Macro-objectivation: a challenge in quantum field theory

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Abstract. Thermodynamics of irreversible processes is taken as the phenomenological starting point for the description of macroscopic systems in quantum mechanics and state parameters, which are amenable to be attributed an objective meaning, are introduced inside non relativistic quantum field theory when the macroscopic system is locally at equilibrium. Conditions for these state parameters to obey a deterministic time evolution are indicated and discussed. The formalism is developed also considering the case of more component systems and bound states. The situation in which the deterministic dynamics of the state parameters breaks down is also envisaged and it is argued that in this case a stochastic generalisation of the theory is called for. First attempts in this direction are outlined, naturally leading to a rooting of the concept of microsystem inside both quantum theory and an objective phenomenological context.

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THE QUEST ABOUT MACRO-OBJECTIVATION

Quantum mechanics is a very satisfactory theory when used to describe the dynamics of particles, with no substantial difficulty in a non relativistic context, and an extraordinary wealth of experimental confirmations. Since particles are prepared and detected by macroscopic apparata a connection between such objects is obviously necessary to give any meaning to the previous sentence. Such connection is always established on the basis of a description, very rough but however practically completely sufficient, of these objects, based on the idea that they are built just by the particles we are talking about, providing in a cooperative way to the macroscopic properties which characterize sources, detectors and intermediate devices as mirrors, screens, targets etc. These systems are generally at least locally in thermodynamic equilibrium: they are prepared on the basis of phenomenological experience settled by consolidated technology, also consolidated by some knowledge of equilibrium statistical mechanics of the particles by which they are built up; here often the classical limit of quantum theory is adequate. This was obviously the case when quantum mechanics was born, since at that time people building and setting apparata could be helped in their work only by classical physics. This explains the only slowly decaying prejudice about "a classical macroworld". In fact the whole experimental setting of an experiment about particles is a system in a metastable macroscopic situation, that just due to the particles one is investigating undergoes different macroscopic evolutions: the generally stochastic character of the experiment is just associated to the quantum theory of the particles and gives the concrete realization

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of quantum mechanical probabilities. So quantum theory is "de facto" a description of pieces of macroscopic dynamics in particular cases of failure of local equilibrium dynamics: just this stochasticity escapes from the realm that can be understood by classical mechanics of the elementary components of systems. Nowadays macroscopic devices are built knowing about quantum mechanics, so quantum theory is clearly present inside phenomenology of macroscopic systems, even if quite obviously quantum theory at space-time scales which are presently investigated (e.g. standard model of elementary particles) is not necessary in order to plan and construct sources, accelerators and particle detectors. Quantum theory works by separating the "relevant" particles from the macroscopic background, which can influence them by effective external fields and by possible "renormalisations" of particles parameters, and furthermore by suitable "guessing" of what "state" in a few particle Hilbert space has been produced by the source and what "observables" are measured by the detectors. If one tries to make stricter these connections between "macroscopic parts" of an experiment as sources and detectors with the mathematical elements of quantum theory as: Hilbert space $\mathscr{H}$ of the relevant particles, states $\psi \in \mathscr{H}$ as representatives of the preparation, selfadjoint operators A in $\mathscr{H}$ as representative of detectors, one is scarcely successful. More precisely one can put quantum theory on an axiomatic basis, introducing e.g. the concept of the "properties" of a system, which can be prepared or detected: these properties can be associated with the subspaces of the Hilbert space $\mathscr{H}$. However if the macroscopic properties of a detector are represented in this way one runs into difficulties if one describes the time evolution of this system coupled to a particle to be detected by it, just due to the linearity of Schrödinger time evolution equation, as it has been shown in a very general way by Bassi and Ghirardi [1]. Macroscopic properties which are necessary to give a concrete meaning to measurement in quantum mechanics, if they are described as indicated by axiomatic quantum mechanics, lead to a departure from Schrödinger equation, e.g., as proposed in the GRW modification of quantum theory, by a spontaneous collapse process. In this modified quantum theory an objective meaning can be given to the center of mass of macroscopic regions of a many particle system. It seems however hardly possible to describe in terms of these objective elements the concrete, physically irreversible, behaviour of a measuring device.
A great effort was done by G. Ludwig [2] to investigate the general compatibility of axiomatic of quantum theory with the phenomenological description of the experimental settings which give physical evidence of microsystems. Since the basic novelty of quantum physics is its essentially statistical character the following question was settled by Ludwig: starting with physical systems with objective properties, assume that when they are coupled together interaction provides events with a reproducible probability due to a carrier directed from system 1 to system 2; does the general mathematical representation of this probabilistic situation reproduce the formalism of quantum theory? Is it possible to motivate in this way the Hilbert space structure of quantum theory? Ludwig answered in a positive way this question, discovering a slightly more general formal structure, in which states are statistical operators on a Hilbert space, transformations are affine maps on them and events are associated to "effects" leading in the case of observables to POV measures: expectations of these observables are given by "expectations" of symmetric operators. This is the structure of modern axiomatic of quantum theory; it was obtained independently by Holevo [3], who succeeded in recovering in the quantum formalism

important aspects already developed inside classical statistical theory. The phenomenological description of the macroscopic components of an experiment by which particles are investigated deals, as long as these components are not coupled together, with situations where non statistical *state parameters* are in evidence, just as local equilibrium thermodynamics is teaching: e.g. in the case of a fluid, temperature and velocity fields and also chemical potentials of the different components. These are classical fields $\zeta_j(\mathbf{x},t)$: they have nothing to do with "mechanics" either classical or quantum; they simply represent our basic experience, related to a suitable space-time scale. Therefore we shall not associate quantum mechanical properties to these objective elements and will not have to worry about their accommodations with respect to the Hilbert space of the system: the contrary would be quite unphysical and incorrect from the point of view of quantum theory; in fact, no measuring process by a measuring apparatus is assumed, even if we assume that the state parameters can be checked without perturbing the system. Compatibility with quantum theory becomes now a far less challenging, but much more realistic job. One starts with an established phenomenological "pretheory"; this also means that a suitable physical content must be delimited in order to make such a "pretheory" feasible: typically a space region Ω must be taken suitably isolated e.g. by walls; a time interval $[t_0,T]$ must be assigned during which such isolation and the concept of "walls" make sense. Taking some suitable model of interacting quantum fields one tries to give a "first principle" derivation of the mentioned "pretheory": e.g. phenomenological equations and characteristic constants are obtained to sufficient accuracy in terms of masses, charges and structure of short range interaction potentials. Then if this pretheory allows to produce an experimental setting giving evidence of particles, there must be a breakdown of the previously established deterministic regime and quantum mechanics of the particles must emerge, firmly rooted to the objectively given parts of the system.

Let us stress that due to the preeminence of the concept of local thermodynamical equilibrium, quantum field theory becomes most useful even in the non relativistic case, where the field aspect is not strictly necessary in the construction of quantum mechanics of the particles: it becomes essential in the relativistic case; anyway also in a non relativistic context, locality of the quantum field description will play a fundamental role.

Let us finally observe that the present construction of a local equilibrium quantum theory with a stochastic regime due to particles can only be an essentially approximated description of the system, available on a suitable space-time scale. It must be superseded by a deeper description if smaller or larger space-time scales are considered. So in this paper we are not giving a quantum mechanical solution of the basic item of macro-objectivation, but relying on a "pretheory" which catches in a phenomenological way some physical domain of reality by suitable state parameters, we discuss its compatibility with quantum theory and indicate how one can substantiate in this way "a first principle" foundation of this phenomenological description. Then we indicate how the concepts of particles emerge, when the microstructure of the system becomes responsible for a breakdown of deterministic evolution of state parameters. Giving a role to a phenomenological pretheory, without superseding it by quantum theory, makes this approach in a sense less fundamental as one would generally expect when a quantum field theory is proposed; this point, which is related to what one expects from theoretical physics was briefly addressed in ref. [4].

MACROSTATES IN QUANTUM FIELD THEORY

We formalise now how a phenomenological pretheory is embedded in a quantum field theory. Let us associate to a system a set of quantum fields. For simplicity, the present very schematic description is non-relativistic, consists of two type of components with zero spin; in Schrödinger representation we have two quantum fields $\hat{\psi}_a(\mathbf{x})$ $a = 1,2$ confined inside a region $\Omega \subset R^3$. The phenomenological state parameters consists of a suitable set of classical fields $\zeta_j(\mathbf{x})$, in the simplest case a field description of temperature, velocity and two chemical potentials; as typical of thermodynamics they are associated with the expectations of a suitable finite set of linearly independent observables $\hat{A}_j(\mathbf{x})$, built with the fields $\hat{\psi}_a(\mathbf{x})$, $\hat{\psi}_a^\dagger(\mathbf{x})$, which will be called "relevant" observables. It will turn out that a basic property of $\hat{A}_j(\mathbf{x})$ will be their locality, or quasi-locality in the sense that $\hat{A}_j(\mathbf{x})$ only depends on $\hat{\psi}_a(\mathbf{y})$, $\hat{\psi}_a^\dagger(\mathbf{y}')$ for $|\mathbf{x}-\mathbf{y}| \ll r_0, |\mathbf{x}-\mathbf{y}'| \ll r_0$; the length r_0 can be taken to characterize the short-distance limit of the physical description, its typical scale space being $\lambda \gg r_0$; we shall call M the linear span of the relevant variables $\hat{A}_j(\mathbf{x})$. The theory is characterised by an Hamiltonian operator $\hat{H}$: we shall not generally assume that $\hat{H} \in M$. It is almost obvious that such assumption is necessary if one aims to a theory for the long-time description of the system: e.g. if one is interested in discussing global equilibrium and approach to it. The point is that assuming for $\hat{H}$ a quasi-local density can be too restrictive at least in a non-relativistic context. The relevant observables can refer to components of the system explained as composed with the elementary ones: only in this way this kind of description can gain theoretical depth and can be extended over decreasing length λ.
Let us now assume that

$$\begin{aligned}
\hat{H} &= \int_\Omega d^3\mathbf{y} \left(\frac{\hbar^2}{2m_1} \nabla \hat{\psi}_1^\dagger(\mathbf{y}) \cdot \nabla \hat{\psi}_1(\mathbf{y}) + V_1^{\text{ext}}(\mathbf{y}) \hat{\psi}_1^\dagger(\mathbf{y}) \hat{\psi}_1(\mathbf{y}) \right. \\
&\qquad \left. + \frac{\hbar^2}{2m_2} \nabla \hat{\psi}_2^\dagger(\mathbf{y}) \cdot \nabla \hat{\psi}_2(\mathbf{y}) + V_2^{\text{ext}}(\mathbf{y}) \hat{\psi}_2^\dagger(\mathbf{y}) \hat{\psi}_2(\mathbf{y}) \right) \\
&+ \frac{1}{2} \int_\Omega d^3\mathbf{y}\, d^3\mathbf{y}' \left(\hat{\psi}_1^\dagger(\mathbf{y}) \hat{\psi}_1^\dagger(\mathbf{y}') V_{11}(|\mathbf{y}-\mathbf{y}'|) \hat{\psi}_1(\mathbf{y}') \hat{\psi}_1(\mathbf{y}) \right. \\
&\qquad + \hat{\psi}_2^\dagger(\mathbf{y}) \hat{\psi}_2^\dagger(\mathbf{y}') V_{22}(|\mathbf{y}-\mathbf{y}'|) \hat{\psi}_2(\mathbf{y}') \hat{\psi}_2(\mathbf{y}) \\
&\qquad \left. + 2 \hat{\psi}_1^\dagger(\mathbf{y}) \hat{\psi}_2^\dagger(\mathbf{y}') V_{12}(|\mathbf{y}-\mathbf{y}'|) \hat{\psi}_2(\mathbf{y}') \hat{\psi}_1(\mathbf{y}) \right) \qquad (1)
\end{aligned}$$

A composed field operator $\hat{\Psi}_g(\mathbf{x})$ can be defined by:

$$\hat{\Psi}_g(\mathbf{x}) = \int_\Omega d^3\mathbf{x}_1\, d^3\mathbf{x}_2 \delta \left(\mathbf{x} - \frac{m_1\mathbf{x}_1 + m_2\mathbf{x}_2}{m_1+m_2} \right) \hat{\psi}_1(\mathbf{x}_1) \hat{\psi}_2(\mathbf{x}_2) g(|\mathbf{x}_1 - \mathbf{x}_2|). \qquad (2)$$

