Contents

Part I Foundations of Quantum Mechanics

1 Quantum Mechanics as Quantum Probability 3
1.1 Introduction 3
1.2 Classical and Quantum Probability 4
1.2.1 Classical and Quantum Description of a Finite system 7
1.2.2 Classical and Quantum Logic 9
References 14

2 Statistical Structure of Quantum Mechanics 15
2.1 Introduction 15
2.2 States and Effects 16
2.2.1 Pure States and Mixtures 18
2.2.2 Trace Class, Hilbert-Schmidt and Compact Operators 21
2.2.3 Purity and Entropy 26
2.2.4 Maximum-Entropy State 36
2.2.5 Decompositions 42
2.2.6 Effects 46
2.2.7 Generalized Probability Measures 53
2.2.8 Dispersion-Free States 54
2.3 Observables 56
2.3.1 Positive Operator-Valued Measures 56
2.3.2 Naimark's Dilation 65
2.3.3 Coexistence 68
2.3.4 Covariance 69
2.3.5 Position and Momentum Observables 72
References 84

3 Maps 87
3.1 Introduction 87
3.2 Transformations of States and Observables 88
3.2.1 Complete Positivity 91
3.2.2 Operations 97

3.2.3 Measurement and State Transformations 100
3.2.4 Mathematical Characterization of Operations 105
3.2.5 ~~Information Loss and Distinguishability Quantifiers~~ 120
3.2.6 ~~Measurement Disturbance~~ 132
3.2.7 Instruments 134
3.2.8 Measurement Models 144
3.3 Quantum Maps and Their Representation 150
3.3.1 Linear Operators on Hilbert-Schmidt Space 151
3.3.2 Standard Basis Representation 152
3.3.3 Map Basis Representation 155
References 165

Part II Open Quantum Systems Theory

4 Composite Systems 169
4.1 Introduction 169
4.2 Reduced State 171
4.3 Bipartite States and Entanglement 172
4.3.1 Schmidt Decomposition and Purifications 173
4.3.2 Pure and Mixed Entangled States 176
4.3.3 Positive Maps and Entanglement Detection 182
4.4 Choi-Jamiołkowski Isomorphism 184
4.4.1 Choi Matrix 185
4.4.2 Isomorphism Between Maps and Operators 191
References 195

5 Dynamical Evolution 197
5.1 Introduction 197
5.2 Reduced Dynamics 199
5.3 Quantum Dynamical Semigroups 201
5.3.1 Structure of the Generator 204
5.3.2 Collisional Decoherence and Quantum Brownian Motion 220
5.3.3 Quantum Test Particle in a Gas 232
5.4 Exactly Solvable Models 239
5.4.1 Spin-Boson Dephasing Model 240
5.4.2 ~~Hyperfine Couplings~~ 263
5.4.3 ~~Spin-Star Decoherence Model~~ 273
5.5 Projection Operator Techniques 280
5.5.1 Standard and Correlated Projections 282
5.5.2 Projected Equations of Motion 286
5.5.3 Nakajima-Zwanzig Master Equation 288
References 290

6 Weak-Coupling Master Equation 295
6.1 Introduction 295
6.2 Born–Markov Approximation 296
6.2.1 Dressed Interaction Operators and Correlation Functions 298
6.2.2 Lindblad form and Complete Positivity 302
6.3 General Properties of the Master Equation 306
6.3.1 Detailed Balance and Stationary State 306
6.3.2 Dynamics of Populations and Coherences 309
6.4 Quantum Optical Master Equation 311
6.4.1 Coupling to the Electromagnetic Field 311
6.4.2 Thermal Environment 316
6.5 Bloch Equations 320
6.5.1 Rotating Frame 322
6.5.2 Matrix Representation 324
6.5.3 Coherent Evolution 326
6.5.4 General Solution and Stationary State 327
6.5.5 Bloch Equations for Nuclear Magnetic Resonance 329
6.5.6 Explicit Solution for Zero Detuning 333
6.6 ~~System Correlation Functions~~ 336
6.6.1 Multi-time Correlation Functions and Regression 336
6.6.2 Resonance Fluorescence 339
6.6.3 Photon Anti-bunching 349
References 351

7 ~~Memory Effects~~ 355
7.1 Introduction 355
7.2 Classical Memoryless Stochastic Dynamics 357
7.2.1 Classical Markov Process 359
7.2.2 Consequences of the Classical Markov Condition 360
7.2.3 Classical Semigroup Evolution 363
7.3 Behavior in Time of Quantum Distinguishability 366
7.3.1 Quantum Markovian Behavior 367
7.3.2 Interpretation in Terms of Information Backflow 373
7.3.3 Bound on Distinguishability Revivals 375
7.3.4 Measure of Distinguishability Revivals 376
7.4 Characterization of Quantum Dynamical Maps 377
7.4.1 Composition Law and Divisibility Properties 378
7.4.2 Time-Local Master Equations 381
7.4.3 Memory-Kernel Master Equations 383

7.5 Analysis of Memory Effects in Exactly Solvable Models 384
7.5.1 Markovian and Non-Markovian Regimes 384
7.5.2 Memory-Kernel Versus Time-Local Master Equations 394
References 408

Index 413