

SHAPE COEXISTENCE AND PHASE TRANSITIONS IN ATOMIC NUCLEI

Int'l workshop on Physics with AGATA
March 22-24 Orsay

AIM OF THIS TALK

EXPLORE CONDITIONS NEEDED TO FIND
SHAPE COEXISTENCE STARTING FROM
ESSENTIAL INGREDIENTS OF THE N-N
FORCE

EXPLORE POWER AND LIMITATIONS OF
THE NUCLEAR SHELL MODEL

POSSIBILITIES USING SYMMETRY IDEAS-
SAMPLE COLLECTIVE MODES

EXPLORE MEAN-FIELD METHODS

STUDY QUADRUPOLE SHAPE VARIABLES
FOR SHAPE TRANSITIONS

OUTLOOK

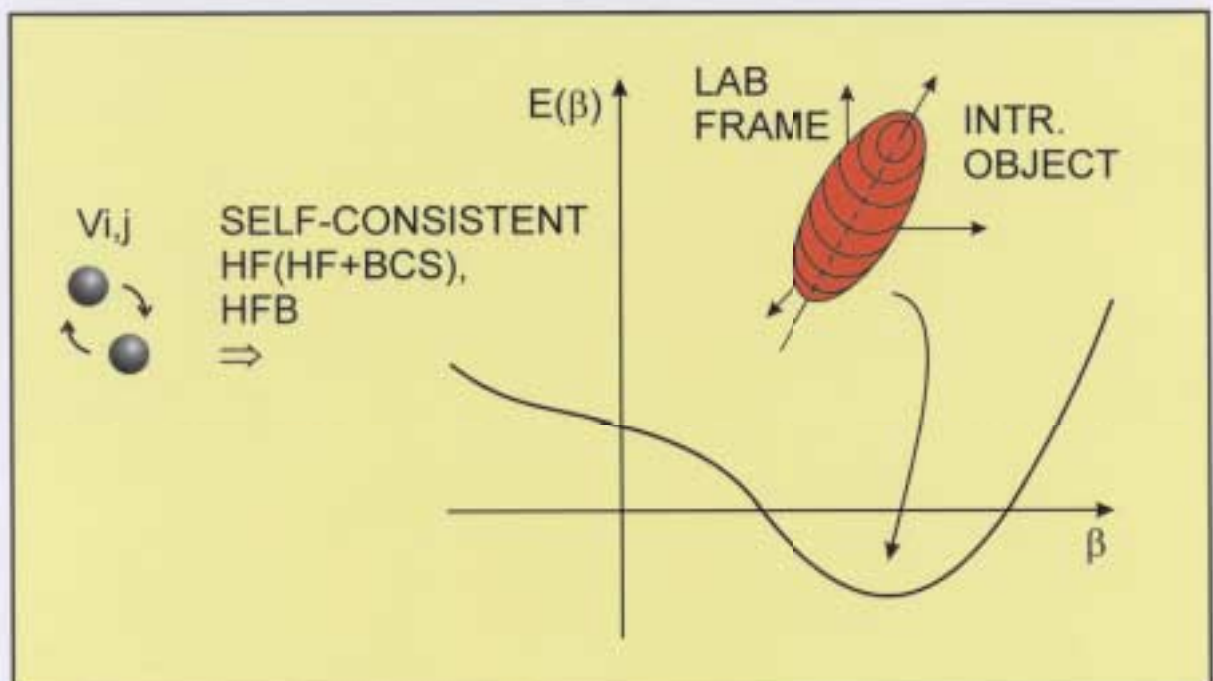
- NUCLEONS INTERACTING THROUGH $V(i,j)$ GENERATE MEAN-FIELDS WITH SHELL STRUCTURE

HF, HF + pairing (BCS), HFB

- SELF - CONSISTENT CALCULATIONS IMPLY THAT ENERGY MINIMA CORRESPOND WITH DEFORMED MEAN - FIELDS MAINLY

└ DISADVANTAGE : NOT IN LAB FRAME (no good $J^\pi, T, N, Z, \dots$)

└ ADVANTAGE : GIVES "INTRINSIC" STRUCTURES → INT. FOR MODELS WITH SHAPE VARIABLES



* VERY - LIGHT NUCLEI ($A \lesssim 10$). AB - INITIO CALCULATIONS

* SHELL - MODEL

1p shell : Cohen, Kurath,...

2s, 1d shell : Brown, Wildenthal,...

2p, 1f shell : Madrid - Strasbourg, (restricted)

: GXPF1 : Honma, Otsuka, Brown, Mizusaki

Heavier ?

MAIN CONCEPT

BUILDING UP OF A (VERY) LARGE MODEL SPACE
dim (N) + METHODS TO DETERMINE THE LOWEST
ENERGY EIGENVALUES + WAVE FUNCTIONS

OXBASH code

ANTOINE code : $\sim 10^9$ states in fp shell

RESTRICTIONS

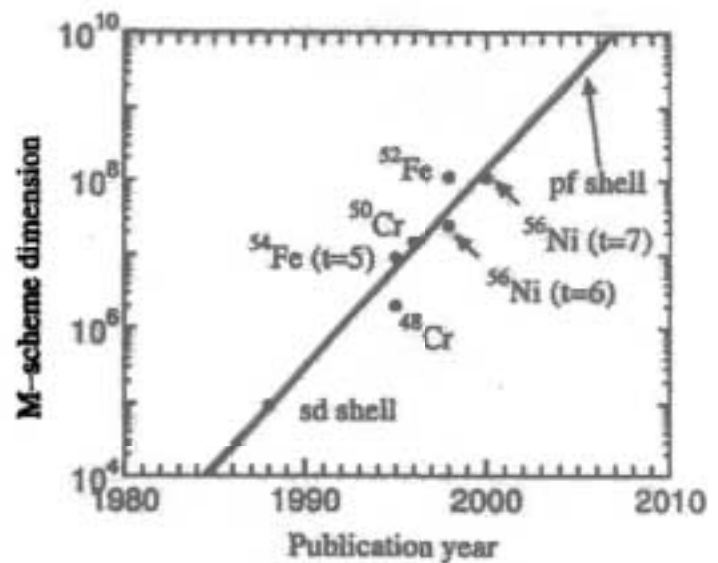
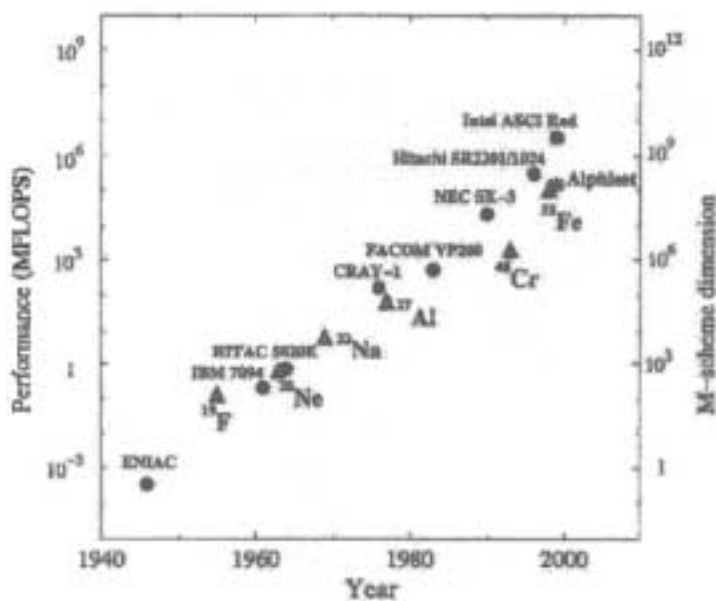
$$C_{N_s}^n$$

ex. $n \cong 100$ nucleons

$N_s = 200$ basis states, $\dim \approx 10^{60} !$

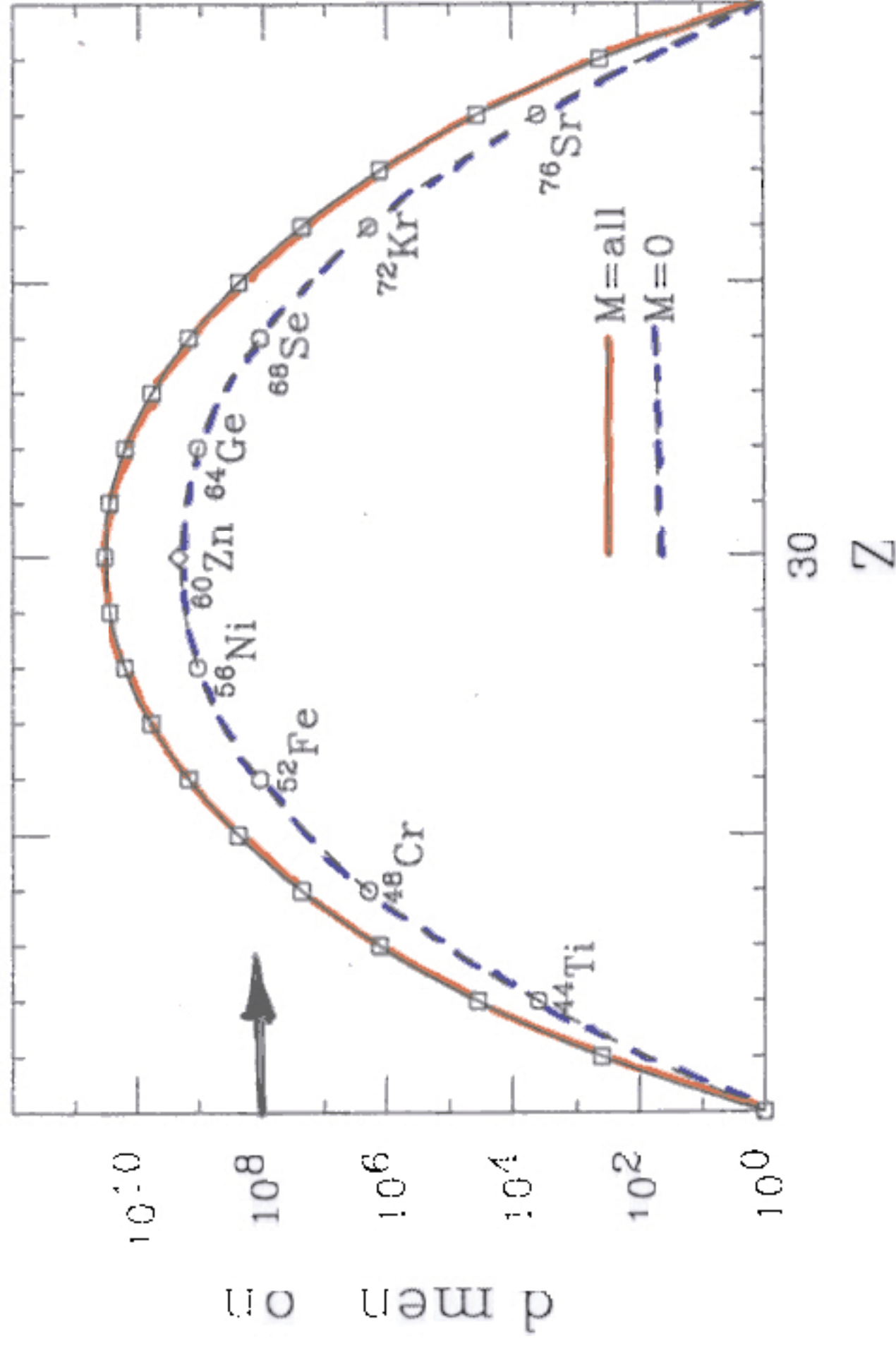
NUCLEAR SHELL-MODEL

- LIMITED NUMBER OF S.P. ORBITALS
(p-shell, sd-shell, fp(g)-shell)
- LIMITED NUMBER OF VALENCE PARTICLES
(12-middle sd shell (16-40))
(20-middle fp shell (40-80))

