DECOHERENCE VERSUS THE IDEALIZATION OF MICROSYSTEMS AS CORRELATION CARRIERS BETWEEN MACROSYSTEMS

Ludovico Lanz and Bassano Vacchini
Dipartimento di Fisica dell'Università di Milano and INFN, Sezione di Milano
Via Celoria 16, I-20133, Milano, Italy
lanz@mi.infn.it, vacchini@mi.infn.it

Olaf Melsheimer
Fachbereich Physik, Philipps–Universität
Renthof 7, D-35032, Marburg, Germany
melsheim@mailer.uni-marburg.de

Keywords: decoherence, macroscopic system, non-equilibrium statistical mechanics

Abstract It is argued that the appropriate framework to describe a microsystem as a correlation carrier between a source and a detector is non-equilibrium statistical mechanics for the compound source-detector system. An attempt is given to elucidate how this idealized notion of microsystem might arise inside a field theoretical description of isolated macrosystems: then decoherence appears as the natural limit of this idealization.

1. PREEMINENT ROLE OF FIELD THEORY

Even if it is generally accepted that quantum field theory must be used in high energy physics, questions on foundations of quantum mechanics, description of measuring process and discussion of decoherence are usually addressed to in the context of the N particle generalization of the Schrödinger equation, while in that context quantum field theory is often only appreciated as a more refined tool to accommodate relativity and to account for particlelike aspects of electromagnetism. This is deeply rooted in mechanics and in the atomistic picture of matter. However one runs into difficulties and puzzles: objective properties for particles cannot be reconciled with quantum mechanics, quantum mechanical models of the measuring process are hardly compatible with objective description of macrosystems, decoherence must be supplied to the Schrödinger equation, either due to lack of isolation in any system, or by some additional stochasticity. We stress the point of view that the concept of a physical process running inside a suitably prepared isolated system and displayed by a certain set of relevant variables must be the starting point and that the theoretical description should be based on quantum field theory of finite systems. This point of view is much closer to thermodynamics than to mechanics: the basic ideas linked to atomistic structure of matter are however kept into account in a more subtle way by the quantization of the fields underlying the physical model and by the locality or quasi-locality of their interaction. The concept of a particle arises only in an unsharp way when one is pursuing universal features arising from locality of field theory and polishing away what comes from boundary conditions and residual interactions. In the most striking way a particle emerges when a process can be performed in which a *source part* of a macrosystem affects a *detecting part* of it through a microchannel consisting of a one- or few-particle system produced by the first part and directed to the second one. Looking at the problem this way, decoherence is obviously already inside the description: actually it is a hard theoretical job to drive it back from the microchannel, completely in tune with what experimentalists do. On the contrary the usual theoretical setting seems strange since it makes theorists work putting decoherence in, while experimentalists work hard to drive it back. This point of view about particles goes back to Ludwig's approach to quantum mechanics. By suitable axiomatization of general features of particular processes which give evidence of particles he succeeded in obtaining as a description of these processes quantum mechanics already in the modern form [1] (p.o.v. measures, operations, instruments) that is now generally recognized [2] as the formalism adequate to describe in a realistic way processes due to microsystems. Obviously when highly idealized schematizations can be applied, typically if decoherence can be neglected, and space-time symmetry for the microsystem can be assumed, the more schematic Dirac's book axiomatics emerges in all its geometrical neatness. While Ludwig pointed to a new theory encompassing microsystems and macrosystems in order to set the duality micro-macrosystems, we try to do this remaining inside quantum field theory, only improving somehow non-equilibrium theory for isolated systems. In § 2 we simply describe how a microchannel can arise, in § 3 the general structure of non-equilibrium theory is recalled and compatibility of the general dynamics of the system with the presence of the microchannel is indicated. The physical model we will use is a self-interacting spinless Schrödinger field confined inside a finite region ω. It can be trivially improved using several interacting fields with spin and should be amenable to the treatment of bound states and resonances between them. However all this is a very primitive stage since no intermediate gauge fields are introduced. To the space region ω a set of *normal modes* $\{u_r(\mathbf{x})\}$ is associated. They are an orthonormal, complete set of solutions of the stationary state equation:

$$-\frac{\hbar^2}{2m}\Delta_2 u_n(\mathbf{x})+\mathcal{V}(\mathbf{x})u_n(\mathbf{x}) = W_n u_n(\mathbf{x}) \quad u_n(\mathbf{x})=0, \mathbf{x}\in\delta\omega, u_n\in L^2(\omega); \tag{1.1}$$

