boldsymbol{\xi}',-(t-t'))], & t_0 \leqslant t' \leqslant t, \end{cases} \tag{26}$$

so that (25) is in fact an integrodifferential equation for $\zeta_j(\boldsymbol{\xi},t)$. It appears that memory of the macrostate through the whole interval $T \leqslant t' \leqslant t$ is expressed in the first-order approximation with respect to (23) by the following two point Kubo correlation functions:

$$\left\langle \frac{i}{\hbar}[\hat{H},\hat{A}_j(\boldsymbol{\xi})], \hat{A}_l(\boldsymbol{\xi}',-(t-t')) \right\rangle_{\hat{w}[\zeta(t)]},$$

$$\left\langle \frac{i}{\hbar}[\hat{H},A_j(\boldsymbol{\xi})], \hat{\mathbf{J}}_l(\boldsymbol{\xi}',-(t-t')) \right\rangle_{\hat{w}[\zeta(t)]}, \quad T \leqslant t' \leqslant t,$$

higher order correlation functions appearing in higher order approximations, which can consistently be neglected in most applications of non-equilibrium thermodynamics. A crucial point, in order to actually calculate the dynamics of a complex system, is linked to *the decaying behaviour of correlation functions*. This is here not achieved by means of a thermodynamic limit, the finite size of the system leading to a quasiperiodical behaviour of correlation functions, but depends on their integration with respect to time and configuration space with suitable smooth functions, as provided by $h_{j\alpha}(t')$ and the parameters $\zeta_j(\boldsymbol{\xi},t')$ themselves, which should be homogeneous enough, provided one has done a suitable choice of the space–time variation scale of the relevant observables. Calling τ the characteristic correlation decay time, the integration over history in (25) can be restricted to the interval $[t-\tau,t]$, so that the integrodifferential equations governing the dynamics depend only on the state parameters and not on the preparation parameters, provided $t_0 - T > \tau$, thus acquiring a universal character. In this situation increase of entropy defined by $-k \operatorname{Tr} \hat{w}[\zeta(t)] \log \hat{w}[\zeta(t)]$ and approach to an equilibrium Gibbs state can be shown. A particularly simple but relevant situation arises if in fact τ settles a typical time scale of the slow dynamics of the relevant observables over which the parameters $\{\zeta(t)\}$ are practically constant. In this case the role of the statistical operator $\hat{\varrho}_t$ is played by the Gibbs state $\hat{w}[\zeta(t)]$ and one obtains that the expectations of the relevant observables are driven by a dynamical semigroup of mappings acting on the linear space generated by these observables. In this way the formalism of the *master equation* appears again, although in a new light. One is not looking for the time evolution of some coarse-grained statistical operator, but rather works in the Heisenberg picture on a subset of relevant slow variables.

4. Memory and microsystems

Inside the general scheme presented in section 3 a dramatic simplification was introduced by the concept of *simple dynamics* and by relying on *decaying behaviour* of Kubo correlation functions. These correlation functions then become the driving element of dynamical evolution: in *simple dynamics* two-point correlations already do the job. Together with typical field theoretical locality this leads to locality of macroscopic dynamics, loosely speaking, evolution at a space–time point depends on the local macroscopic state in a short time interval before. In a relativistic context this point becomes still sharper due to micro-causality. However, memory loss is only a general possibility related to an interplay between the quasiperiodical behaviour of correlation functions and the more or less smooth space–time behaviour of macroscopic state parameters: it is not a necessary feature. If you consider a system composed by the source of a microsystem, some intermediate device lingering within the microsystem for some time Δt and a detector, you have immediate evidence of a system completely outside the previous description. Assume that dynamics described by $\hat{\varrho}_t$ in (22) was *normal* (i.e. memory confined inside a very small time interval τ) until time $\bar{t}$. Let us now pose a seed of non-normal behaviour in some future time interval and describe, in not too technical a way, how this can be done. In the history part

$$\int_{\bar{t}-\tau}^{\bar{t}} \mathrm{d}t' \hat{\mathcal{S}}(t')$$

at the exponent of the expression of $\hat{\varrho}_t$ given by (22), we cut out (setting e.g. $\zeta(t') = \zeta_n(t') + \tilde{\zeta}(t')$) a part $\hat{\mathcal{S}}_{\bar{t}}(t')$ which possibly will not simply decay: $\hat{\mathcal{S}}(t') = \hat{\mathcal{S}}_n(t') + \hat{\mathcal{S}}_{\bar{t}}(t')$. Then $\hat{\varrho}_{\bar{t}}$ has a structure of the form

$$\hat{\varrho}_{\bar{t}} = \exp\left\{ -\hat{\mathcal{F}}(\bar{t}) + \int_{\bar{t}-\tau}^{\bar{t}} \mathrm{d}t' \hat{\mathcal{S}}_n(t') + \int_{\bar{t}-\tau}^{\bar{t}} \mathrm{d}t' \hat{\mathcal{S}}_{\bar{t}}(t') \right\}. \tag{27}$$