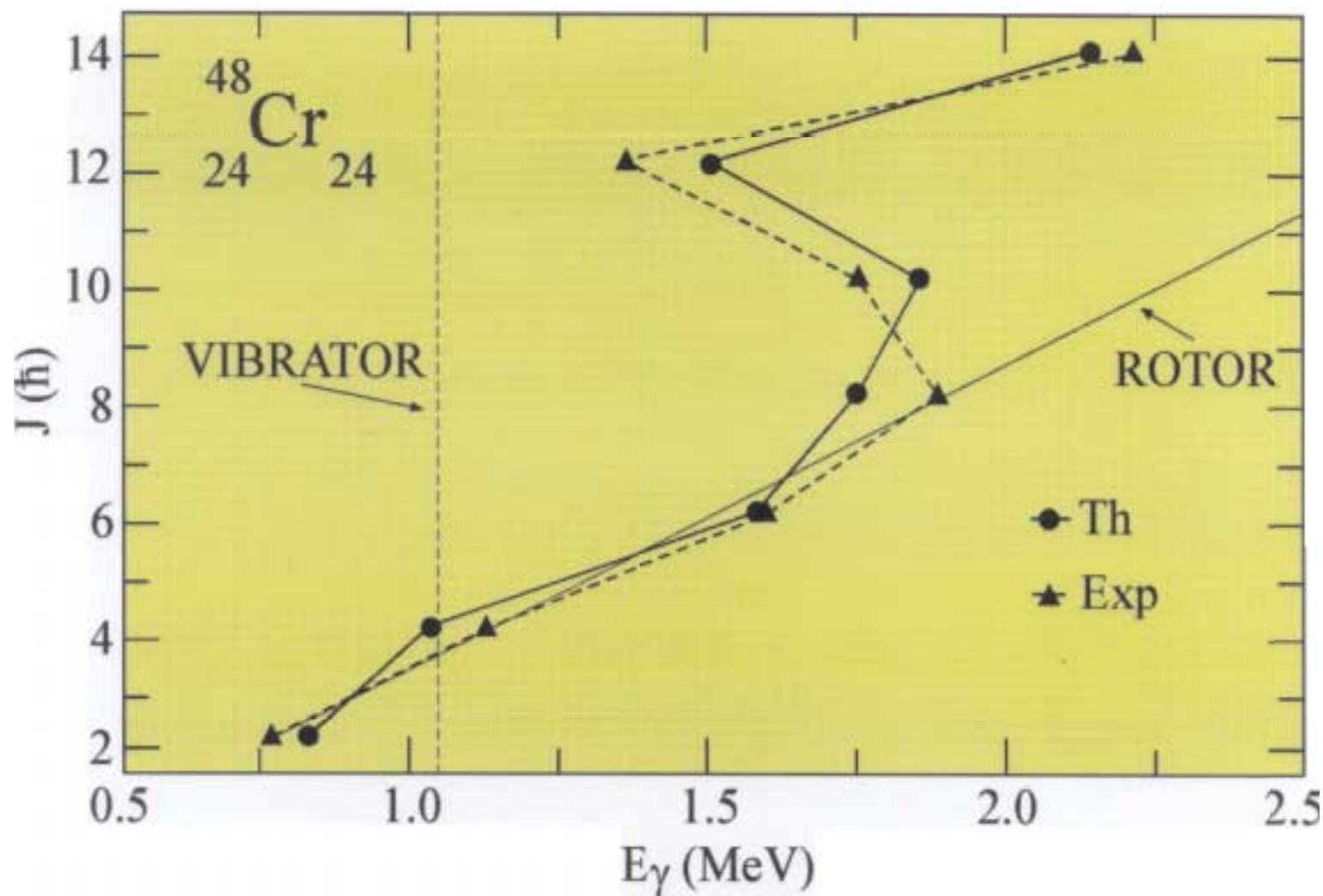


CODES: ANTOINE, NATHAN (Strasbourg, Madrid)
VECSSE, MSHELL (Tokyo)
OXBASH (Oxford, Buenos-Aires)

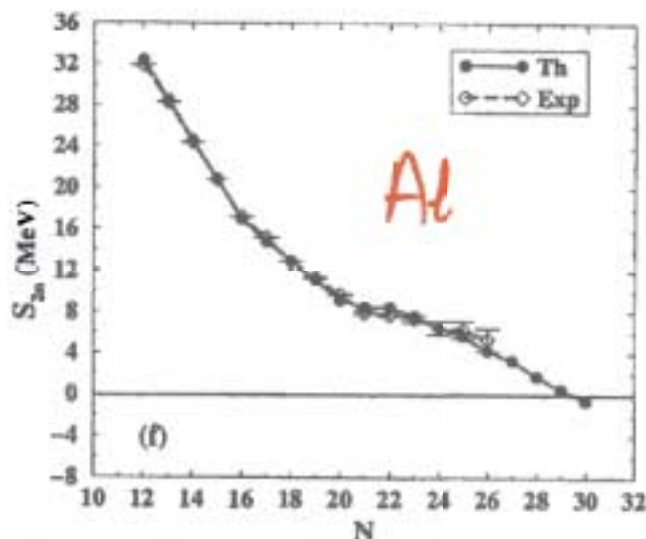
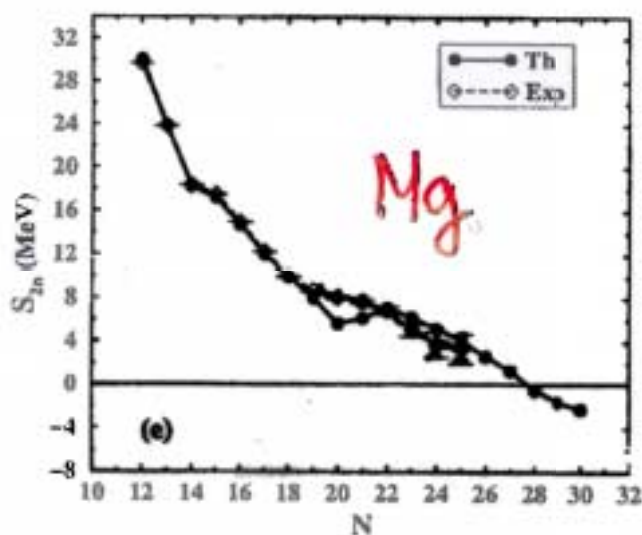
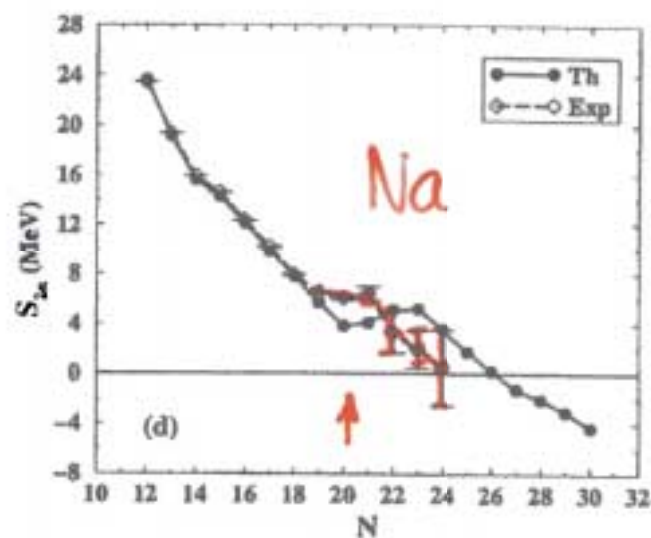
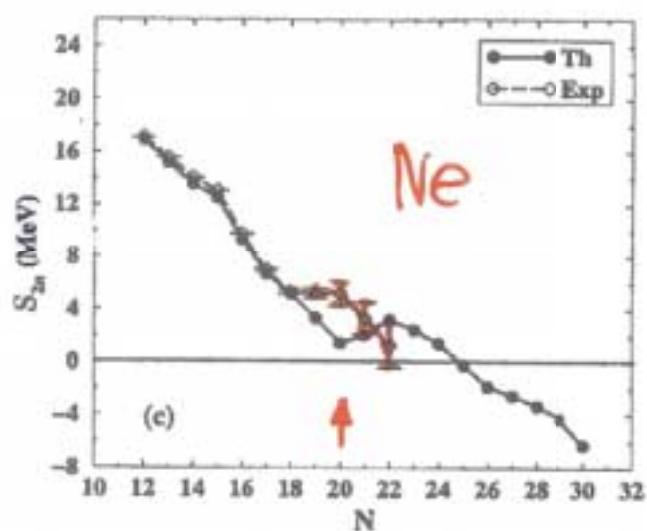
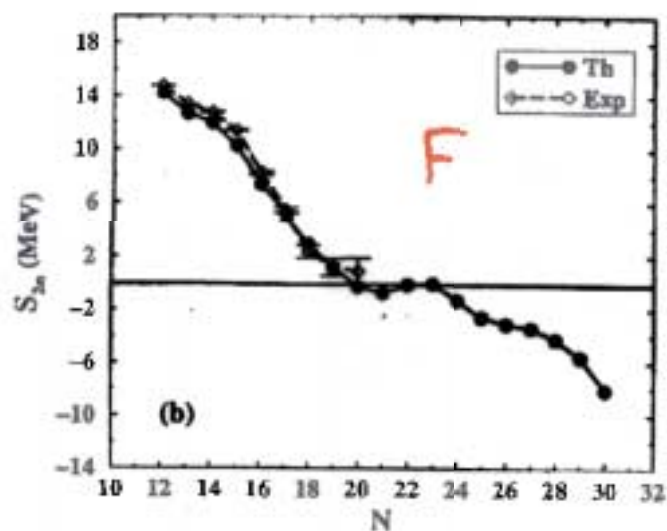
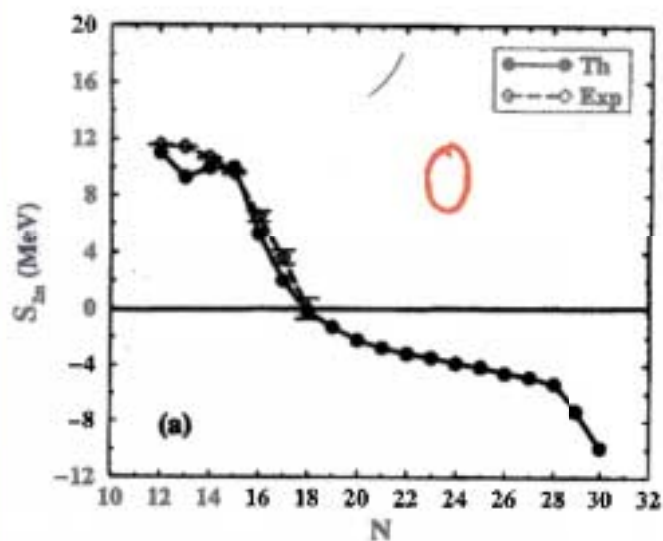
N - Z nuclei in pf shell



