

Quantum theory: the role of microsystems and macrosystems

Ludovico Lanz¹, Bassano Vacchini¹ and Olaf Melsheimer²

¹ Dipartimento di Fisica dell'Università di Milano and INFN, Sezione di Milano Via Celoria 16, I-20133, Milan, Italy

² Fachbereich Physik, Philipps-Universität Renthof 7, D-35032, Marburg, Germany

Received 29 June 2006, in final form 30 October 2006

Published 7 March 2007

Online at stacks.iop.org/JPhysA/40/3123

Abstract

We stress the notion of statistical experiment, which is mandatory for quantum mechanics, and recall Ludwig's foundation of quantum mechanics, which provides the most general framework to deal with statistical experiments giving evidence for particles. In this approach particles appear as interaction carriers between preparation and registration apparatuses. We further briefly point out the more modern and versatile formalism of quantum theory, stressing the relevance of probabilistic concepts in its formulation. At last we discuss the role of macrosystems, focusing on quantum field theory for their description and introducing objective state parameters for them.

PACS numbers: 03.65.Ta, 03.65.Ca, 03.65.Yz

1. Introduction

Quantum theory is an increasingly successful theory of matter and some typical features that have appeared paradoxical, such as EPR correlations, are now on the way to becoming a technological resource in the realm of quantum communication. There is however a fundamental difficulty: it appears as a theory of measurements that runs into troubles if one describes in the most naive quantum mechanical way a measuring device at work. A precise proof of this was recently given by Bassi and Ghirardi [1], further pointing to unitary quantum mechanical evolution as the basic flaw and proposing the GRW modification of the Schrödinger equation by a universal stochastic process [2]. Many other proposals appeared to make quantum theory less measuring device dependent; just to mention a few let us recall the Bohmian interpretation (see [3] and literature therein for recent review and [4] for present developments), together with a new more general framework suggested by Adler [5], also motivated by open problems in high energy physics, by which stochastic modifications to the Schrödinger equation can become a more natural low energy effective theory. Our aim in this paper is to recall and refresh the, in our opinion, very deep reformulation of foundations

of quantum mechanics that was given by Ludwig: in this approach the very concept of microsystems is investigated and quantum theory turns out to be a naturally incomplete theory of it, completely satisfactory at a non-relativistic level. Its extension to general systems cannot dispense with thermodynamics and, in our opinion, provides a natural opening to quantum field theory. Section 2 stresses the essential role played by statistical experiments in quantum mechanics; in section 3 the concept of microsystems as proposed by Ludwig is recalled; in section 4 we briefly review the modern formulation of quantum mechanics, which besides Ludwig's approach also arose quite independently in other contexts; in section 5 the generalization to many types of microsystems is considered together with the role played by quantum field theory in order to cope with this situation; in section 6 we finally briefly summarize the contents and main message of the paper, and discuss possible future developments.

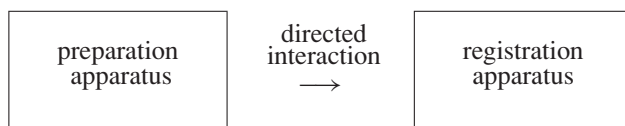
2. Statistical experiments

Quantum theory marked a turning point in physics since the basic Galilean concept of reproducible experiments encountered a basic crisis and it was necessary to weaken and clarify the notion of *reproducibility*. After Galileo, in the pre-quantum era, the concept of reproducible experiment allowed us to delimit a part of the world ruled by physics, arriving for it to an atomistic underlying model, the interactions between the elementary components being the universal unifying core of the huge phenomenological complexity. The quantum era is characterized by the evidence that naive reproducibility fails in experiments focused on microsystems and must be generalized by the much subtler concept of reproducibility of statistical experiments, so that just the most fundamental physics becomes *essentially statistical*. An experiment deals with a system reprepared in a well fixed way for a large number of independent experimental runs and only the frequencies of the well-defined events one is looking for in these runs are the result of the experiment and have a counterpart in the underlying theory. What is going on during a single run obviously belongs to reality, but not in all aspects of physics. To physics belongs what we have described as *fixable* in the repetitions and the frequencies of well-identified changes. In suitable conditions the essentially statistical character we have pointed out can be neglected and then the previously mentioned classical description of matter appears and provides a conceptual structure leading through a procedure called *quantization* to the actual quantum theory. This could feed the belief that *quantization* of classical frameworks is fundamental enough to catch the extremely vast phenomenology of physical systems. In contrast, based on Ludwig's foundations of quantum theory we shall take such phenomenology as the starting point of quantum theory and its natural completion. It is the present day technology, offering high vacuum techniques, highly efficient detectors and highly controllable sources and devices for trapping and handling single microsystems, that provides most direct evidence for the peculiarity of microsystems. In the more commonly observed phenomenology what happens inside a certain space region until a certain time influences in future times the adjacent space regions involving simultaneously an infinity of space points. In contrast if an elementary microsystem is prepared in this region (e.g. an attenuated source is located there) a process is generated starting at some future time point from *one* single space point at an appreciable distance (may be also an enormous distance in astrophysical extrapolations) which then expands inside its future light cone. In these preparations an effect is triggered only at one spacetime point in the universe even if detectors are placed everywhere. The position of this point and often the whole process contained in its future light cone is a stochastic variable; by many runs frequencies can be established and repeating this whole procedure the reproducibility of these frequencies can be

controlled and one gets evidence of a statistical law. This statistical law depends in general strongly also on the devices placed between the source and the points from which macroscopic processes can start. Still more impressive is the most direct generalization of a microsystem consisting of two elementary components: a source of such system causes two processes inside the future light cones of two spacetime points, with stochasticity as before but with the restriction of absolutely strict correlations between the two parts of the whole processes, associated with conservation rules, quite independently of the distance of the two points. This is a preparation such that only at two spacetime points in the universe a process begins due to preparation, consisting of two parts, extremely strongly correlated each other, one part showing a stochastic character. Often the processes started by microsystems behave as further sources of elementary and composed microsystems randomly involving a finite number of spacetime points correlated among themselves and starting points of further correlated processes. The fundamental consequence of quantum theory is the statistical flavour attached to experiments, by this refinement of the concept of reproducibility. Since operatively a statistical experiment is a much more intriguing enterprise, where some conditions must be explicitly selected and then they must be controlled and guaranteed during several runs, one can indeed expect that the mathematical representation of what is done in an experiment must have a higher complexity as it was in pre-quantum physics. Ludwig's work can be understood as a fundamental justification of the new mathematical tools. In setting up an experiment aiming to establish results in some physical context, previous achievements of physics are taken as consolidated, some pretheories are taken as given and already enter in the language that is used by people setting the experiment. So physicists, engineers and technicians building apparatuses for a high energy experiment aiming e.g. to verify aspects of the standard model, use experience and knowledge coming from a well-established and much simpler phenomenology: it turns out that they have under complete control a huge part of what goes on in the experiment, but obviously not these aspects of reality that the experiment is challenging. In experiments in the quantum era the technology of suitable triggering of apparatuses with correlation and anticorrelation settings is superposed to more classical arrangements related e.g. to Euclidean geometry, time determinations, phase space of classical mechanics and so on.

3. The notion of microsystems

In his axiomatic approach to the foundations of quantum mechanics Ludwig proposed to take as fundamental domain of the theory the statistical experiments with single microsystems and the frequencies of the related phenomena. Instead of the particles themselves one considers the macroscopic setup of any real experiment, which can be divided into a preparation procedure and a registration procedure, both to be described in terms of pretheories. A simple example of a preparation apparatus could be an accelerator plus target, while a typical registration apparatus could be a bubble chamber. Once this experimental setup is suitably described, one considers the rate according to which microsystems prepared with the given preparation apparatus trigger the assigned registration apparatus: these are the frequencies to be compared with the quantum mechanical laws. The general scheme of a statistical experiment can be depicted as follows, with preparation and registration apparatuses displayed as boxes (so-called *Ludwig's Kisten*), the latter acted upon by the former by means of a directed interaction brought about by a microsystem



In this spirit we want to introduce the notion of microsystems as something which has been prepared by a preparation apparatus and registered by a registration apparatus. To do this we need a statistical theory, in terms of which the general structures of preparation and registration, which can be applied both to microsystem and macrosystem, can be described. We will almost verbatim follow the introductory reference [6] when recalling the basic axioms, but we shall proceed in a less detailed way with respect to the full axiomatic approach initiated in the 1960s and described in [7].