$\mathcal{V}(\mathbf{x})$ being a suitable potential for external and internal effective forces. The Schrödinger field is defined by:

$$\hat{\Phi}(\mathbf{x}) = \sum_n \hat{a}_n u_n(\mathbf{x}), \qquad [\hat{a}_n, \hat{a}^\dagger_{n'}]_\mp = \delta_{nn'} \tag{1.2}$$

with $\hat{a}_n$ Bose or Fermi annihilation operators on the Fock-space of the system. The Hamiltonian of the system is assumed as:

$$\hat{H} = \sum_n W_n \hat{a}^\dagger_n \hat{a}_n + \frac{1}{2}\int_\omega d^3\mathbf{x} d^3\mathbf{y}\, \hat{\Phi}^\dagger(\mathbf{x})\hat{\Phi}^\dagger(\mathbf{y})V(|\mathbf{x}-\mathbf{y}|)\hat{\Phi}(\mathbf{y})\hat{\Phi}(\mathbf{x}), \tag{1.3}$$

$V(r)$ being the basic microphysical input, a short-range function which gives the quasi-local form of interaction and will finally represent two-body interaction between the particles of the system.

2. THE MICROCHANNEL

Postponing a more technical sketch of the treatment of a non-equilibrium system, we now come to the main point: the microchannel. For this issue we choose a bundle of normal modes $r \in M$, M being a suitable subset of the indexes n: M are the normal modes, M^C the remaining ones. The field operator $\hat{\Phi}(\mathbf{x})$ contains them both

$$\hat{\Phi}(\mathbf{x}) = \hat{\Phi}_M(\mathbf{x}) + \hat{\Phi}_{M^C}(\mathbf{x}), \tag{2.1}$$

$$\hat{\Phi}_M(\mathbf{x}) = \sum_{r\in M} \hat{a}_r u_r(\mathbf{x}), \qquad \hat{\Phi}_{M^C}(\mathbf{x}) = \sum_{s\in M^C} \hat{a}_s u_s(\mathbf{x}).$$

The idea of a microchannel is formalized assuming that during a time interval $[t_0, t_1]$ the channel modes are depleted, so that the contribution to the dynamics of the system due to interaction between channel-modes is negligible. Then there is a possible dynamics of the system with unfeeded channel, described by a statistical operator $\hat{\rho}^0_t$ satisfying:

$$\hat{a}_r \hat{\rho}^0_t = 0, \qquad \forall r \in M \tag{2.2}$$

i.e., without excitations of M-modes and evolving according to $d\hat{\rho}_t^0/dt = -i/\hbar[\hat{H}, \hat{\rho}_t^0]$. There is however also a possible dynamics with feeded channel, described by a statistical operator

$$\hat{\rho}_t^{(1)} = \sum_{r,r' \in M} w_{rr'}(t) \hat{a}_r^\dagger \hat{\rho}_t^0 \hat{a}_{r'}, \tag{2.3}$$

with one excitation related to M. $\hat{\rho}_t^{(1)}$ is a positive operator if $w_{rr'}(t)$ is a positive matrix and by (2.2) it is normalized if $\sum_r w_{rr}(t) = 1$. For $t \in [t_0, t_1]$ the following representation of the statistical operator for a system endeavored with a microchannel should hold:

$$\hat{\rho}_t = (1-\lambda)\,\hat{\rho}_t^0 + \lambda \sum_{r,r' \in M} w_{rr'}(t) \hat{a}_r^\dagger \hat{\rho}_t^0 \hat{a}_{r'}, \qquad 0 < \lambda < 1, \tag{2.4}$$

