

Order to chaos transition and extreme deformations

Araceli Lopez-Martens



- Random Matrix Theory + quantum « chaos »
- level statistics + decay widths
- « chaos »-assisted tunneling
- rotational damping
- motional narrowing + ergodic bands



Random Matrix Theory:

developed to explain complex fission resonance spectra

Wigner: resonance energies = eigenvalues of a random matrix sharing the same properties as the Hamiltonian of the nucleus

no preferred basis for the random matrix $\Rightarrow$ Gaussian probability densities

Dyson & Mehta: time-reversal invariant systems are described by GOE and GSE

GOE:

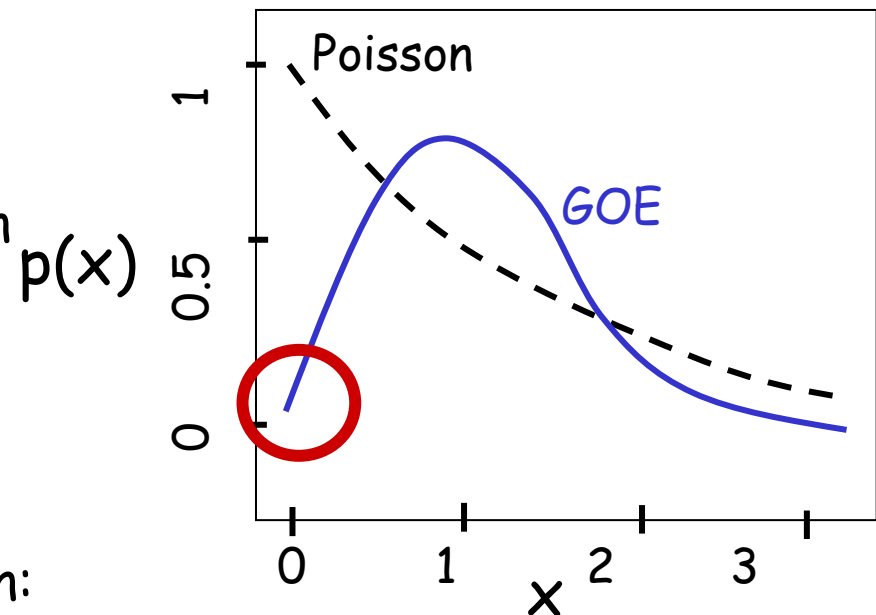
Nearest Neighbour Spacing distribution
(for sequence with same I, π):

$$p(x) = \pi x / 2 \exp(-\pi x^2 / 4)$$

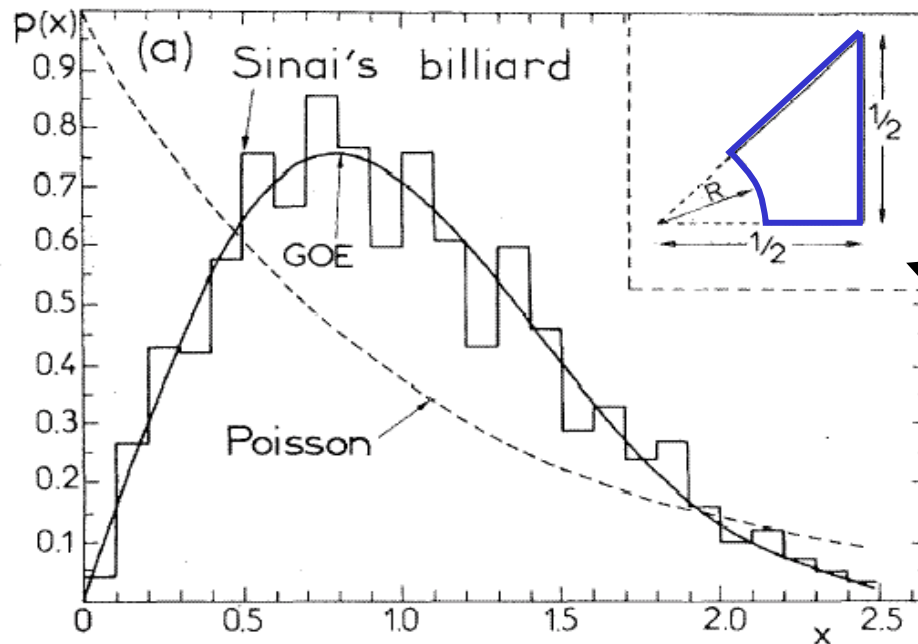
$x = s/d$ $\leftarrow$ unfolding procedure

reduced transition strength distribution:

Porter-Thomas distribution: $p(\xi) = (\xi/2\pi)^{-1/2} \exp(-1/2 \xi)$
 $\xi = B / \langle B \rangle$



Quantum « chaos »:



1-particle quantum system
exhibits random matrix
statistics !

2D billiard = classically
chaotic : hypersensitive
to initial conditions

O. Bohigas et al., PRL 52 (1984)

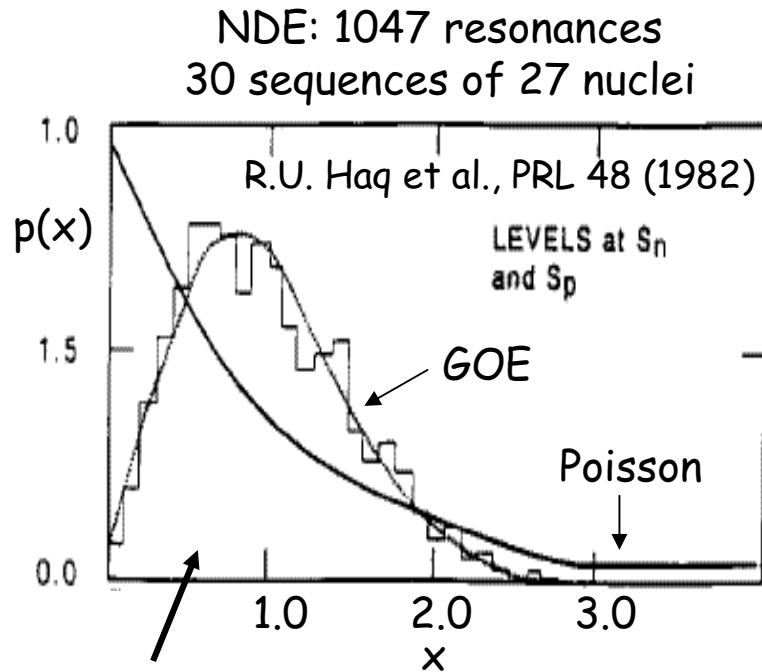
Bohigas, Giannoni and Schmit conjecture:

Spectra of systems whose classical analogues are fully chaotic
show correlation properties as modeled by the Gaussian Ensembles

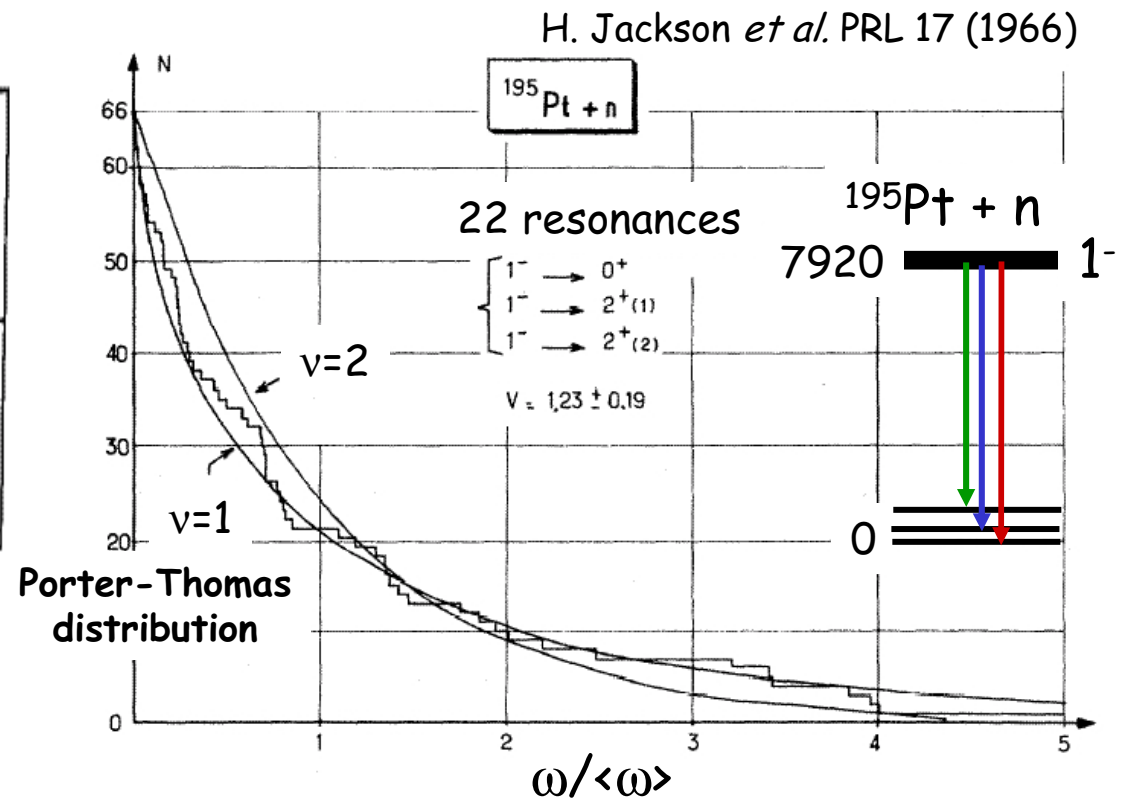
=> association of RMT with « chaos »

=> universality of the laws of level fluctuations ?

Resonance region : « chaotic » regime



for each nucleus, the
sequence of levels must
be **complete** and **pure**



What happens at lower E^* ?

Does « chaos » coexist with « order » ?

Is there a transition from « order » to « chaos » ?

When does the transition occur ?

What is the spin, deformation & mass dependence of the transition ?