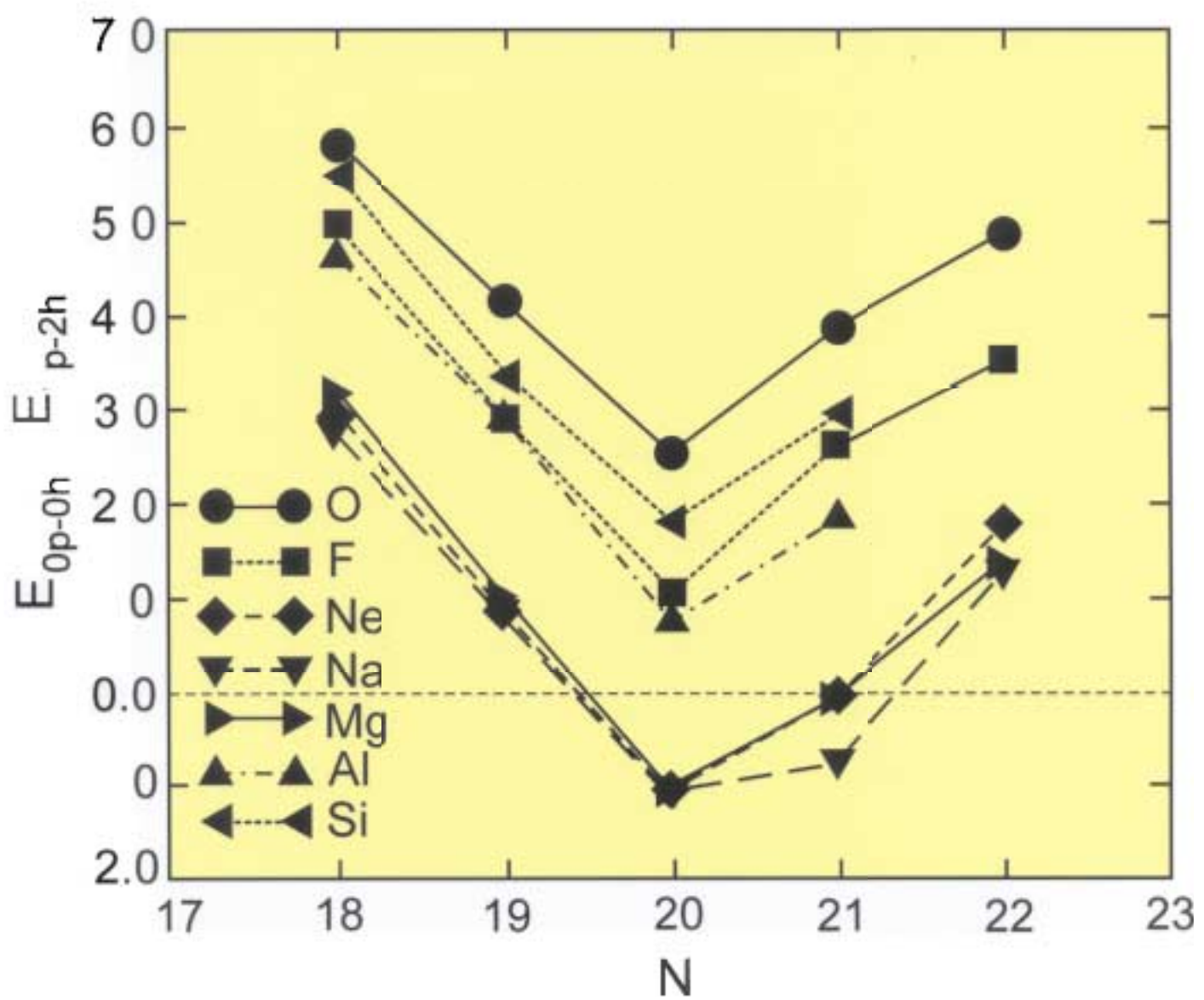
STATE-OF-THE ART SHELL-MODEL CALCULATION IN fp SHELL



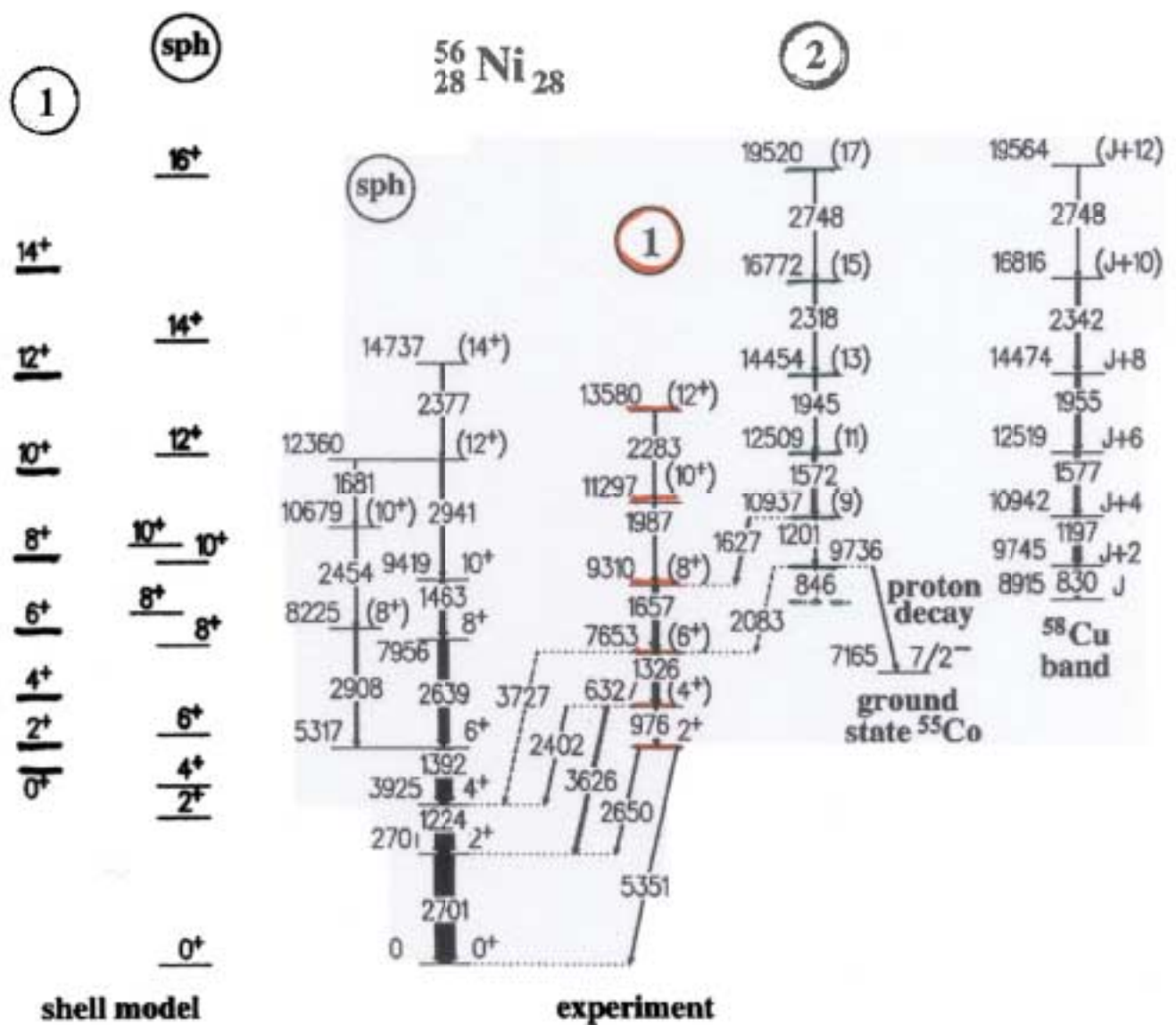
Strasbourg-Madrid
(CAURIER, NOWACKI, ZUKER,
POVES et al.)



Theoretical and experimental S_{2n} values
 E. Caurier et al, PRC 58 (1998) 2033



E. Courrier et al., PRC58 (1998), 2033



D. Rudolph et al., PRL 82 (1999), 3763

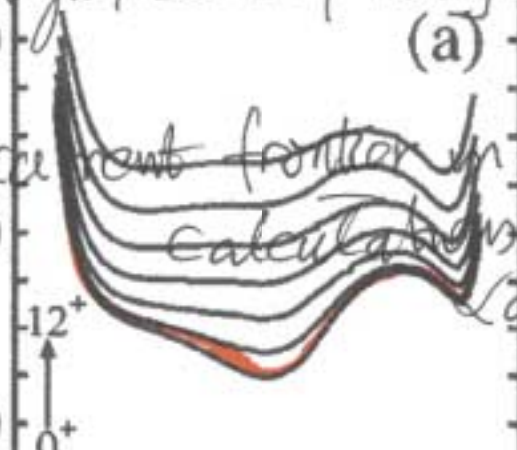
Band 1 Main components 4p-4h
Calculation in 6p-6h basis
: 25×10^6 states

Will be more det dnc $N=2$ nuclei
 to m γ rays, Lenzi, Manganeau

fp shell

$\langle H \rangle$ (MeV)

80
190
-200

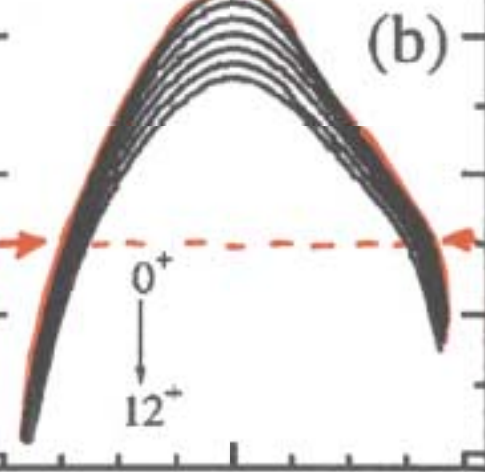


(a)

current frontier in o. m
 Calculations: standard
 Lenzi's methods

$n >$

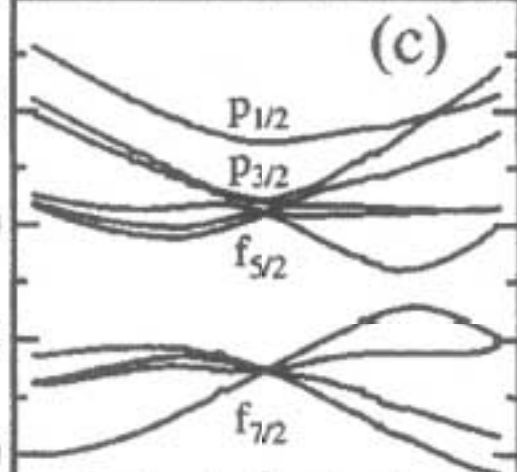
15
12
9
6



(b)

ϵ MeV

-5
-10
-15
-20



(c)

100 -50 0 50 100
 q (efm)

M Zusaki et al PRC59 (1999) R4846

Constrained Hartree Fock "CHF"