It is seen that the operator $\dot{\hat{\psi}}_a(\mathbf{x}) = \frac{1}{i\hbar} \left[\hat{H}, \hat{\psi}_a(\mathbf{x}) \right]$, which yields the evolution law in the Heisenberg description, assumes a very significant structure if the function g in (2) is the eigenfunction of the "center of mass" two-particle energy:

$$-\frac{\hbar^2}{2\mu} \Delta g_\kappa(\mathbf{r}) + V_{12}(r) g_\kappa(\mathbf{r}) = u_\kappa g_\kappa(\mathbf{r}), \qquad \mu = \frac{m_1 m_2}{m_1+m_2}. \qquad (3)$$

From

$$
\begin{aligned}
i\hbar\dot{\hat{\psi}}_1(\mathbf{x}) &= -\frac{\hbar^2}{2m_1}\Delta\hat{\psi}_1(\mathbf{x}) + \Big[V_1^{\rm ext}(\mathbf{x}) + \int_\Omega d^3\mathbf{y}' \ \Big(\hat{\psi}_1^\dagger(\mathbf{y}')V_{11}(|\mathbf{x}-\mathbf{y}'|)\hat{\psi}_1(\mathbf{y}') \\
&+ \hat{\psi}_2^\dagger(\mathbf{y}')V_{12}(|\mathbf{x}-\mathbf{y}'|)\hat{\psi}_2(\mathbf{y}')\Big)\Big]\hat{\psi}_1(\mathbf{x}) \qquad (4)
\end{aligned}
$$

and a similar equation for $\dot{\hat{\psi}}_2(\mathbf{x})$ one has easily:

$$
\begin{aligned}
i\hbar\dot{\hat{\Psi}}_g(\mathbf{x}) &= -\frac{\hbar^2}{2(m_1+m_2)}\Delta\hat{\Psi}_g(\mathbf{x}) \\
&+ \int d^3\mathbf{r}\ \hat{\psi}_1(\mathbf{x}+\frac{m_2}{m_1+m_2}\mathbf{r})\left(-\frac{\hbar^2}{2\mu}\Delta_r g(\mathbf{r}) + V_{12}(r)g(\mathbf{r})\right)\hat{\psi}_2(\mathbf{x}-\frac{m_1}{m_1+m_2}\mathbf{r}) \\
&+ \int d^3\mathbf{r}\ \left(V_1^{\rm ext}(\mathbf{x}+\frac{m_2}{m_1+m_2}\mathbf{r}) + V_2^{\rm ext}(\mathbf{x}-\frac{m_2}{m_1+m_2}\mathbf{r})\right) \\
&\qquad \hat{\psi}_1(\mathbf{x}+\frac{m_2}{m_1+m_2}\mathbf{r})g(\mathbf{r})\hat{\psi}_2(\mathbf{x}-\frac{m_1}{m_1+m_2}\mathbf{r}) \\
&+ \int d^3\mathbf{y}\int d^3\mathbf{r}\ \Big[\hat{\psi}_1^\dagger(\mathbf{y})\hat{\psi}_1(\mathbf{y})\Big(V_{11}(|\mathbf{y}-\mathbf{x}-\frac{m_2}{m_1+m_2}\mathbf{r}|) \\
&\qquad + V_{21}(|\mathbf{y}-\mathbf{x}+\frac{m_1}{m_1+m_2}\mathbf{r}|)\Big) \\
&+ \hat{\psi}_2^\dagger(\mathbf{y})\hat{\psi}_2(\mathbf{y})\Big(V_{21}(|\mathbf{y}-\mathbf{x}-\frac{m_2}{m_1+m_2}\mathbf{r}|) + V_{22}(|\mathbf{y}-\mathbf{x}+\frac{m_1}{m_1+m_2}\mathbf{r}|)\Big)\Big] \\
&\quad \hat{\psi}_1(\mathbf{x}+\frac{m_2}{m_1+m_2}\mathbf{r})g(\mathbf{r})\hat{\psi}_2(\mathbf{x}-\frac{m_1}{m_1+m_2}\mathbf{r}). \qquad (5)
\end{aligned}
$$

(Note that $V_{12}(r)$ in the second line of the above formula arises from the requirement of normal-ordering for the operators). An important simplification occurs if one assumes that $g(\mathbf{r})$ obeys to equation (3): then the first two lines of eq. (5) become

$$
i\hbar\dot{\hat{\Psi}}_\kappa(\mathbf{x}) = -\frac{\hbar^2}{2(m_1+m_2)}\Delta\hat{\Psi}_\kappa(\mathbf{x}) + u_\kappa\hat{\Psi}_\kappa(\mathbf{x}),
$$

where $\hat{\Psi}_\kappa(\mathbf{x}) \equiv \hat{\Psi}_{g_\kappa}(\mathbf{x})$. Let us assume that eq. (3) has a typical "bound state" solution if $g_\kappa(\mathbf{r}) \in L^2(R^3)$ and that $g_\kappa(\mathbf{r})$ for bound states with $|u_\kappa|$ large enough, is practically negligible for $r > \lambda_b$ and assume that $V_a^{\rm ext}(\mathbf{x}), a = 1,2$ are practically constant on the space scales $\frac{m_1}{m_1+m_2}\lambda_b, \frac{m_2}{m_1+m_2}\lambda_b$, then the third and the fourth lines of eq. (5) become:

$$
\left(V_1^{\rm ext}(\mathbf{x}) + V_2^{\rm ext}(\mathbf{x})\right)\hat{\Psi}_g(\mathbf{x}).
$$

The last part of (5) has a more complex many body structure, however one can expect that a power expansion of the $\mathbf{r}$ dependence of the functions $V_{il}, i,l = 1,2$ can be significant; then the zero order contribution of such expansion becomes:

$$
\hat{V}^{(0)}(\mathbf{x})\hat{\Psi}_g(\mathbf{x}), \qquad \hat{V}^{(0)}(\mathbf{x}) = \int d^3\mathbf{y}\Big[\hat{\psi}_1^\dagger(\mathbf{y})\hat{\psi}_1(\mathbf{y})\left(V_{11}(|\mathbf{y}-\mathbf{x}|) + V_{21}(|\mathbf{y}-\mathbf{x}|)\right)
$$

$$+\hat{\psi}_2^\dagger(\mathbf{y})\,\hat{\psi}_2(\mathbf{y})\,(V_{21}(|\mathbf{y}-\mathbf{x}|)+V_{22}(|\mathbf{y}-\mathbf{x}|))\Big]\,; \quad (6)$$

we skip here the explicit expression of the corrections $V_{il}^{(1)}(\mathbf{x})$. Let us observe that in this way one recovers to a remarkable extent in Heisenberg representation a simple Schrödinger field dynamics for the composed fields $\hat{\Psi}_\kappa(\mathbf{x})$. As a consequence also the fundamental balance equation are reobtained, apart from certain corrective contributions, allowing to define also for the composed component the typical hydrodynamical fields. One has e.g. a mass density observable for the composed components

$$\hat{\rho}_\kappa(\mathbf{x}) = (m_1+m_2)\hat{\Psi}_\kappa^\dagger(\mathbf{x})\hat{\Psi}_\kappa(\mathbf{x}) \quad (7)$$

and one obtains from (5) the balance equation:

$$\begin{aligned} \dot{\hat{\rho}}_\kappa(\mathbf{x}) &= -\nabla\cdot\mathbf{J}_\kappa(\mathbf{x}) + S_\kappa(\mathbf{x}) \quad (8)\\ \mathbf{J}_\kappa(\mathbf{x}) &= \frac{1}{2}\hat{\Psi}_\kappa^\dagger(\mathbf{x})\,(-i\hbar\nabla)\,\hat{\Psi}_\kappa(\mathbf{x}) + h.c. \end{aligned}$$

The contribution $S_\kappa(\mathbf{x})$ is zero if the approximation (6) of the last terms in (5) is taken into account, otherwise one has a "source" contribution. Starting with (5) the whole structure of quantum field hydrodynamics is reobtained with correction terms linked to the many body structure of the last term of (5). Even if in our treatment the concept of particles as elementary constituents of systems is no longer in the foreground, however the basic quantum mechanical structure arising from bound state wave functions and bound states energies is still in full evidence.
Furthermore one has the following nonzero (anti)commutation relations:

$$\begin{aligned} \left[\hat{\Psi}_\kappa(\mathbf{x}),\hat{\psi}_1^\dagger(\mathbf{y})\right]_\mp &= \frac{m_1+m_2}{m_2}g_\kappa\left(\frac{m_1+m_2}{m_1}|\mathbf{x}-\mathbf{y}|\right)\hat{\psi}_2\left(\frac{m_1+m_2}{m_2}\mathbf{x}-\frac{m_1}{m_2}\mathbf{y}\right) \quad (9)\\ \left[\hat{\Psi}_\kappa(\mathbf{x}),\hat{\psi}_2^\dagger(\mathbf{y})\right]_\mp &= \frac{m_1+m_2}{m_1}g_\kappa\left(\frac{m_1+m_2}{m_2}|\mathbf{x}-\mathbf{y}|\right)\hat{\psi}_1\left(\frac{m_1+m_2}{m_2}\mathbf{x}-\frac{m_2}{m_1}\mathbf{y}\right)\\ \left[\hat{\Psi}_{\kappa'}(\mathbf{x}),\hat{\Psi}_\kappa^\dagger(\mathbf{y})\right]_- &= \delta_3(\mathbf{x}-\mathbf{y})\delta_{\kappa'\kappa}\mathbf{I} \quad (10)\\ &+ \int d^3\mathbf{r}\,g_\kappa\left(\mathbf{r}-\frac{m_1+m_2}{2m_1}(\mathbf{x}-\mathbf{y})\right)g_{\kappa'}\left(\mathbf{r}+\frac{m_1+m_2}{2m_2}(\mathbf{x}-\mathbf{y})\right)\\ &\left[\left(\frac{m_1+m_2}{m_2}\right)^3\hat{\psi}_2^\dagger\left(\mathbf{y}-\frac{m_1}{m_1+m_2}\mathbf{r}-\frac{m_1}{2m_2}(\mathbf{x}-\mathbf{y})\right)\right.\\ &\hat{\psi}_2\left(\mathbf{x}-\frac{m_1}{m_1+m_2}\mathbf{r}+\frac{m_1}{2m_2}(\mathbf{x}-\mathbf{y})\right)+\\ &\left(\frac{m_1+m_2}{m_1}\right)^3\hat{\psi}_1^\dagger\left(\mathbf{y}+\frac{m_2}{m_1+m_2}\mathbf{r}-\frac{m_2}{2m_1}(\mathbf{x}-\mathbf{y})\right)\\ &\left.\hat{\psi}_1\left(\mathbf{x}+\frac{m_2}{m_1+m_2}\mathbf{r}+\frac{m_2}{2m_1}(\mathbf{x}-\mathbf{y})\right)\right]; \end{aligned}$$

by the first term at the r.h.s. of (10) the usual situation is represented, the other terms, due to the behaviour of $g_\kappa(\mathbf{r}), g_{\kappa'}(\mathbf{r})$, are negligible if either $|\mathbf{x}-\mathbf{y}|\frac{1}{2}(1+\frac{m_2}{m_1}) > \lambda_b$ or