Low-lying level statistics

light -> heavy

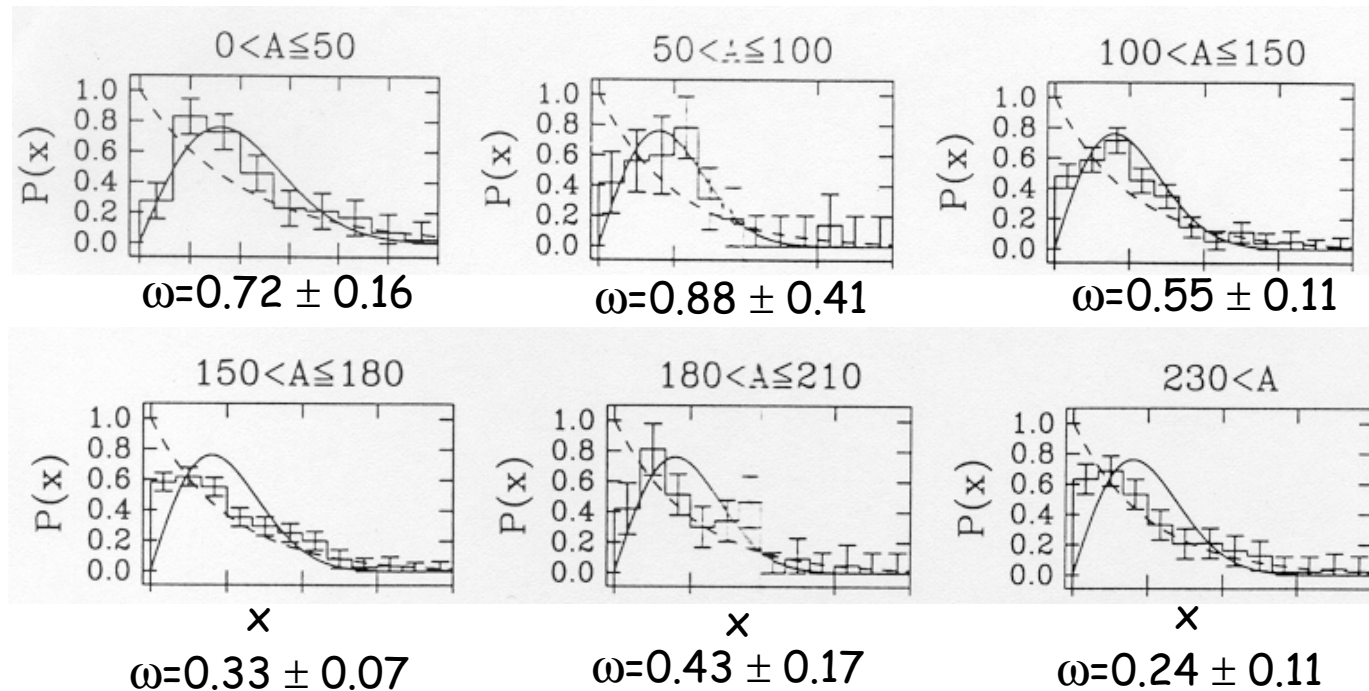
988 spacings from 90 nuclides from ^{24}Na to ^{244}Am

J.F. Shriner et al., ZPA 339 (1991)

Brody parameter ω : interpolation between 2 extreme distributions

$\omega = 0$: Poisson

$\omega = 1$: GOE



=> trend from GOE to Poisson as the mass increases

spherical -> deformed

$$\beta_2/\beta_{2(sp)} > 12 \Rightarrow \text{deformed}$$

$$\omega = 0.30 \pm 0.06$$

$$\beta_2/\beta_{2(sp)} < 7 \Rightarrow \text{spherical}$$

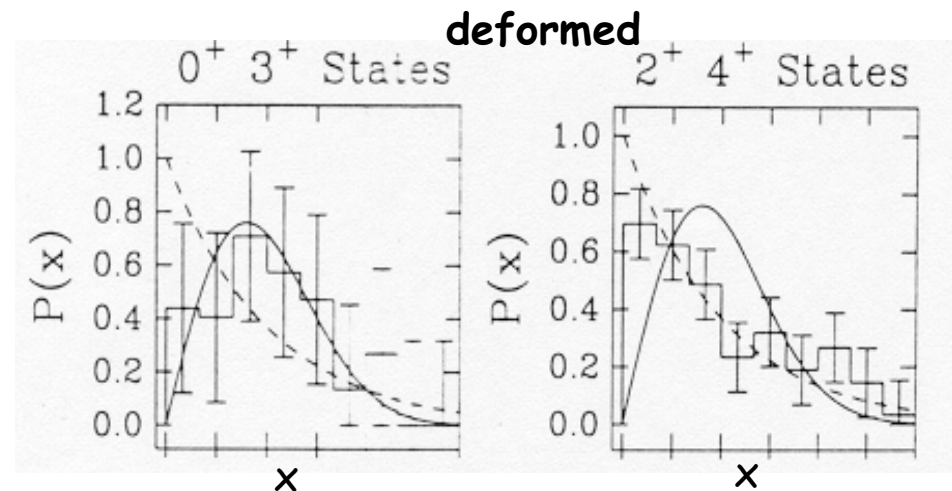
$$\omega = 0.60 \pm 0.08$$

=> NNS distribution of deformed nuclei closest to Poisson

spin dependence (e-e nuclei)

spherical $0^+, 3^+$: $\omega = 0.52 \pm 0.21$
 $2^+, 4^+$: $\omega = 0.52 \pm 0.15$

deformed $0^+, 3^+$: $\omega = 0.74 \pm 0.52$
 $2^+, 4^+$: $\omega = 0.16 \pm 0.13$



shortage of data => no definite conclusions

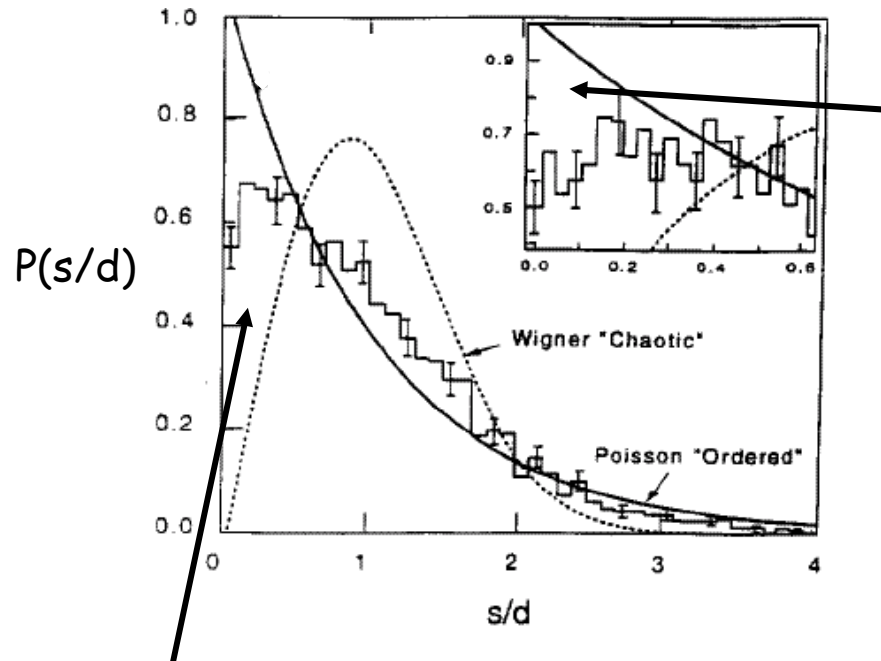
=> more extensive data in a single nuclide is required:

- phase transition
- neglected symmetries ?

Towards regularity or influence of K quantum number ?

3130 near yrast spacings of e-e and odd A rare earth nuclei

J. Garrett et al., PLB 392 (1997)



deviation from Poisson (independent of I, π and nuclear type):

mixing of closely spaced states with the same quantum numbers $\Rightarrow$ level repulsion ?

80% of closely spaced levels (with known K-values) correspond to states that differ in K by 2 or more units

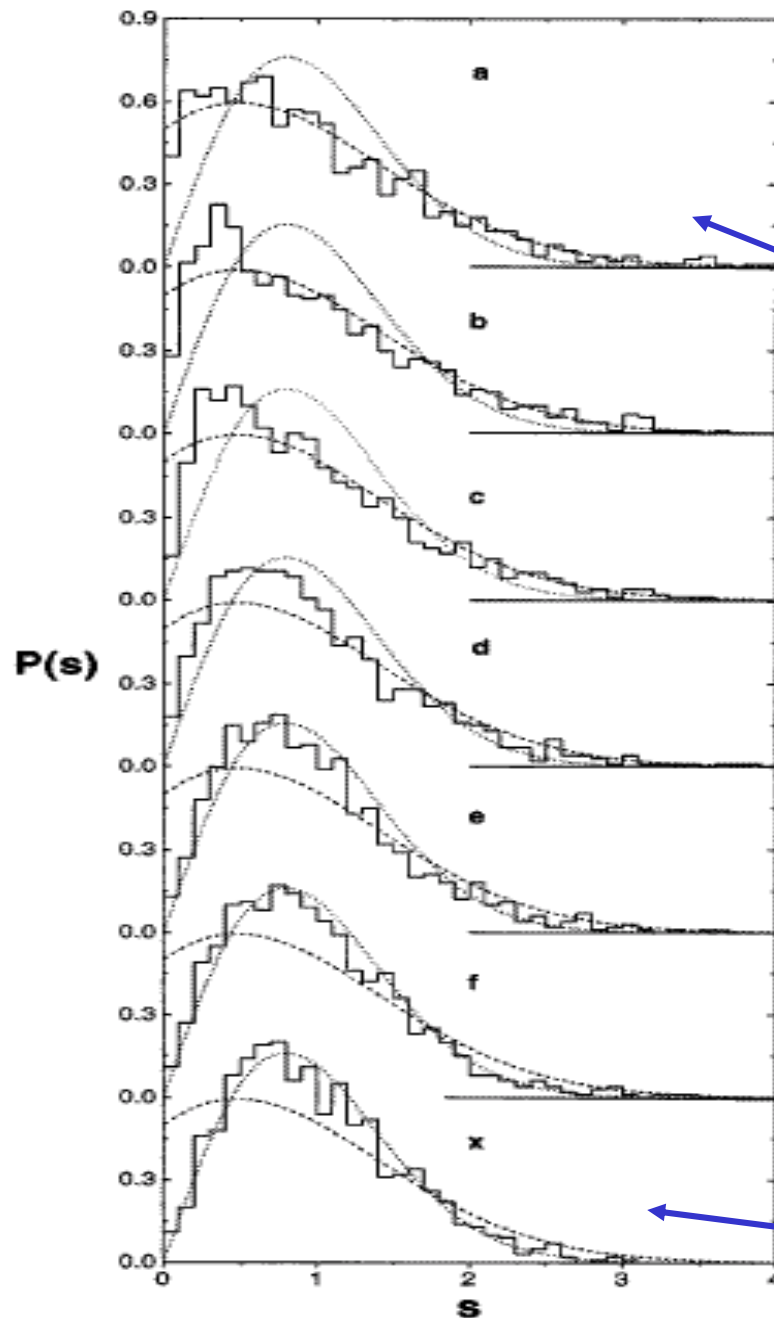
$\Rightarrow$ K quantum number = good quantum number for classifying deformed states

$\Rightarrow$ superposition of unrelated sequences ?