3.1. Statistical selection procedures

Let M be the set having as elements the representatives of the physical features whose statistics we want to describe (in the present case we shall concentrate on microsystems). The statistics is related to selection procedures, by which special features may be selected. A selection procedure is to be described by a subset $a \subset M$, corresponding to the subset of features (microsystems) that satisfy the given selection procedure. We define as *selection procedure* the following mathematical structure: a set M and a subset $\mathcal{S} \subset P(M)$ (where $P(M)$ denotes the power set of M) such that

$$\mathbf{S\ 1.1} \quad a, b \in \mathcal{S}, a \subset b \Rightarrow b \setminus a \in \mathcal{S}$$

$$\mathbf{S\ 1.2} \quad a, b \in \mathcal{S} \Rightarrow a \cap b \in \mathcal{S}.$$

We call selection procedure both \mathcal{S} and an element a of \mathcal{S} . **S 1.2** says that the selection procedure consisting in selecting both according to a and b exists. If $a \subset b$ we say that a is *finer* than b . **S 1.1** says that if we use two selection procedures a and b , where a is finer than b , the rest of the objects $x \in b$, which do not satisfy the finer criterion a , still constitute a selection procedure. Note that in this construction it is not necessarily $M \in \mathcal{S}$. Only if some selection has been done on its elements M acquires a physical meaning; therefore, we shall not assume that M itself belongs to \mathcal{S} . In fact $a \in \mathcal{S}, M \in \mathcal{S}$ would lead to $M \setminus a \in \mathcal{S}$ contrary to physical meaningfulness. Let us consider in fact a given source for a certain type of particles. For the prepared particles we can make important assertions about the experiments for which the particles are used, but we cannot make any definite statement for all other particles of the same type not prepared from this source. Thus it is meaningful not to require that M be a selection procedure. Let us note that $\mathcal{S}(a) \equiv \{b \in \mathcal{S} | b \subset a\}$ is a Boolean ring, while \mathcal{S} need not even be a lattice, because $a, b \in \mathcal{S}$ does not automatically imply $a \cup b \in \mathcal{S}$. In particular two selection procedures $a, b \in \mathcal{S}$ are called *coexistent* relative to c if both $a \subset c$ and $b \subset c$.

It often happens in applications that two selection procedures a and b , where b is finer than a , are not statistically independent. Consider for example an experiment in which of the N systems prepared according to the selection procedure a , N_1 also satisfy the selection criterion of b : we say that N_1/N is the relative frequency of b relative to a . If this frequency is shown to be reproducible and it is confirmed by experiments with great number of systems, we say that b is statistically dependent on a . Let $\mathcal{S} \subset P(M)$ be a selection procedure, for which **S 1.1** and **S 1.2** hold, and let $\mathcal{T} \equiv \{(a, b) | a, b \in \mathcal{S}, b \subset a, a \neq \emptyset\}$: we say that \mathcal{S} is a *statistical selection procedure* whenever a real function $\lambda(a, b)$ with $0 \leq \lambda(a, b) \leq 1$ is defined on \mathcal{T} such that

$$\mathbf{S\ 2.1} \quad a_1, a_2 \in \mathcal{S}, a_1 \cap a_2 = \emptyset, a_1 \cup a_2 \in \mathcal{S} \Rightarrow \lambda(a_1 \cup a_2, a_1) + \lambda(a_1 \cup a_2, a_2) = 1$$

$$\mathbf{S\ 2.2} \quad a_1, a_2, a_3 \in \mathcal{S}, a_1 \supset a_2 \supset a_3, a_2 \neq \emptyset \Rightarrow \lambda(a_1, a_3) = \lambda(a_1, a_2)\lambda(a_2, a_3)$$

$$\mathbf{S\ 2.3} \quad a_1, a_2 \in \mathcal{S}, a_1 \supset a_2, a_2 \neq \emptyset \Rightarrow \lambda(a_1, a_2) \neq 0.$$

$\lambda(a, b)$ is usually called the *conditional probability* of b relative to a and represents the frequency with which systems selected by a also satisfy b . If $a_1 \cup a_2$ is a selection procedure, both a_1 and a_2 are finer than $a_1 \cup a_2$; if $a_1 \cap a_2 = \emptyset$ they exclude each other. If N systems are selected according to $a_1 \cup a_2$, of which N_1 also satisfy a_1 and N_2 satisfy a_2 , because of $a_1 \cap a_2 = \emptyset$ we have $N_1 + N_2 = N$: this explains **S 2.1**. If for three selection procedures we have $a_1 \supset a_2 \supset a_3$ and N_1 systems are selected according to a_1 , between these N_2 according to a_2 , between these again N_3 according to a_3 , we simply have $N_3/N_1 = (N_2/N_1)(N_3/N_2)$, that is to say **S 2.2**. If $a_1 \supset a_2 \neq \emptyset$, of the N systems chosen according to a_1 certainly finitely many will also satisfy a_2 , which is **S 3.3**. From these axioms follows $\lambda(a_1, 0) = 0$ and $\lambda(a_1, a_1) = 1$; moreover, if $a_2 \cap a_3 = \emptyset$, $a_2, a_3 \subset a_1$, we have $\lambda(a_1, a_2 \cup a_3) = \lambda(a_1, a_2) + \lambda(a_1, a_3)$.

Note that $\mu(b) = \lambda(a, b)$ is an additive measure on the Boolean ring $\mathcal{S}(a)$ and for $a \supset a_1 \supset a_2$ we have $\lambda(a_1, a_2) = \mu(a_2)/\mu(a_1)$. On the Boolean ring $\mathcal{S}(a)$ one can therefore recover all the conditional probabilities $\lambda(a, b)$ from the probability function $\mu(b)$.

Such structure is very general and only the additional criteria by which the family \mathcal{S} is selected out from M allows us to recognize relationship with physical procedures. These procedures come from phenomenology, from known sectors of physics and technology. They can be implemented in laboratories since appropriate language and techniques have been developed and it is rather obvious that apparatuses used in experimental settings are related to the whole technical evolution by which materials were produced, which can be sorted and adequately transformed. General thermodynamical concepts such as local equilibrium are immediately of relevance, non-equilibrium being producible by putting different components together; basic physical indexes, such as temperature, were recognized and led to the feasibility of increasingly sophisticated selections. Families \mathcal{S} of subsets satisfying only the requirements **S 1**, which we have simply called selection procedures, acquire the fundamental properties **S 2** only if an adequate degree of selection has been attained, generally including some suitable isolation or shielding device. Then when a sufficiently selected subset $a \in \mathcal{S}$ has been obtained, further partitions of a into disjoint subsets show the statistical regularity expressed by **S 2**, so that \mathcal{S} is a statistical selection procedure and physics can start to explain the probability function $\lambda(a, b)$, $b \subset a$. Condition **S 1.1** means that once we are able to perform selection a and selection b , it is possible to build equipment which produces selections a and b together, i.e. we are considering only compatible selection procedures: this is the practical way to produce, starting with two selection procedures a and b , another one $a \cap b$ finer than a and b since $a \cap b \subset a$, $a \cap b \subset b$. Phenomenology used in setting experiments seems to satisfy this very simple and general criterion; this is often but misleadingly described as a *classical* character of macroscopic world. Taking the concepts of time and space as already established, associating selection procedures with spacetime regions one is led, by relativistic causality, to assume that selection procedures associated with two spacetime regions at spacelike separation from each other are compatible. It is a selection procedure to prepare a physical system during a certain initial time interval inside a finite space region; then finer selections can be done over a longer time interval and if these are statistical selection procedures a very general statistical description of dynamics is achieved, i.e. control of the system in the initial time interval is often enough to allow a statistical regularity during the time evolution of the system.

We now introduce a mathematical expression for the notion of experimental mixture. Considering a selection procedure \mathcal{S} , a partition of $a \in \mathcal{S}$ of the form $a = \cup_{i=1}^n b_i$, with $b_i \in \mathcal{S}$ and mutually disjoint is called a *decomposition* of a in the b_i , and a is called a *mixture* of b_i . Since the set $\mathcal{S}(a)$ is a Boolean ring a decomposition of a is simply a disjoint partition of the unit element a of $\mathcal{S}(a)$. With the above-defined additive measure $\mu(b)$ over $\mathcal{S}(a)$ we have $\sum_{i=1}^n \mu(b_i) = 1$, $\mu(b_i) = \lambda(a, b_i)$ being the weights of b_i in a . If we experimentally choose

N systems according to a , and of these N_i are further selected according to b_i , the relations $N_i/N \approx \mu(b_i)$ must be verified in physical approximation. This should however not induce the reader to confuse the notion of selection procedure with that of ensemble, which will be introduced later on.