λ giving the probability that the microchannel is feeded. For the statistical operator (2.4) by (2.2) the interaction between modes in M is negligible. We assume at first that also the interaction between a mode $r \in M$ and the modes $s \in M^C$ can be neglected at least in the time interval $[t_0, t_1]$: then Liouville von Neumann equation for $\hat{\rho}_t$ implies that

$$\frac{dw_{rr'}}{dt}(t) = -\frac{i}{\hbar}(W_r - W_{r'}) w_{rr'}(t). \tag{2.5}$$

Eq. (2.5) can be considered as the evolution equation of a statistical operator $W^{(1)}(t)$ defined as

$$W^{(1)}(t) = \sum_{r,r' \in M} |r\rangle w_{rr'}(t) \langle r'|, \tag{2.6}$$

describing the microsystem inside the microchannel:

$$\frac{dW^{(1)}}{dt}(t) = -\frac{i}{\hbar}[H_0^{(1)}, W^{(1)}(t)], \qquad H_0^{(1)} = \sum_r |r\rangle W_r \langle r|,$$

while $|r\rangle$ is a basis in the one-particle Hilbert space $\mathcal{H}^M \subset L_2(\omega)$ spanned by $r \in M$, $\langle \mathbf{x}|r\rangle = u_r(\mathbf{x})$. Taking an observable $\hat{A}$ of the system or more in particular an element $\hat{E}^{\mathbf{A}}(S)$ of the spectral measure of some commuting set of self-adjoint operators on some σ-algebra of sets S, expectations or probability measures are given by expressions:

$$\mathrm{Tr}(\hat{A}\hat{\rho}_t) = (1-\lambda)\mathrm{Tr}(\hat{A}\hat{\rho}_t^0) + \lambda \sum_{r,r' \in M} w_{rr'}(t) \mathrm{Tr}(\hat{a}_{r'} \hat{A} \hat{a}_r^\dagger \hat{\rho}_t^0) \tag{2.7}$$

$$\mathrm{Tr}(\hat{E}^{\mathbf{A}}(S)\hat{\rho}_t) = (1-\lambda)\mathrm{Tr}(\hat{E}^{\mathbf{A}}(S)\hat{\rho}_t^0) + \lambda \sum_{r,r' \in M} w_{rr'}(t) \mathrm{Tr}(\hat{a}_{r'} \hat{E}^{\mathbf{A}}(S) \hat{a}_r^\dagger \hat{\rho}_t^0).$$

Setting $\mathrm{Tr}(\hat{a}_{r'}\hat{A}\hat{a}^\dagger_r\hat{\rho}^0_t) = \langle r'|A^{(1)}_t|r\rangle$, $\mathrm{Tr}(\hat{a}_{r'}\hat{E}^{\mathbf{A}}(S)\hat{a}^\dagger_r\hat{\rho}^0_t) = \langle r'|F^{(1)}_t(S)|r\rangle$, the r.h.s. of eq. (2.7) related to the microchannel can be written:

$$\begin{aligned}\sum_{r,r'\in M} w_{rr'}(t)\langle r'|A^{(1)}_t|r\rangle &= \mathrm{Tr}_{\mathcal{H}^M}(W^{(1)}(t)A^{(1)}_t) \qquad (2.8)\\ \sum_{r,r'\in M} w_{rr'}(t)\langle r'|F^{(1)}_t(S)|r\rangle &= \mathrm{Tr}_{\mathcal{H}^M}(W^{(1)}(t)F^{(1)}_t(S)),\end{aligned}$$

showing the typical mathematical structure of *one-particle* quantum mechanics formulated in the Hilbert space $\mathcal{H}^M$. Let us notice that even if $\hat{E}^{\mathbf{A}}(S)$ is a Fock-space projection valued measure, the related $\hat{F}^{\mathbf{A}}(S)$ is in general a p.o.v. normalized measure (positive operator valued or *effect* valued), according to modern axiomatics. Let us assume that $[\hat{E}^{\mathbf{A}}, \hat{N}_M]$, with $\hat{N}_M = \sum_{r\in M}\hat{a}^\dagger_r\hat{a}_r$ and furthermore