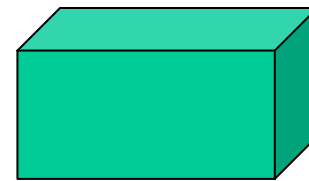
Deviations from GOE & hidden symmetries

1400 acoustic resonances in quartz blocks (with 1 point-group symmetry)

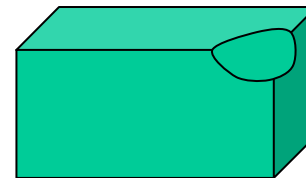
C. Ellegaard et al., PRL 77 (1996)



2 noninteracting
spectra from 2
GOE matrices



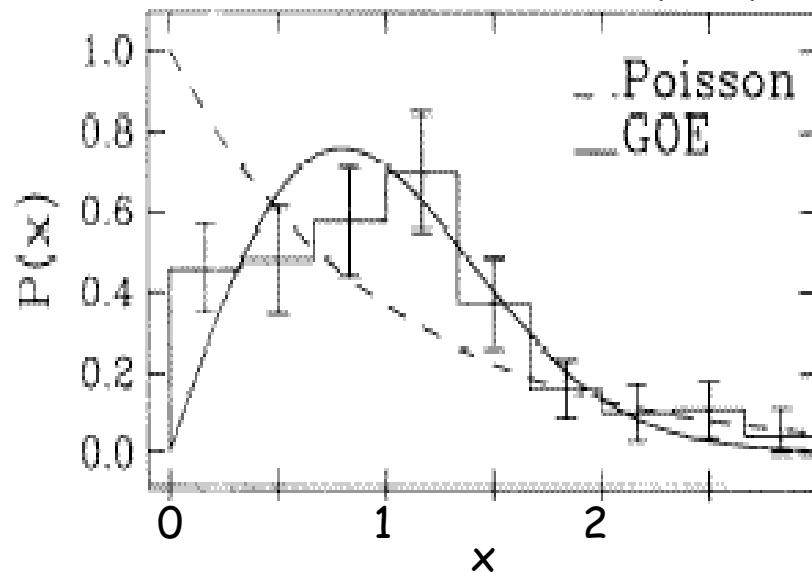
Symmetry
breaking



GOE

1st 100 +parity levels in ^{26}Al ($E^* = 0-8$ MeV)

G. E. Mitchell et al., PRL 61 (1988)



N=Z odd-odd nucleus: T=0 & T=1 states coexist at all energies

=> NNS distribution intermediate between 1 & 2 GOE

Coupled random matrix calculations:

=> 3% isospin mixing

=> <Coulomb matrix element> ~ 20 keV

T. Guhr, H.A. Weidenmüller, Ann. Phys. (1990)

M.S. Hussein, M.P. Pato, PRC 47 (1993)

distribution of electromagnetic transition strengths in ^{26}Al & ^{30}P

=> deviation from Porter-Thomas

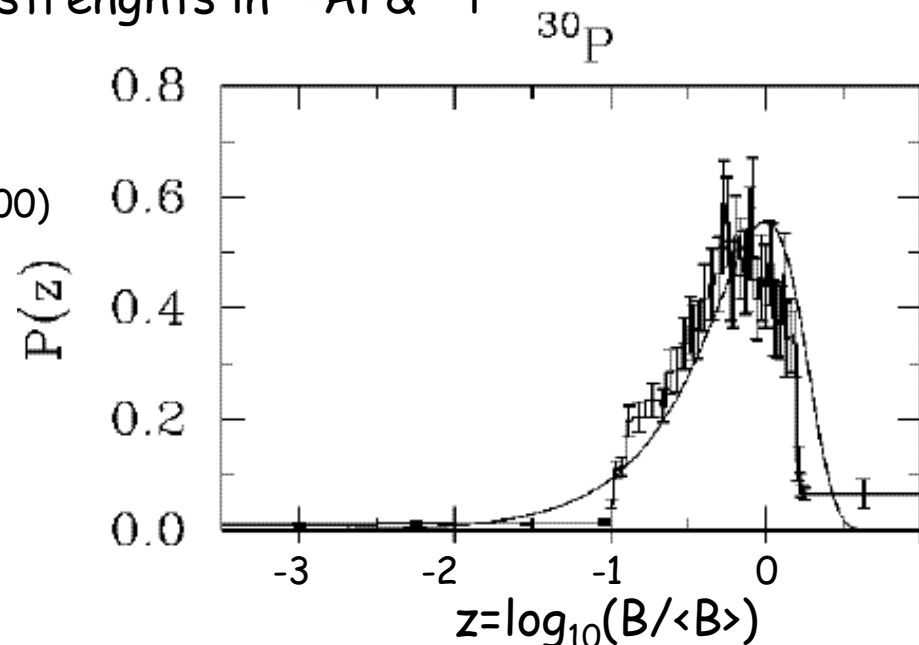
A.A. Adams et al., PLB 422 (1998)

J.F. Shriner, C.A. Grossmann, G.E. Mitchell, PRC62 (2000)

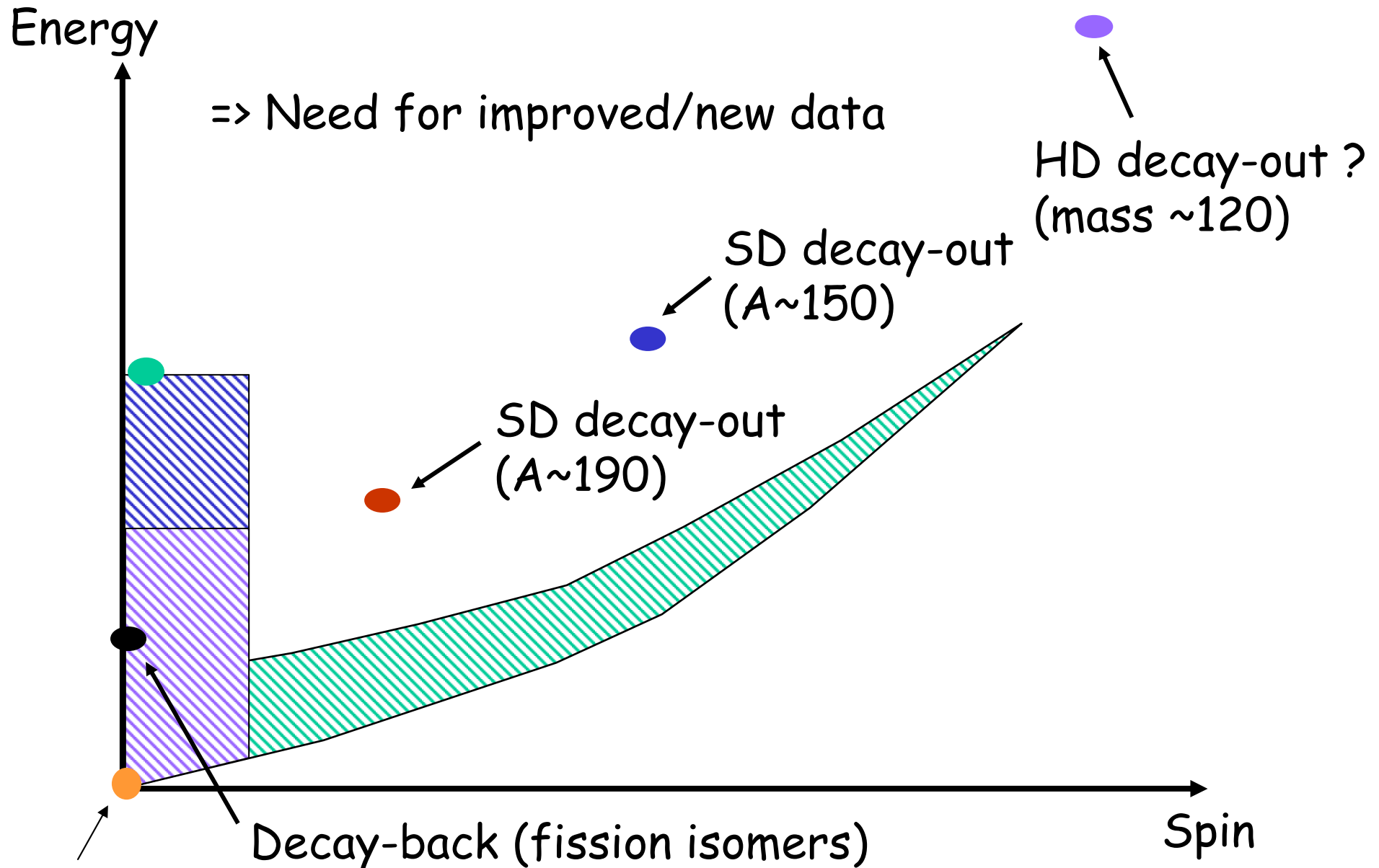
Consistent with recent theoretical work

C.I. Barbosa, T. Guhr, H.L. Harney, PRE 62 (2000)

M.S. Hussein, M.P. Pato, PRL 84 (2000)