3.2. Preparation and registration

Exploiting the above-defined notions of selection procedure and of statistical selection procedure we want to introduce on M (which is expected to become the set of microsystems) suitable mathematical structures, so as to interpret its elements as physical systems, in that they can be prepared and registered. Let a structure $\mathcal{Q} \subset P(M)$ be given on M , which we call *preparation procedure*, such that (**A** standing for axiom)

A 1 \mathcal{Q} is a statistical selection procedure.

The elements of \mathcal{Q} are representatives of well-defined technical processes, to be described by pretheories and not by quantum mechanics itself, due to which microsystems can be produced in large numbers. The mathematical relation $x \in a$ ($\in \mathcal{Q}$) means: x has been obtained according to the preparation procedure a . There are many examples of preparation procedures, e.g., an ion-accelerator together with the apparatus which generates the ion-beam. We denote by $\lambda_{\mathcal{Q}}(a, b)$ the probability function defined over \mathcal{Q} . We now consider a specific physical example, in order to make this construction clearer. We take an experimental apparatus which generates couples (1, 2) of spin-1/2 particles with total spin 0 and emits them in opposite directions. As a preparation procedure for system 1 we consider the apparatus consisting of the preparation apparatus for the couple (1, 2) and an apparatus detecting the z component of the spin of system 2. This apparatus gives us three different preparation procedures for system 1. Preparation procedure a_1^3 : all prepared systems 1 independent of the detection on system 2; preparation procedure a_1^{3+} : all systems 1, by which a positive z component has been detected for system 2; preparation procedure a_1^{3-} : all systems 1, by which a negative z component has been detected for system 2. We obviously have $a_1^{3+} \subset a_1^3, a_1^{3-} \subset a_1^3, a_1^{3+} \cap a_1^{3-} = \emptyset$ and $a_1^3 = a_1^{3-} \cup a_1^{3+}$ represents a decomposition of a_1^3 . In this particular case the weights are given by $\mu(a_1^{3\pm}) = \lambda(a_1^3, a_1^{3\pm}) = 1/2$.

We now want to introduce the notion of registration. Let there be on M two further structures, the set of *registration procedures* $\mathcal{R} \subset P(M)$ and the set of *registration methods* $\mathcal{R}_0 \subset P(M)$, satisfying

A 2 \mathcal{R} is a selection procedure.

A 3 \mathcal{R}_0 is a statistical selection procedure.

A 4.1 $\mathcal{R}_0 \subset \mathcal{R}$.

A 4.2 From $b \in \mathcal{R}$ and $\mathcal{R}_0 \ni b_0 \subset b$ follows $b \in \mathcal{R}_0$.

A 4.3 To each $b \in \mathcal{R}$ there exists a $b_0 \in \mathcal{R}_0$ for which $b \subset b_0$.

These two structures correspond to the two steps of a typical registration process: the construction and utilization of the registration apparatus and the selection according to the changes which have occurred or not occurred in the registration apparatus. Let us consider for example a proportional counter: $b_0 \in \mathcal{R}_0$ is the set of all microsystems which have been applied to the counter; the elements of \mathcal{R}_0 characterize therefore the construction of the registration apparatus and its application to microsystems. For a particular microsystem $x \in b_0$ the counter may or may not respond: let b_+ (with $b_+ \subset b_0$) be the selection procedure of all $x \in b_0$ for

which the counter has responded and b_- the set of all $x \in b_0$ for which the counter has not responded. b_+ and b_- are elements of \mathcal{R} . **A 3** accounts for the fact that the apparatus, apart from triggering by microsystems, is a macrosystem with statistically reproducible features. It is instead extremely important that we do not require \mathcal{R} to be a statistical selection procedure. To understand this point let us come back to the previous example. The counter characterized by b_0 may respond or not, so that b_0 is decomposed into the two sets b_+ and b_- , such that $b_0 = b_+ \cup b_-$ and $b_+ \cap b_- = \emptyset$. There is however in nature no reproducible frequency $\lambda_{\mathcal{R}}(b_0, b_+)$; in fact if in a real experiment N microsystems x_1, x_2, \dots, x_N are applied to the counter, i.e., $x_1 \in b_0, x_2 \in b_0, \dots, x_N \in b_0$, and for N_+ of these the counter has responded, the frequency N_+/N depends in an essential way on the previous history of the microsystems. It cannot be reproduced on the basis of the registration procedure alone.

Let us call \mathcal{S} the smallest set of selection procedures containing all $a \cap b$ with $a \in \mathcal{Q}$ and $b \in \mathcal{R}$ (remember that $a \cap b$ is the set of all microsystems that have been prepared according to a and registered according to b). We have $\mathcal{S} \subset P(M)$, but in the general case neither $\mathcal{Q} \subset \mathcal{S}$ nor $\mathcal{R} \subset \mathcal{S}$ will be true. We now come to a most important statement, according to which preparation and registration procedures together give reproducible frequencies

A 5 \mathcal{S} is a statistical selection procedure.

Of course there will be some relations between the statistics in \mathcal{S} and those in \mathcal{Q} and \mathcal{R}_0 . We now want to express the fact that preparation procedures and registration methods are independent of each other; denoting with $\lambda_{\mathcal{S}}(c, c')$ the probability function in \mathcal{S} we have

A 6.1 If $a, a' \in \mathcal{Q}$, $a' \subset a$ and $b_0 \in \mathcal{R}_0$, then $\lambda_{\mathcal{S}}(a \cap b_0, a' \cap b_0) = \lambda_{\mathcal{Q}}(a, a')$.

A 6.2 If $a \in \mathcal{Q}$ and $b_0, b'_0 \in \mathcal{R}_0$, $b'_0 \subset b_0$, then $\lambda_{\mathcal{S}}(a \cap b_0, a \cap b'_0) = \lambda_{\mathcal{R}_0}(b_0, b'_0)$.

In contrast, in general, $\lambda_{\mathcal{S}}(a \cap b, a' \cap b) \neq \lambda_{\mathcal{Q}}(a, a')$, where $\lambda_{\mathcal{Q}}(a, a')$ is the frequency with which microsystems prepared according to a satisfy the finer selection a' . **A 6.1** and **A 6.2** mean that, except for the microsystem, the preparation and registration apparatuses do not interact. Thus **A 6.1** and **A 6.2** express the directedness of the interaction of the preparation on the registration apparatus.

A set M with three structures $\mathcal{Q} \subset P(M)$, $\mathcal{R} \subset P(M)$, $\mathcal{R}_0 \subset P(M)$ satisfying **A 1** to **A 6** is a set of physical systems selected by a measuring process. As stressed at the beginning of this section the structures we have used to introduce the notion of physical system are not restricted to the case of microsystems; they can describe measurements on macroscopic systems as well. Thanks to axioms **A 1** to **A 6**, implying the independence of the preparation procedure with respect to the registration procedure, the facts that we have called physical systems have some reality beyond that of the direct interpretation in terms of preparation and registration procedures. Intuitively this means that in the preparation *something* is produced which can be detected afterwards by the registration apparatus. Nevertheless the physical systems that we have introduced are still closely related to the associated production and detection methods; it does not seem that they can be described in terms of the objective properties that we are accustomed to ascribe to physical systems. Speaking of self-existing objects which do not suffer or exert any influence on the rest of the world would be physically meaningless and, from a logical point of view, self-contradictory. Nevertheless in physics one seeks to describe portions of the world as if they were isolated, in the sense that on a given description level their interactions with the rest of the world may be neglected. To the extent that this is possible one may attribute objective properties to the considered system. The introduced scheme is so far very general, being applicable both to macrosystems and microsystems: the selection procedures in \mathcal{S} describe a conventional *classical statistics*, not exhibiting the *typical* quantum

mechanical structure. The transition to quantum statistics will be made only later with axiom **QM**, thus coming to the notion of microsystem.

3.3. Equivalence classes

From **S 2** and **A 6** one can prove that the probability function $\lambda_S(c, c')$ is uniquely determined by λ_Q and by the special values

$$\lambda_S(a \cap b_0, a \cap b), \quad (1)$$

with $a \in Q, b \in \mathcal{R}, b_0 \in \mathcal{R}_0$ and $b \subset b_0$. $\lambda_S(a \cap b_0, a \cap b)$ gives the frequencies with which microsystems prepared by a and applied to the apparatus characterized by b_0 trigger it according to b . The values (1) are just the values the experimental physicist obtains to compare with the theory: N systems are prepared according to the preparation procedure a and applied to the registration method specified by b_0 ; then one counts the number N_+ of microsystems which trigger the registration apparatus in a definite way, specified by b . Within physical approximations the number N_+/N should agree with (1): the whole statistics of experiments with microsystems is contained in (1).