This positive operator can be represented as the normal statistical operator $\hat{\varrho}_{\bar{t},n}$ (consisting of (27) with normal history part $\hat{\mathcal{S}}_n(t')$, properly normalized) with a suitable correction in the following form:

$$\hat{\varrho}_{\bar{t}} = \lambda \left[\hat{\mathbf{1}} + \int_{\bar{t}-\tau}^{\bar{t}} \mathrm{d}t' \hat{\mathbf{S}}_{\bar{t}}(t') \right] \hat{\varrho}_{\bar{t},n} \left[\hat{\mathbf{1}} + \int_{\bar{t}-\tau}^{\bar{t}} \mathrm{d}t' \hat{\mathbf{S}}^{\dagger}_{\bar{t}}(t') \right], \tag{28}$$

where $\hat{\mathbf{S}}_{\bar{t}}(t')$ is essentially constructed with $\hat{S}_{\bar{t}}(t')$. Assume now that, due to some strong dishomogeneity of the classical parameters in $\hat{\varrho}_{\bar{t},n}$, there is a region ω_I such that

$$\hat{\psi}(\mathbf{y},\sigma)\hat{\varrho}_{\bar{t},n} \approx 0, \quad \mathbf{y} \in \omega_I; \tag{29}$$

then the simplest representation of $\hat{\mathbf{S}}_{\bar{t}}(t')$ is:

$$\hat{\mathbf{S}}_{\bar{t}}(t')\hat{\varrho}_{\bar{t},n} = \sum_{\sigma} \int_{\omega_I} \mathrm{d}^3\mathbf{y} \hat{\psi}^{\dagger}(\mathbf{y},\sigma)\hat{A}(\mathbf{y},\sigma,t')\hat{\varrho}_{\bar{t},n}, \tag{30}$$

where $\hat{A}(\mathbf{y},\sigma,t')$ acts as a destruction operator only in a region ω_S outside $\omega_I (\omega_S \cap \omega_I = \emptyset)$. In general by (29) a strongly dishomogeneous (i.e. non-equilibrium) situation of the system is stated. Typically one can think of a *source* located in ω_S which feeds ω_I with one microsystem, taking it in some complicated way from region ω_S. These correction factors in (28) give a decaying contribution to expectations of observables which are influenced by the structure of $\hat{\varrho}_{\bar{t},n}$ inside ω_S: typically the dynamics of the source itself is *normal*. Assume however that there are observables not influenced by the structure of $\hat{\varrho}_{\bar{t},n}$ inside ω_S. This is obviously a very particular situation: consider an observable $\hat{B}$ of the system that is shielded against the region ω_S, except for the microsystem; one can formalize this by $\mathrm{Tr}\,(\hat{B}\hat{U}(t-\bar{t})\hat{\varrho}_{\bar{t},n}\hat{U}^{\dagger}(t-\bar{t})) \approx 0$. With this condition one can see that just a part of (28) bilinear is $\hat{\mathbf{S}}_{\bar{t}}(t')$, becomes important and provides a contribution of the form:

$$\mathrm{Tr}\,(\hat{B}\hat{\varrho}_t) = (1-\lambda)\,\mathrm{Tr}\,(\hat{B}\hat{U}(t-\bar{t})\hat{\varrho}_{\bar{t},a}\hat{U}^{\dagger}(t-\bar{t})), \quad 0<\lambda<1,$$

$$\hat{\varrho}_{\bar{t},a} = \sum_{\sigma,\sigma'}\int_{\omega_I} d^3\mathbf{y}\int_{\omega_I} d^3\mathbf{y}'\hat{\psi}^{\dagger}(\mathbf{y},\sigma)\hat{\varrho}_{\bar{t},n}\hat{\psi}(\mathbf{y}',\sigma')\langle\mathbf{y},\sigma|\varrho^{(1)}_{\bar{t}}|\mathbf{y}',\sigma'\rangle \tag{31}$$

where

$$\langle\mathbf{y},\sigma|\varrho^{(1)}_{\bar{t}}|\mathbf{y}',\sigma'\rangle = \frac{\mathrm{Tr}\,(\hat{A}(\mathbf{y},\sigma)\hat{\varrho}_{\bar{t},n}\hat{A}^{\dagger}(\mathbf{y}',\sigma'))}{\sum_{\sigma}\int_{\omega_I} d^3\mathbf{y}\,\mathrm{Tr}\,(\hat{A}(\mathbf{y},\sigma)\hat{\varrho}_{\bar{t},n}\hat{A}^{\dagger}(\mathbf{y},\sigma'))},$$

$$\hat{A}(\mathbf{y},\sigma) = \int_{\bar{t}-\tau}^{\bar{t}} dt'\hat{A}(\mathbf{y},\sigma t').$$