FIND PRECISE BALANCE BETWEEN

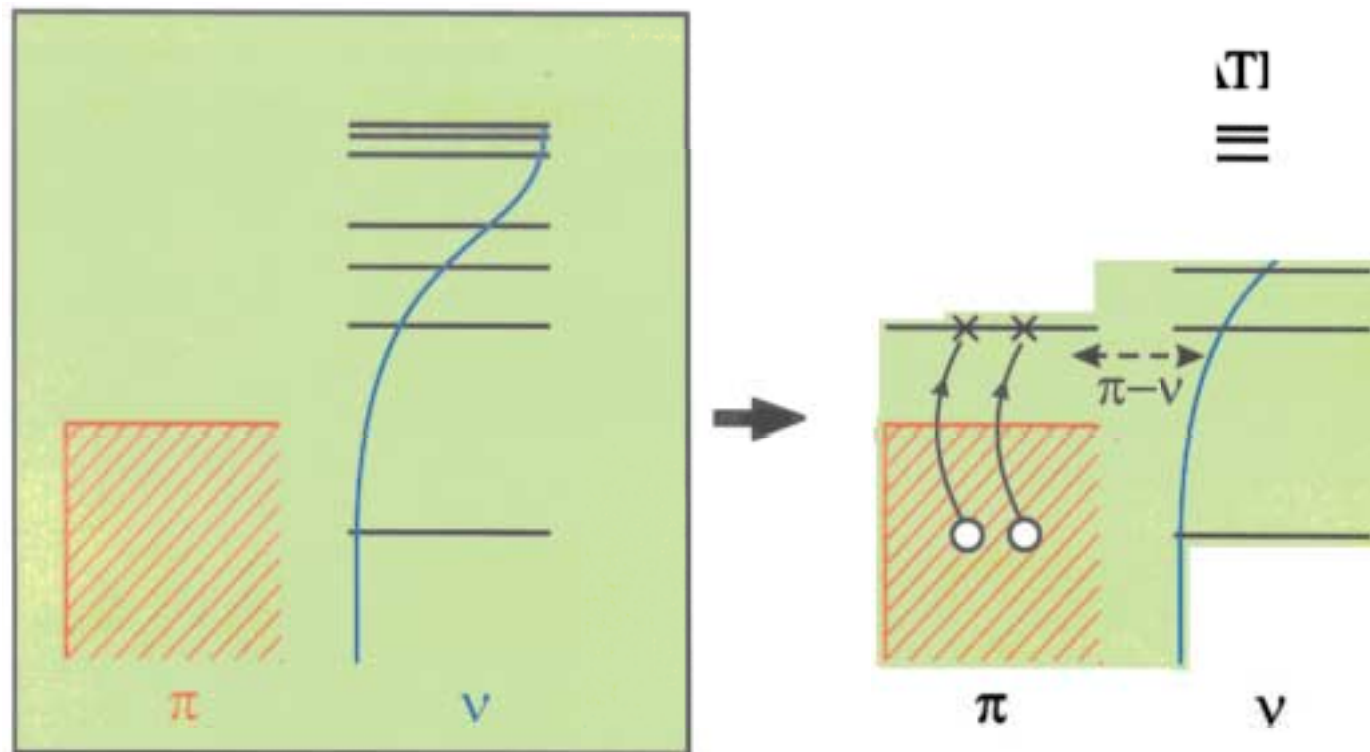
VALENCE PROTONS - “CLOSED” NEUTRON SHELL
OR SUBSHELL

IN ORDER TO MEET CONDITIONS APPROPRIATE FOR
HAVING SHAPE COEXISTENCE DEVELOP

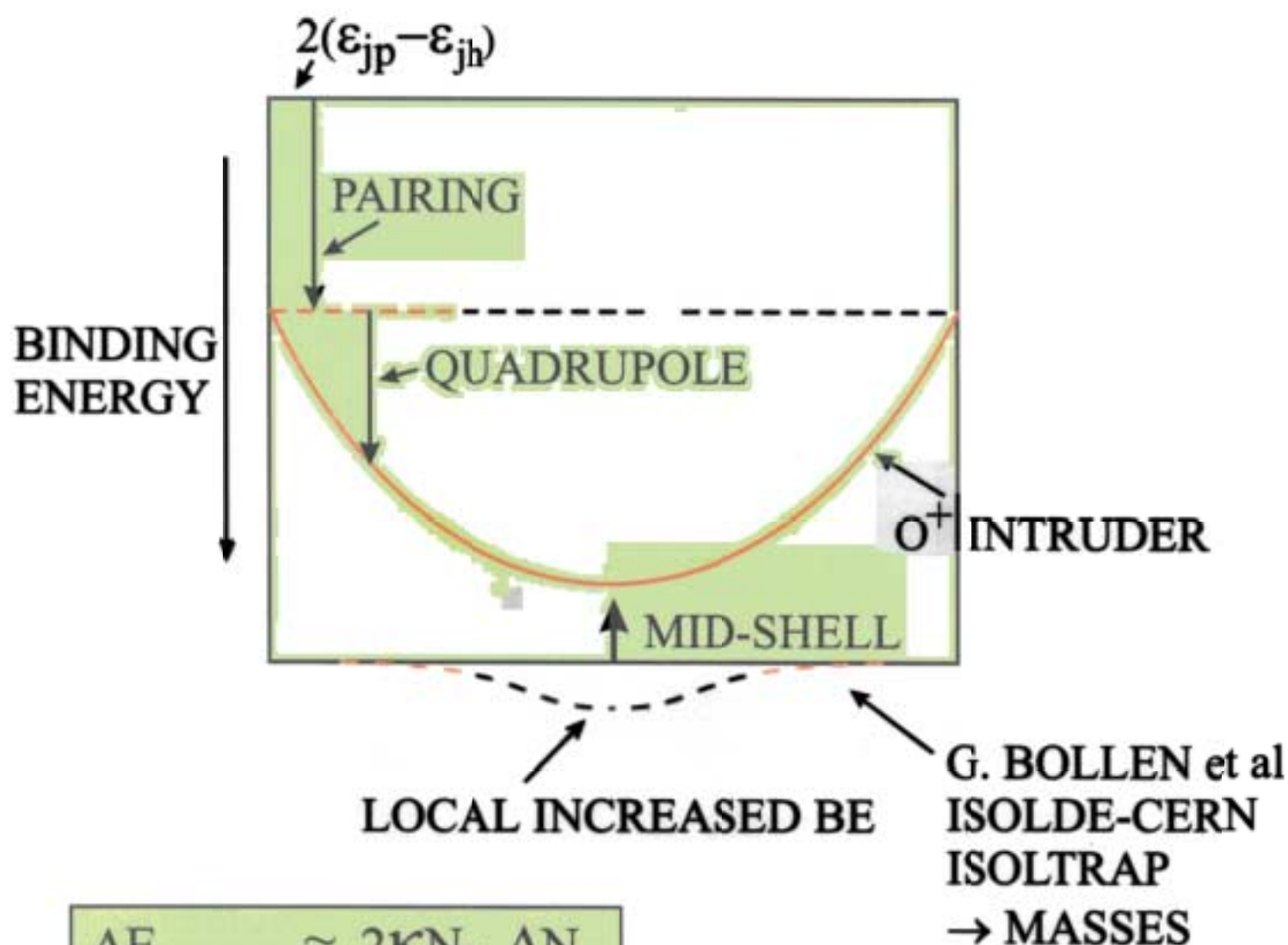
NUCLEI WITH $N=Z$ AT CLOSED SHELLS
OR SUB SHELLS

e.g. ■ Ne, Na, Mg $N=20$
■ region near ^{56}Ni

NEED FOR “IMPORTANCE” SAMPLING OF
BOTH THE $N-N$ INTERACTION AND MODEL SPACE

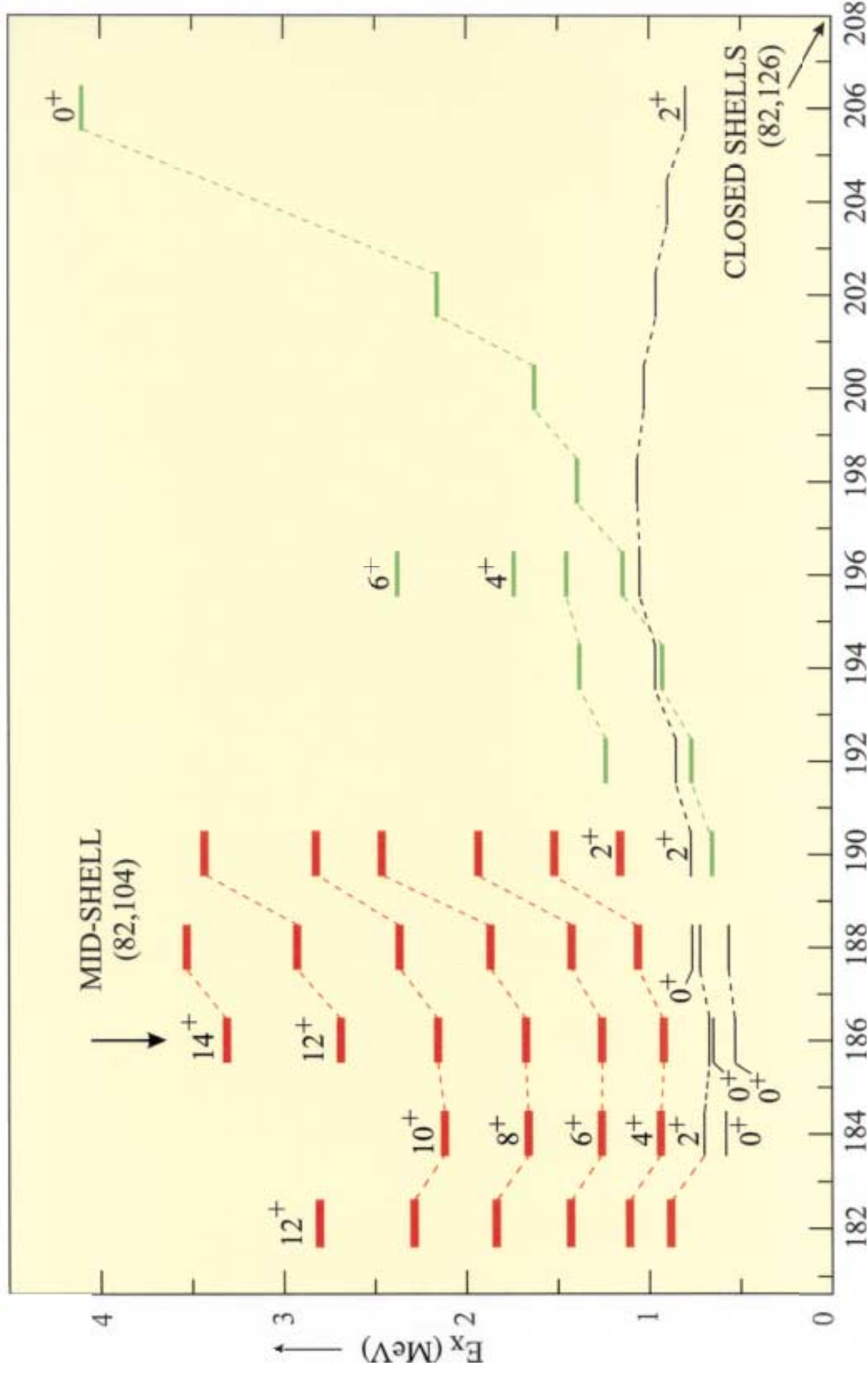


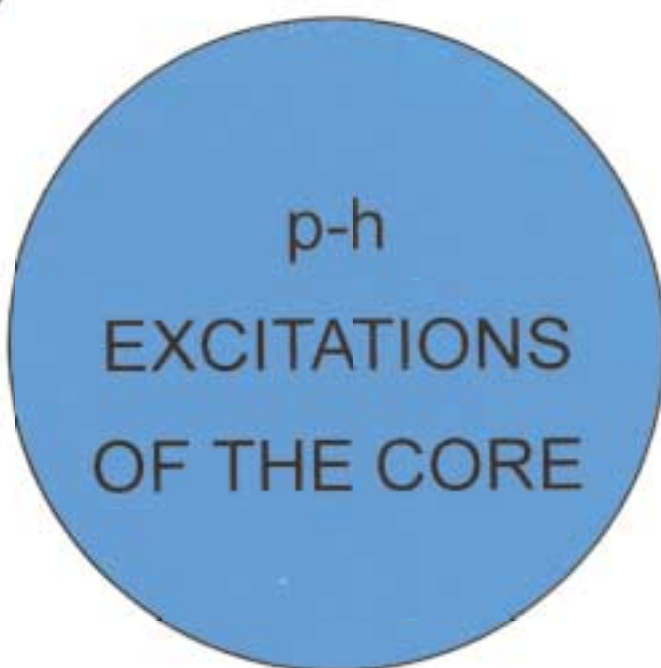
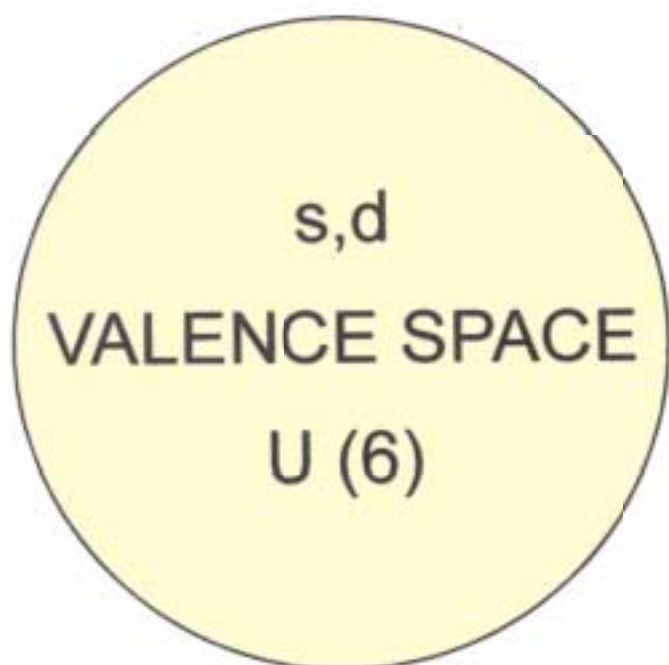
$$\Delta E = \langle O_I^+ | \hat{H} | O_I^+ \rangle - \langle O_{GS}^+ | \hat{H} | O_{GS}^+ \rangle$$