$|\mathbf{x}-\mathbf{y}|\frac{1}{2}(1+\frac{m_1}{m_2}) > \lambda_b$, which ensures commutations of local observables related with the composed components at fixed time, for space separation $|\mathbf{x}-\mathbf{y}|\frac{1}{2}(1+\frac{m_2}{m_1}) > \lambda_b$, where we take $m_1 \leq m_2$; by (9) similar considerations are immediately extended to local observables built with the elementary fields $\hat{\psi}_a(\mathbf{x}), \hat{\psi}_a^\dagger(\mathbf{x})$.
In our case a meaningful choice of relevant observables could be the mass densities $m_1\hat{\psi}_1^\dagger(\mathbf{x})\hat{\psi}_1(\mathbf{x})$, $m_2\hat{\psi}_2^\dagger(\mathbf{x})\hat{\psi}_2(\mathbf{x})$, $(m_1+m_2)\hat{\psi}_\kappa^\dagger(\mathbf{x})\hat{\psi}_\kappa(\mathbf{x})$, momentum and kinetic densities, some effective short-range and therefore quasilocal interaction energy.
A set of state parameters consists of functions $\zeta_j(\mathbf{x})$ such that the operator

$$\hat{\Phi}(\zeta) \equiv \sum_j \int_\Omega d^3\mathbf{x}\,\zeta_j(\mathbf{x})\hat{A}_j(\mathbf{x}) \tag{11}$$

is essentially self-adjoint with a point spectrum and $\exp[-\hat{\Phi}(\zeta)]$ is trace class, so that a reference generalized Gibbs state

$$\hat{w}_\zeta = \frac{\mathrm{e}^{-\hat{\Phi}(\zeta)}}{\mathrm{Tr}\,\mathrm{e}^{-\hat{\Phi}(\zeta)}} \tag{12}$$

can be defined. The pretheory which has been formulated in terms of the state parameters ζ presents for them, at least in suitable time intervals, a deterministic time evolution: there are typical classical differential equations by which a trajectory $\zeta(t)$ can be calculated; existence of this deterministic stage allows the phenomenological individuation of the system itself. In this situation a basic relationship between $\zeta(t)$ and the relevant variables $\hat{A}_j(\mathbf{x})$ is given by the assumption that the expectation values of them is given by $\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\hat{w}_{\zeta_t}$, establishing in this way that $\hat{w}_{\zeta_t}$ is equivalent to the statistical operator $\hat{\rho}_t$ of the system relatively to the relevant variables. See also (29).
The expectations of the relevant variables, being linked to the deterministically evolving state parameters $\zeta(t)$, are “reproducible” physical attributes of statistical collection of systems, prepared in an appropriate way. Very often the statistical character of the relevant variables can be neglected and one can talk about their values $A_j(\mathbf{x},t) =< \hat{A}_j(\mathbf{x}) >_t$ for the dispersion however $< \left(A_j(\mathbf{x},t) - < \hat{A}_j(\mathbf{x}) >_t\right)^2 >$ which decides for this, the expression Tr $\left(\hat{A}_j(\mathbf{x}) - < \hat{A}_j(\mathbf{x}) >_t\right)^2 \hat{w}_{\zeta_t}$ gives only a partial account, when $\hat{A}_j^2(\mathbf{x}) \neq M$: anyway $A_j(\mathbf{x},t)$ can be taken as a shortened notation for $< \hat{A}_j(\mathbf{x}) >_t$.
By this relative equivalence of $\hat{w}_{\zeta_t}$ to $\hat{\rho}_t$ one has $< \hat{A}_j(\mathbf{x}) >_t = -\frac{\delta}{\delta\zeta_j(x)} \log Z(\zeta_l)$, where the partition function:

$$Z(\zeta) = \mathrm{Tr}\,\mathrm{e}^{-\hat{\Phi}(\zeta)} \tag{13}$$

has been introduced. Due to locality the expectations $< \hat{A}_j(\mathbf{x}) >_t$ depend on the local state $\zeta_t(\mathbf{x})$. In fact if $\mathbf{x}$ is at a distance from the boundary $\partial\Omega$ of the confining region Ω much larger than λ, taking around $\mathbf{x}$ a sphere $\omega_\delta(\mathbf{x})$ with centre in $\mathbf{x}$ and radius δ with $r_0 \ll \delta \ll \lambda$,

$$< \hat{A}_j(\mathbf{x}) > \simeq \frac{\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\mathrm{e}^{-\sum_j \int_{\omega_\delta(\mathbf{x})\cup\omega^c_{\delta+r_0}(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}}{\mathrm{Tr}\,\mathrm{e}^{-\sum_j \int_{\omega_\delta(\mathbf{x})\cup\omega^c_{\delta+r_0}(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}}, \tag{14}$$

where in the integration region the spherical sheet $\delta \leq |\mathbf{x}| \leq \delta + r_0$ has been eliminated, by an approximation reliable, due to the condition $r_0^3 \ll V(\Omega)$; then by the locality property of the observables $\hat{A}_j(\mathbf{x})$ the factorization occurs:

$$\frac{\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\mathrm{e}^{-\int_{\omega_\delta(\mathbf{x})\cup\omega^c_{\delta+r_0}(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}}{\mathrm{Tr}\,\mathrm{e}^{-\int_{\omega_\delta(\mathbf{x})\cup\omega^c_{\delta+r_0}(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}} = \frac{\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\mathrm{e}^{-\int_{\omega_\delta(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}}{\mathrm{Tr}\,\mathrm{e}^{-\int_{\omega_\delta(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}} \frac{\mathrm{Tr}\,\mathrm{e}^{-\int_{\omega^c_{\delta+r_0}(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}}{\mathrm{Tr}\,\mathrm{e}^{-\int_{\omega^c_{\delta+r_0}(\mathbf{x})} d^3\mathbf{y}\,\zeta_{jt}(\mathbf{y})\hat{A}_j(\mathbf{y})}}; \tag{15}$$

splitting away the last factor, replacing $\zeta_{jt}(\mathbf{y})$ with $\zeta_{jt}(\mathbf{x})$, which is reliable since $\delta \ll \lambda$, one has approximate representation:

$$< \hat{A}_j(\mathbf{x}) > \simeq \frac{\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\mathrm{e}^{-\int_\Omega d^3\mathbf{y}\,\zeta_{jt}(\mathbf{x})\hat{A}_j(\mathbf{y})}}{\mathrm{Tr}\,\mathrm{e}^{-\int_\Omega d^3\mathbf{y}\,\zeta_{jt}(\mathbf{x})\hat{A}_j(\mathbf{y})}} \tag{16}$$

This approximation can be improved by a series expansion of $\zeta_{jt}(\mathbf{y})$ around $\mathbf{x}$, so that corrections to $< \hat{A}_j(\mathbf{x}) >$ arise depending on $\nabla\zeta_{jt}(\mathbf{x})$: in such a way one actually controls whether on the relevant space scale these corrections can be neglected. All this shows the subtle interplay of locality of relevant variables (where in the case of composed field observables this means $\lambda_b \sim r_0$), space scale λ, and macroscopic scale of the system by $r_0^3 \ll V(\Omega)$.
The key point that makes the connection of phenomenological description of systems with an underlying field theory highly non trivial is the fact that the set M generated by the relevant variables is not invariant under time evolution: even if one assumes $\hat{H} \in M$, due to the fact that M is not an operator algebra, for $\hat{A}_j(\mathbf{x}) \in M$ generally $\dot{\hat{A}}_j(\mathbf{x}) = \frac{i}{\hbar}\left[\hat{H}, \hat{A}_j(\mathbf{x})\right] \neq M$: in a loose way $\dot{\hat{A}}_j(\mathbf{x})$ is less local than $\hat{A}_j(\mathbf{x})$; by inspection of the typical hydrodinamical fields $\dot{\hat{A}}_j(\mathbf{x})$ presents higher order space derivatives of $\hat{\psi}(\mathbf{x}), \hat{\psi}^\dagger(\mathbf{x})$ as $\hat{A}_j(\mathbf{x})$.
In the model we are referring to, since we assumed that the mass densities $\hat{\rho}_1(\mathbf{x}), \hat{\rho}_2(\mathbf{x}), \hat{\rho}_\kappa(\mathbf{x})$ and the momentum densities $\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2, \hat{\mathbf{p}}_\kappa$ belong to M, one has that concerning the corresponding commutators with $\hat{H}$ only $\dot{\hat{\rho}}_1(\mathbf{x}), \dot{\hat{\rho}}_2(\mathbf{x})$ still belong to M, whereas $\dot{\hat{\rho}}_\kappa(\mathbf{x})$ does only if the source term $\hat{S}_\kappa(\mathbf{x})$ in (8) can be neglected.
To better appreciate the relevance of this point, let us assume for a moment that M is invariant under the time evolution map in Heisenberg representation. Then there is a quite obvious choice for the initial statistical operator $\hat{\rho}_{t_0}$: it is the Gibbs state (12) $\hat{\rho}_{t_0} = \hat{w}_{\zeta_{t_0}}$ with state parameters determined by the initial values of the expectations of the relevant variables; this is the "least biased" choice of a statistical operator $\hat{\rho}$ under the conditions $< \hat{A}_j(\mathbf{x}) >_{t_0} = \mathrm{Tr}\left(\hat{A}_j(\mathbf{x})\hat{\rho}_{t_0}\right)$ for all j and for any $\mathbf{x} \in \Omega$.
Indicating by $\mathscr{U}^t_{t_0}$ the unitary time evolution group determined by the Hamiltonian one has

$$\hat{\rho}_t = \mathscr{U}^t_{t_0}\hat{\rho}_{t_0} = \mathscr{U}^t_{t_0}\hat{w}_{\zeta_{t_0}} = Z_t^{-1}\mathrm{e}^{-[\mathscr{U}^t_{t_0}\hat{\Phi}(\zeta_{t_0})]}, \tag{17}$$

setting

$$\mathscr{U}_{t_0}^t \hat{A}_j(\mathbf{x}) = \sum_{j'} \int d^3\mathbf{y}' \gamma_{jj'}(\mathbf{y},\mathbf{y}',(t-t_0))\hat{A}_j(\mathbf{y}') \tag{18}$$

one has immediately:

$$\hat{\rho}_t = \hat{w}_{\zeta_t}, \qquad \zeta_{jt}(\mathbf{y}) = \sum_{j'} \int d^3\mathbf{y}' \zeta_{j'}(\mathbf{y}')\gamma_{j'j}(\mathbf{y}',\mathbf{y},(t-t_0)) \tag{19}$$

giving in terms of the dynamical coefficients defined by (19), the deterministic evolution of the state parameters.
The entropy $S(\zeta_t) = -k \,\mathrm{Tr}\, \hat{w}_{\zeta_t} \log \hat{w}_{\zeta_t}$ associated with the Gibbs states $\hat{w}_{\zeta_t}$, coinciding with the Von-Neumann entropy $-k \,\mathrm{Tr}\, \hat{\rho}_t \log \hat{\rho}_t$ is independent of time and a reversible dynamics of the relevant variables would be found, quite contrary to significant phenomenology: let us observe that if this treatment were realistic all quantum features should be inside (19) and there would be no evidence even for the concept of microsystem. Indeed the choice of $\hat{\rho}_{t_0}$ must be biased by the fact that a deterministic evolution of the state parameters exists. We shall assume that $\hat{\rho}_t$ must be represented as

$$\hat{\rho}_t = Z_t^{-1} \mathrm{e}^{-[\hat{\Phi}(\zeta_t)+\hat{\mathscr{S}}(t)]} \qquad Z_t = \mathrm{Tr}\, \mathrm{e}^{-[\hat{\Phi}(\zeta_t)+\hat{\mathscr{S}}(t)]}, \tag{20}$$

$\hat{\mathscr{S}}(t)$ being a small correction term built with “irrelevant observables”. In order to find the structure of $\hat{\mathscr{S}}(t)$, let us write

$$\hat{\rho}_t = \mathscr{U}_{t_0}^t \hat{\rho}_{t_0} = Z_t^{-1} \mathrm{e}^{-[\mathscr{U}_{t_0}^t \hat{\Phi}(\zeta_{t_0}) + \mathscr{U}_{t_0}^t \hat{\mathscr{S}}(t_0)]}, \tag{21}$$

and observe that one can trivially rewrite $\mathscr{U}_{t_0}^t \hat{\Phi}(\zeta_{t_0})$ in terms of $\hat{\Phi}(\zeta_t)$ by:

$$\begin{aligned}\mathscr{U}_{t_0}^t \hat{\Phi}(\zeta_{t_0}) &= \hat{\Phi}(\zeta_t) - \int_{t_0}^t dt' \frac{d}{dt'} [\mathscr{U}_{t'}^t \hat{\Phi}(t')] \\ &= \hat{\Phi}(\zeta_t) - \sum_j \int_{t_0}^t dt' \int_\Omega d^3\mathbf{x}\, \mathscr{U}_{t'}^t \left[\zeta_{jt'}(\mathbf{x}) \dot{\hat{A}}_j(\mathbf{x}) + \frac{\partial \zeta_{jt'}}{\partial t'}(\mathbf{x}) \hat{A}_j(\mathbf{x}) \right];\end{aligned} \tag{22}$$

then, comparing (20) with (21) and (22) one has

$$\hat{\mathscr{S}}(t) = -\sum_j \int_{t_0}^t dt' \int_\Omega d^3\mathbf{y}\, \mathscr{U}_{t'}^t \left[\zeta_{jt'}(\mathbf{y}) \dot{\hat{A}}_j(\mathbf{y}) + \frac{\partial \zeta_{jt'}}{\partial t'}(\mathbf{y}) \hat{A}_j(\mathbf{y}) \right] + \mathscr{U}_{t_0}^t \hat{\mathscr{S}}(t_0) \tag{23}$$