To proceed further let us introduce the set \mathcal{F} of *effect processes*: $\mathcal{F} \equiv \{(b_0, b) | b_0 \in \mathcal{R}_0, b_0 \neq \emptyset, b \in \mathcal{R}, b \subset b_0\}$. A couple (b_0, b) in \mathcal{F} exactly describes the experimental situation corresponding to the generation of an effect. We may now write in a simpler way the function (1): denoting by $g = (b_0, b)$ a couple in \mathcal{F} we define $\lambda_S(a \cap b_0, a \cap b) = \mu(a, g)$, where the function $\mu(a, g)$ is defined on the whole $Q \times \mathcal{F}$. According to $\mu(a_1, g) = \mu(a_2, g)$ for all $g \in \mathcal{F}$ an equivalence relation $a_1 \sim a_2$ is defined on Q , which allows us to partition it into equivalence classes. We call \mathcal{K} the set of all equivalence classes in Q : an element of \mathcal{K} is called *ensemble* (or *state*) and \mathcal{K} is the set of ensembles. Let us stress the fact that an ensemble $w \in \mathcal{K}$ is not a subset of M , that is to say, an ensemble w is not a set of prepared microsystems: it is a class of sets a of prepared microsystems. The difference between ensembles and preparation procedures is very important. Analogously to what has been done in Q , one can introduce an equivalence relation in \mathcal{F} : $g_1 \sim g_2$ whenever $\mu(a, g_1) = \mu(a, g_2)$ for all $a \in Q$. We denote by \mathcal{L} the set of all equivalence classes in \mathcal{F} : an element $f \in \mathcal{L}$ is called *effect* and \mathcal{L} is the set of all effects. Once again one should not confuse effects and effect processes. Through $\tilde{\mu}(w, f) = \mu(a, g)$ for $w \in \mathcal{K}, f \in \mathcal{L}$ and $a \in w, g \in f$ a function $\tilde{\mu}(w, f)$ is defined on $\mathcal{K} \times \mathcal{L}$ (in the following we will simply write μ instead of $\tilde{\mu}$). For the real function $\mu(w, f)$ on $\mathcal{K} \times \mathcal{L}$ we have

- (1) $0 \leq \mu(w, f) \leq 1$,
- (2) $\mu(w_1, f) = \mu(w_2, f) \forall f \in \mathcal{L} \Rightarrow w_1 = w_2$,
- (3) $\mu(w, f_1) = \mu(w, f_2) \forall w \in \mathcal{K} \Rightarrow f_1 = f_2$,
- (4) $\exists! f_0 \in \mathcal{L}$ (also denoted by 0) such that $\mu(w, f_0) = 0 \forall w \in \mathcal{K}$,
- (5) $\exists! f_1 \in \mathcal{L}$ (also denoted by 1) such that $\mu(w, f_1) = 1 \forall w \in \mathcal{K}$.

Mixtures on Q are transferred on \mathcal{K} , as one can show taking into account $\lambda_S(a \cap b_0, a' \cap b_0) = \lambda_Q(a, a')$: let $a = \cup_{i=1}^n a_i, a_i \in Q, a_i \not\subset \emptyset, a_i \cap a_j = \emptyset (i \neq j)$ then if $a \in w, a_i \in w_i$ one has

$$w = \sum_{i=1}^n \lambda_Q(a, a_i) w_i.$$

By this fundamental statistical property a preparation procedure $a \in Q$ of a microsystem becomes very close to an element $w \in \mathcal{K}$. It is however very important to be aware of the fact that the passage from Q to \mathcal{K} is a step by which new mathematical entities are introduced which have a basic role in describing the physics of a microsystem under *all possible preparation and*

detection procedures: then w does not simply represent one concrete preparation procedure of a microsystem. By the introduction of equivalence classes some universality character of $\mu(w, f)$ has been introduced: in the case of a microsystem these equivalence classes contain a huge number of elements. Actually something of the particular experimental situation described by $a \in \mathcal{Q}$ gets lost when the equivalence class w to which a belongs is considered and the inverse passage from w to a cannot be done if one only relies on quantum theory of microsystems. Some paradox in quantum mechanics, e.g. EPR paradox, have their roots just in neglecting the fact that, typically in an equivalence class $w \in \mathcal{K}$ two preparation procedures a and $a' \in \mathcal{Q}$ can be contained which are *incompatible*: $a \cap a' = \emptyset$, i.e. the two concrete selection procedures cannot be performed together. In Ludwig's point of view the debated question of completeness of quantum mechanics is not an issue from the very beginning.

The introduced partitions into equivalence classes of the sets \mathcal{Q} and \mathcal{F} are most important. These partitions do not simply amount to make the theory of the considered physical systems independent of inessential features in the construction of the apparatuses $a \in \mathcal{Q}$ and $b_0 \in \mathcal{R}_0$. They have a much deeper significance with regard to the physical theory. For example the partition of \mathcal{Q} depends in an essential way on which and how many effect processes are physically realizable. Restricting the set \mathcal{F} to a subset $\tilde{\mathcal{F}}$ could imply a coarser partition of \mathcal{Q} . Axioms about the extension of the sets \mathcal{Q} and \mathcal{F} amount to specify the theory one is dealing with, thus indirectly identifying the described physical systems and the possible realizable experiments.

3.4. Quantum mechanics

So far we have introduced the quantities that connect theory and experiment, that is to say the elements of \mathcal{Q} , \mathcal{R} , \mathcal{R}_0 and the functions $\lambda_{\mathcal{Q}}$, $\lambda_{\mathcal{R}_0}$, $\lambda_{\mathcal{S}}$. Note that contrary to the usual formulations of quantum mechanics, neither the statistical operators (or in particular the pure states) nor the self-adjoint operators (describing the so-called observables) will be used for direct comparison with experiment: the relationship between mathematical description and experiment exclusively rests upon the preparation and the registration procedures and the probability function $\lambda_{\mathcal{S}}$. We now add an axiom connecting this general theoretical scheme to the usual Hilbert space quantum mechanics (**QM** standing for quantum mechanics).

QM *There is a bijective map \mathcal{W} of \mathcal{K} onto the set $\mathcal{K}(\mathcal{H})$ of positive self-adjoint operators W on a Hilbert space \mathcal{H} with $\text{Tr}(W) = 1$ and a bijective map \mathcal{F} of \mathcal{L} onto the set $\mathcal{L}(\mathcal{H}) \subset \mathcal{B}(\mathcal{H})$ of all self-adjoint operators with $0 \leq F \leq \mathbb{1}$, so that $\mu(w, f) = \text{Tr}(WF)$ holds where $W = \mathcal{W}[w]$, $F = \mathcal{F}[f]$.*

Because of **QM** one simply identifies \mathcal{K} with $\mathcal{K}(\mathcal{H})$, \mathcal{L} with $\mathcal{L}(\mathcal{H})$ and $\mu(w, f)$ with $\text{Tr}(WF)$. The convex set $\mathcal{K}(\mathcal{H})$ is the base of the base-norm space $\mathcal{T}(\mathcal{H})$ of trace-class operators on \mathcal{H} , while $\mathcal{L}(\mathcal{H})$ is the order unit interval of the order unit space $\mathcal{B}(\mathcal{H})$ of bounded operators on \mathcal{H} . The Banach space $\mathcal{B}(\mathcal{H})$ is the dual of the Banach space $\mathcal{T}(\mathcal{H})$, the canonical bilinear form being given by $\langle W, A \rangle = \text{Tr}(W^{\dagger}A)$ with $W \in \mathcal{T}(\mathcal{H})$ and $A \in \mathcal{B}(\mathcal{H})$. The axiom **QM** is prepared by introducing the functions $\mu(w, f)$ and their affine dependence on w . It can be guessed when quantum mechanics is introduced in the usual textbook way and the basic statistical interpretation is given. A deeper axiomatic effort has been done by Ludwig [7, 8] in order to obtain the Hilbert space structure based on physically more transparent axioms.