$$\Delta E_{\text{QUAD.}} \simeq 2KN_V \cdot \Delta N_\pi$$

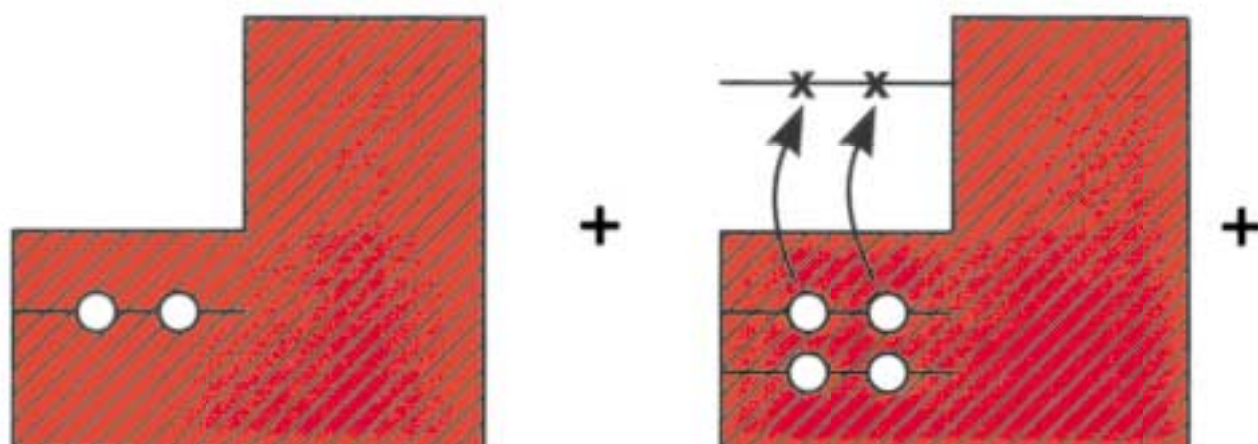
Pb NUCLEI





HOW TO
"COMBINE"
THE "BEST" OF
BOTH SPACES

CONSIDER 2p-2h, 4p-4h,.. EXCITATIONS
AS SEPARATE MODEL SPACE WITH N, N+2,
N+4 BOSONS.

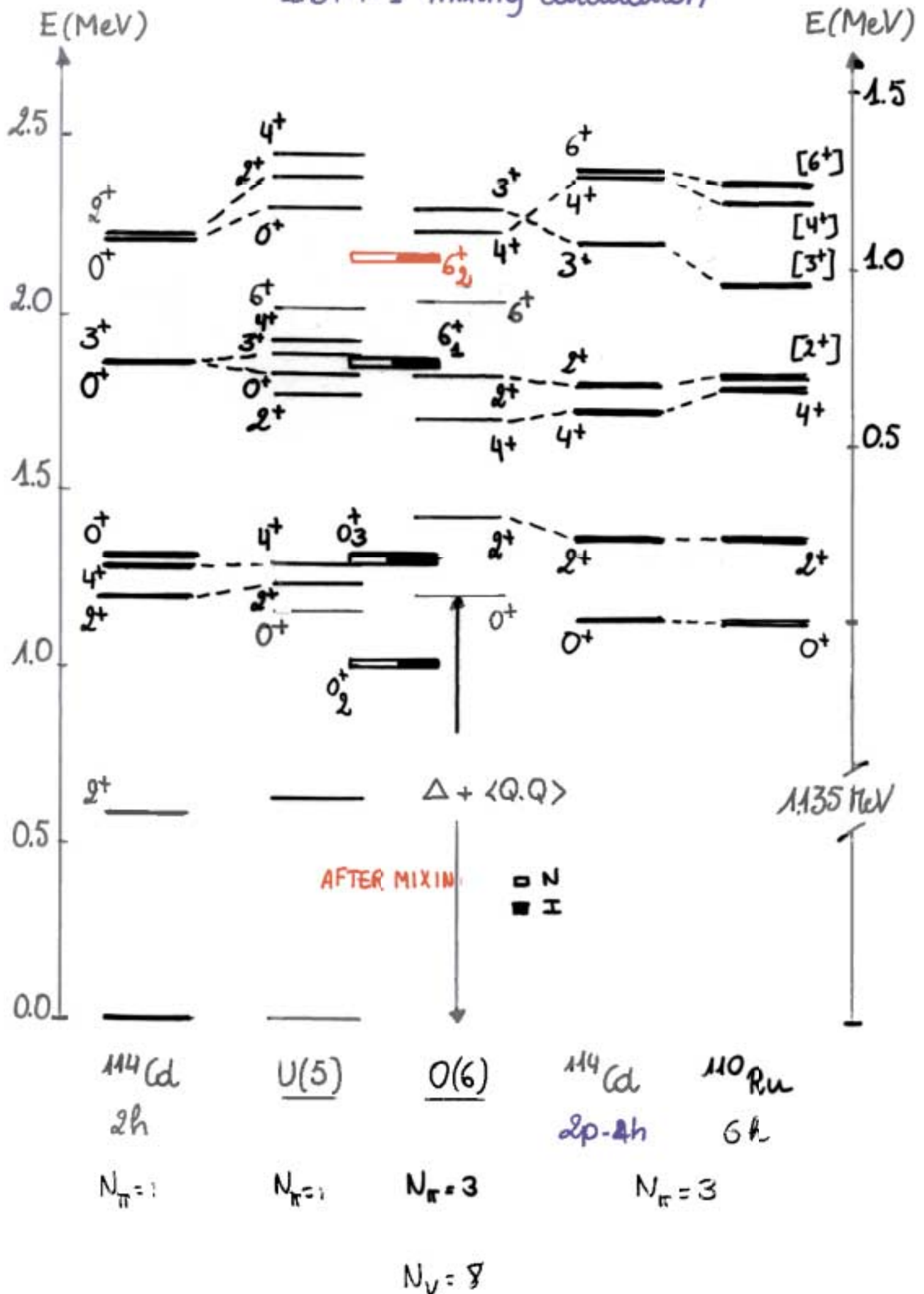


$$\begin{bmatrix} E_i(N) & \langle V \rangle & 0 \\ \langle V \rangle & E_i(N+2)+\Delta & \langle V \rangle \\ 0 & \langle V \rangle & E_i(N+4)+\Delta' \end{bmatrix}$$

$$H_{\text{mix}} = \alpha(s_p^+ s_h^+ + \text{h.c.})^{(0)} + \beta (d_p^+ d_h^+ + \text{h.c.})^{(0)}$$

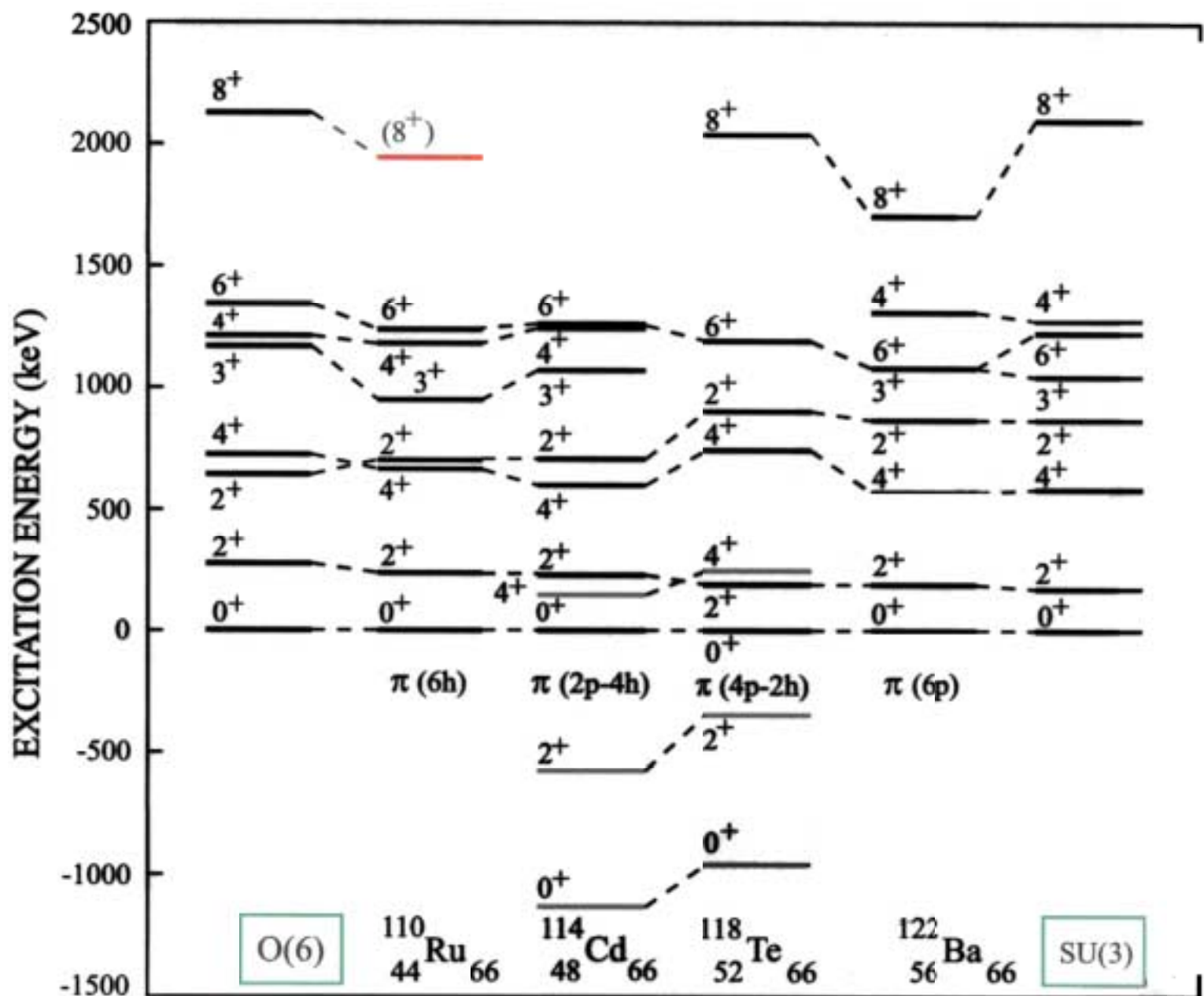
Duval, Barret, NPA 376(1982), 213
Applications in Z=50, Z=82 regions