This structure shows the following composition law for $\hat{\mathscr{S}}(t)$: for $t_0 \leq t_1 \leq t$ one has

$$\hat{\mathscr{S}}(t) = \hat{\mathscr{S}}(t, t-t_1) + \mathscr{U}_{t_1}^t \hat{\mathscr{S}}(t_1), \tag{24}$$

where

$$\hat{\mathscr{S}}(t, t-t_1) = -\sum_j \int_{t-t_1}^t dt' \int_\Omega d^3\mathbf{y}\, \mathscr{U}_{t'}^t \left[\zeta_{jt'}(\mathbf{y}) \dot{\hat{A}}_j(\mathbf{y}) + \frac{\partial \zeta_{jt'}}{\partial t'}(\mathbf{y}) \hat{A}_j(\mathbf{y}) \right], \tag{25}$$

will be called the "head part" of $\hat{\mathscr{S}}(t)$, referring to the decomposition of the time interval $(-\infty,t)$ into $(t-t_1,t)$ and $(-\infty,t-t_1)$.
The structure of $\hat{\mathscr{S}}(t)$ suggests that also $\hat{\mathscr{S}}(t_0)$ should have an analogous form. Taking formally $\lim_{t_0\to-\infty}\hat{\mathscr{S}}(t_0)=0$ (there is no longer a previous deterministic evolution) one has:

$$
\begin{aligned}
\hat{\mathscr{S}}(t) &= -\sum_j \int_{-\infty}^{t} dt' \int_\Omega d^3\mathbf{y}\, \mathscr{U}_{t'}^{t} \left[\zeta_{jt'}(\mathbf{y}) \dot{\hat{A}}_j(\mathbf{y}) + \frac{\partial \zeta_{jt'}}{\partial t'}(\mathbf{y}) \hat{A}_j(\mathbf{y}) \right] \\
&= -\sum_j \int_{-\infty}^{t} dt' \int_\Omega d^3\mathbf{y}\, \mathscr{U}_{t'}^{t} \left[\frac{\partial \zeta_{jt'}(\mathbf{y})}{\partial \mathbf{y}} \cdot \hat{\mathbf{j}}_j(\mathbf{y}) + \frac{\partial \zeta_{jt'}}{\partial t'}(\mathbf{y}) \hat{A}_j(\mathbf{y}) \right. \\
&\quad \left. - \nabla \cdot (\zeta_{jt'}(\mathbf{y}) \hat{\mathbf{j}}_j(\mathbf{y})) + \zeta_{jt'}(\mathbf{y}) \hat{S}_j(\mathbf{y}) \right]
\end{aligned}
\tag{26}
$$

where we have also taken into account the general form of the balance equation of the relevant field observables:

$$
\dot{\hat{A}}_j(\mathbf{x}) = -\nabla \cdot \hat{\mathbf{j}}_j(\mathbf{x}) + \hat{S}_j(\mathbf{x}),
\tag{27}
$$

the source term appearing for the composed field observables.
Eqs (11),(20),(26) represent the statistical operator of the system, carrying one family ζ_t of objective state parameters suggested by the phenomenological pretheory. For $t' \leq t_0$ they describe the preparation of the system, for $t > t_0$ they will be determined by the condition of existence of a deterministic evolution. For $t' \leq t_0$ they must be chosen in order to describe phenomenological elements of the preparation procedure. One expects that their choice is physically significant only for $t' \in [t_0-\tau, t_0]$, τ being the duration of the preparation procedure, having the order of magnitude of the typical time scale of the description we are embedding in the more fundamental field theory; since in this time interval the system is not isolated also the fluxes of the currents $\hat{\mathbf{j}}_j$ across $\partial\Omega$ can be important; for $t > t_0$ these contributions are eliminated if an isolated system is considered. By this structure of $\hat{\mathscr{S}}(t)$ a time arrow $t_0 - \tau < t_0$ becomes explicit and irreversibility comes in.
The treatment of non equilibrium systems proposed by Zubarev [5], leads to a "non equilibrium statistical operator" coinciding with our $\hat{\rho}_t$ if an adiabatic damping factor is introduced to describe the limit $\tau \to -\infty$ after the limit $V(\Omega) \to \infty$. The question if the most naive choice $\zeta_{jt'} = 0$ for $t' < t_0 - \tau$ is appropriate for our purpose requires further analysis of realistic models.

THE CASE OF A DETERMINISTIC DYNAMICS

The idea of phenomenological dynamics in terms of one family of state parameters ζ_t, i.e. a deterministic dynamics, embedded in an underlying quantum field theory, leads to the representation (21) of a statistical operator $\hat{\rho}_t$ carrying these state parameters for $t' \leq t$ as shown by (20), (11), (26); then the following evolution equation for the expectation of relevant variables arises:

$$
\frac{d}{dt}\hat{A}_j(\mathbf{x}) = \mathrm{Tr}\left(\dot{\hat{A}}_j(\mathbf{x})\hat{\rho}_t\right).
\tag{28}
$$

Since the basic characterisation of $\zeta_l(\mathbf{y})$ was given by the equivalence of $\hat{\rho}_t$ to $\hat{w}_{\zeta_t}$ relatively to the relevant variables, i.e.

$$\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\hat{\rho}_t = \mathrm{Tr}\,\hat{A}_j(\mathbf{x})\hat{w}_{\zeta_t} = <\hat{A}_j(\mathbf{x})>_t, \tag{29}$$

by (28), (29) the state variables obey the equation

$$\frac{d}{dt}\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\hat{w}_{\zeta_t} = \mathrm{Tr}\,\left(\dot{\hat{A}}_j(\mathbf{x})\hat{\rho}_t\right) \tag{30}$$

or also

$$\frac{d}{dt}\left(-\frac{\delta \log Z_t(\zeta_l)}{\delta \zeta_j(x)}\right) = \frac{\mathrm{Tr}\,\dot{\hat{A}}_j(\mathbf{x})\mathrm{e}^{-[\hat{\Phi}(\zeta_t)+\hat{\mathscr{S}}(t)]}}{\mathrm{Tr}\,\mathrm{e}^{-[\hat{\Phi}(\zeta_t)+\hat{\mathscr{S}}(t)]}}, \tag{31}$$

where $\hat{\mathscr{S}}(t)$ is given by (26).
By a glance to (26) one becomes aware that (31) has not the form of an evolution equation for ζ_t since there is no separation of $\frac{d\zeta_t}{dt}$ from $\zeta_{t'}$, $t' \leq t$. Such a separation can be achieved on a basis of a perturbative procedure based on the leading idea of a prominence of $\hat{\Phi}(\zeta_t)$ on $\hat{\mathscr{S}}(t)$.
There is a very rough, though important approximation to (31), simply consisting in neglecting $\hat{\mathscr{S}}(t)$ with respect to $\hat{\Phi}(\zeta_t)$: then (31) becomes

$$\frac{d}{dt}\left(-\frac{\delta \log Z_t(\zeta_l)}{\delta \zeta_j(x)}\right) = \mathrm{Tr}\,\left(\dot{\hat{A}}_j(\mathbf{x})\hat{w}_{\zeta_t}\right); \tag{32}$$

taking locality of $\hat{A}_j(\mathbf{x})$ and $\dot{\hat{A}}_j(\mathbf{x})$ into account, by the argument based on $\lambda \gg r_0, \lambda \gg \lambda_b$ which led to (15) the very relevant approximation of (32) holds:

$$\frac{d}{dt}\left(-\frac{\delta \log Z_t(\zeta^0)}{\delta \zeta_j^0(x)}\right)_{\zeta^0=\zeta_{jt}(x)} = \mathrm{Tr}\,\left(\dot{\hat{A}}_j(\mathbf{x})\hat{w}(\zeta_{lt}(\mathbf{x}))\right), \tag{33}$$

where $\hat{w}(\zeta_{lt}(\mathbf{x}))$ is the Gibbs state at fixed state parameters $\zeta_{lt}(\mathbf{x})$, taken at the space point $\mathbf{x}$ and $Z(\zeta_{lt}(\mathbf{x}))$ is the associated partition function. Passing from state variables $\zeta_{lt}(\mathbf{x})$ to expectations $A_j(\mathbf{x},t) = \mathrm{Tr}\,\hat{A}_j(\mathbf{x})\hat{w}_{\zeta_t}$ of the corresponding relevant variables leads to

$$\frac{\partial}{\partial t}A_j(\mathbf{x},t) = \mathrm{Tr}\,\left[\left(-\nabla\cdot\hat{\mathbf{j}}_j(\mathbf{x}) + \hat{S}_j(\mathbf{x})\right)\hat{w}(\zeta_{lt}(\mathbf{x}))\right] \tag{34}$$

these are well known equations, e.g. in the case of fluidodynamics (34) are the equations of an ideal fluid.
Notice that (34) are classical field equations with an impact from underlying quantum theory which here appears formally as a state equation based on a quantum Gibbs state. By eq. (34) an approximate representation of $\frac{\partial}{\partial t}\zeta_{lt}(\mathbf{x})$ can be given in terms of $\zeta_{lt}(\mathbf{x})$:

$$\frac{\partial}{\partial t}\zeta_{lt}(\mathbf{x}) = \alpha_l^0\left(\zeta_t(\mathbf{x})\right), \tag{35}$$

useful only on a time scale smaller than the physical typical time scale of the evolution of ζ_t: for such time scale the approximation consisting in simply neglecting $\hat{\mathscr{S}}(t)$ in (31) would kill all irreversible dynamical features.
Let us now come back to $\hat{\mathscr{S}}(t)$ given by (26) and introducing the typical time scale τ on which the state variables are practically constant, recalling (24), let us split $\hat{\mathscr{S}}(t)$ in an "head part"$\hat{\mathscr{S}}(t,t-\tau)$ and a tail part $\mathscr{U}^t_{t-\tau}\hat{\mathscr{S}}(t-\tau)$:

$$\hat{\mathscr{S}}(t) = \hat{\mathscr{S}}(t,t-\tau) + \mathscr{U}^t_{t-\tau}\hat{\mathscr{S}}(t-\tau), \tag{36}$$

for $\hat{\mathscr{S}}(t,t-\tau)$ we use approximation (35) to represent $\frac{\partial}{\partial t}\zeta_{lt}(\mathbf{y})$, then one has

$$\hat{\mathscr{S}}^{(0)}(t,t-\tau) = -\sum_j \int_{t-\tau}^{t} dt' \int_\Omega d^3\mathbf{y}\, \mathscr{U}^t_{t'} \left[\frac{\partial \zeta_{jt'}(\mathbf{y})}{\partial \mathbf{y}} \cdot \hat{\mathbf{j}}_j(\mathbf{y}) + \alpha^0_j(\zeta_t(\mathbf{y}))\hat{A}_j(\mathbf{y}) + \zeta_{jt'}(\mathbf{y})\hat{S}_j(\mathbf{y}) \right] \tag{37}$$