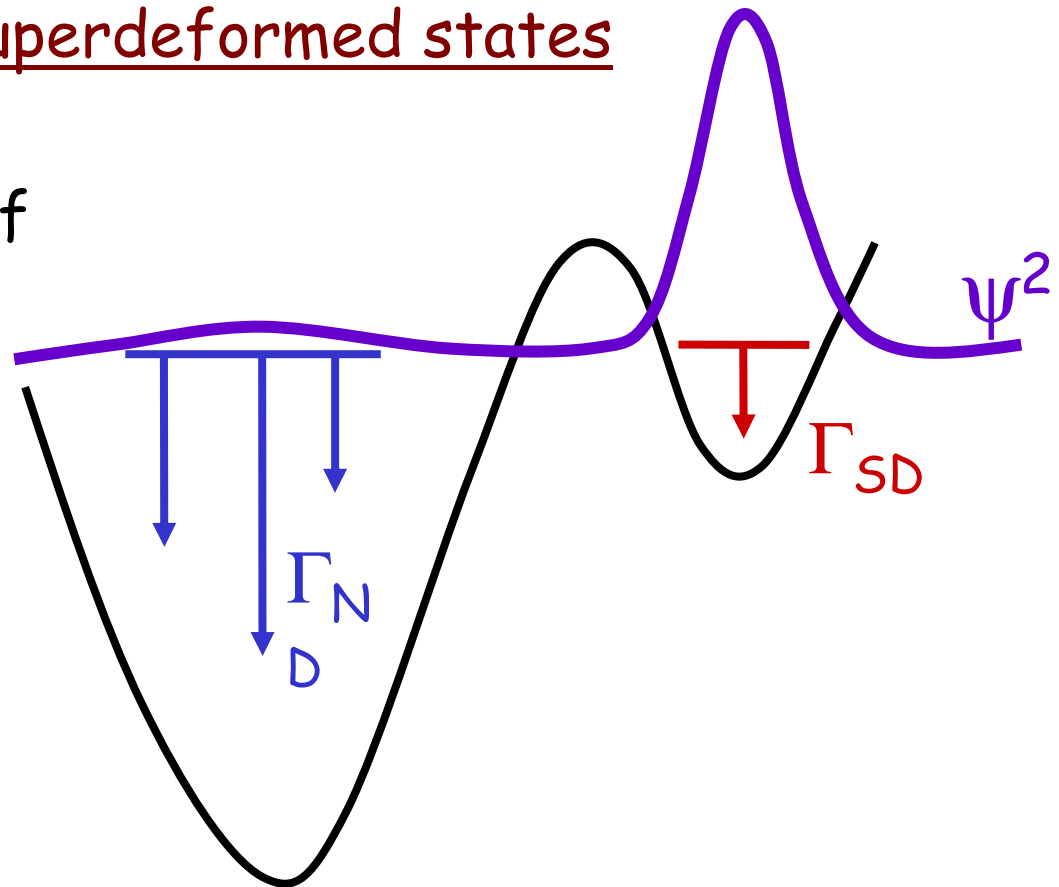


Status of « chaoticity » investigations



Decay from superdeformed states

decay-out occurs because of weak coupling to ND states through a potential barrier



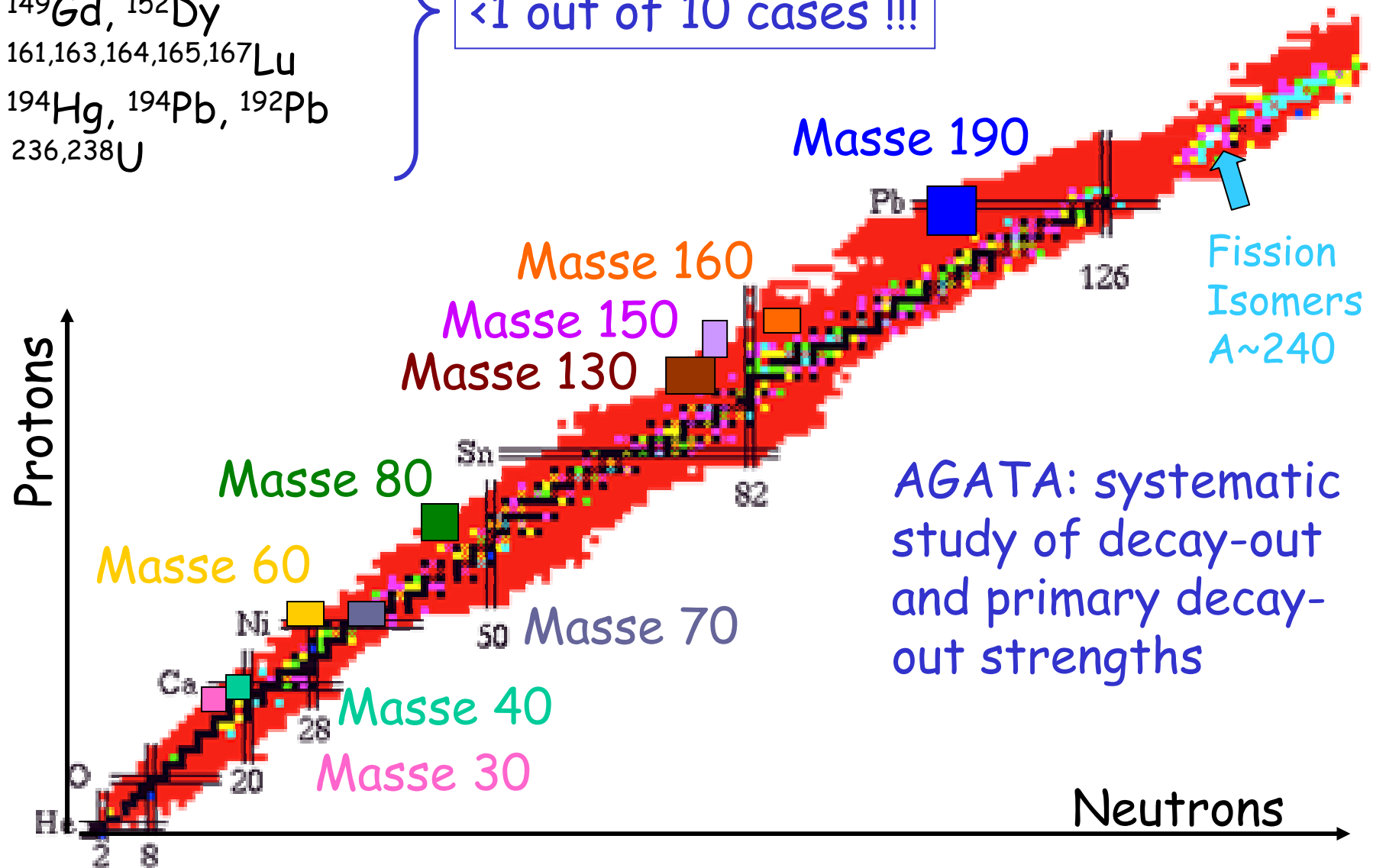
=> what is observed is the decay of excited ND states

highly deformed rotational bands provide states of well defined (E, I, π) which can act as probes of the ND spectrum

Cases for which $(E, I, \pi)_{SD}$ is known from γ -ray spectroscopy:

- ^{36}Ar , ^{40}Ca
- $^{60,61}\text{Zn}$, $^{58,59}\text{Cu}$, ^{56}Ni
- $^{132-137}\text{Nd}$
- ^{149}Gd , ^{152}Dy
- $^{161,163,164,165,167}\text{Lu}$
- ^{194}Hg , ^{194}Pb , ^{192}Pb
- $^{236,238}\text{U}$

<1 out of 10 cases !!!



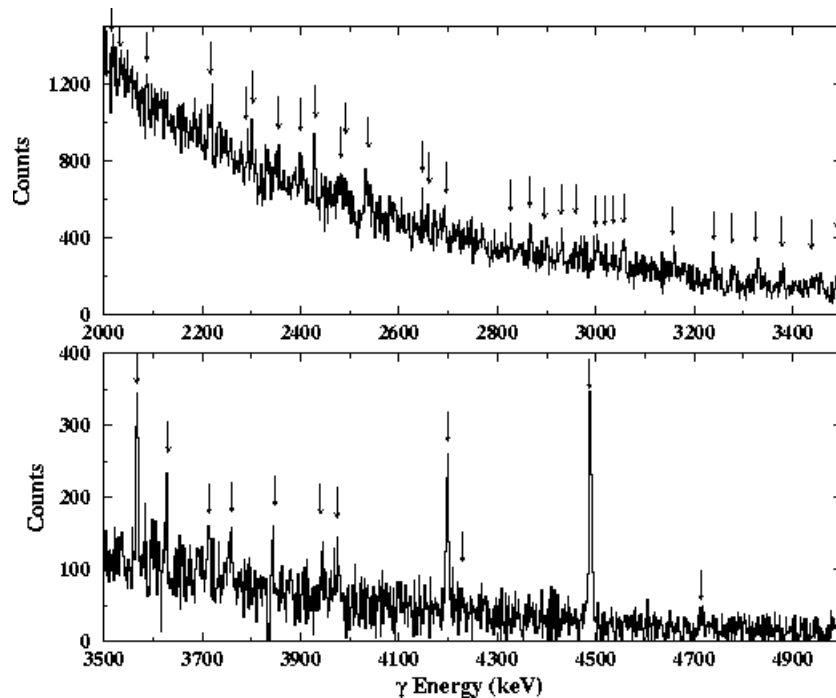
Primary decay-out strength analysis in ^{194}Hg

(high statistics Gammasphere experiment)

ideally, one should identify primary lines from their coincidences and determine their E&M nature and multipolarity