IBM-1 mixing calculation



FULL $I = 3/2$ MULTIPLET

$$I_z = -3/2 \quad -1/2 \quad +1/2 \quad +3/2$$



B(E2) Values in 3/2 MULT PLET

Neutron Number	Cd Intruder $B(E2; 2_3^+ \rightarrow 0_A^+)$	Ru $B(E2; 2_1^+ \rightarrow 0_1^+)$	Ba $B(E2; 2_1^+ \rightarrow 0_1^+)$
62	23^{+27}_{-18}	--	--
64	56^{+17}_{-17}	58^{+5}_{-5}	--
66	61^{+8}_{-8}	70^{+5}_{-5}	154^{+14}_{-14}
68	86^{+24}_{-30}	74^{+7}_{-7}	116^{+6}_{-6}
70	--	--	98^{+16}_{-16}

May 25

Close this window to return to the previous window

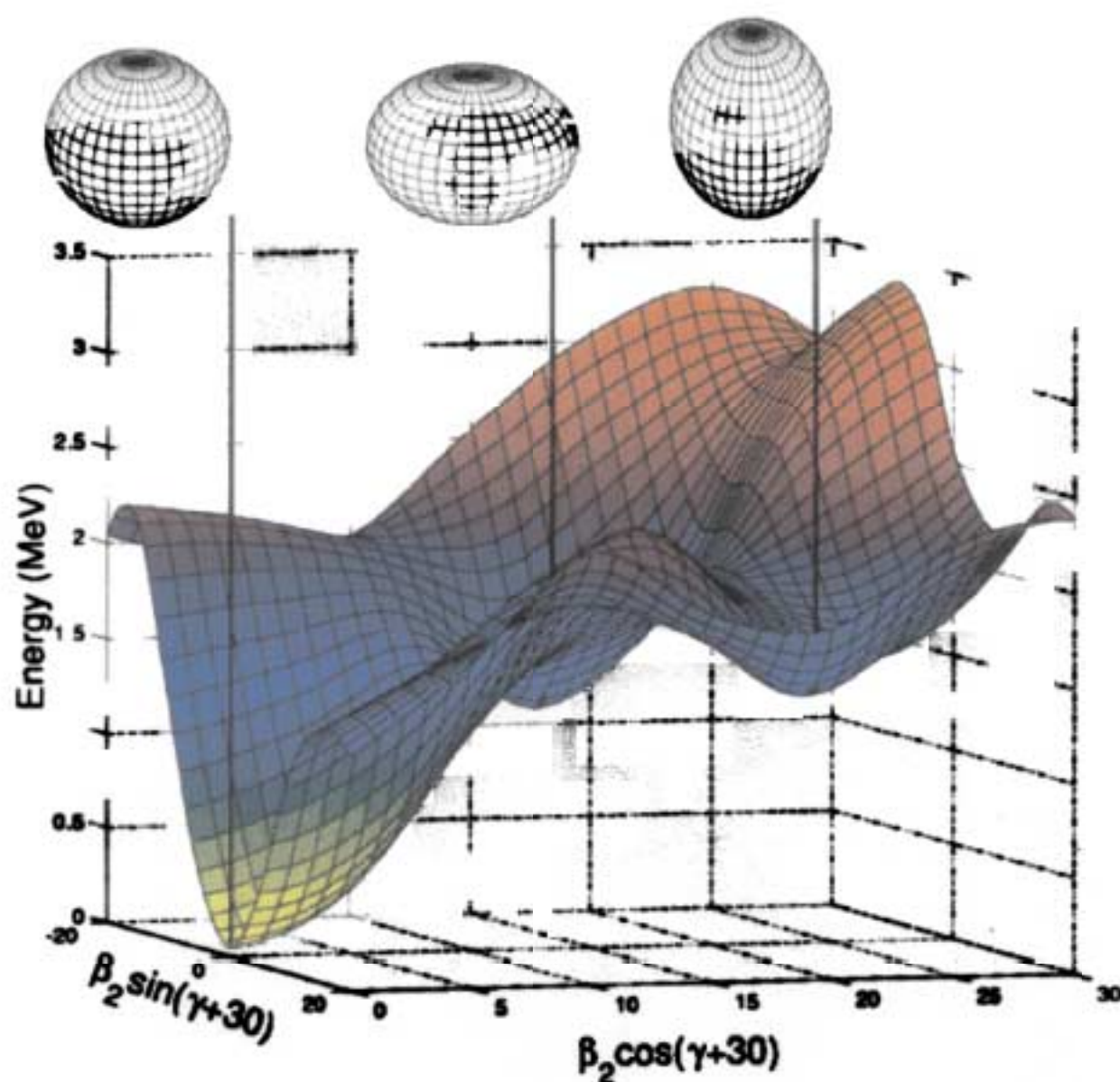


Figure 1 Calculated potential energy surface of ^{186}Pb . Spherical, oblate and prolate minima are indicated by thick vertical black lines. Calculations are performed on the cartesian mesh. The β_2 parameter expresses the elongation of the nucleus along the symmetry axis, while the γ parameter relates to the degree of triaxiality in the deformation (that is, the relative lengths of the three principal axes of the spheroidal nucleus). The γ parameter is defined such that $\gamma = 0$ corresponds to a prolate (cigar-like) shape and $\gamma = 60$ to an oblate (disk-like) shape.

← item →

Note: Figures may be difficult to render in a web browser. In such cases, we recommend

HF + BCS method

STATE $|\psi(q)\rangle$ GENERATED BY A SELF-CONSISTENT MEAN-FIELD CALCULATION, USING A CONSTRAINT ON A COLLECTIVE COORDINATE q , STARTING FROM $V(i,j)$



INTRINSIC STATE



BREAKS A NUMBER OF SYMMETRIES OF "EXACT" MANY-BODY STATES $\rightarrow$ NEED TO GO TO LAB FRAME



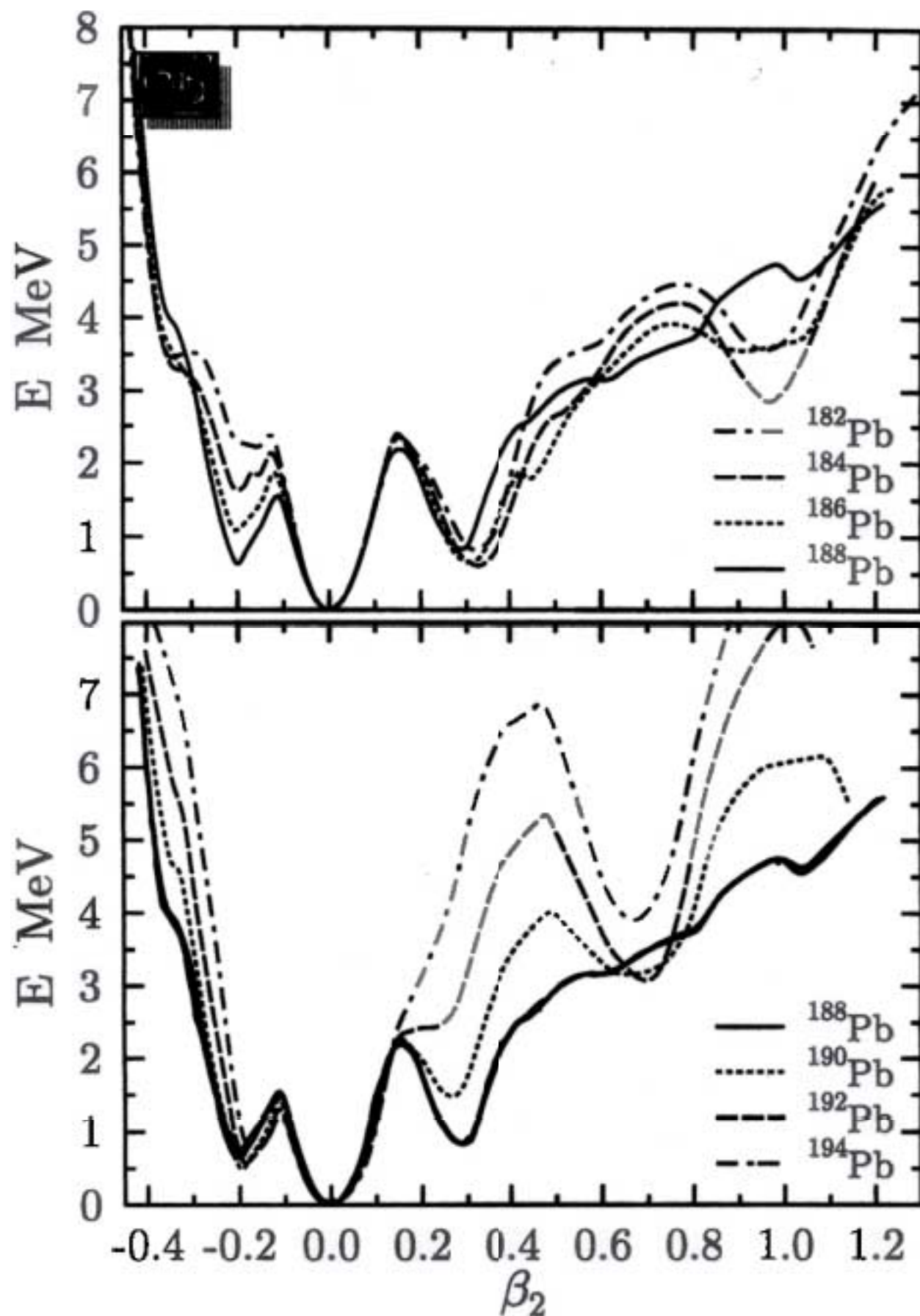
PROJECT ON $J^\pi, N, Z, T, \dots \rightarrow |J, M, q\rangle$



VARIATIONAL CONFIGURATION MIXING IN THE COLLECTIVE COORDINATE q , FOR EACH J^π

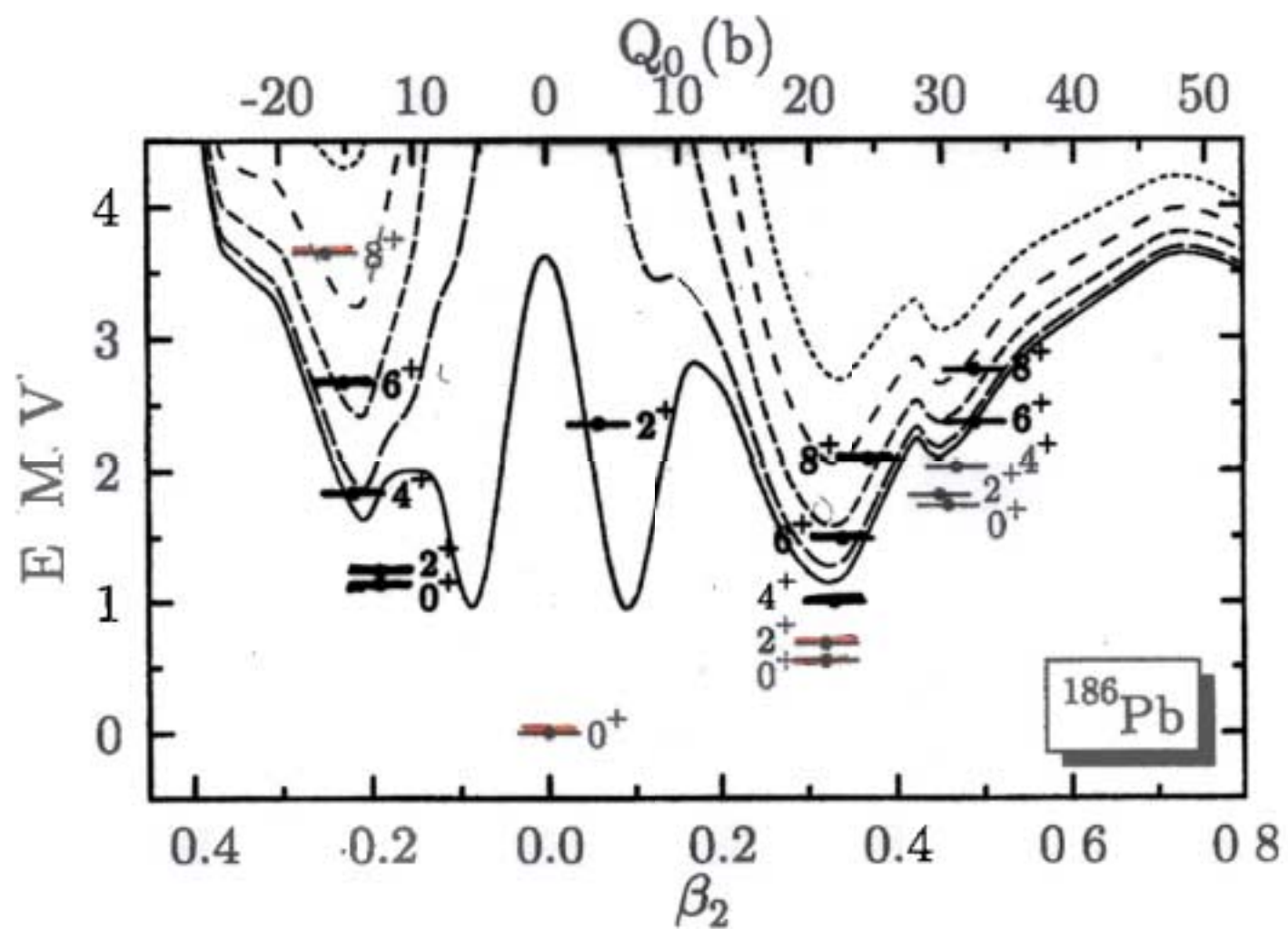
$$|JM, k\rangle = \sum_q f_{J,k}(q) |JM, q\rangle$$

$$E_{J,k} = \frac{\langle JM, k | \hat{H} | JM, k \rangle}{\langle JM, k | JM, k \rangle}$$