Let us note that for an isolated system, due to time translation invariance, $\mathscr{U}_t^{t-\tau'}$ depends only on τ' and can be indicated by $\mathscr{U}(\tau')$; then the dependence of $\hat{\mathscr{S}}^{(0)}(t,t-\tau)$ on t appears only through the state parameters $\zeta_{jt}(\mathbf{y})$, however an explicit dependence on the time scale τ of the description arises: by dimensional reasons it will appear in the form τ_0/τ, τ_0 being a characteristic microphysical time and by an appropriate value of τ_0/τ this explicit dependence on τ should eventually be negligible, like the more simple contributions of the type r_0/λ we have already met. To embed a phenomenological description in a more fundamental theory has this price of the appearance of these τ_0/τ, r_0/λ contributions; it becomes cheaper, lowering τ_0, r_0 with new fields, new choices of M. By this basic step, which involves (35) and requires the application α^0 mapping ζ_t to $\frac{d\zeta_t}{dt}$, eq. (31) becomes an evolution equation, albeit a very complicated one, for the state parameters since $\frac{d\zeta_{lt}(\mathbf{x})}{dt}$ appears at the l.h.s while $\zeta_{l't}(\mathbf{y}')$ and $\frac{d\zeta_{lt'}(\mathbf{y}')}{dt'}$ for $t' \leq t-\tau$ $\mathbf{y}' \in \Omega$ appear at r.h.s; the clue to a solution is the fact that the space-time local part $\hat{\Phi}(\zeta_t)$ should dominate with respect to $\hat{\mathscr{S}}(t)$ which is non local in space-time. As it was shown in [6] and recalled in [4] one can represent the exponential in (21) separating the contribution $\mathrm{e}^{-\hat{\Phi}(\zeta_t)}$, so that (20) becomes:

$$\hat{\rho}_t = \frac{\mathrm{e}^{-\hat{\Phi}(\zeta_t)} + \hat{\mathsf{S}}\mathrm{e}^{-\hat{\Phi}(\zeta_t)} + \mathrm{e}^{-\hat{\Phi}(\zeta_t)}\hat{\mathsf{S}}^\dagger + \hat{\mathsf{S}}\mathrm{e}^{-\hat{\Phi}(\zeta_t)}\hat{\mathsf{S}}^\dagger}{\mathrm{Tr}\,[\mathrm{e}^{-\hat{\Phi}(\zeta_t)} + \hat{\mathsf{S}}\mathrm{e}^{-\hat{\Phi}(\zeta_t)} + \mathrm{e}^{-\hat{\Phi}(\zeta_t)}\hat{\mathsf{S}}^\dagger + \hat{\mathsf{S}}\mathrm{e}^{-\hat{\Phi}(\zeta_t)}\hat{\mathsf{S}}^\dagger]} \tag{38}$$

where

$$\hat{\mathsf{S}} \equiv \int_0^{\frac{1}{2}} du\, \mathrm{e}^{-u[\hat{\Phi}(\zeta_t)+\hat{\mathscr{S}}(t)]}\hat{\mathscr{S}}(t)\mathrm{e}^{+u\hat{\Phi}(\zeta_t)}; \tag{39}$$

dividing numerator and denominator of (38) by $\mathrm{Tr}\,\mathrm{e}^{-\hat{\Phi}(\zeta_t)}$, one relates $\hat{\rho}_t$ to $\hat{w}_{\zeta_t}$ by

$$\hat{\rho}_t = \frac{\hat{w}_{\zeta_t} + \hat{\mathsf{S}}\hat{w}_{\zeta_t} + \hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger + \hat{\mathsf{S}}\hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger}{1 + \mathrm{Tr}\,(\hat{\mathsf{S}}\hat{w}_{\zeta_t} + \hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger + \hat{\mathsf{S}}\hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger)} = \hat{w}_{\zeta_t} + \hat{\hat{\Gamma}}(t), \tag{40}$$

where $\hat{\mathring{\Gamma}}(t)$ is a traceless operator given by:

$$\hat{\mathring{\Gamma}}(t) = \gamma(t)[\hat{\mathsf{S}}\hat{w}_{\zeta_t} + \hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger + \hat{\mathsf{S}}\hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger - \hat{w}_{\zeta_t}\mathrm{Tr}\,(\hat{\mathsf{S}}\hat{w}_{\zeta_t} + \hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger + \hat{\mathsf{S}}\hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger)] \tag{41}$$

with $\gamma(t) = 1/[1 + \mathrm{Tr}\,(\hat{\mathsf{S}}\hat{w}_{\zeta_t} + \hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger + \hat{\mathsf{S}}\hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger)]$. In correspondence to the splitting of $\hat{\mathscr{S}}(t)$ one has a corresponding splitting of $\hat{\mathsf{S}}(t)$ which takes the form

$$\hat{\mathsf{S}} = \hat{\mathsf{S}}(t, t-\tau) + \hat{\mathsf{T}}(t) \tag{42}$$

where

$$\begin{aligned} \hat{\mathsf{S}}(t,t-\tau) \;&=\; \sum_j \int_\Omega d^3\mathbf{y} \int_0^{\frac{1}{2}} du\, \mathrm{e}^{-u[\hat{\Phi}(\zeta_t) + \hat{\mathscr{S}}(t)]} \\ &\int_\tau^0 d\tau' \mathscr{U}(\tau') \left(\frac{\partial \zeta_{jt}(\mathbf{y})}{\partial \mathbf{y}} \cdot \hat{\mathbf{j}}_j(\mathbf{y}) + \alpha_j^0(\zeta_t(\mathbf{y}))\hat{A}_j(\mathbf{y}) \right) \mathrm{e}^{u[\hat{\Phi}(\zeta_t)]} \end{aligned} \tag{43}$$

and by (26)

$$\hat{\mathsf{T}}(t) = \int_\Omega d^3\mathbf{y} \int_0^{\frac{1}{2}} du\, \mathrm{e}^{-u[\hat{\Phi}(\zeta_t) + \hat{\mathscr{S}}(t)]} \int_{-\infty}^{t-\tau} dt'\, \mathscr{U}^t_{t'} \sigma_{\zeta_{t'}}(\mathbf{y}) \mathrm{e}^{+u\hat{\Phi}(\zeta_t)} \tag{44}$$

where $\sigma_{\zeta_{t'}}(\mathbf{y}) = \nabla \cdot (\zeta_{jt'}(\mathbf{y})\hat{\mathbf{j}}_j(\mathbf{y})) - \zeta_{jt'}(\mathbf{y})\hat{S}_j(\mathbf{y})$. Note that $\hat{\rho}_t$ in (40) depends on ζ_t through $\hat{w}_{\zeta_t}, \hat{\mathsf{S}}(t, t-\tau)$ whereas it depends on the previous history $\zeta(t')$, $t' \leq t - \tau$ through $\hat{\mathsf{T}}(t)$ directly inside (41) and also, presumably weakly, on $\hat{\mathsf{T}}(t)$ inside $\hat{\mathscr{S}}(t)$ appearing at the r.h.s of eq. (43). Usual deterministic phenomenology however gives no evidence of such a situation: so one can expect that by rather general reasons $\hat{\mathsf{T}}(t)$ can be neglected; e.g. just in this way fluidodynamics can be derived in a field theoretical context [5, 7]. However one should not expect this to be a general formal feature: in fact if this were the case, there would be no conceptual reason to introduce microsystems and just the breakdown of deterministic dynamics, which gives experimental evidence of them, would be rather mysterious; only instabilities related to the new structure of evolution equations for $\zeta_t(\mathbf{x})$ or problems in relations between $A(\mathbf{x},t)$ and $\zeta_t(\mathbf{x})$ could require statistical cures. Let us now pursue a moment the idea that $\hat{\mathsf{T}}(t)$ is negligible and that $\hat{\Phi}(\zeta_t)$ is prevailing on $\hat{\mathscr{S}}(t, t-\tau)$. Correspondingly we take the following approximation of $\hat{\mathring{\Gamma}}(t)$ in (40)

$$\begin{aligned} \hat{\mathring{\Gamma}}^{(0)}(t) = \Bigg\{ \Bigg[&\sum_j \int_\Omega d^3\mathbf{y} \int_0^{\frac{1}{2}} du\, \mathrm{e}^{u\hat{\Phi}(\zeta_t)} \int_0^\tau d\tau' \mathscr{U}(\tau') \\ &\left(-\frac{\partial \zeta_{jt}(\mathbf{y})}{\partial \mathbf{y}} \cdot \hat{\mathbf{j}}_j(\mathbf{y}) - \alpha_j^0(\zeta_t(\mathbf{y}))\hat{A}_j(\mathbf{y}) \right) \mathrm{e}^{-u\hat{\Phi}(\zeta_t)} \hat{w}_{\zeta_t} \\ &+ \hat{w}_{\zeta_t} \sum_j \int_\Omega d^3\mathbf{y} \int_0^{\frac{1}{2}} du\, \mathrm{e}^{u\hat{\Phi}(\zeta_t)} \int_\tau^0 d\tau' \mathscr{U}(\tau') \end{aligned} \tag{45}$$

$$\left(-\frac{\partial \zeta_{jt}(\mathbf{y})}{\partial \mathbf{y}} \cdot \hat{\mathbf{j}}_j(\mathbf{y}) - \alpha_j^0(\zeta_t(\mathbf{y}))\hat{A}_j(\mathbf{y})\right) \mathrm{e}^{-u\hat{\Phi}(\zeta_t)}\Big] - \hat{w}_{\zeta_t}\mathrm{Tr}[-]\Big\} \frac{1}{1+\mathrm{Tr}[-]},$$

where the operator inside $[-]$ is understood to be the one written between square brackets above. By this approximation the evolution equation (30) become much simpler, one has:

$$\frac{d}{dt}\mathrm{Tr}\,\hat{A}_j(\mathbf{x})\hat{w}_{\zeta_t} = \mathrm{Tr}\,\dot{\hat{A}}_j(\mathbf{x})\hat{w}_{\zeta_t} + \mathrm{Tr}\,\dot{\hat{A}}_j(\mathbf{x})\hat{\mathring{\Gamma}}^{(0)}(t). \tag{46}$$

In the last term of (46) the non local structure of $\hat{\mathring{\Gamma}}^{(0)}(t)$ with respect to the state parameters $\zeta_{jt}(\mathbf{y})$ and their derivatives $\frac{\partial \zeta_{jt}(\mathbf{y})}{\partial \mathbf{y}}$ can be neglected, replacing them respectively with $\zeta_{jt}(\mathbf{x})$, $\frac{\partial \zeta_{jt}(\mathbf{x})}{\partial \mathbf{x}}$; in this way the last term of (46) becomes a linear combination of these terms with coefficients being the Kubo correlation functions involving the field variables $\dot{\hat{A}}_j(\mathbf{x}),\hat{A}_j(\mathbf{y}),\hat{\mathbf{j}}_j(\mathbf{y})$; in this way (46) becomes a closed, non linear, local evolution equation for the state parameters ζ_t, or for the expectations $A_l(\mathbf{x},t) = \mathrm{Tr}\,\hat{A}_j(\mathbf{x})\hat{w}_{\zeta_t}$ of the relevant observables. It represents the central equation inside linear thermodynamics of irreversible processes, and provides for this phenomenological description a first principle basis, linking the aforementioned coefficients (generally called phenomenological coefficients, e.g. viscosity, thermal conductivity in fluidodynamics), to the Kubo correlation functions of suitable quantum field observables. Eq. (46) provides a zero order solution $\zeta_t^{(0)}$ to the dynamics of the state variables ζ_t (e.g. Navier Stokes equation in fluidodynamics) and our treatment also indicates this description has precise limitations and the way to improve it. First of all we must recall the basic role of (35) to transform the formal identity (31) into an evolution equation. Eq. (46) modifies (34), due to the last term of (46), representing irreversible effects; it implies a modification of (35) which becomes $\frac{\partial}{\partial t}\zeta_{lt}(\mathbf{x}) = \alpha_l^1(\zeta_t(\mathbf{x}))$. This changes eq. (45) and in turn provides the same substitution of $\alpha_l^0(\zeta_t(\mathbf{x}))$ with $\alpha_l^1(\zeta_t(\mathbf{x}))$ inside the last term of (46), where this factor multiplies the Kubo correlation functions related to the pairs $\dot{\hat{A}}_j(\mathbf{x})$ and $\hat{A}_j(\mathbf{x})$. If such a modification is small enough one can expect that by a recursive procedure starting with $\alpha_l^0(\zeta_t(\mathbf{x}))$ a function $\alpha_l(\zeta_t(\mathbf{x}))$ can be determined by which eq. (46) is established; however at this point one could have also evidence of a failure of the hypothesis of a deterministic dynamics in terms of the state variables ζ_t; physically one knows about turbulence when gradients of state variables are large enough.
One can easily imagine how this procedure can be extended in order to take into account higher order contributions in $\mathscr{S}(t,t-\tau)$ from the non linear contribution in (41) and from the exponent $\mathrm{e}^{-u\hat{\mathscr{S}}(t)}$ inside eq. (43).