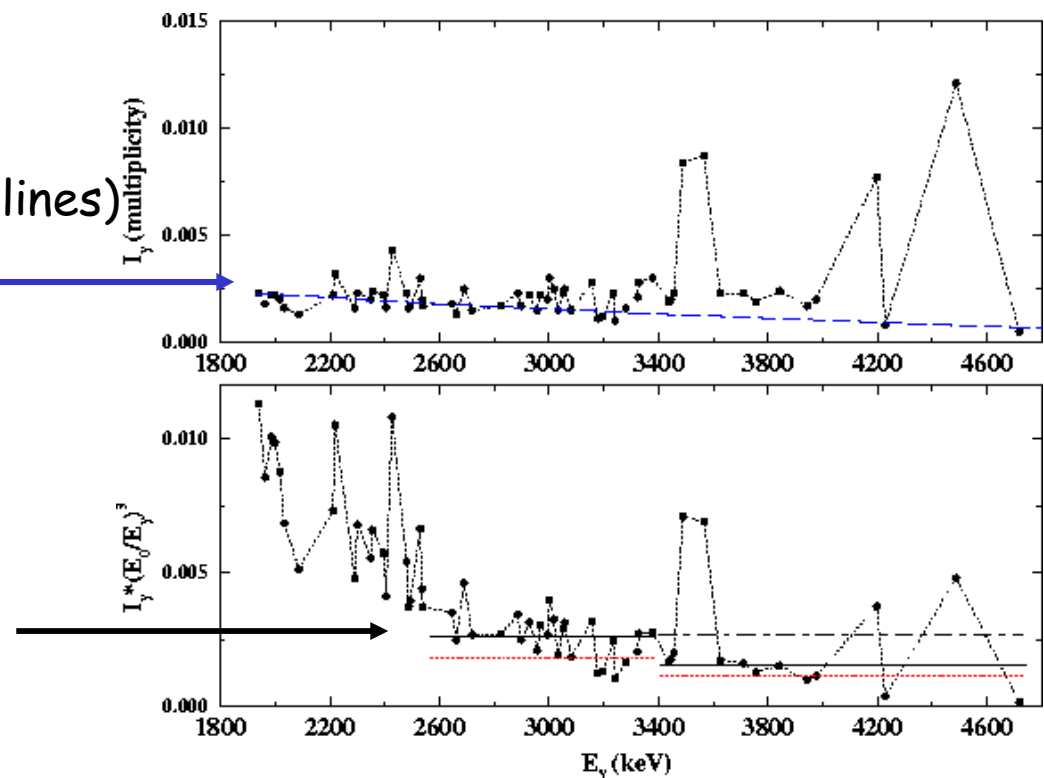
lack of statistics in ^{194}Hg

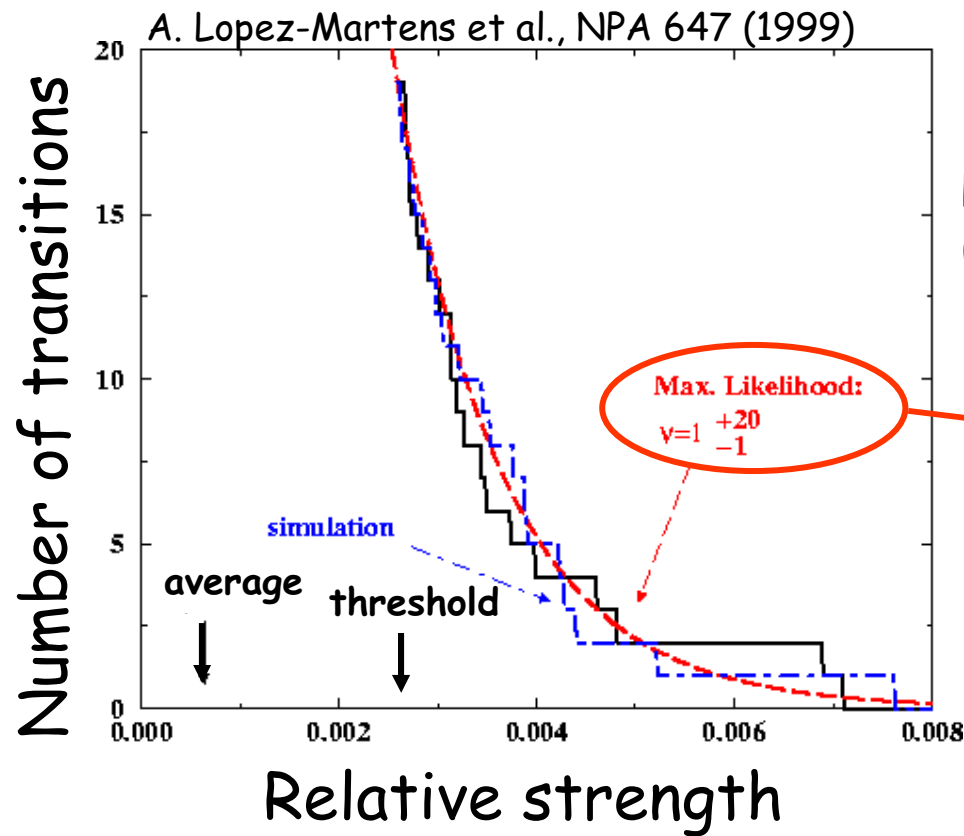
=> all discrete lines > 2.6 MeV = primary E1's



40 lines identified
(Fluctuation Analysis => 600 primary lines)
32 > 3σ limit

19 > strength threshold



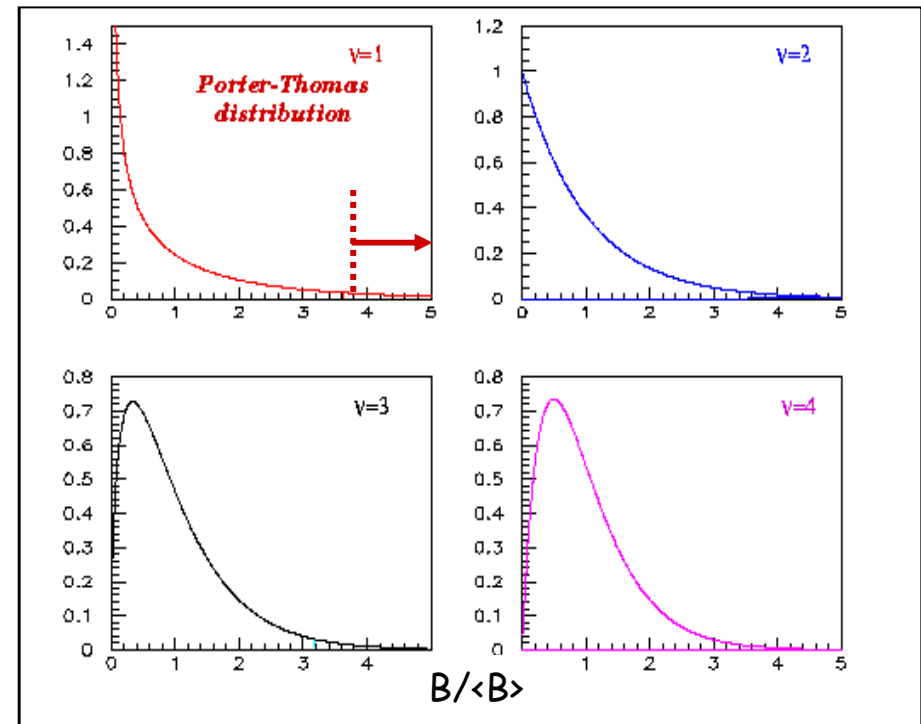


Porter-Thomas distribution: most likely
(extrapolation of most likely distribution
to the origin $\Rightarrow$ 400 primary lines)

Large error bar !

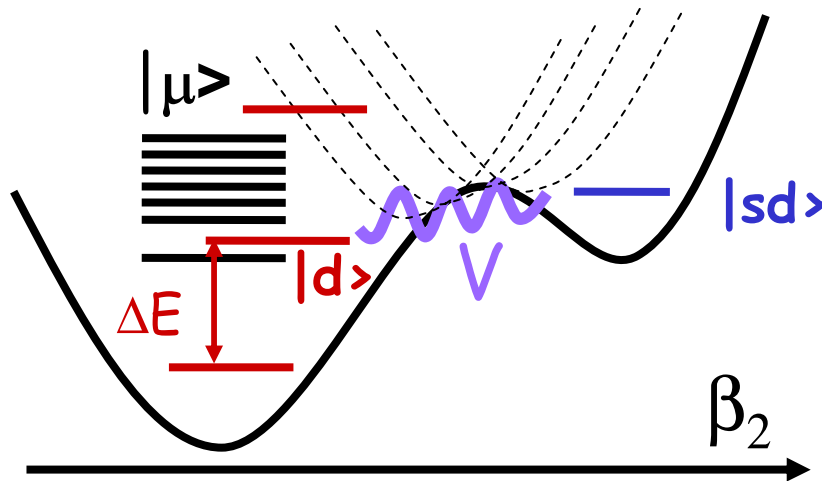
threshold = $3.8 \times$ average strength

$\Rightarrow$ Only the tail of the distribution
is accessible

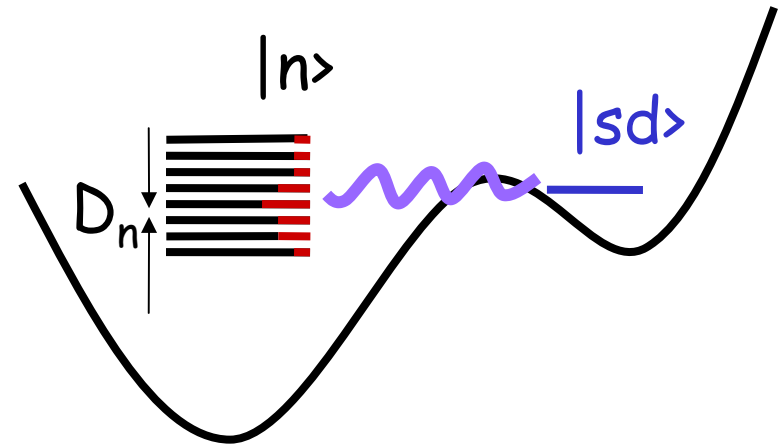


Onset of « Chaos » & tunneling enhancement

S. Åberg, PRL 82 (1999)



off diagonal elements = 0



off diagonal elements = GOE

$$H = \begin{pmatrix} \blacksquare & & & & & \\ & \blacksquare & & & & \\ & & \blacksquare & & & \\ & & & \blacksquare & & \\ & & & & \text{V} & \\ & & & & & \text{V} \\ & & & & & & \blacksquare \end{pmatrix}$$

size = $N \times N$

$\Delta = 0$



$\Delta = 1$

$$H = \begin{pmatrix} \blacksquare & \blacksquare & \square & \square & \square & \square & \square \\ \square & \blacksquare & \square & \square & \square & \square & \square \\ \square & \square & \blacksquare & \square & \square & \square & \square \\ \square & \square & \square & \blacksquare & \square & \square & \square \\ \square & \square & \square & \square & \blacksquare & \text{V} & \square \\ \square & \square & \square & \square & \text{V} & \blacksquare & \square \\ \square & \square & \square & \square & \square & \square & \blacksquare \end{pmatrix}$$