Particle number projected deformation energy curves for $^{182}-^{194}\text{Pb}$

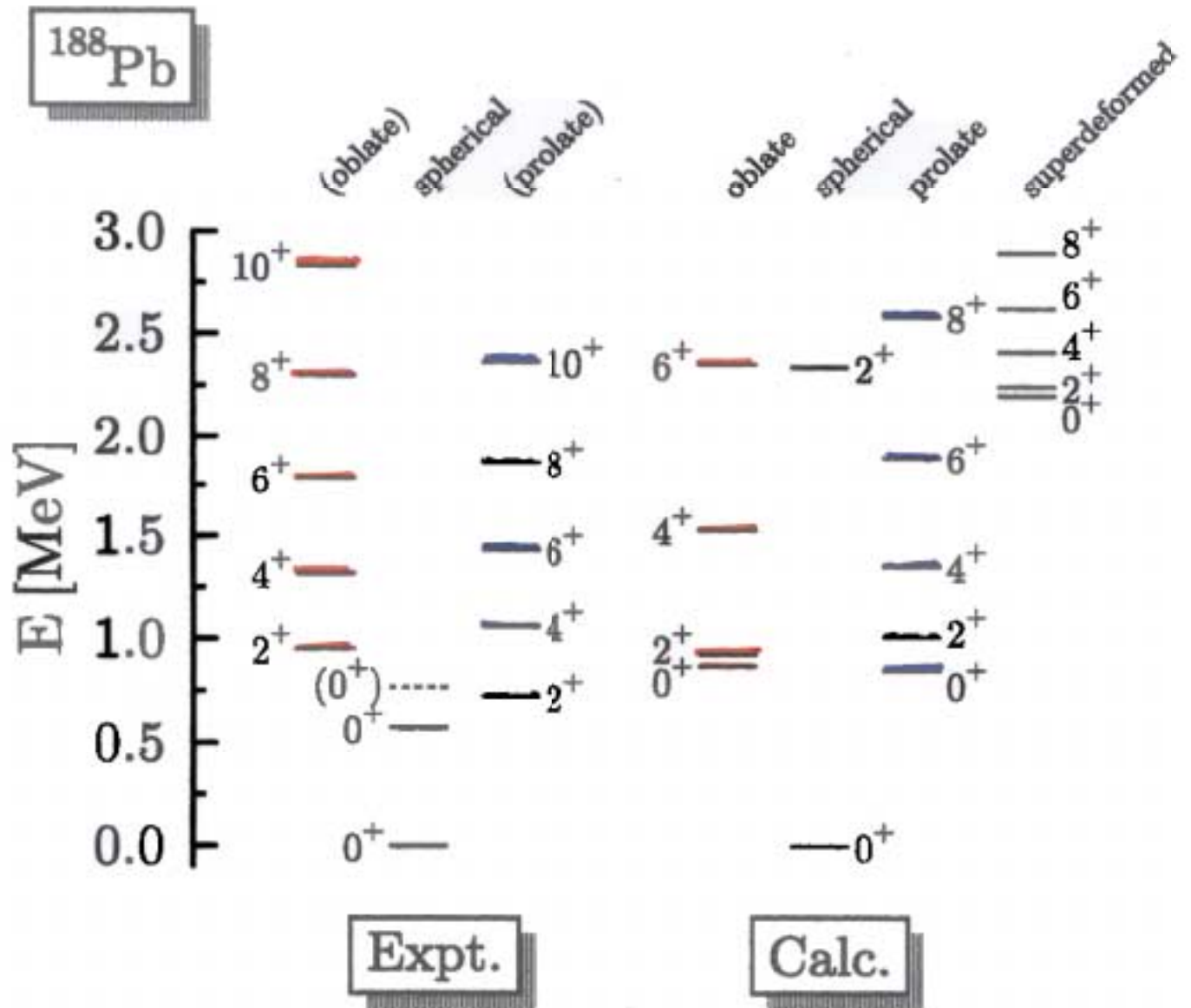
M Bender et al, t.b.p



spectrum of the lowest positive parity bands with even and $K=0$

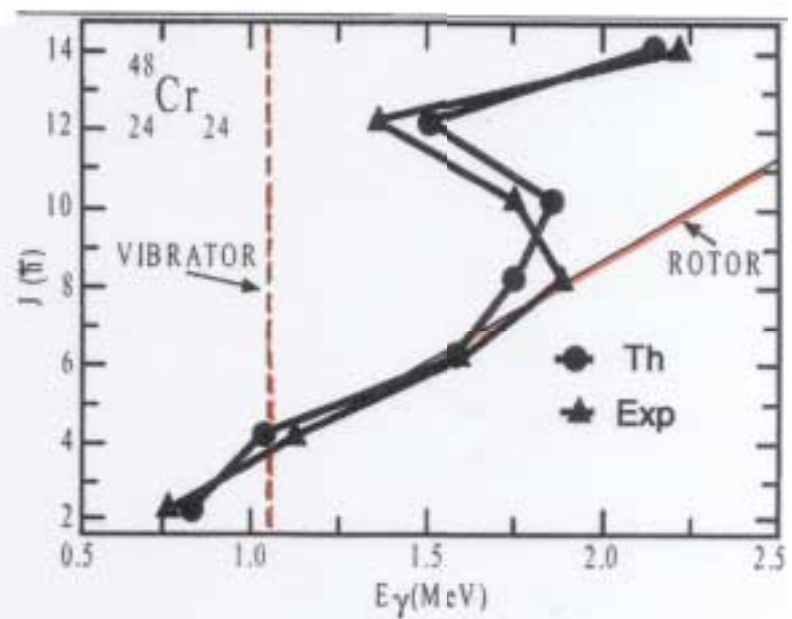
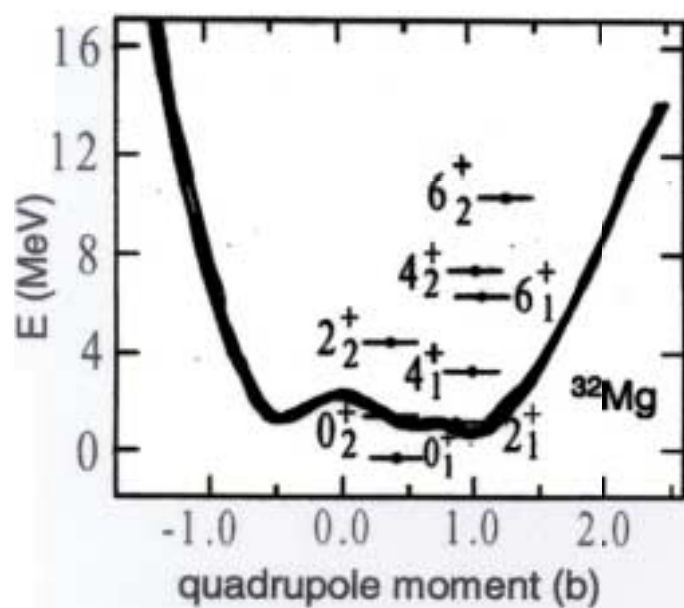
The angular momentum projected energy curves are also given

Duquet et al. (PLB 559 (2003), 20)



6. Fender et al. 196 p

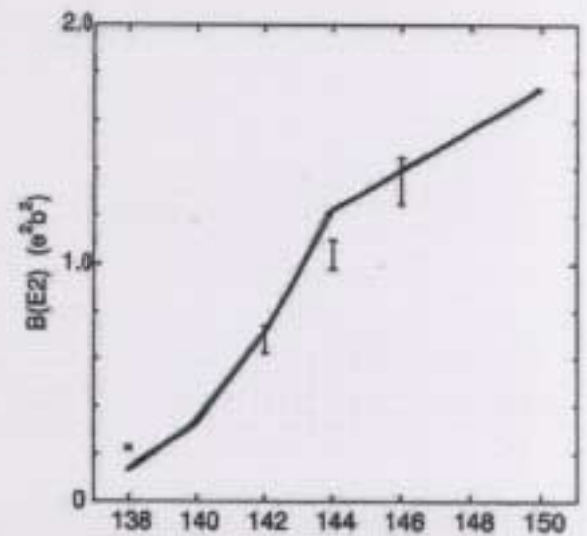
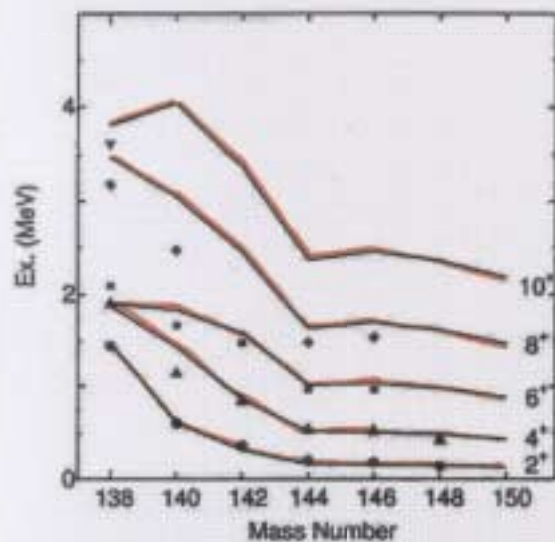
MERGING MEAN-FIELD AND SHELL-MODEL APPROACHES



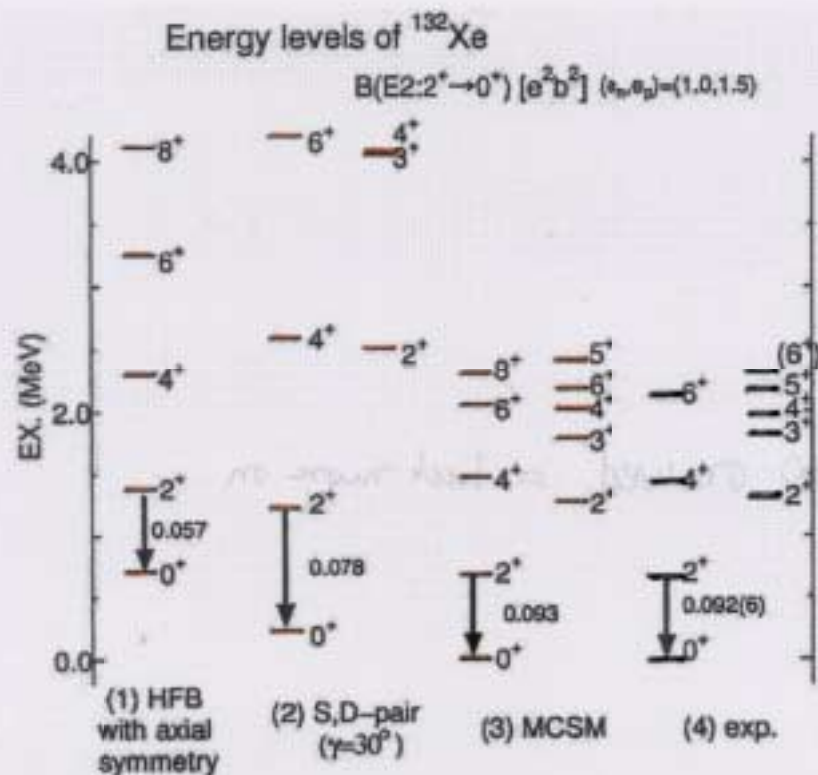
METHOD FOR DIAGONALIZING HAMILTONIANS FOR NUCLEAR MANY-BODY SYSTEMS USING QUANTUM MONTE CARLO METHODS (SMMC / Otsuka et al.) : CALL IT AN "IMPORTANCE TRUNCATION SCHEME" (SIMILARITIES WITH SELF-CONSISTENT MEAN-FIELD METHODS + CONFIG. MIXING CALC.)