After the introduction of **QM** we call M the set of *microsystems*. So far we have considered only one type of microsystems, a more general situation will be considered later on. It seems very stimulating that the simple physical fact of essentially statistical regularity of processes

leads in Ludwig's point of view to a concept of microsystem which goes much beyond the classical concept of atom brought in by chemistry: it is no longer so strictly associated with *smallness* and with the role of *component* of matter. We shall recall in the next section how naturally mathematics of quantum theory is born if one describes this concept of microsystem. On the basis of the above formulation of the foundations of quantum mechanics it is clear that the Hilbert space does not directly describe a physical structure. It is a mathematical tool which permits us to cleverly handle the structure of the convex set $\mathcal{K}(\mathcal{H})$. Since the positive affine functionals on $\mathcal{K}(\mathcal{H})$ are identical to the elements of the positive cone of $\mathcal{B}(\mathcal{H})$ (of which $\mathcal{L}(\mathcal{H})$ is the basis), it is the structure of $\mathcal{K}(\mathcal{H})$ alone which determines the physical structure of microsystems.

4. The modern formulation of quantum mechanics

In the introduction we have tried to give a brief exposition of the main ideas behind Ludwig's axiomatic approach to quantum mechanics. One of his aims was to put aside the ill-defined notions of state and observable, primarily focusing on a proper description of the statistical experiments one is actually faced with in quantum mechanics. In a typical experiment a macroscopic apparatus realizing a classically described preparation procedure triggers some detector which gives as output a macroscopic signal, according to a suitably devised registration procedure. The notion of microsystem is only recovered as a convenient way to describe the most simple among such statistical experiments, in which a preparation procedure triggers with a definite reproducible frequency some registration procedure, the microsystem acting as correlation carrier from the former to the latter. The mathematical entity describing an equivalence class of preparation apparatuses is then identified with the state of the microsystem, while the mathematical entity corresponding to an equivalence class of registration apparatuses, originally called *Effekt* by Ludwig, contains the information about what has been experimentally measured [9]. The spaces in which these objects live are the latter the dual of the former, the relative frequency with which the preparation triggers the registration is obtained by using the canonical bilinear form among the two spaces. This frequency characterizes the yes–no answer of the registration or measuring apparatus when affected by the preparation apparatus. As a result of Ludwig's analysis in the quantum case states, to be seen as mathematical representatives of equivalence classes of actual preparation procedures, are given by statistical operators, while observables, to be seen as mathematical representatives of equivalence classes of actual registration or measuring procedures, are given by effects. Taking into account the fact that registration apparatuses associated with effects are naturally endowed with a scale (e.g. an interval on the real line for a positive measurement) the notion of effect immediately leads to the concept of observable as positive operator-valued measure [10]. Note that the consideration of equivalence classes is actually a key point. Utterly different and incompatible (in the sense that they cannot be performed together) preparation procedures might lead to one and the same state, i.e. statistical selection procedure. The different preparation procedures in the same equivalence class are related to the, generally infinite, possible decompositions of a given statistical operator, corresponding to generally incompatible macroscopic procedures, as stressed by the EPR paradox. In the present section we will give a very brief presentation of the more general and more flexible formulation of quantum mechanics, which naturally comes out of Ludwig's approach. This modern formulation of quantum mechanics, giving the most general description of statistical experiments and transformation of states, is obviously the result of research work by many authors, often starting from quite different standpoints. Among the many possible references on the subject we recall the work by Ludwig [7] and by Holevo [11, 12], referring to these

books for a more extensive bibliography. Let us mention that the modern formulation of quantum mechanics can also be recovered within the Bohmian approach [4].

4.1. Description of quantum measurements

A state in quantum mechanics, to be understood as the mathematical representative of an equivalence class of preparation procedures, is given by a statistical operator, i.e. a trace class operator, positive and with trace equal to 1. We recall that the set $\mathcal{T}(\mathcal{H})$ of trace class operators on a Hilbert space \mathcal{H} forms a Banach space and is in particular an ideal of the Banach space of bounded operators $\mathcal{B}(\mathcal{H})$, which is the dual space of $\mathcal{T}(\mathcal{H})$, the duality form being given by the trace. In particular the set of statistical operators $\mathcal{K}(\mathcal{H})$,

$$\mathcal{K}(\mathcal{H}) = \{\rho \in \mathcal{T}(\mathcal{H}) | \rho \geq 0 \text{ Tr } \rho = 1\}$$

is a convex subset of the space of self-adjoint elements in $\mathcal{T}(\mathcal{H})$ and is the base of the cone of positive elements which generates the space of self-adjoint elements in $\mathcal{T}(\mathcal{H})$. The convex structure of the set naturally accounts for the possibility of considering statistical mixtures, i.e.

$$\rho_i \in \mathcal{K}(\mathcal{H}), \quad \lambda_i \geq 0 \quad \sum_i \lambda_i = 1 \Rightarrow \sum_i \lambda_i \rho_i \in \mathcal{K}(\mathcal{H}),$$

while pure states in the sense of one-dimensional projections appear as extreme points of the convex set $\mathcal{K}(\mathcal{H})$, i.e. elements which do not admit any further demixture

$$\rho = \lambda \rho_1 + (1 - \lambda) \rho_2 \quad 0 < \lambda < 1 \quad \rho_1, \rho_2 \in \mathcal{K}(\mathcal{H}) \Rightarrow \rho = \rho_1 = \rho_2,$$

corresponding to the highest control in the preparation procedure. Being compact and self-adjoint any statistical operator can be represented as a convex combination of pure states

$$\rho = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i| \quad \lambda_i \geq 0 \quad \sum_i \lambda_i = 1 \quad \|\psi_i\| = 1.$$

One such representation is given by the spectral representation of ρ ; however in general, infinitely many such representations are possible, not necessarily involving orthogonal vectors; these different representations do generally correspond to different and incompatible preparation procedures, in the sense that they cannot be performed together (think e.g. of a device preparing spin-1/2 particles in terms of their spin states, fully unpolarized states can be obtained by observing the spin along any axis, no device however can simultaneously measure the spin along two different axes). An important confirmation that statistical operators give the most general mathematical representative of a preparation comes from the following highly nontrivial theorem by Gleason [13]. Let us consider the set $\mathcal{P}(\mathcal{H})$ of orthogonal projections in \mathcal{H} , in one-to-one correspondence with the closed subspaces of \mathcal{H} , building up the so-called quantum logic of events [14]. We first define a probability measure on $\mathcal{P}(\mathcal{H})$ as a real function $\mu : \mathcal{P}(\mathcal{H}) \rightarrow \mathbb{R}$ such that $0 \leq \mu(E) \leq 1 \forall E \in \mathcal{P}(\mathcal{H})$ and $\mu(\sum_i E_i) = \sum_i \mu(E_i)$ for $\{E_i\} \subset \mathcal{P}(\mathcal{H})$, $E_i E_j = 0, i \neq j$ (i.e. $\{E_i\}$ are compatible projections corresponding to orthogonal subspaces). Then according to Gleason for $\dim \mathcal{H} \geq 3$ any such probability measure has the form $\mu(E) = \text{Tr } \rho E \forall E \in \mathcal{P}(\mathcal{H})$, with ρ a statistical operator.

4.2. Generalized notion of observable

In order to describe the statistics of a given experiment, once the state has been characterized one needs to specify the probability that the registered value of the quantity one is trying to measure lies in a given interval within the physically allowed range (in the following \mathbb{R} for the sake of simplicity). This amounts to define an affine mapping (i.e. preserving convex