$$\begin{aligned}&\sum_r \langle r_1|F^{(1)}_t(S)|r\rangle\langle r|F^{(1)}_t(S)|r_2\rangle\\ &\quad= \sum_r \mathrm{Tr}(\hat{a}_{r_1}\hat{E}^{\mathbf{A}}(S)\hat{a}^\dagger_r\hat{\rho}^0_t)\mathrm{Tr}(\hat{a}_r\hat{E}^{\mathbf{A}}(S)\hat{a}^\dagger_{r_2}\hat{\rho}^0_t)\\ &\quad\approx \sum_r \mathrm{Tr}(\hat{a}_{r_1}\hat{E}^{\mathbf{A}}(S)\hat{a}^\dagger_r\hat{a}_r\hat{E}^{\mathbf{A}}(S)\hat{a}^\dagger_{r_2}\hat{\rho}^0_t)\\ &\quad= \mathrm{Tr}(\hat{a}_{r_1}\hat{E}^{\mathbf{A}}(S)\hat{N}_M\hat{a}^\dagger_{r_2}\hat{\rho}^0_t) = \mathrm{Tr}(\hat{a}_{r_1}\hat{E}^{\mathbf{A}}(S)\hat{a}^\dagger_{r_2}\hat{\rho}^0_t).\end{aligned}$$

Then $\hat{F}^{(1)}_t = (\hat{F}^{(1)}_t)^2$ turns out to be the spectral measure of a self-adjoint operator in $\mathcal{H}^M$, representing an observable $A^{(1)}_t$ of the microsystem. Let us stress that in this construction an explicit time dependence of $F^{(1)}_t$ ($A^{(1)}_t$ in the more particular case) arises in a quite natural way, since in addition to the microchannel a macrosystem dependent dynamics cannot be in general avoided. However right now the concept of a *good detecting part* inside the system can be easily formulated assuming that in the relevant time interval $[t_0, t_1]$ the explicit time dependence of $F^{(1)}_t$ ($A^{(1)}_t$) is either negligible or well-known on the basis of macroscopic phenomenology. We shall simply forget this time dependence setting $\hat{F}^{(1)}_t(S) \approx \hat{F}^{(1)}_{t_0}(S) \approx \hat{F}^{(1)}(S)$ ($\hat{A}^{(1)}_t \approx \hat{A}^{(1)}_{t_0} \approx \hat{A}^{(1)}$). Then r.h.s. of (2.8) becomes the basic formula for probability distribution of an observable given in general by a p.o.v. measure or by a self-adjoint operator $A^{(1)}$ in a more idealized situation, related to a microsystem associated with the statistical operator $W^{(1)}(t)$ and produced, living and detected inside the macrosystem: $\mathcal{H}^M$ is its Hilbert space and $H^{(1)} = \sum_{r\in M}|r\rangle W_r\langle r|$ its Hamiltonian. In this neat picture there is however a fundamental flaw: interaction with M^C modes has been neglected. Experimental particle physics shows us that this is indeed allowed when experimental physicists have been clever enough, but what we have described can never be more than an approximation. Corrections to this picture can be calculated: when they are small enough to preserve the basic picture, the concept of a microsystem undergoing an unavoidable decoherence arises in a very natural way. Let us take a statistical operator $\hat{\rho}^0_{t_0}$ of the form (2.4)

$$\hat{\rho}_t = (1-\lambda)\hat{\rho}^0_t + \lambda \sum_{r,r' \in M} w_{rr'}(t_0) e^{-\frac{i}{\hbar}\hat{H}(t-t_0)} \hat{a}^\dagger_r e^{+\frac{i}{\hbar}\hat{H}(t-t_0)} \hat{\rho}^0_t e^{-\frac{i}{\hbar}\hat{H}(t-t_0)} \hat{a}_{r'} e^{+\frac{i}{\hbar}\hat{H}(t-t_0)},$$