Also corrections due to the non local space-time structure of $\hat{\mathring{\Gamma}}(t)$ can be obtained in principle starting with the local solution of (46), substituting it inside $\hat{\mathsf{T}}(t)$ and evaluating the resulting contribution to the expectation values of $\dot{\hat{A}}_j(\mathbf{x})$. One wonders however that contributions from $\hat{\mathsf{T}}(t)$ can remain small on increasing times t: qualitative arguments for this were given in ref. [6], based on the interplay of the locality of the operator fields $\hat{A}_j(\mathbf{x})$, $\dot{\hat{A}}_j(\mathbf{x})$ entering the relevant expectations, in front of the typically non local behaviour induced by $\mathscr{U}_{t'}^{t}$ for $t-t' \geq \tau$. In particular looking at (40) the positive term $\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^\dagger(t)$ coming from $\hat{\mathsf{S}}\hat{w}_{\zeta_t}\hat{\mathsf{S}}^\dagger$ represents a "subcollection" produced by time evolution, associated with $\hat{w}_{\zeta_t}$: as discussed in [6], one can expect that this is equivalent to $\hat{w}_{\zeta_t}$ in the sense that for local observables $\hat{O}(\mathbf{x})$ the following typical clustering property holds:

$$\mathrm{Tr}(\hat{O}(\mathbf{x})\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^\dagger(t)) \approx \mathrm{Tr}(\hat{O}(\mathbf{x})\hat{w}_{\zeta_t})\mathrm{Tr}(\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^\dagger(t)), \tag{47}$$

then one can understand that $\hat{w}_{\zeta_t}$ can maintain its prevalence also for $t \gg \tau$, producing the deterministic dynamics ruled by $\hat{w}_{\zeta_t}$ together with the part of $\hat{\hat{\Gamma}}(t)$ related to $\hat{\mathsf{S}}(t,t-\tau)$. However more complicated patterns than (47) for $\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^\dagger(t)$ can arise, changing in a profound way the dynamical evolution.

STOCHASTICITY BY MICROSYSTEMS

We can explain in a schematic way the persistence of a deterministic dynamics for $t \gg \tau$, if we look at eq. (40) with $\hat{\mathsf{S}}(t) = \hat{\mathsf{S}}(t,t-\tau)+\hat{\mathsf{T}}(t)$, as follows: $\hat{w}_{\zeta_t}$ is the macrostate determined by the expectations of the relevant variables which leads the time evolution; the terms containing only operators $\hat{\mathsf{S}}(t,t-\tau)$, $\hat{\mathsf{S}}^\dagger(t,t-\tau)$ with irrelevant variables, still retaining some locality, contribute essentially to irreversible dynamics through the expectations of $\dot{\hat{A}}_j(\mathbf{x})$; the terms in which $\hat{\mathsf{S}}, \hat{\mathsf{T}}$ appear together are negligible for the expectations of local observables; the subcollection $\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^\dagger(t)$ which could represent a new contribution, having increasing relevance for t large enough, can behave relatively to local observables in an equivalent way to $\hat{w}_{\zeta_t}$ by (47).
To investigate more accurately the last point the structure of $\hat{\mathscr{S}}(t)$ given by (26) becomes important and in particular time evolution in Heisenberg representation of the operator fields $\hat{\mathbf{j}}_j(\mathbf{y}), \hat{A}_j(\mathbf{y}), \hat{S}_j(\mathbf{y})$ associated with the corresponding state parameters and derivatives of them. For $t > \tau$ the general expression of $\hat{\mathscr{S}}(t)$ becomes exceedingly complicated and it is just the extremely important clustering (47) that often makes this long time behaviour essentially irrelevant, so that eqs. (46) make all the job. The structure of $\hat{\mathscr{S}}(t)$ accounts for all quantum processes associated in the time interval $[0,t]$, with any possible state. The fact that $\hat{\mathscr{S}}(t)$ is applied to $\hat{w}_{\zeta_t}$, allows for adequate choices of ζ_t some simplifications. Let us assume that at least locally a "low energy" situation is considered: e.g. initially, i.e. $t \in [t_0-\tau, t_0]$, the only non zero state parameters are those referring to the composed field in the fundamental state $\kappa = 0$; then only operators $\hat{\Psi}_0(\mathbf{x}), \hat{\Psi}_0^\dagger(\mathbf{x})$ appear inside $\hat{\Phi}(\zeta_t)$ and inside the square brackets in eq. (26), except for the related source terms $\hat{S}_j(\mathbf{y})$, that for simplicity we did not make explicit; they can be easily found in terms of the fundamental fields $\hat{\psi}_1(\mathbf{x}), \hat{\psi}_1^\dagger(\mathbf{x}), \hat{\psi}_2(\mathbf{x}), \hat{\psi}_2^\dagger(\mathbf{x})$: just the circumstance that one needs here these operators represents the non "fundamental" character of the composed fields; by the action of $\mathscr{U}_{t'}^{t}$ such very simple structure is anyway lost since interaction is described by the fundamental fields; then $\hat{\mathscr{S}}(t)$ will generally be a very complicated linear combination of products of the fundamental operators, however satisfying the condition that all terms must conserve the "total number" of "particles" 1 and 2 since these observables are constants of motion in this non relativistic description. At time $t=0$ all terms

are built in terms of products of factors $\hat{\psi}_1(\mathbf{x})\hat{\psi}_2(\mathbf{x}), \hat{\psi}_1^\dagger(\mathbf{x})\hat{\psi}_2^\dagger(\mathbf{x})$, this peculiarity is conserved under time evolution. So it is still possible to express $\hat{\mathscr{S}}(t)$ in terms of the full set of fields $\hat{\Psi}_\kappa(\mathbf{x})$, taking into account also the continuous spectrum of the "center of mass" two-particle energy: then one can expect that the most important contributions are those involving only products like $\hat{\Psi}_0^\dagger(\mathbf{x}_1)\ldots\hat{\Psi}_0^\dagger(\mathbf{x}_n)\hat{\Psi}_0(\mathbf{y}_n)\ldots\hat{\Psi}_0(\mathbf{y}_1)$ related to multiple elastic scattering for the fundamental composed state, next to these also the contributions of the form $\hat{\Psi}_1^\dagger(\mathbf{x}_1)\hat{\Psi}_2^\dagger(\mathbf{x}_2)\hat{\Psi}_0^\dagger(\mathbf{x}_3)\hat{\Psi}_0(\mathbf{y}_1)\hat{\Psi}_0(\mathbf{y}_2)$ and the more complicated ones $\hat{\Psi}_1^\dagger(\mathbf{x}_1)\hat{\Psi}_2^\dagger(\mathbf{x}_2)\hat{\Psi}_0^\dagger(\mathbf{x}_3)\ldots\hat{\Psi}_0^\dagger(\mathbf{x}_n)\hat{\Psi}_0(\mathbf{y}_1)\hat{\Psi}_0(\mathbf{y}_2)\hat{\Psi}_0(\mathbf{y}_3)\ldots\hat{\Psi}_0(\mathbf{y}_{n-1}), n > 3$ related to one inelastic process will be important.
For the contributions related to the sole operators $\hat{\Psi}_0(\mathbf{x}), \hat{\Psi}_0^\dagger(\mathbf{x})$ one can expect that the typical condition (47) holds. Instead more complicated clustering properties can be expected for the contributions showing $\hat{\Psi}_1^\dagger(\mathbf{x}_1), \hat{\Psi}_2^\dagger(\mathbf{x}_2)\ldots$. To this contribution a formal discussion in ref. [4] is now adapted in a sketchy way. Let us describe before an ideal strongly non equilibrium system by which this new situation can be expected. Imagine that a very diluted gas in an electric field becomes a possible source of single electron-ion pairs by very local and impulsive heating; the electric field accelerates the charged particles, which then can start further ionization processes and finally produce correlated spatially separated currents in the gas; then these currents are a highly stochastic breakdown of the deterministic dissipation of the local heating.
Let us represent $\hat{\mathsf{T}}(t)$ as:

$$\hat{\mathsf{T}}(t) = \int_\Omega d^3\mathbf{x}_1\, d^3\mathbf{y}_1\, \hat{\psi}_1^\dagger(\mathbf{x}_1)\hat{\psi}_2^\dagger(\mathbf{y}_1)\hat{D}_{12\,t}(\mathbf{x}_1,\mathbf{y}_1); \tag{48}$$

the operator $\hat{D}_{12\,t}(\mathbf{x}_1,\mathbf{y}_1)$ has the general structure:

$$\begin{aligned}\hat{D}_{12\,t}(\mathbf{x}_1,\mathbf{y}_1) &= \int_\Omega d^3\eta_1 \int_\Omega d^3\xi_1 \int_\Omega d^3\xi_2\, \hat{\Psi}_0^\dagger(\eta_1)\hat{\Psi}_0(\xi_1)\hat{\Psi}_0(\xi_2)\sigma_{1\,t}(\mathbf{x}_1,\mathbf{y}_1;\eta_1,\xi_1,\xi_2)\\ &+ \int_\Omega d^3\eta_1 \int_\Omega d^3\eta_2 \int_\Omega d^3\xi_1 \int_\Omega d^3\xi_2 \int_\Omega d^3\xi_3 \\ &\quad \hat{\Psi}_0^\dagger(\eta_1)\hat{\Psi}_0^\dagger(\eta_2)\hat{\Psi}_0(\xi_1)\hat{\Psi}_0(\xi_2)\hat{\Psi}_0(\xi_3)\sigma_{2\,t}(\mathbf{x}_1,\mathbf{y}_1;\eta_1,\eta_2,\xi_1,\xi_2,\xi_3)\\ &+ \ldots\end{aligned} \tag{49}$$

it operates on $\hat{w}_{\zeta_t}$ regarding the composed field features of this state, acts as a generalised destruction operator for pairs of "particles" 1 and 2.
If in (49) the integration region Ω are replaced with $\Omega\backslash\omega_\delta(\mathbf{x})$, by the fact that $\delta^3 \ll V(\Omega)$, since no particular concentration for the functions $\sigma_{1\,t}, \sigma_{2\,t}$ should arise with respect to the region $\omega_\delta(\mathbf{x})$, one expects that a negligible modification is done. However if the expectation of a local observable $\hat{O}(\mathbf{x})$ is considered taking this modified expression of $\hat{D}_{12\,t}(\mathbf{x}_1,\mathbf{y}_1)$ by the same considerations which led to the local expression (16), one obtains:

$$\begin{aligned}&\mathrm{Tr}(\hat{O}(\mathbf{x})\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^\dagger(t)) - \mathrm{Tr}(\hat{O}(\mathbf{x})\hat{w}_{\zeta_t})\mathrm{Tr}(\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^\dagger(t)) = \\ &\int_\Omega d^3\mathbf{x}_1\, d^3\mathbf{y}_1\, d^3\mathbf{x}'_1\, d^3\mathbf{y}'_1\, [\mathrm{Tr}\hat{O}(\mathbf{x})\hat{\psi}_1^\dagger(\mathbf{x}_1)\psi_2^\dagger(\mathbf{y}_1)\hat{w}_{\zeta_t}\hat{\psi}_2(\mathbf{y}'_1)\hat{\psi}_1(\mathbf{x}'_1) - \mathrm{Tr}(\hat{O}(\mathbf{x})\hat{w}_{\zeta_t})\\ &\times\mathrm{Tr}(\psi_1^\dagger(\mathbf{x}_1)\psi_2^\dagger(\mathbf{y}_1)\hat{w}_{\zeta_t}\hat{\psi}_2(\mathbf{y}'_1)\hat{\psi}_1(\mathbf{x}'_1))]\mathrm{Tr}(\hat{D}_{12\,t}(\mathbf{x}_1,\mathbf{y}_1)\hat{w}_{\zeta_t}\hat{D}_{12\,t}^\dagger(\mathbf{x}'_1,\mathbf{y}'_1))\end{aligned} \tag{50}$$

The expression $g_t(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}'_1,\mathbf{y}'_1) = \mathrm{Tr}(\hat{D}_{12\,t}(\mathbf{x}_1,\mathbf{y}_1)\hat{w}_{\zeta t}\hat{D}^{\dagger}_{12\,t}(\mathbf{x}'_1,\mathbf{y}'_1))$ defines a positive operator g_t on $L^2(\Omega\times\Omega)$ by:

$$(\mathrm{g}_t f)(\mathbf{x}_1,\mathbf{y}_1) = \int_{\Omega\times\Omega} d^3\mathbf{x}'_1\, d^3\mathbf{y}'_1\, g_t(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}'_1,\mathbf{y}'_1) f(\mathbf{x}'_1,\mathbf{y}'_1). \tag{51}$$