linear combinations) from the convex set $\mathcal{K}(\mathcal{H})$ of statistical operators to the set of probability measures on $\mathcal{B}(\mathbb{R})$ (the Borel σ -algebra on the space of outcomes \mathbb{R}). In full generality such mappings take the form $\text{Tr } \rho F(M)$, where ρ is the statistical operator, and $F(M)$ is a uniquely defined positive operator-valued measure [12]. (M being an element in the Borel σ -algebra $\mathcal{B}(\mathbb{R})$.) As is well known a positive operator-valued measure is a mapping defined on the σ -algebra $\mathcal{B}(\mathbb{R})$ and taking values in the space of positive bounded operators such that $0 \leq F(M) \leq \mathbb{1}$, $F(\mathbb{R}) = \mathbb{1}$ so that one has the normalization necessary for the probabilistic interpretation, and σ -additivity holds, in the sense that $F(\cup_i M_i) = \sum_i F(M_i)$ for any disjoint partition $\{M_i\}$ of \mathbb{R} . For fixed $M \in \mathcal{B}(\mathbb{R})$ the operator $F(M)$ is an effect, i.e. a positive operator between 0 and $\mathbb{1}$, and $\text{Tr } \rho F(M)$ tells us the probability that in an experiment, whose preparation procedure is described by ρ , we will actually find that our registration procedure gives a positive answer to the question whether the measured quantity lies in the fixed interval M . The above-introduced structures in which registrations are naturally associated with points or intervals in \mathbb{R} are straightforward in Ludwig's construction of effects. Note that the statistical nature of the experiment only requires $\text{Tr } \rho F(M)$ to be a number between zero and one. There is no reason to ask $F(M)$ to be a projection-valued measure, and therefore that for any fixed $M \in \mathcal{B}(\mathbb{R})$ the operator $F(M)$ is an orthogonal projection (also called *decision effect* by Ludwig), this will only happen for a subset of the possible registration procedures, corresponding to most *sensitive* measurements, which moreover do not generally exhaust the set of extreme points of the convex set of positive operator-valued measures. If $F(M)$ is a projection-valued measure, it then uniquely corresponds to the spectral measure of a self-adjoint operator in \mathcal{H} . The usual notion of observable in the sense of a self-adjoint operator is thus recovered for particular observables. What one is really interested in is the probability distribution of the different possible outcomes of an experiment, once the state has been fixed, and only in particular, though often very relevant, cases this can be done by identifying a self-adjoint operator and its associated spectral measure. Note that contrary to classical mechanics different generalized observables have different probability distributions and not all observables can be described in terms of a joint probability distribution, leading to a structure known as quantum probability, generalizing the classical notion of probability theory [15]. In fact it has been argued that the passage from classical to quantum theory is actually a generalization of probability theory [16]. Note that inside Ludwig's point of view coexistence of observables is related to the actual possibility of constructing concrete measuring apparatuses. This (most concise) presentation of how to express the quantum mechanical theoretical predictions for a statistical experiment, leading to the notion of state as a statistical operator and generalized observable as a positive operator-valued measure (also called non-orthogonal resolution of identity in the mathematical literature), is certainly not in the spirit of textbooks on quantum mechanics (even not very recent ones); it is much closer to the presentation of quantum mechanics one finds in the introductory chapters of books concerned with quantum information and communication, e.g. [17]. Note however that in quantum information and communication one is often only concerned with finite-dimensional Hilbert spaces, so that the range of the positive operator-valued measure is given by a denumerable set of operators $0 \leq F_i \leq \mathbb{1}$ summing up to the identity, $\sum_i F_i = \mathbb{1}$.

4.3. Measurements as mappings on states

Up to now we have only given the general description of the statistics of the outcomes of a possible measurement. More generally one might be interested in how a state is transformed as a consequence of a given measurement. Note that the shift from pure states, corresponding

to state vectors, to statistical operators from a mathematical standpoint shifts the attention from operators in \mathcal{H} to affine mappings on the convex set $\mathcal{K}(\mathcal{H})$. The way in which a state is changed as a consequence of some registration procedure applied to it is generally described in terms of an instrument, a notion first introduced by Davies and Lewis [18]. An instrument is a mapping \mathcal{M} defined on the σ -algebra $\mathcal{B}(\mathbb{R})$ giving the possible outcomes of an experiment and taking values in the set of operations, i.e. of contracting, positivity preserving affine mappings on $\mathcal{K}(\mathcal{H})$, first introduced by Haag and Kastler [19] and called *Umpraeparierung* by Ludwig in his axiomatic construction. In particular an instrument \mathcal{M} is such that: $\mathcal{M}(M)$ is an operation $\forall M \in \mathcal{B}(\mathbb{R})$, i.e. $\mathcal{M}(M)[\rho] \geq 0$ and $\text{Tr } \mathcal{M}(M)[\rho] \leq 1$; $\text{Tr } \mathcal{M}(\mathbb{R})[\rho] = 1$, accounting for normalization; $\mathcal{M}(\cdot)$ is σ -additive, i.e. $\mathcal{M}(\cup_i M_i) = \sum_i \mathcal{M}(M_i)$ for any collection of pairwise disjoint sets $\{M_i\}$. The interpretation is as follows, $\mathcal{M}(M)[\rho]$ gives the statistical subcollection obtained by selecting the prepared state described by ρ according to the fact that the measurement outcome lies in M , $\mathcal{M}(\mathbb{R})[\rho]$ is the transformed state obtained if no selection is made according to the measurement outcome. Of course knowledge of the instrument corresponding to a certain state transformation related to a given measurement also provides the full statistics of the outcomes, obtained by the positive operator-valued measure given by $\mathcal{M}'(M)[\mathbb{1}]$, where the prime denotes the adjoint with respect to the trace operation. However different instruments actually lead to the same positive operator-valued measure, according to the fact that the very same quantity can be actually measured in different ways, leading to states which transformed differently, depending on the actual experimental apparatus used in order to implement the measurement. Knowledge of the transformed state allows us to deal with subsequent measurements, both discrete and continuous, and in fact the notion of instrument leads to a formulation of continual measurement in quantum mechanics. The field of continual measurement is by now well established, providing the necessary theoretical background for important experiments in quantum optics (see [20] for a recent review mainly in the spirit of quantum stochastic differential equations and [21, 22] for earlier work). Once again this description of a measurement as a reparation of the incoming state depending on the measurement outcome is certainly not emphasized in quantum mechanics textbooks, but is a natural and fruitful standpoint in quantum information and communication theory. Actually the very notion of microsystem as something which is prepared by a macroscopic apparatus and subsequently registered in a registration apparatus, i.e. as correlation carrier between macroscopically operated apparatuses, naturally emerging from Ludwig's axiomatic studies, is a very pregnant and fertile viewpoint in quantum mechanics and in particular quantum information and communication, as advocated by Werner [23]; not by chance key concepts like preparation and registration are naturally renamed as sender and receiver.

4.4. Open systems and irreversibility

As a last remark we note that the operational approach we have most briefly and incompletely sketched, stressing the relevance of mappings acting on states living in the space of trace class operators, corresponding to transformation of states (Schrödinger picture), together with the adjoint mappings acting in the space of bounded operators (Heisenberg picture), does not only apply to the description of a measurement process. These mappings also generally describe the spontaneous reparations of a system with elapsing time, i.e. its dynamics. If the system is closed, so that one has reversibility, then it can be shown that the time evolution mapping necessarily has the form $\mathcal{L}[\rho] = U(t)\rho U^\dagger(t)$, with $U(t)$ a unitary mapping, and no measuring decomposition applies, consisting in sorting statistical subcollections on the basis of a certain measurement outcome. In the general case of an open system however irreversibility comes

in, either due to the interaction with some environment or to the effect of some measuring apparatus, so that more general mappings appear in order to describe this wider class of transformations of quantum states and observables. This is an open and very active field of research, of interest both to mathematicians and physicists [12, 24], where the relevance of concepts and techniques inherited and generalized or inspired by the classical theory of probability and stochastic processes cannot be overstressed. A general characterization of such mappings has been obtained only in a few cases, exploiting their property of being completely positive. For example in the description of irreversible and Markovian dynamics a landmark result has been obtained by Gorini, Kossakowski, Sudarshan and Lindblad [25], leading to the so-called Lindblad structure of a master-equation, very useful in applications [26, 24]. Important hints and restrictions on the structure of such mappings come from the requirement of covariance under the action of some symmetry group relevant for the system at hand [12]. Biased by our interests and work let us quote recent results in this framework, dealing with quantum Brownian motion [27] and decoherence due to momentum transfer events [28], where the relevance of covariance and probabilistic concepts appear at work.

5. From microsystems to macrosystems

In Ludwig's point of view spacetime symmetries arise as follows: let us consider an experiment with preparation part \mathcal{Q} and registration part $\mathcal{R}_0\mathcal{R}$, then placing a reference frame on \mathcal{Q} one gets a family of symmetry transformed registration parts $g\mathcal{R}_0g\mathcal{R}$ for any reference frame transformation g belonging to the relevant symmetry group and a new experiment $\mathcal{Q}, g\mathcal{R}_0g\mathcal{R}$ can be considered. By an appropriate treatment [7] one recovers the typical results of usual symmetry theory based on Wigner's theorem, where the Hilbert space \mathcal{H} associated with a single microsystem carries a unitary projective representation of the Galilei group: if the microsystem is elementary such a representation is irreducible [29].