by the assumption of a *good detecting part* we can replace $\hat{\rho}^0_t$ with $\hat{\rho}^0_{t_0}$ in the second term. To take interaction with M^C modes into account a suitable time scale $\tau \approx \frac{\hbar}{\Delta W}$, where ΔW is the width of the energy band of the microchannel, must be considered and τ must be large enough to allow for a treatment of M, M^C interaction by a formalism similar to scattering theory, in which states are replaced by operators, the Hamiltonian by the Liouvillean $\mathcal{H} = -\frac{i}{\hbar}[\hat{H}, \cdot]$ and the scattering operator by a scattering map $\mathcal{T}(z)$. In fact setting $\mathcal{H} = \mathcal{H}_0 + \mathcal{V}$, with $\mathcal{H}_0 = \frac{i}{\hbar}[\hat{H}_0, \cdot]$, $\hat{H}_0 = \sum_r W_r \hat{a}^\dagger_r \hat{a}_r$ one has:

$$e^{-\frac{i}{\hbar}\hat{H}(t-t_0)} \hat{a}^\dagger_r e^{+\frac{i}{\hbar}\hat{H}(t-t_0)} = \int_{-i\infty+\eta}^{+i\infty+\eta} \frac{dz}{2\pi i}\, e^{z\tau} (z-\mathcal{H})^{-1} \hat{a}^\dagger_r$$
$$= e^{-\frac{i}{\hbar}W_r(t-t_0)} \hat{a}^\dagger_r + \int_{-i\infty+\eta}^{+i\infty+\eta} \frac{dz}{2\pi i}\, e^{z\tau} \left[(z-\mathcal{H}_0)^{-1} \mathcal{T}(z) (z-\mathcal{H}_0)^{-1} \right] \hat{a}^\dagger_r$$

and similarly for the adjoint operator. The part depending on $\mathcal{T}(z)$ is responsible for decoherence. If $\hat{\rho}^0_{t_0}$ is an equilibrium state the treatment of this part gives in a perspicuous way the theory of Brownian motion [3] and in the limit of small momentum transfers the typical dynamics of a particle undergoing friction and position and momentum diffusion is found. One can expect that also in the case of a non-equilibrium $\hat{\rho}^0_{t_0}$ of the kind that will be discussed in § 3 a similar approach can be fruitful.

3. EMBEDDING OF MICROCHANNEL IN THE DYNAMICS OF A MACROSYSTEM

In our approach microsystems are derived entities and are no longer the basic elementary starting point of the physical description: then this description must stand on its own legs by a suitable reformulation of quantum mechanics of finite isolated non-equilibrium system. Let us briefly recall some main points about this general description of macrosystems [4]. The very claim that a physical system is isolated implies the choice of a subset of observables that are *under control* by a suitable preparation procedure performed on the system during a preparation time interval $[T, t_0]$. These observables are suitable slowly varying quantities, typically densities of conserved charges $\hat{A}_j(\boldsymbol{\xi})$ related to symmetry properties of the underlying local field theoretical structure. By their expectations $\{\langle \hat{A}_j(\boldsymbol{\xi})\rangle_t\}_j$ a set of classical fields $\{\zeta_j(\boldsymbol{\xi}, t)\}_j$ is determined when these expectations are reproduced by means of a maximal von Neumann entropy state $\hat{w}[\zeta(t)]$. This is a generalized Gibbs state, induced at any time t by the statistical operator $\hat{\varrho}_t$ via the expectations $\langle \hat{A}_j(\boldsymbol{\xi})\rangle_t = \mathrm{Tr}\,(\hat{A}_j(\boldsymbol{\xi})\hat{\varrho}_t)$. It depends on the operators $\hat{A}_j(\boldsymbol{\xi})$ and the fields $\zeta_j(\boldsymbol{\xi}, t)$ and provides an entropy for the classical state $\{\zeta_j(\boldsymbol{\xi}, t)\}_j$. Such classical state, though related to statistical properties of the system that has been prepared in the time interval $[T, t]$, is taken as an objective property of the system at time t. This is already done, perhaps without complete awareness, when a velocity, a temperature or a chemical potential field is associated to a massive continuum. The statistical operator $\hat{\varrho}_t$, representing preparation until time $t \geq t_0$, shows the spontaneous dynamics of the isolated system in the time interval $[t_0, t]$, given by $\hat{\varrho}_t = e^{-\frac{i}{\hbar}\hat{H}(t-t_0)}\hat{\varrho}_{t_0} e^{+\frac{i}{\hbar}\hat{H}(t-t_0)}$, and can be written in the form