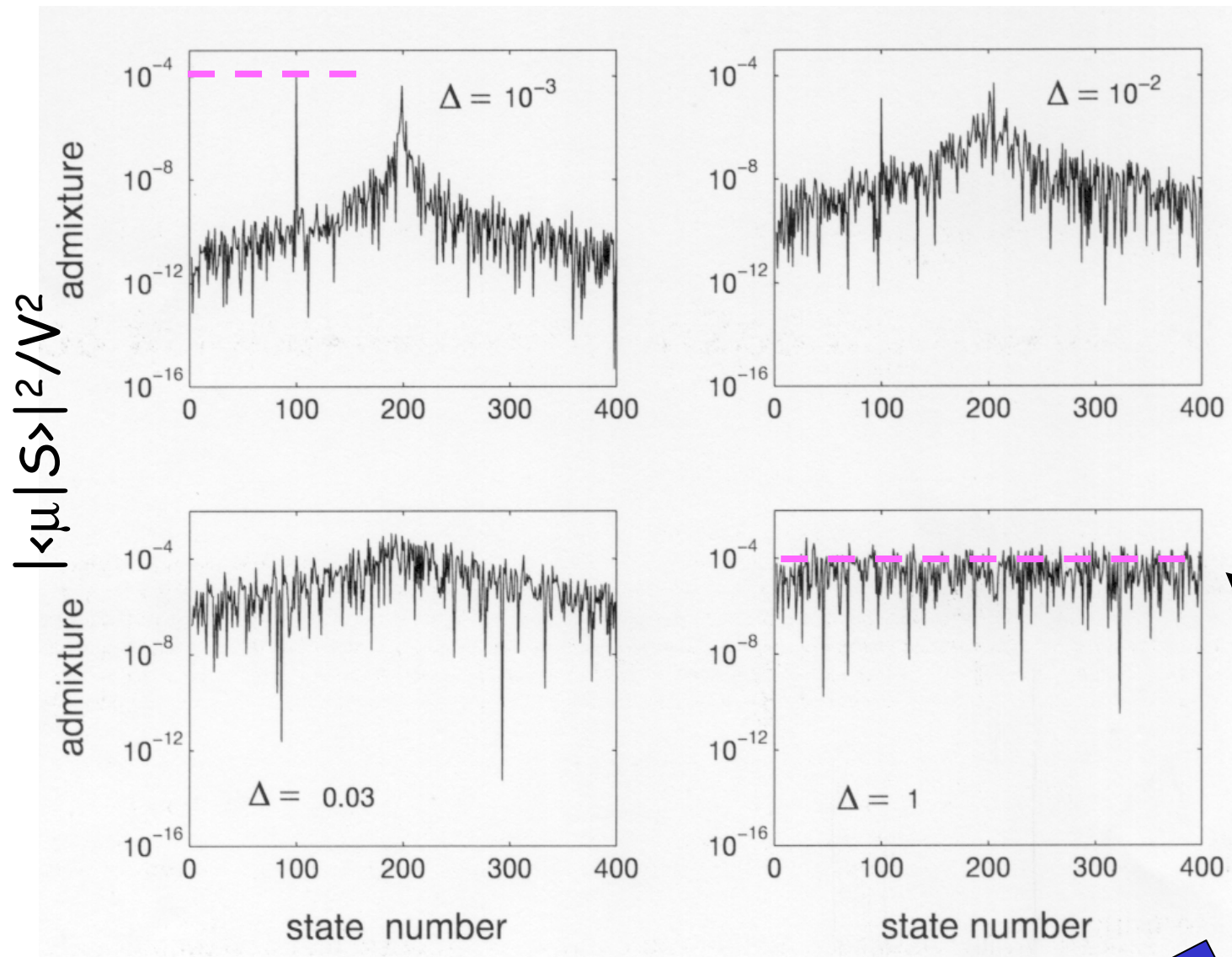


$|S\rangle$



$|S\rangle$

T. Døssing et al.



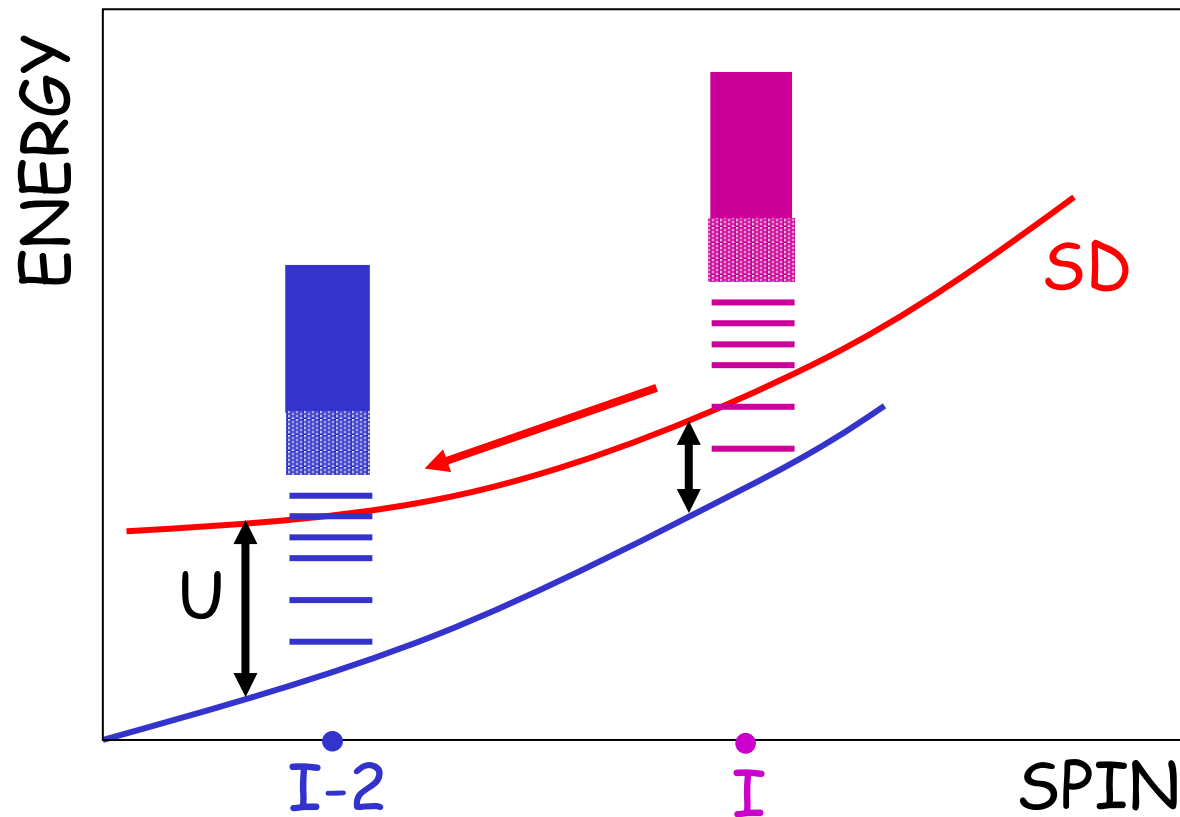
$$N_{SD} = N/2$$

$$N_{door} = N/4$$

N states
contribute
with the
same
admixture

« Chaos » assisted tunneling

Consequences of the onset of « chaos »



- increasing tunneling probability as $I \rightarrow I-2$
- large decay-out strength fluctuations

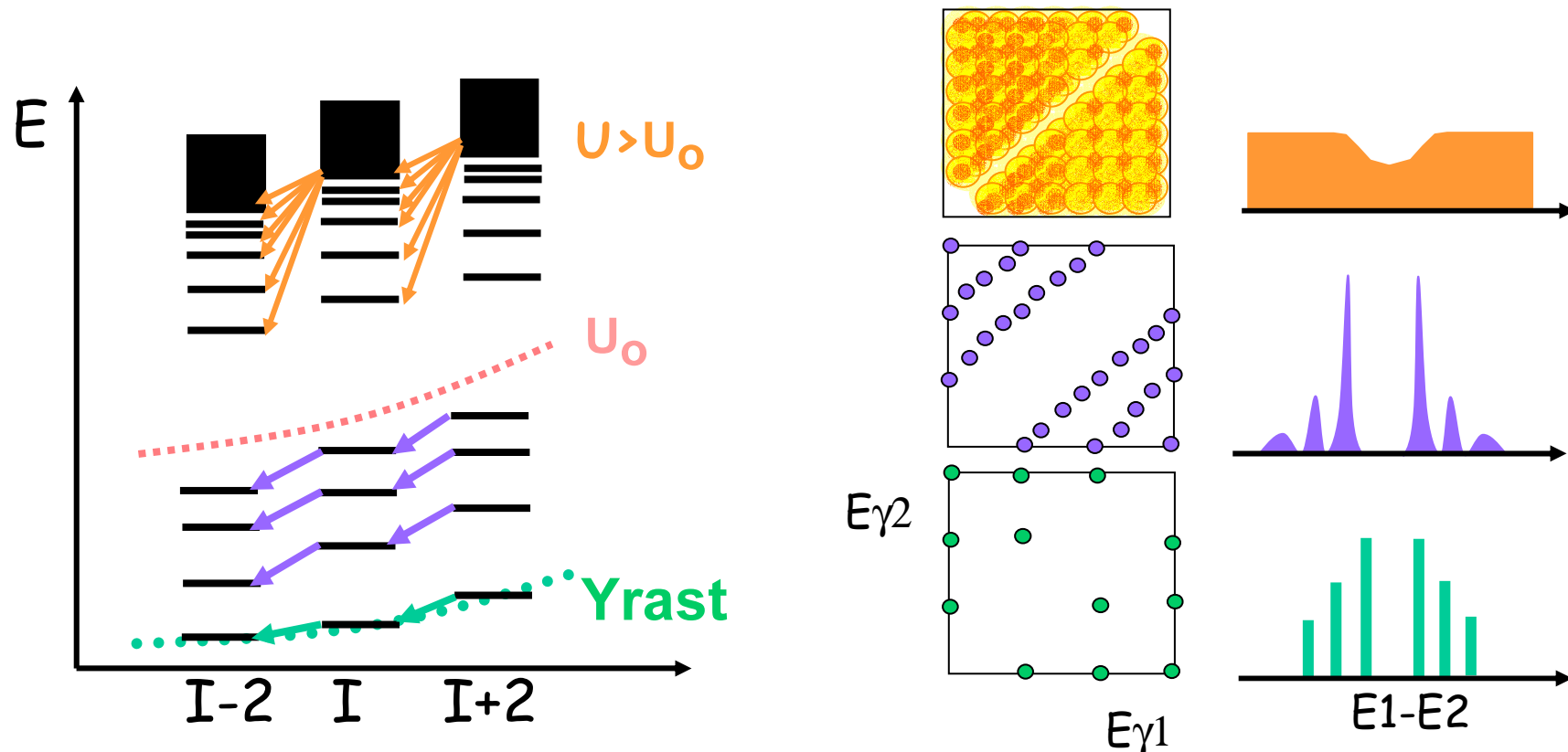
=> analysis of decay-out strengths at 2 consecutive spins

=> alternative explanation for the sudden decay out ?