So far only a single microsystem has been treated, however one has evidence of different elementary microsystems and of a huge set of non elementary ones. The general description of different types of microsystems, labelled as $1, 2, \dots, n$ requires an n -uple of Hilbert spaces $\mathcal{H}_1, \dots, \mathcal{H}_n$, an element of \mathcal{K} being an n -uple of positive trace class operators W_1, \dots, W_n , normalized according to $\sum_{i=1}^n \text{Tr} W_i = 1$, an element of \mathcal{L} being an n -uple F_1, \dots, F_n of operators on $\mathcal{H}_1, \dots, \mathcal{H}_n$ such that $0 \leq F_i \leq \mathbb{1}_i, i = 1, \dots, n$, then finally $\mu(w, f) = \sum_{i=1}^n \text{Tr}(W_i F_i)$. The projection $F_i = (0, \dots, 0, \mathbb{1}_i, 0, \dots, 0)$ with probability $\mu(w, f_i) = \text{Tr}(W_i)$ corresponds to the registration of the microsystem of type i . It turns out that non elementary microsystems are described in Hilbert spaces

$$\mathcal{H}_i = h_{\alpha_1}^{(e)} \otimes \dots \otimes h_{\alpha_{\kappa_i}}^{(e)}, \quad (2)$$

where $h_{\alpha_j}^{(e)}$ is the Hilbert space of an elementary microsystem (the superscript (e) standing for elementary) and κ_i is the number of elementary components: the basic simplification is the restricted number of the latter ones. A large variety of non elementary microsystems is understood having as elementary microsystem electron and nuclei in the context of electromagnetism; by deeper understanding it was discovered that nuclei are not elementary microsystems and a smaller number of more fundamental microsystems is introduced in present day subnuclear physics. In the tensor product (2) many factors are repeated: of these repeated factors only the completely symmetric or antisymmetric part must be taken. This is a very important correction on the simple structure (2), that we indicate simply by \mathcal{H}_i^σ , the superscript σ standing for the aforementioned symmetrizations.

In the non-relativistic case symmetry transformations for non elementary microsystems can be obtained from those associated with the elementary components. Here the self-adjoint

generators of the one-parameter subgroups acquire an outstanding importance and the projection-valued measures associated with them provide observables with a straightforward physical interpretation like position, momentum, angular momentum and energy. Apart from the energy they have an additive structure and their expectation values take a simple form:

$$\langle A \rangle = \sum_i \text{Tr}_{\mathcal{H}_i^\sigma} (A_i W_i) \quad A_i = \sum_{l=1}^{\kappa_i} A_l^{(e)}. \quad (3)$$

In the case of the energy a non additive contribution generally arises called *interaction energy*, responsible for binding elementary microsystems to composed microsystems.

When a classical limit holds the picture of elementary microsystems appears as elementary classical particles, and that of composed microsystems as structures of these interacting particles, showing quantities built up collectively by elementary contributions: then a composed system with many components is a macrosystem. A phase space Γ emerges, a state of the macrosystem being a point P in this space, a selection procedure can be represented by suitable subsets of Γ and it becomes a statistical selection procedure when a probability density $\rho(P)$ is given on Γ . Of course Γ is a huge space, to give $\rho(P)$ and calculate the functions $\lambda(a, b)$ can be very difficult. Actually in this way experimental settings are invented, realized and finally work, also a feeling is established which helps in correct guessing of W and F associated with $a \in \mathcal{Q}$ and $b_0 b \in \mathcal{R}_0 \mathcal{R}$. However all this is an approximation which e.g. cannot really grasp the typical quantum feature of \mathcal{H}_i^σ replacing \mathcal{H}_i : it is the absence of Γ that makes it difficult to represent statistical selection procedures. Then a problem appears in a consistent way close to Ludwig's point of view inside present day quantum theory. Ludwig aims to a more comprehensive theory, which should provide in a natural way a state space for a macroscopic system. We shall now conclude this discussion indicating briefly a way we have taken to face this problem [30]. First of all let us stress a peculiar role that quantum field theory can have with respect to macrosystems. A macrosystem is the physical support of all possible types of microsystems; we shall not depend on the naive atomistic point of view that it is composed of them, instead it is the carrier of all of them. If we consider the microsystems prepared when a macrosystem evolves until a time t , the $W_t \in \mathcal{K}$ shows by the structure $W_{1t}, W_{2t}, \dots, W_{nt}$ which types of microsystems have been prepared; this typology varies with time t : the number of microsystems N_{it} becomes an interesting quantity. The question immediately arises of an underlying Hilbert space such that \mathcal{H}_i^σ are isomorphic to subspaces of it and possibly the connection $\mathcal{H}_i \rightarrow \mathcal{H}_i^\sigma$ becomes natural; then an observable arises to be interpreted as number of microsystems of type α . It is well known how quantum field theory solves in a brilliant way this question: for each elementary microsystem a Fock space $\mathcal{H}_{F\alpha}$ is defined and the Hilbert space is given by

$$\mathfrak{H} = \prod_{\alpha} \otimes \mathcal{H}_{F\alpha} \quad (4)$$

where the factors are the Fock spaces associated with each type of elementary microsystem. In this setting (3) is replaced by $\langle A \rangle = \text{Tr}(AW)$, W being a statistical operator on \mathfrak{H} and A a self-adjoint operator in \mathfrak{H} .

5.1. Quantum field theory and macrosystems

While the operators on the Hilbert spaces \mathcal{H}_i are constructed in terms of fundamental operators x and p having the meaning of position and momentum with a clear classical limit, so that quantum theory appears close to classical atomistic physics via a *quantization* procedure, in this new setting related to \mathfrak{H} given by (4) fundamental operators by which

W and A are constructed appear that just connect the subspaces characterized by a fixed number of elementary microsystems, acting as creation and annihilation operators of the elementary microsystems. Therefore the Hilbert space \mathfrak{H} and the related set of statistical operators appear as natural candidates for a quantum theory of a macrosystem and one can expect that just focusing quantum field theory to macrosystems one can both improve the characterization of physically meaningful statistical operators in \mathfrak{H} and account for the objectivity elements which should characterize macrosystems. One is immediately confirmed in this idea by the fact that just this new framework provides fields observables $A(\mathbf{x})$ as densities of generators of symmetry transformations, which obey to typical balance equations, so that $\text{Tr } A(\mathbf{x})W = \langle A(\mathbf{x}) \rangle$ can be interpreted as expectation of the physical quantities that one needs in the phenomenological description of macroscopic systems. Furthermore there is the well-known example of a macrosystem at equilibrium. It is described by the statistical operator:

$$W \equiv \frac{e^{-\beta(H_\Omega - \mu N)}}{\text{Tr } e^{-\beta(H_\Omega - \mu N)}}, \quad (5)$$

which is built in terms of the relevant observables energy H_Ω and the new observable N typical for the passage $h_\alpha^{(e)} \rightarrow \mathcal{H}_{F_\alpha}$ in the simplest case of only one type of elementary microsystem. H_Ω is the operator in \mathfrak{H} constructed in terms of an energy density $H(\mathbf{x})$: $H_\Omega = \int_\Omega d^3\mathbf{x} H(\mathbf{x})$, where $H(\mathbf{x})$ is obtained in terms of the fundamental field operator $\psi(\mathbf{x})$ as it is established by quantum field technique, taking also in account boundary conditions on Ω . The input of all this is the Hamilton operator which comes from time translations of a microsystem; actually by this resetting one ends up with a self-adjoint operator H_Ω having a point spectrum, so that the trace class operators W can be constructed, when the parameters β and μ are in appropriate ranges. These parameters label the different equilibrium macrosystems and have a precise phenomenological meaning as temperature and chemical potential, entering in a primary way in any macroscopic selection procedure.

5.2. The role of non-equilibrium states

The statistical operator given in (5) is an element of $\mathcal{K}(\mathcal{H})$, constructed as a function of the observables H_Ω and N , so that the total mass is related to a superselection rule. The impact from thermodynamics at equilibrium is so fruitful that one wonders whether one can generalize it outside the very particular and in a sense too strongly idealized situation described as *equilibrium*, still satisfying the superselection rule for the total mass. Once a suitable set of *relevant* linearly independent field observables $A_j(\mathbf{x})$ is given in \mathfrak{H} one considers a set of classical fields $\zeta_j(\mathbf{x})$ such that the operator $\Phi(\zeta) \equiv \sum_j \int_\Omega d^3\mathbf{x} \zeta_j(\mathbf{x}) A_j(\mathbf{x})$ is essentially self-adjoint and $e^{-\Phi(\zeta)}$ is trace class so that a statistical operator, that we call macroscopic reference state, can be defined:

$$W_\zeta = \frac{e^{-\Phi(\zeta)}}{\text{Tr } e^{-\Phi(\zeta)}}. \quad (6)$$

The classical fields represent a *local generalization* of the previous equilibrium parameters β , μ : the field operators $A_j(\mathbf{x})$ have a quasi-local character in the sense that they depend on $\psi(\mathbf{x})$, $\psi^\dagger(\mathbf{y})$ for $|\mathbf{x} - \mathbf{y}| \ll \delta$ with δ much smaller than the typical variation scale of the state parameters $\zeta_j(\mathbf{x})$. Such a quasi-local character emerges if one considers the fundamental mechanical densities that we recall in the non-relativistic case: mass density, momentum density, kinetic energy density, where higher derivatives inside the different expressions loosely mean less locality. The field $\psi(\mathbf{x}) = \int d^3\mathbf{x}_1 \dots d^3\mathbf{x}_k g(\mathbf{x}, \mathbf{x}_1, \dots, \mathbf{x}_k) \psi_1(\mathbf{x}_1) \dots \psi_k(\mathbf{x}_k)$ refers to a field composed of elementary ones, $(\mathbf{x}, \mathbf{x}_1, \dots, \mathbf{x}_k)$ being a suitable structure function

of microsystems, concentrated for $|\mathbf{x} - \mathbf{x}_i| \ll \delta, i = 1, 2, \dots, k$. The reference state (6), that we call a *macrostate* with state parameters $\zeta(\mathbf{x})$, provides a geometrical structure [31] which replaces lacking phase-space and the expression $-k \text{Tr}(W_\zeta \log W_\zeta)$ acquires the role of thermodynamical entropy. The subtlety with relevant variables is that their linear span is not invariant under time evolution. The far reaching consequence of this is that the general dynamics of a macrosystem cannot be described only with a family of *macrostates* W_ζ , by a suitable choice of time-dependent state parameters ζ_t : a more general framework is necessary and in addition to relevant variables, *irrelevant* ones impose themselves. One succeeds however in constructing statistical operators ρ_t , solutions of the Liouville-von Neumann equation, having as input the reference state (6) and displaying the whole history $\zeta_{t'}(\mathbf{x})$ for $t' < t$ of the state parameters and also introducing irreversibility in a fundamental way. This is also the philosophy behind the formalism of non-equilibrium statistical operator initially proposed by Zubarev [32] and extensively used in non-equilibrium thermodynamics [33]. Now we come to the main difficulty: the construction of ρ_t by means of *one* state W_ζ , i.e. ρ_t carrying only *one* family of state parameters, is in general successful only for suitable time intervals. Mixtures of several reference states with the structure (6) but modified through the creation of microsystems in suitable states $\psi_{\alpha_t} \in \mathcal{H}_i$ also appear, providing a much richer parametrization of ρ_t in terms of states ψ_{α_t} of microsystems with certain statistical weights and state parameters ζ_{α_t} influenced by the microsystem. In conclusion on the space \mathfrak{H} state parameters, having a direct relevance in macroscopic phenomenology, can be introduced in a natural way. Deterministic evolution of one set of state parameters means that a selection has been done fine enough to avoid for them a statistical description; in general however, at least piecewise, dynamics is not deterministic and this is described by the appearance of microsystems.

Looking at quantum field theory in this way, microsystems become intertwined with the reference macrostate, which in a sense replaces the vacuum state of the usual field theoretical treatment of microsystems: this can have an impact also for the general description of interaction in the context of relativistic quantum field theory.

6. Conclusions and outlook

We have recalled the modern formulation of quantum theory with observables given by positive operator-valued measures and evolution of possibly open systems given by mappings on states, recalling in particular the starting point of Ludwig's approach. In this framework the essentially statistical character of the phenomenology of microsystems appears as a universal feature of typical non-equilibrium systems. We have argued that quantum field theory, which emerges as underlying theory of all microsystems, appears as a natural framework in which statistical operators can be constructed carrying objective state parameters consisting in classical fields, which generalize the well-known parametrization in terms of temperature and chemical potential. Evidence of microsystems is related to the breakdown of the deterministic time evolution of these state parameters: then by facing stochasticity in their dynamics, quantum theory of microsystems can emerge in a natural way from quantum field theory, initially focused on macrosystems, putting in a new light the question of a proper separation of their dynamics from the macroscopic background.

Acknowledgments

The authors gratefully acknowledge financial support by MIUR under FIRB and PRIN05.

References

- [1] Bassi A and Ghirardi G 2000 *Phys. Lett. A* **275** 373
- [2] Bassi A and Ghirardi G 2003 *Phys. Rep.* **379** 257
- [3] Dürr D 2001 *Bohmsche Mechanik als Grundlage der Quantenmechanik* (Berlin: Springer)
- [4] Dürr D, Goldstein S and Zanghi N 2004 *J. Stat. Phys.* **116** 959
- [5] Adler S L 2004 *Quantum Theory as an Emergent Phenomenon* (Cambridge: Cambridge University Press)
- [6] Ludwig G 1976 *Einführung in die Grundlagen der Theoretischen Physik* (Braunschweig: Vieweg)
- [7] Ludwig G 1983 *Foundations of Quantum Mechanics* (Berlin: Springer)
- [8] Ludwig G 1985 *An Axiomatic Basis for Quantum Mechanics* (Berlin: Springer)
- [9] Kraus K 1983 *States, Effects and Operations (Lecture Notes in Physics vol 190)* (Berlin: Springer)
- [10] Busch P, Grabowski M and Lahti P J 1997 *Operational Quantum Physics (Lecture Notes in Physics vol m31)* 2nd edn (Berlin: Springer)
- [11] Holevo A S 1982 *Probabilistic and Statistical Aspects of Quantum Theory* (Amsterdam: North-Holland)
- [12] Holevo A S 2001 *Statistical Structure of Quantum Theory (Lecture Notes in Physics vol m67)* (Berlin: Springer)
- [13] Gleason A M 1957 *J. Math. Mech.* **6** 885
- [14] Beltrametti E G and Cassinelli G 1981 *The Logic of Quantum Mechanics* (Reading, MA: Addison-Wesley)
- [15] Fagnola F 1999 *Proyecciones* **18** 1
Alicki R and Fannes M 2001 *Quantum Dynamical Systems* (Oxford: Oxford University Press)
Strocchi F 2005 *Mathematical Structure of Quantum Mechanics* (Singapore: World Scientific)
- [16] Streater R F 2000 *J. Math. Phys.* **41** 3556
- [17] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
- [18] Davies E B and Lewis J T 1970 *Commun. Math. Phys.* **17** 239
- [19] Haag R and Kastler D 1964 *J. Math. Phys.* **5** 848
- [20] Barchielli A 2006 *Continual Measurements in Quantum Mechanics Open Quantum Systems III* ed S Attal, A Joye and C-A Pillet (*Lecture Notes in Mathematics* vol 1882) (Berlin: Springer)
- [21] Davies E B 1976 *Quantum Theory of Open Systems* (London: Academic)
- [22] Barchielli A, Lanz L and Prosperi G M 1982 *Nuovo Cimento* **72B** 79
Barchielli A, Lanz L and Prosperi G M 1983 *Found. Phys.* **13** 779
- [23] Alber G et al 2001 *Quantum Information* (Berlin: Springer)
- [24] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)
- [25] Gorini V, Kossakowski A and Sudarshan E C G 1976 *J. Math. Phys.* **17** 821
Lindblad G 1976 *Rep. Math. Phys.* **10** 393
- [26] Alicki R and Lendi K 1987 *Quantum Dynamical Semigroups and Applications (Lecture Notes in Physics vol 286)* (Berlin: Springer)
- [27] Vacchini B 2000 *Phys. Rev. Lett.* **84** 1374
Vacchini B 2001 *J. Math. Phys.* **42** 4291
Lanz L and Vacchini B 2002 *Int. J. Mod. Phys. A* **17** 435
Vacchini B 2002 *Phys. Rev. E* **66** 027107
Petruccione F and Vacchini B 2005 *Phys. Rev. E* **71** 046134
- [28] Vacchini B 2004 *J. Mod. Opt.* **51** 1025
Vacchini B 2005 *Phys. Rev. Lett.* **95** 230402
- [29] Mackey G W 1963 *Mathematical Foundations of Quantum Mechanics* (New York: Benjamin)
- [30] Lanz L, Melsheimer O and Vacchini B 2000 *Rep. Math. Phys.* **46** 191
Lanz L, Melsheimer O and Vacchini B 2004 *Quant. Inf. Comput.* **4** 513
- [31] Streater R F 1995 *Statistical Dynamics* (London: Imperial College Press)
- [32] Zubarev D N 1974 *Non-equilibrium Statistical Thermodynamics* (New York: Consultant Bureau)
- [33] Zubarev D N, Morozov V and Roepke G 1996 *Statistical Mechanics of Non-Equilibrium Processes* (Berlin: Akademie)