$$\hat{\varrho}_t = \exp\{-\zeta_0(t)\hat{\mathbf{1}} - \sum_j \int d\boldsymbol{\xi}\, \zeta_j(\boldsymbol{\xi}, t)\hat{A}_j(\boldsymbol{\xi}) + \int_T^t dt'\, \hat{\mathcal{S}}_t(t')\} \tag{3.1}$$

where the history part $\hat{\mathcal{S}}_t(t')$ for $t' \in [T, t_0]$ describes the preparation procedure and for $t' \in [t_0, t]$ can be simply given in terms of state variables $\zeta_j(\boldsymbol{\xi}, t')$, $\dot{\zeta}_j(\boldsymbol{\xi}, t')$ and the related density and current operators given at time $-(t - t')$ in Heisenberg picture. Since the first term alone in the exponent already exactly gives $\langle \hat{A}_j(\boldsymbol{\xi})\rangle_t$, an expansion with respect to the history term becomes very natural and e.g., at first order leads to an evolution equation

$$\frac{d}{dt}\langle \hat{A}_j(\boldsymbol{\xi})\rangle_t = \mathrm{Tr}\,(\dot{\hat{A}}_j(\boldsymbol{\xi})\hat{w}[\zeta(t)]) + \int_T^t dt'\, \langle \dot{\hat{A}}_j(\boldsymbol{\xi}), \hat{\mathcal{S}}_t(t')\rangle_{\hat{w}[\zeta(t)]} + \ldots \tag{3.2}$$

where $\hat{w}[\zeta(t)]$ is the generalized Gibbs state associated to the classical state at time t. To the expectation values of the operators $\dot{\hat{A}}_j(\boldsymbol{\xi})$ calculated with $\hat{w}[\zeta(t)]$ corrections responsible for irreversibility arise by the history term, which brings in foreground an integral over t' of the two point Kubo correlation functions between operators $\dot{\hat{A}}_j(\boldsymbol{\xi})$ and operators $\dot{\hat{A}}_j(\boldsymbol{\xi}', -(t - t'))$, $\hat{A}_j(\boldsymbol{\xi}', -(t - t'))$ in the macrostate $\hat{w}[\zeta(t)]$. Now the possibility of a great simplification imposes on our attention: as at equilibrium, these correlation functions, at least inside a time integral with well shaped classical state parameters, could practically vanish if

$t' < t - \tau$, τ being a characteristic decay time; then $\int_T^t dt' \to \int_{t-\tau}^t dt'$, thus eliminating memory of the preparation procedure for $t > t_0 + \tau$ and memory of previous classical state if it variates slowly enough during a time interval τ. We call such a situation *simple dynamics*: it dominates a large part of equilibrium thermodynamics. However we also discover a large arena where a behavior different from *simple dynamics* can arise. One expects that when the fields $\zeta_j(\boldsymbol{\xi}, t_1)$ are inhomogeneous enough around time t_1, depletion of certain modes can arise: $\hat{a}_r \hat{w}[\zeta(t_1)] \approx 0$ if $r \in M$, then the part of previous history related to creation of these modes might present a slowly decaying contribution. Let us write:

$$\hat{\mathcal{S}}_{t_1}(t') = \hat{\mathcal{S}}_{t_1}^{(S)}(t') + \hat{\mathcal{S}}_{t_1}^{(M)}(t') \tag{3.3}$$

where $\hat{\mathcal{S}}_{t_1}^{(S)}(t')$ does not create particles in the M modes, thus yielding through (3.1) (with $\hat{\mathcal{S}}_{t_1}^{(S)}(t')$ at place of $\hat{\mathcal{S}}_t(t')$) a statistical operator $\hat{\varrho}_{t_1}^{(S)}$ with *simple dynamics*, while the full statistical operator $\hat{\varrho}_{t_1}$ can be written by an expansion with respect to $\hat{\mathcal{S}}_{t_1}^{(M)}(t')$, preserving positivity:

$$\hat{\varrho}_{t_1} = \lambda \left[\hat{\mathbf{1}} + \int_{t_1-\tau}^{t_1} dt' \, \hat{\mathrm{S}}_{t_1}^{(M)}(t') \right] \hat{\varrho}_{t_1}^{(S)} \left[\hat{\mathbf{1}} + \int_{t_1-\tau}^{t_1} dt' \, \hat{\mathrm{S}}_{\bar{t}}^{(M)\dagger}(t') \right], \tag{3.4}$$

where $\hat{\mathrm{S}}_{t_1}^{(M)}(t')$ is essentially determined by $\hat{\mathcal{S}}_{t_1}^{(M)}(t')$. Let us write:

$$\int_{t_1-\tau}^{t_1} dt' \, \hat{\mathrm{S}}_{t_1}^{(M)}(t') = \sum_{r \in M} \hat{a}_r^\dagger \int_{t_1-\tau}^{t_1} dt' \int_{\omega_s} d\boldsymbol{\xi}' \, \hat{A}_r(-(t_1 - t'), \boldsymbol{\xi}'),$$

where the field operator $\hat{A}_r(-(t_1 - t'), \boldsymbol{\xi}')$ acts as annihilation operator typically for $\boldsymbol{\xi}'$ inside some space region ω_s. If a local observable $\hat{B}(\boldsymbol{\xi}, t)$ is considered at time t, such that correlations between the space-time point $(\boldsymbol{\xi}, t)$ and region ω_s are negligible, one can write

$$\begin{aligned} \mathrm{Tr}\,[\hat{B}(\boldsymbol{\xi}, t) \sum_{r,r' \in M} \hat{a}_r^\dagger \int_{t_1-\tau}^{t_1} dt' \!\int_{\omega_s} d\boldsymbol{\xi}' \, \hat{A}_r(-(t_1 - t'), \boldsymbol{\xi}') \hat{\varrho}_{t_1}^{(S)} \\ \cdot \int_{t_1-\tau}^{t_1} dt' \!\int_{\omega_s} d\boldsymbol{\xi}' \, \hat{A}_r^\dagger(-(t_1 - t'), \boldsymbol{\xi}') \hat{a}_r] \approx \mathrm{Tr}\,[\hat{B}(\boldsymbol{\xi}, t) \sum_{r,r' \in M} \hat{a}_r^\dagger \hat{\varrho}_{t_1}^{(S)} \hat{a}_r] \sigma_{r'r} \end{aligned}$$

with

$$\sigma_{r'r} = \mathrm{Tr}\,[\int_{t_1-\tau}^{t_1} dt' \!\int_{\omega_s} d\boldsymbol{\xi}' \, \hat{A}_r(-(t_1 - t'), \boldsymbol{\xi}') \hat{\varrho}_{t_1}^{(S)} \int_{t_1-\tau}^{t_1} dt' \!\int_{\omega_s} d\boldsymbol{\xi}' \, \hat{A}_r^\dagger(-(t_1 - t'), \boldsymbol{\xi}')],$$

being a positive matrix describing the way in which the normal modes of M are feeded by destruction of particles in ω_s: in this way we are recovering the starting point of § 2.

Acknowledgments

L. L. and B. V. gratefully acknowledge financial support by MURST under Cofinanziamento. B. V. also acknowledges support by MURST under Progetto Giovani.

References

[1] Ludwig, G. (1985). *An Axiomatic Basis for Quantum Mechanics*, Springer, Berlin; (1983). *Foundations of Quantum Mechanics*, Springer, Berlin.

[2] Davies, E. B. (1976). *Quantum Theory of Open Systems*, Academic Press, London; Holevo, A. S. (1982). *Probabilistic and Statistical Aspects of Quantum Theory*, North Holland, Amsterdam; Kraus, K. (1983). *States, Effects and Operations*, in Lecture Notes in Physics, Volume 190, Springer, Berlin; Busch, P., Grabowski, M., and Lahti, P. J. (1995). Operational Quantum Physics, in *Lecture Notes in Physics*, Vol. m31, Springer, Berlin.

[3] Vacchini, B. (2000). *Phys. Rev. Lett.* **84**, 1374; (2000). submitted for pubblication.

[4] Lanz, L., Melsheimer, O., Vacchini, B. (2000). *Rep. Math. Phys.* to appear.