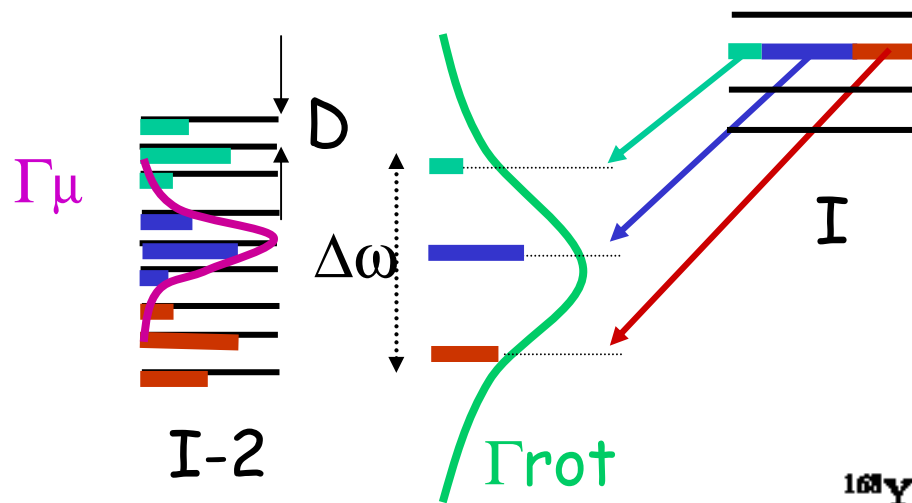
Onset of « chaos » & rotational damping

quasicontinuum γ -ray spectroscopy:

- « E2 bump » studies $\Rightarrow$ rotational motion with full collective strength extends to very high I
- 2-D $E_{\gamma 1}.E_{\gamma 2}$ correlations: weaker correlations in the quasicontinuum $\Rightarrow$ damping of rotational motion
- * 3-D correlations + count fluctuation + covariance analyses $\Rightarrow$ better picture of the γ -decay of rotating nuclei



Microscopic origin of rotational damping



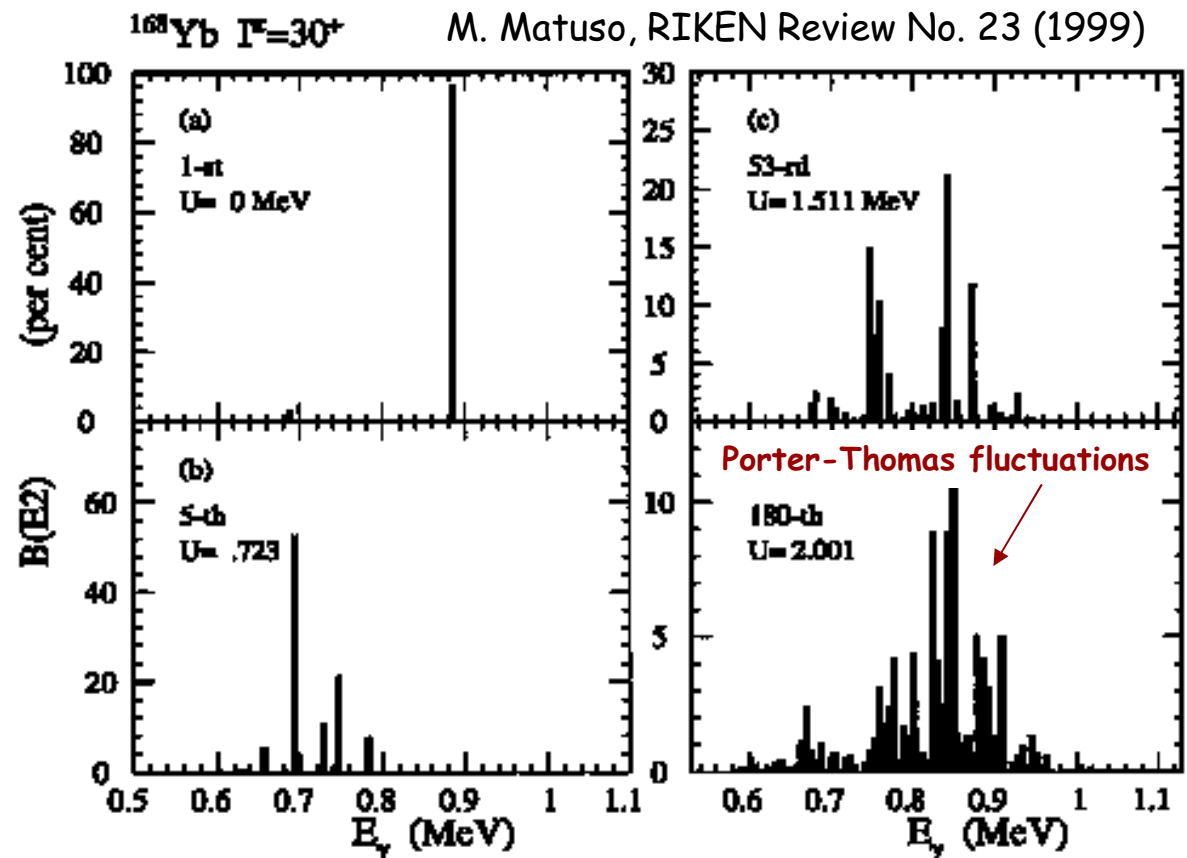
As U increases, $\rho(U) \nearrow$

=> residual interaction mixes np-nh states with different rotational properties

=> fragmentation of the E2 strength

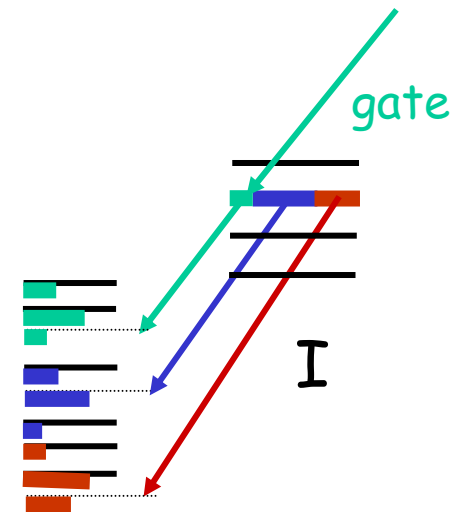
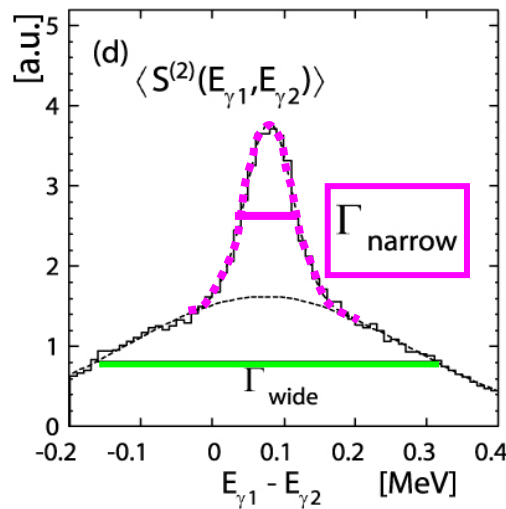
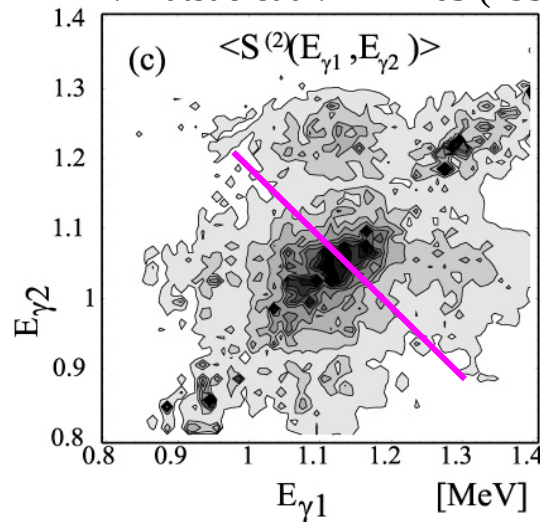
Cranked Nilsson
mean field
+
SDI
(residual 2-body
interaction force)

=> Γ_{μ} and Γ_{rot} have
different physical
origins



Evidence for the effect of Γ_μ in γ - γ correlations

M. Matsuo et al. PLB 465 (1999)



doorway states maintain the rotational correlation ($\Gamma_\mu < \Gamma_{\text{rot}}$)
but the correlation is smeared by Γ_μ

=> can measure both Γ_{rot} and Γ_μ

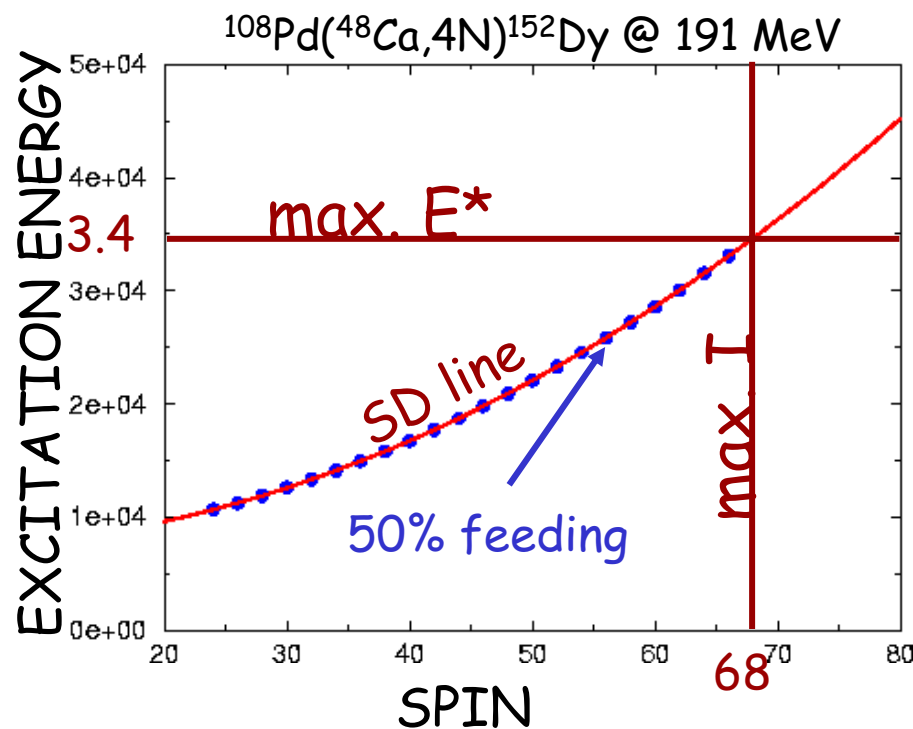
recent example of ^{168}Yb

S. Leoni et al EPJ A4 (1999)

F. Stephens et al., PRL 88 (2002), Frontiers of Nuclear Structure, AIP (2003)

=> can simulate the γ -decay of the warm rotating nucleus with
a measured entry distribution

(AGATA=total absorption & multiplicity spectrometer)