Let us represent it using orthonormal eigenstates $g_{jt}(\mathbf{x}_1,\mathbf{y}_1)$ and the positive eigenvalues:

$$\begin{aligned}&\int_{\Omega\times\Omega} d^3\mathbf{x}'_1\, d^3\mathbf{y}'_1\, g_t(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}'_1,\mathbf{y}'_1) f(\mathbf{x}'_1,\mathbf{y}'_1)\\ &= \sum_j \lambda_j(t) g_{jt}(\mathbf{x}_1,\mathbf{y}_1) \int_{\Omega\times\Omega} d^3\mathbf{x}'_1\, d^3\mathbf{y}'_1\, g^*_{jt}(\mathbf{x}'_1,\mathbf{y}'_1) f(\mathbf{x}'_1,\mathbf{y}'_1).\end{aligned} \tag{52}$$

Setting $\hat{\psi}^{(1,2)}_{jt} = \int_{\Omega\times\Omega} d^3\mathbf{x}'_1\, d^3\mathbf{y}'_1\, g^*_{jt}(\mathbf{x}'_1,\mathbf{y}'_1)\hat{\psi}_1(\mathbf{x}'_1)\hat{\psi}_2(\mathbf{y}'_1)$ equation (50) becomes

$$\begin{aligned}&\mathrm{Tr}(\hat{O}(\mathbf{x})\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^{\dagger}(t)) - \mathrm{Tr}(\hat{O}(\mathbf{x})\hat{w}_{\zeta_t})\mathrm{Tr}(\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^{\dagger}(t)) =\\ &\sum_j \lambda_j(t)\mathrm{Tr}\hat{O}(\mathbf{x})\hat{\psi}^{(1,2)\,\dagger}_{jt}\hat{w}_{\zeta t}\hat{\psi}^{(1,2)}_{jt} - \mathrm{Tr}(\hat{O}(\mathbf{x})\hat{w}_{\zeta_t})\mathrm{Tr}(\hat{\psi}^{(1,2)\,\dagger}_{jt}\hat{w}_{\zeta_t}\hat{\psi}^{(1,2)}_{jt})\end{aligned} \tag{53}$$

where the typical mechanism by which $\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^{\dagger}(t)$ is “equivalent” to $\hat{w}_{\zeta_t}$ as given by (47) can be stated as:

$$\mathrm{Tr}\hat{O}(\mathbf{x})\hat{\psi}^{(1,2)\,\dagger}_{jt}\hat{w}_{\zeta_t}\hat{\psi}^{(1,2)}_{jt} \approx \mathrm{Tr}(\hat{O}(\mathbf{x})\hat{w}_{\zeta_t})\mathrm{Tr}(\hat{\psi}^{(1,2)\,\dagger}_{jt}\hat{w}_{\zeta_t}\hat{\psi}^{(1,2)}_{jt}). \tag{54}$$

Thinking about the previously sketched physical situation taking the case of variables $\hat{A}(\mathbf{x}),\dot{\hat{A}}(\mathbf{x})$ which are related to the components 1 and 2 of the system, one just expects that (54) can be violated and that together with the macrostate $\hat{w}_{\zeta_t}$ also the new states

$$\frac{\hat{\psi}^{(1,2)\,\dagger}_{jt}\hat{w}_{\zeta_t}\psi^{(1,2)}_{jt}}{\mathrm{Tr}\hat{\psi}^{(1,2)\,\dagger}_{jt}\hat{w}_{\zeta_t}\psi^{(1,2)}_{jt}} \tag{55}$$

should have a role, as macrostates perturbed by the “two particle” microsystems $1+2$ in the state $g_{jt}(\mathbf{x}_1,\mathbf{y}_1)$. By these considerations one becomes aware that our treatment of deterministic dynamics must fail if in a long time limit such a situation arises: contextually also a straightforward generalisation appears, consisting in a more general form of the reference state $\hat{w}_{\zeta_t}$, showing a more general parameterization of the dynamics, shifting in a sense components of $\hat{\mathsf{T}}(t)\hat{w}_{\zeta_t}\hat{\mathsf{T}}^{\dagger}(t)$ to the new states (55).
Let us introduce a stochastic generalisation of $\hat{w}_{\zeta_t}$, that we write assuming “one kind” of microsystem. In our case the “two particle” system 1,2:

$$\hat{w}_t = \lambda_t\hat{w}_{\zeta_t} + \sum_\alpha \lambda_{\alpha t}\frac{\hat{\psi}^{\dagger}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}_{\alpha t}}{\mathrm{Tr}\hat{\psi}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}^{\dagger}_{\alpha t}} \quad \lambda_t,\lambda_{\alpha t}\geq 0, \quad \lambda_t + \sum_\alpha \lambda_{\alpha t} = 1 \tag{56}$$

with an obvious stochastic interpretation. It carries the parameters $\zeta_{\alpha t}, g_{\alpha t}$, the last ones associated with the microstate $\hat{\psi}_{\alpha t}$ characterising each subcollection:

$$\hat{w}_{\alpha,\zeta_{\alpha t}} = \frac{\hat{\psi}^{\dagger}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}_{\alpha t}}{\mathrm{Tr}\hat{\psi}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}^{\dagger}_{\alpha t}}, \tag{57}$$

here the correlation appears between state parameters $\zeta_{\alpha t}$ and state $g_{\alpha t}$ of the perturbing microsystem. A straightforward generalisation of the procedure by which $\hat{w}_{\zeta_t}$ was associated to a solution of the Liouville Von Neumann equation gives:

$$\begin{aligned}
\hat{\rho}_t \;=\; & \Bigg[\xi(t)\exp[-\sum_j\int_\Omega d^3\mathbf{x}\,\zeta_j(\mathbf{x})\hat{A}_j(\mathbf{x}) + \sum_j\int_\Omega d^3\mathbf{x}\int_{-\infty}^{t} dt'\,\frac{d}{dt'}(\mathscr{U}^t_{t'}\zeta_{jt'}(\mathbf{x})\hat{A}_j(\mathbf{x}))] \\
&+ \sum_\alpha\int_{\Omega\times\Omega} d^3\mathbf{x}_1\,d^3\mathbf{x}_2\,\Big(\xi_{\alpha t}(\mathbf{x}_1,\mathbf{x}_2)\hat{\psi}^{\dagger}_1(\mathbf{x}_1)\hat{\psi}^{\dagger}_2(\mathbf{x}_2) \\
&\qquad - \int_{-\infty}^{t} dt'\,\frac{d}{dt'}(\mathscr{U}^t_{t'}\xi_{\alpha t'}(\mathbf{x}_1,\mathbf{x}_2)\hat{\psi}^{\dagger}_1(\mathbf{x}_1)\hat{\psi}^{\dagger}_2(\mathbf{x}_2))\Big) \\
&\times \exp[-\sum_j\int_\Omega d^3\mathbf{x}\,\zeta_{j\alpha t}(\mathbf{x})\hat{A}_j(\mathbf{x}) + \sum_j\int_\Omega d^3\mathbf{x}\int_{-\infty}^{t} dt'\,\frac{d}{dt'}(\mathscr{U}^t_{t'}\zeta_{j\alpha t'}(\mathbf{x})\hat{A}_j(\mathbf{x}))] \\
&\times \sum_\alpha\int_{\Omega\times\Omega} d^3\mathbf{x}_1\,d^3\mathbf{x}_2\,\Big(\xi^*_{\alpha t}(\mathbf{x}_1,\mathbf{x}_2)\hat{\psi}_1(\mathbf{x}_1)\hat{\psi}_2(\mathbf{x}_2) \\
&\qquad - \int_{-\infty}^{t} dt'\,\frac{d}{dt'}(\mathscr{U}^t_{t'}\xi^*_{\alpha t'}(\mathbf{x}_1,\mathbf{x}_2)\hat{\psi}_1(\mathbf{x}_1)\hat{\psi}_2(\mathbf{x}_2))\Big)\Bigg]\frac{1}{\mathrm{Tr}[-]}
\end{aligned} \tag{58}$$

which is related to the stochastic reference state

$$\begin{aligned}
\hat{w}_t \;=\; & \Bigg[\xi(t)\exp[-\sum_j\int_\Omega d^3\mathbf{x}\,\zeta_j(\mathbf{x})\hat{A}_j(\mathbf{x})] + \sum_\alpha\int_{\Omega\times\Omega} d^3\mathbf{x}_1\,d^3\mathbf{x}_2\,\xi_{\alpha t}(\mathbf{x}_1,\mathbf{x}_2)\hat{\psi}^{\dagger}_1(\mathbf{x}_1)\hat{\psi}^{\dagger}_2(\mathbf{x}_2) \\
&\times\exp[-\sum_j\int_\Omega d^3\mathbf{x}\,\zeta_{\alpha jt}(\mathbf{x})\hat{A}_j(\mathbf{x})]\int_{\Omega\times\Omega} d^3\mathbf{x}_1\,d^3\mathbf{x}_2\,\xi^*_{\alpha t}(\mathbf{x}_1,\mathbf{x}_2)\hat{\psi}_1(\mathbf{x}_1)\hat{\psi}_2(\mathbf{x}_2)\Bigg]\frac{1}{\mathrm{Tr}[-]} \\
=\; & \frac{\xi(t)Z(\zeta_t)\hat{w}_{\zeta_t} + \sum_\alpha\|\xi_{\alpha t}\|^2 Z(\zeta_t,\xi_{\alpha t})\frac{\hat{\psi}^{\dagger}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}_{\alpha t}}{\mathrm{Tr}\hat{\psi}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}^{\dagger}_{\alpha t}}}{\xi(t)Z(\zeta_t) + \sum_\alpha\|\xi_{\alpha t}\|^2 Z(\zeta_t,\xi_{\alpha t})}
\end{aligned} \tag{59}$$

where $Z(\zeta_t,\xi_{\alpha t}) = \mathrm{Tr}\hat{\psi}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}^{\dagger}_{\alpha t}$ easily leading to λ_t and $\lambda_{\alpha t}$ which appear in (56).
To determine the coefficients $\zeta_{\alpha t}$ and $\xi_{\alpha t}$ we begin stating as before that $\hat{w}_t$ is equivalent to $\hat{\rho}_t$ with respect to the relevant observables,

$$\mathrm{Tr}\hat{A}_j(\mathbf{x})\hat{\rho}_t = \mathrm{Tr}\hat{A}_j(\mathbf{x})\hat{w}_t, \qquad \forall\hat{A}\in M; \tag{60}$$

now however a larger set of parameters has been introduced; correspondingly a larger set of conditions must be given. We assume that in order to determine $\hat{w}_{\alpha,\zeta_{\alpha t}}$ also "typical" observables of the microsystem must be considered; in our case these observables are a suitable subset $M^{(1,2)}$ of observables of the form:

$$\hat{A}^{(1,2)} = \int d^3\mathbf{y}_1\,d^3\mathbf{y}_2\,d^3\mathbf{y}'_1\,d^3\mathbf{y}'_2\,\hat{\psi}^{\dagger}_1(\mathbf{y}_1)\hat{\psi}^{\dagger}_2(\mathbf{y}_2)A^{(1,2)}(\mathbf{y}_1,\mathbf{y}_2,\mathbf{y}'_1,\mathbf{y}'_2)\hat{\psi}_1(\mathbf{y}'_1)\hat{\psi}_2(\mathbf{y}'_2) \tag{61}$$

One can expect that completing (60) by the additional conditions:

$$\text{Tr}\hat{A}\hat{\rho}_t = \text{Tr}\hat{A}\hat{w}_t, \qquad \forall \hat{A} \in M^{(1,2)}, \tag{62}$$