The study of the time evolution of the statistical operator $\hat{U}(t-\bar{t})\hat{\varrho}_{\bar{t},a}\hat{U}^{\dagger}(t-\bar{t})$ was just the subject of section 2, where this operator was simply called $\hat{\varrho}_t$ (while $\hat{\varrho}'_t = \hat{U}(t-\bar{t})\hat{\varrho}_{\bar{t},n}\hat{U}^{\dagger}(t-\bar{t})$). The mixture parameter λ has the obvious interpretation as probability that region ω_S does not become active as a source of a microsystem. The *anomalous* dynamics of the system around time $\bar{t}$, consisting in the structure $\hat{\varrho}_{\bar{t}} = \lambda\hat{\varrho}_{\bar{t},n} + (1-\lambda)\hat{\varrho}_{\bar{t},a}$ is also indicated by the fact that one cannot longer state that entropy increases around time $\bar{t}$. One could say that an *event* can happen with probability $(1-\lambda)$ in the time interval $[\bar{t}-\tau,\bar{t}]$, by which a new structure, the microsystem, is seeded, which propagates inside ω_I and thus produces a more complex dynamics which conserves memory of what was going on inside ω_S in the time interval $[\bar{t}-\tau,\bar{t}]$, to be detected by observables $\hat{B}$ at time t; the value $|t-\bar{t}|$ depending on how successful the fight against decoherence was (see section 2).

5. Conclusions and outlook

In sections 3 and 4 we have indicated a possible way of tackling quantum field theory to construct a strongly non-equilibrium dynamics. A natural opening seems to appear indicating microsystems as representatives of fundamental and universal local properties of matter and as systems storing the physics of *source* parts and transmitting this memory to detecting parts. In this way entanglement and interaction with other microsystems can be achieved giving an outlook over more complex memory storing structures and *anomalous* (that is non-entropy increasing) dynamics. What was described in section 4 happening around time $\bar{t}$ could be called an *event* and some relation with ideas recently expressed by Haag [10] can be expected. Fortunately enough quantum mechanics exists independently: one has only to *guess* properly what element of $\mathcal{H}^{(1)}$ represents concrete sources and detectors. If this is skillfully done and also decoherence is taken into account by some correct weak coupling to the macroscopic environment, all this works very well in any practical application. No fundamental problem appears but it should become clear that *Dirac's book* quantum mechanics is an exceedingly smart short pass across this quantum field theory of non-equilibrium systems. If one goes hiking on a smart short pass across a non-trivial landscape, believing to be on a main road, to encounter puzzling situations is a common experience. So one does not wonder that quantum mechanics is so puzzling and mysterious that Bell claimed that it contains the seeds of its own dissolution.

Acknowledgments

BV thanks the Alexander von Humboldt-Stiftung for financial support.

References

[1] Anderson, M., *et al.*, 1995, *Science,* **269,** 198.
[2] Bell, J., 1990, *Phys. World*, **2,** 33.
[3] Englert, B.-G., 1999, *Z. Naturforsch.*, **54a,** 11.
[4] Ludwig, G., 1985, *An Axiomatic Basis for Quantum Mechanics* (Berlin: Springer); 1983, *Foundations of Quantum Mechanics* (Berlin: Springer).
[5] Lanz, O., Melsheimer, O., and Vacchini, B., 2000, *Rep. Math. Phys.*, to appear.
[6] Giulini, D., *et al.*, 1996, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Berlin: Springer).
[7] Zubarev, D. N., 1974, *Non-equilibrium Statistical Thermodynamics* (New York: Consultant Bureau); Zubarev, D. N., Morozov, V., and Roepke, G., 1996, *Statistical Mechanics of Non-equilibrium Processes* (Berlin: Akademie-Verlag).
[8] Barchielli, A., Lanz, L., and Prosperi, G. M., 1982, *Nuovo Cimento*, **72B,** 79; 1983, *Found. Phys.*, **13,** 779.
[9] Robin, W. A., 1990, *J. Phys.* A, **23,** 2065.
[10] Haag, R., 1996, *Commun. Math. Phys.,* **180,** 733; Blanchard, Ph., and Jadczyk, A. (eds), 1999, *Lecture Notes in Physics*, Vol. 517 (Berlin: Springer).