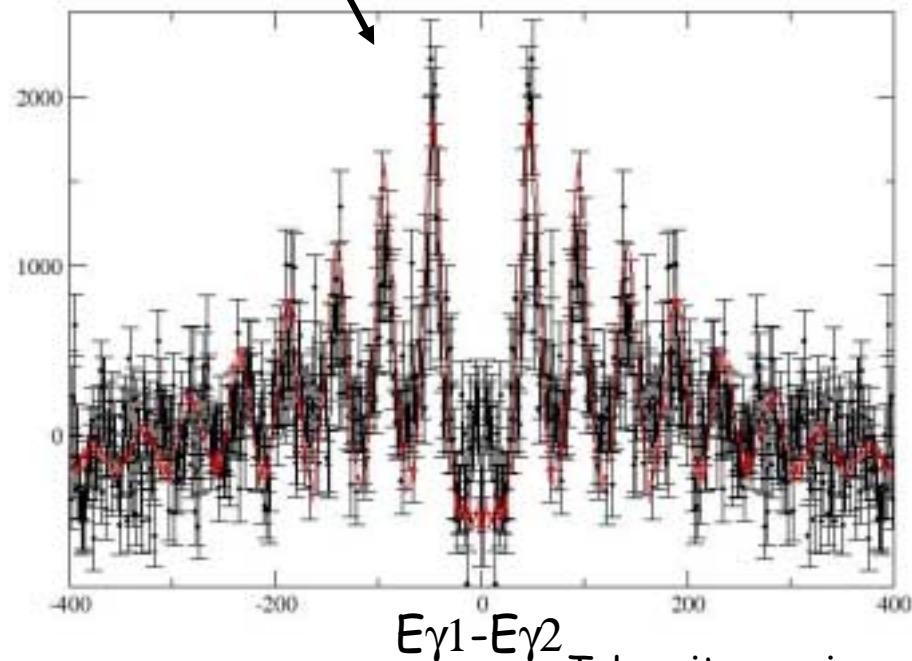


Feeding of yrast superdeformed states in ^{152}Dy

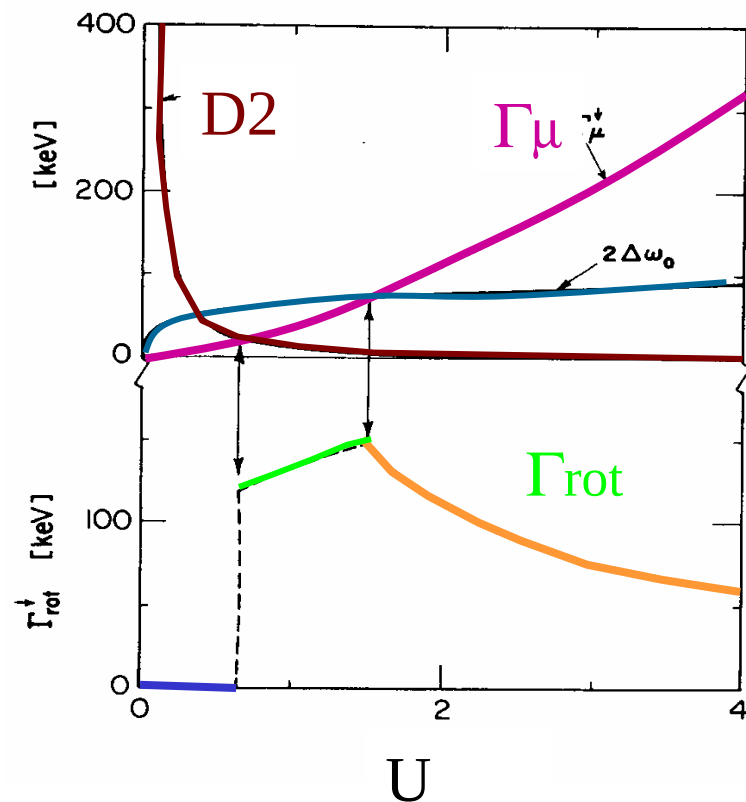
=> would not expect many ridges in coincidence with the yrast SD band

4-5 ridges sitting on a wider structure are observed in coincidence with the SD band !!

=> need to know the entry distribution to understand the population + ridge/valley structure



T. Lauritsen priv. comm.



$$\Gamma_\mu < D2$$

$$\Gamma_\mu > D2$$

$$\Gamma_\mu > 2\Delta\omega$$

$$\Gamma_\mu/D \gg 1 \text{ \& } \Gamma_{\text{rot}}/D < 1$$

discrete bands

onset of mixing

motional narrowing

Ergodic bands

B. R. Mottelson, NPA 557 (1993)

S. Åberg, ZPA 358 (1997)

Where to look for ergodic bands ?

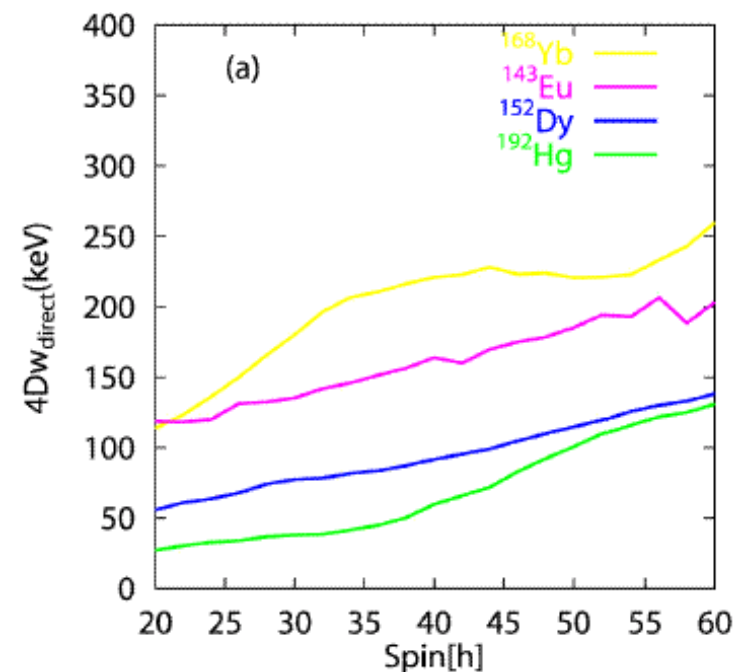
shell effects (level density & alignments)

=> hyperdeformation = favourable case ?

S. Åberg ZPA 358 (1997)

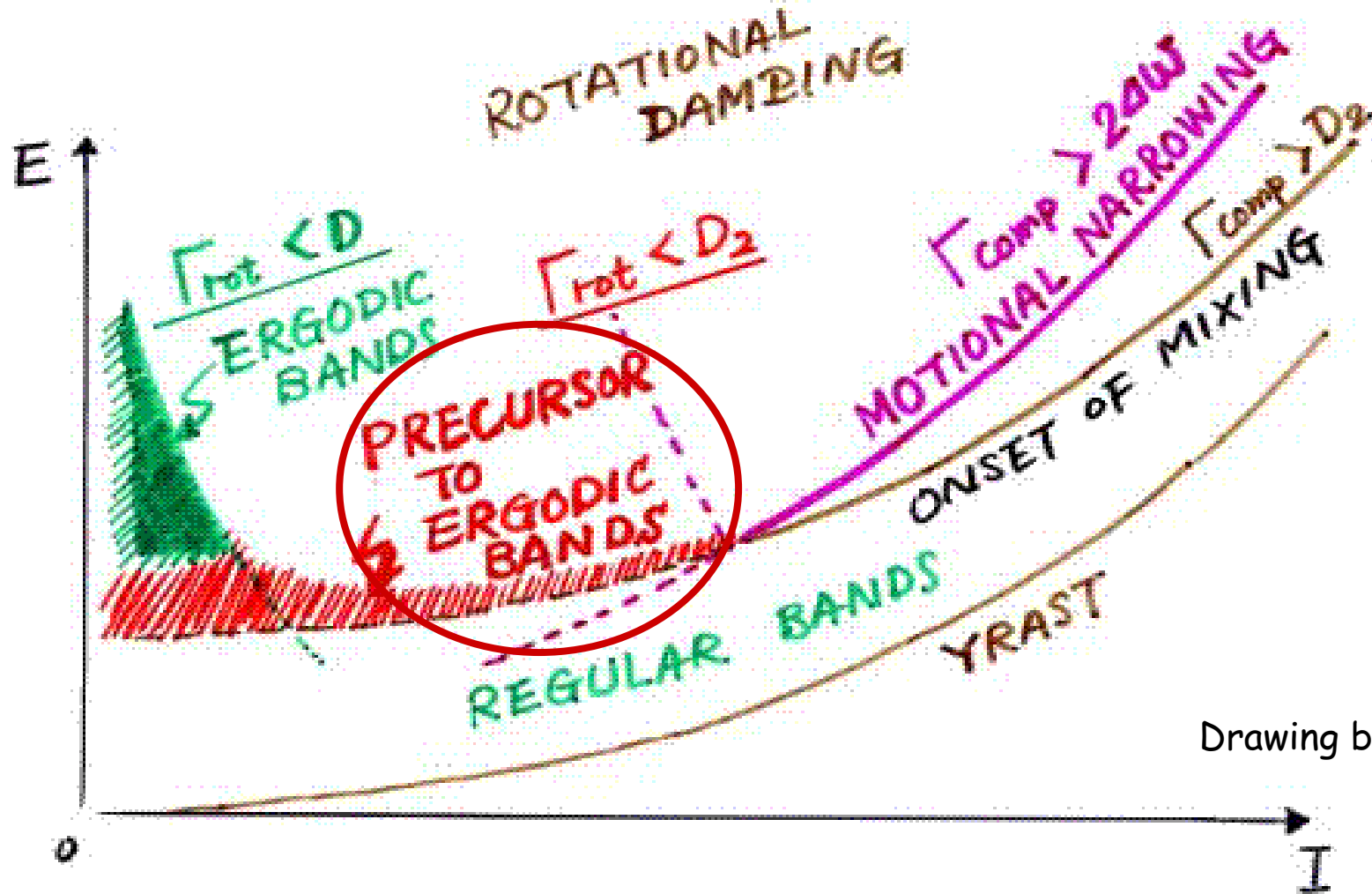
spin effects

K. Yoshida and M. Matsuo, NPA A636 (1998)



Precursor to ergodic bands in ^{194}Hg ?

Gammasphere experiment: 2-3 ridges in coincidence with the yrast SD band
100-150 bands contribute to the ridge
narrow ridges, no or little E2 branching



Drawing by M. Matsuo

Conclusions & perspectives

- ❑ Need for Improved technology to perform COMPLETE spectroscopy
=> wealth of new data on level spacings and transition strengths
- ❑ Fluctuation properties => order-> « chaos » transition symmetry breaking, microscopic origin of quantum « chaos »,...
- ❑ Decay-out of SD (and HD) bands : new regions of (E,I) to study transition strength statistics & possibility of « chaos »-assisted tunneling
- ❑ Damping of rotational motion: extraction of the compound width & rotational damping width
- ❑ Understand and simulate the features of the γ -cascades emitted from fast rotating nuclei
- ❑ Motional narrowing and ergodic rotational bands