$\hat{w}_t$ is completely determined. Then the whole procedure that led in the deterministic case to evolution equation for the parameters can be repeated: the starting point are the equations

$$\text{Tr}\dot{\hat{A}}\hat{\rho}_t = \frac{d}{dt}\text{Tr}\hat{A}\hat{w}_t, \qquad \forall \hat{A} \in M, \, \forall \hat{A} \in M^{(1,2)}, \tag{63}$$

like (35) for the state parameters which we now indicate by $\zeta_\alpha(t)$, also a deterministic evolution of the quantum state $g_{\alpha t}$ of the microsystem will be necessary for each subcollection α. Also in this more general framework due to the tail contributions, dynamics can indicate that a more stochastic description must be given: this means that an increasing complexity of the "microsystem" structure of $\hat{w}_t$ can be necessary; all these questions require further investigations.
We concentrate now on (62) and relate it to results obtained in ref. [4]. Recalling that $\hat{\psi}^{(1,2)}_{jt} = \int_{\Omega\times\Omega} d^3\mathbf{x}'_1 \, d^3\mathbf{y}'_1 \, g^*_{jt}(\mathbf{x}'_1, \mathbf{y}'_1) \hat{\psi}_1(\mathbf{x}'_1) \hat{\psi}_2(\mathbf{y}'_1)$, one has:

$$\begin{aligned} \text{Tr}\hat{A}^{(1,2)}\hat{w}_{\alpha,\,\zeta_\alpha(t)} = & \left(\int d^3\mathbf{y}_1 \, d^3\mathbf{y}_2 \, d^3\mathbf{y}'_1 \, d^3\mathbf{y}'_2 \, g^*_{\alpha t}(\mathbf{y}_1, \mathbf{y}_2) A^{(1,2)}(\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}'_1, \mathbf{y}'_2) g_{\alpha t}(\mathbf{y}'_1, \mathbf{y}'_2) \right. \\ & + \left. \text{Tr}\left(\hat{A}^{(1,2)}_g \hat{w}_{\alpha_j \zeta_\alpha(t)} \right) \right) \frac{1}{1 \pm \text{Tr}\hat{\psi}_{\alpha t} \hat{w}_{\zeta_{\alpha t}} \hat{\psi}^\dagger_{\alpha t}} \end{aligned} \tag{64}$$

where

$$\begin{aligned} \hat{A}^{(1,2)}_g &= \hat{A}^{(1,2)} + \hat{\psi}^\dagger_{\alpha t} \hat{A}^{(1,2)} \hat{\psi}_{\alpha t} \\ &\pm \int d^3\mathbf{y}_1 \, d^3\mathbf{y}_2 \, d^3\mathbf{y}'_1 \, d^3\mathbf{y}'_2 \, g^*_{\alpha t}(\mathbf{y}_1, \mathbf{y}_2) A^{(1,2)}_g(\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}'_1, \mathbf{y}'_2) g_{\alpha t}(\mathbf{y}'_1, \mathbf{y}'_2), \end{aligned} \tag{65}$$

with

$$\begin{aligned} & A^{(1,2)}_g(\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}'_1, \mathbf{y}'_2) = \\ & g(\mathbf{y}_1, \mathbf{y}_2) \int_{\Omega\times\Omega} d^3\mathbf{y}''_1 \, d^3\mathbf{y}''_2 \, g^*(\mathbf{y}''_1, \mathbf{y}''_2) A^{(1,2)}(\mathbf{y}''_1, \mathbf{y}''_2, \mathbf{y}'_1, \mathbf{y}'_2) \\ & + \int_{\Omega\times\Omega} d^3\mathbf{y}''_1 \, d^3\mathbf{y}''_2 \, A^{(1,2)}(\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}''_1, \mathbf{y}''_2) g(\mathbf{y}''_1, \mathbf{y}''_2) g^*(\mathbf{y}'_1, \mathbf{y}'_2). \end{aligned}$$

Now we come to a main point of our treatment: by a suitable choice of $M^{(1,2)}$ and a corresponding choice of the states $g(\mathbf{x}_1, \mathbf{x}_2)$ eq. (64) reduces simply to the first term inside the bracket

$$\text{Tr}\hat{A}^{(1,2)}_g \hat{w}_{\alpha,\,\zeta_\alpha(t)} \simeq < g_{\alpha t} | \hat{A}^{(1,2)} | g_{\alpha t} >_{\mathscr{H}^{(1,2)}} \tag{66}$$

Here the formalism of quantum mechanics naturally appears for a microsystem (1,2), described by the normalized wavefunction $g_{\alpha t}(\mathbf{x}_1, \mathbf{x}_2)$, belonging to the Hilbert space

$\mathscr{H}^{(1,2)} = L^2(\Omega)\otimes L^2(\Omega)$ associated at time t with this microsystem; $\hat{A}^{(1,2)}$ is the operator in $\mathscr{H}^{(1,2)}$, with the function $A^{(1,2)}(\mathbf{x}_1,\mathbf{y}_2,\mathbf{x'}_1,\mathbf{y'}_2)$ being its representative in the position basis $|\mathbf{x},\mathbf{y}>$. This very simple result is a consequence of the local structure of the relevant variables: only the operators $\Psi(\mathbf{x}),\Psi^\dagger(\mathbf{x})$ of the composed fields allow in $\hat{w}_\zeta$ a quasi local structure related to $\hat{\psi}_1,\hat{\psi}_2$ factors but, by the "bound state" structure of $g_\kappa(\mathbf{r})$ given by eq. (3), such a non locality is restricted to a distance λ_b. Then let us set on the functions $g_{\alpha t}(\mathbf{x},\mathbf{y})$ by which the states $\psi_{\alpha t}$ are constructed the condition that they are "not too local", i.e. they are concentrated on values of $\mathbf{r}=\mathbf{x}-\mathbf{y}$ with $|\mathbf{r}|>\lambda_b$. By this condition the expression $\mathrm{Tr}\hat{\psi}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\hat{\psi}^\dagger_{\alpha t}$, $\mathrm{Tr}\hat{\psi}_{\alpha t}\hat{A}^{(1,2)}\hat{\psi}^\dagger_{\alpha t}$ and

$$\mathrm{Tr}\{\big[\hat{\psi}^\dagger_{\alpha t}\int d^3\mathbf{y}'_1\,d^3\mathbf{y}'_2\int d^3\mathbf{y}''_1\,d^3\mathbf{y}''_2\,g^*_\alpha(\mathbf{y}''_1,\mathbf{y}''_2)A^{(1,2)}(\mathbf{y}''_1,\mathbf{y}''_2,\mathbf{y}'_1,\mathbf{y}'_2)\hat{\psi}_2(\mathbf{y}'_2)\hat{\psi}_1(\mathbf{y}'_1)$$
$$+\int d^3\mathbf{y}_1\,d^3\mathbf{y}_2\,\hat{\psi}^\dagger_1(\mathbf{y}_1)\hat{\psi}^\dagger_2(\mathbf{y}_2)\int d^3\mathbf{y}''_1\,d^3\mathbf{y}''_2\,A^{(1,2)}(\mathbf{y}_1,\mathbf{y}_2,\mathbf{y}''_1,\mathbf{y}''_2)g_\alpha(\mathbf{y}''_1,\mathbf{y}''_2)\hat{\psi}_{\alpha t}\big]\hat{w}_{\zeta_t}\}$$

arising in (64) by (65) are negligible; loosely this amounts to take: $\hat{\psi}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\simeq 0$, or $\hat{\psi}^\dagger_{\alpha t}\hat{\psi}_{\alpha t}\hat{w}_{\zeta_{\alpha t}}\sim 0$ meaning that $\hat{\psi}^\dagger_{\alpha t}$ creates a microsystem that has in $\hat{w}_{\zeta_{\alpha t}}$ a negligible occupation; in our example such a condition looks very clear, but it seems to be reasonable also in a general context; then by (64):

$$\mathrm{Tr}\left(\hat{\psi}^\dagger_{\alpha t}\hat{A}^{(1,2)}\hat{\psi}_{\alpha t}\hat{w}_{\alpha,\zeta_{\alpha t}}\right) = < g_{\alpha t}|\hat{A}^{(1,2)}|g_{\alpha t}> + \mathrm{Tr}\left(\hat{A}^{(1,2)}\hat{w}_{\alpha,\zeta_{\alpha t}}\right), \tag{67}$$

here the last contribution can be neglected if we assume that the two particle observables $\hat{A}^{(1,2)}$ are insensitive to the two particle system if they are at a distance $|\mathbf{r}|\leq\lambda_b$; i.e. for the function

$$A^{(1,2)}(\mathbf{y}_1,\mathbf{y}_2,\mathbf{y}'_1,\mathbf{y}'_2)\approx 0 \tag{68}$$

holds if $|\mathbf{y}_1-\mathbf{y}_2|\leq\lambda_b$ and $|\mathbf{y}'_1-\mathbf{y}'_2|\leq\lambda_b$; by this choice eq. (64) becomes (66). Then by (56) and by (62) one has:

$$\begin{aligned}\mathrm{Tr}\hat{A}^{(1,2)}\hat{\rho}_t &= \mathrm{Tr}\hat{A}^{(1,2)}\hat{w}_t = \sum_\alpha \lambda_\alpha(t) < g_{\alpha t}|\hat{A}^{(1,2)}|g_{\alpha t}>_{\mathscr{H}^{(1,2)}}\\ &= \mathrm{Tr}_{\mathscr{H}^{(1,2)}}\hat{A}^{(1,2)}\hat{\rho}^{(1,2)}_t,\end{aligned} \tag{69}$$

here $\hat{\rho}^{(1,2)}_t = \sum_\alpha \lambda_\alpha(t)|g_{\alpha t}><g_{\alpha t}|$ is a positive operator on $\mathscr{H}^{(1,2)}$ with $\mathrm{Tr}\hat{\rho}^{(1,2)}_t = 1-\lambda(t)$ describing the microsystem (1,2) which exists at time t with probability given by $\mathrm{Tr}\hat{\rho}^{(1,2)}_t = \sum_\alpha \lambda_\alpha(t) = 1-\lambda(t)$. This operator is defined by the expectations of $\hat{\rho}_t$ on $M^{(1,2)}$; its spectralisation provides the states $g_{\alpha t}(\mathbf{x}_1,\mathbf{x}_2)$ and the coefficients $\lambda_\alpha(t)$, characterising the subcollection α; then by (60) one determines the corresponding state parameters $\zeta_\alpha(t)$; they can differ between each other, due to the different states $g_{\alpha t}$; thinking about the example of a source creating one electron-ion pair, one has an initial situation described by state parameters $\zeta_{lt}(\mathbf{x})$, providing negligible values for the relevant field observables for the fundamental components 1 and 2. In this condition $\hat{w}_t$ and $\hat{w}_{\zeta_t}$ are equivalent for the relevant variables, dynamics is deterministic, the microphysical structure of $\hat{w}_t$ is only a redundant complication; in the long time limit the "shadow" subcollection $\mathsf{T}(t)\hat{w}_\zeta\mathsf{T}^\dagger(t)$ might no longer behave as indicated by (47),

then $\hat{w}_{\zeta(t)}$ must be replaced by $\hat{w}_t$ and quantum theory of a microsystem naturally related to the constitution of the composed component, becomes an essential ingredient of the stochastic dynamics of the system.

CONCLUSIONS AND OUTLOOK

Models of interacting quantum fields in the non relativistic case were primarily focused to give a "first principle" foundations of phenomenological thermodynamics of irreversible processes extending the framework already treated in [7, 8] to the case of a two components system, allowing also the description of binding these components by means of a composed field. The state parameters on which the phenomenological description is based appear as a useful mathematical parameterisation of the statistical operator of the system, providing an approximate solution of the Liouville Von Neumann equation, adequate on a suitable space-time scale. The persistence of a deterministic regime has been linked to the typical clustering property (47) induced by long time evolution; however also more complicated clustering patterns can arise (cf. eq. (50)): they lead to a non deterministic regime, that can be naturally described basing on the concept of a "microsystem", which provides an additional parameterisation of the statistical operator consisting in the possible state vectors describing the microsystem, their statistical weights and the state parameters of the macrosystem perturbed by the microsystem for each of such states: now the leading part of the statistical operator changes from $\hat{w}_{\zeta_t}$, cf. eq. (12) to $\hat{w}_t$, cf. eq. (56). Together with the relevant variables associated with the state parameters, also the "observables of the microsystem", cf. eqs. (61), (67), have a basic role in driving the dynamics, the statistical operator $\hat{\rho}_t$, restricted to these observables providing the statistical operator of the microsystem, cf. eq. (69). In this way quantum theory of a microsystem becomes the essential ingredient to face the breakdown of the simple deterministic regime. The outlook is towards particle physics inside quantum field thermodynamics of irreversible processes.

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