RECENT DEVELOPMENT : OTSUKA et al. MONTE-CARLO SELECTION OF BASIS STATES REDUCTION OF BASIS TO 30~50 STATES

MONTE-CARLO SHELL-MODEL (MCSM)



BA NUCLEI



GENERAL SOLUTION TO COLLECTIVE QUADRUPOLE MOTION - BOHR HAMILTONIAN SOLUTIONS

$$\hat{H}\psi = E\psi$$

$$\hat{H} = \frac{-\hbar^2}{2B} \left[\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} - \frac{1}{4\beta^2} \sum_{\kappa} \frac{Q_{\kappa}^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)} \right] + V(\beta, \gamma)$$

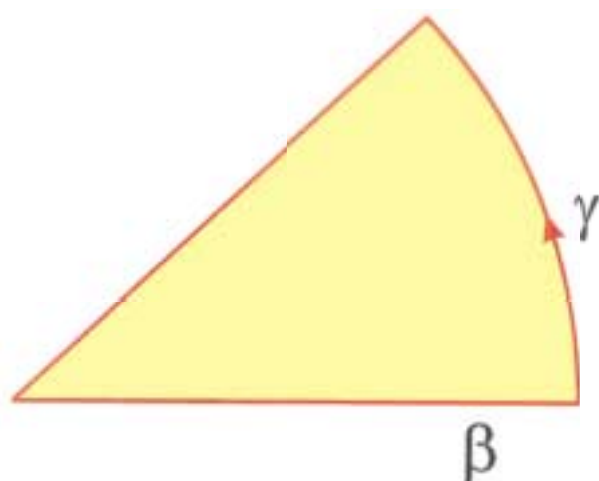
$$\psi(\beta, \gamma, \theta_i) \Rightarrow$$

RECENT STUDIES

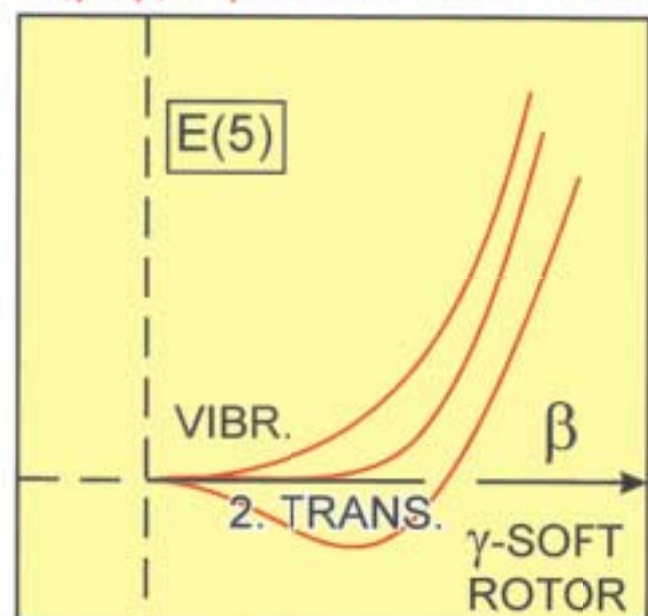
F. Iachello, PRL 85 (2000), 3580; PRL 87 (2001) 052502; PRL 91 (2003), 132502

DISCUSS 3 INTERESTING EXACT SOLUTIONS FOR APPROXIMATE POTENTIALS TO $V(\beta, \gamma)$

CRITICAL POINTS

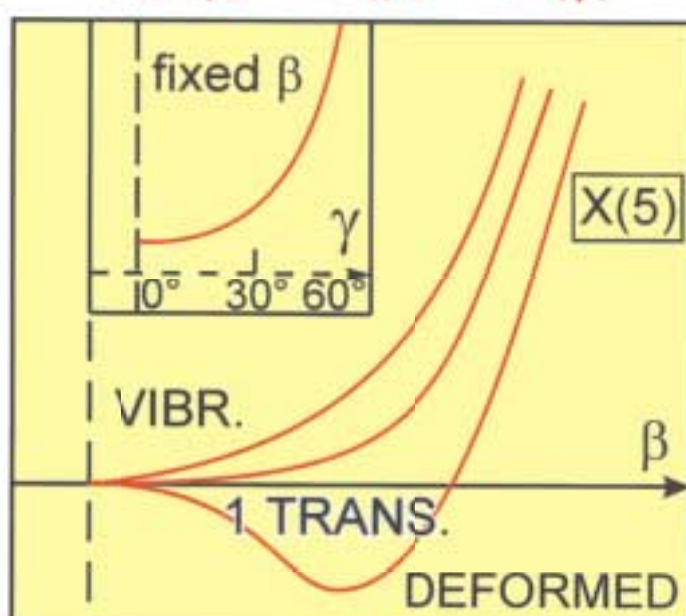


$V(\beta, \gamma) : \gamma$ INDEPENDENT

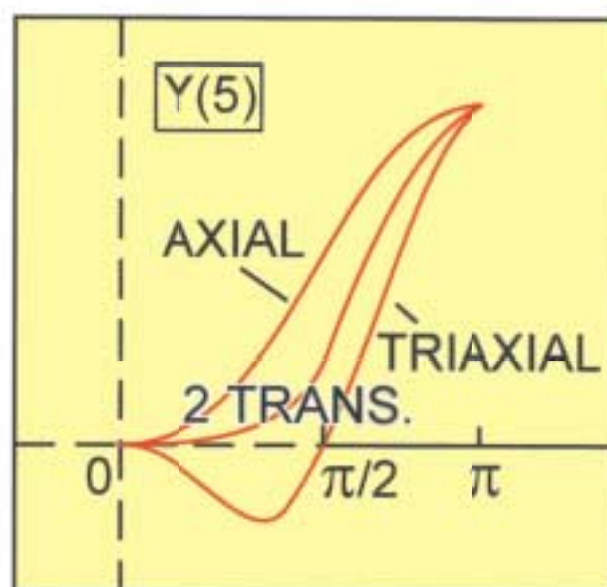


VIBRATIONAL $\rightarrow$ γ - SOFT ROTOR

$V(\beta, \gamma) = V'(\beta) + V''(\gamma)$



VIBRATIONAL $\rightarrow$ DEFORMED

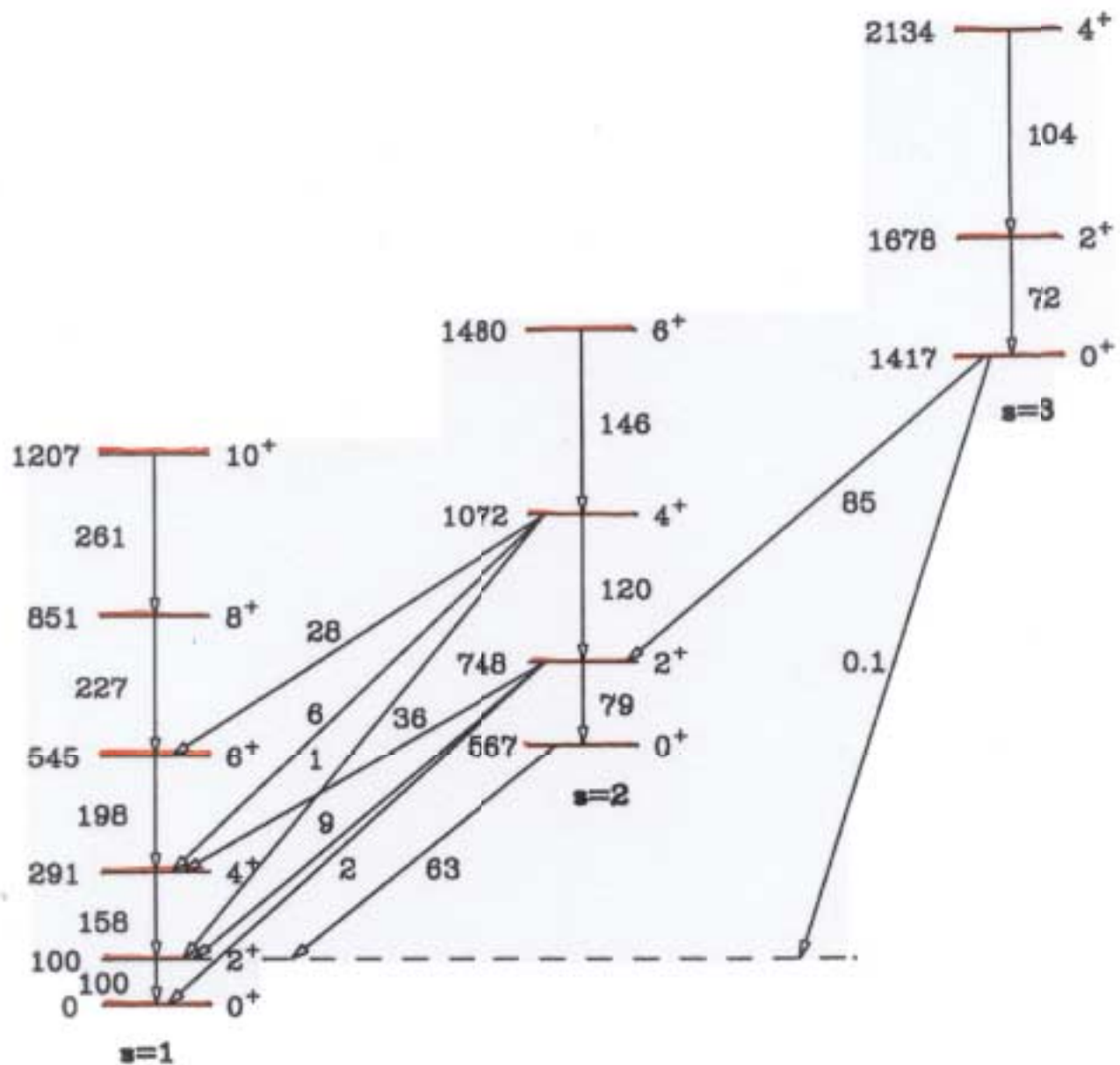


AXIAL $\rightarrow$ TRIAXIAL

$V(\beta, \gamma) = V'(\beta) + V''(\gamma)$

$$\frac{u_0}{2}(\beta - \beta_0)^2$$

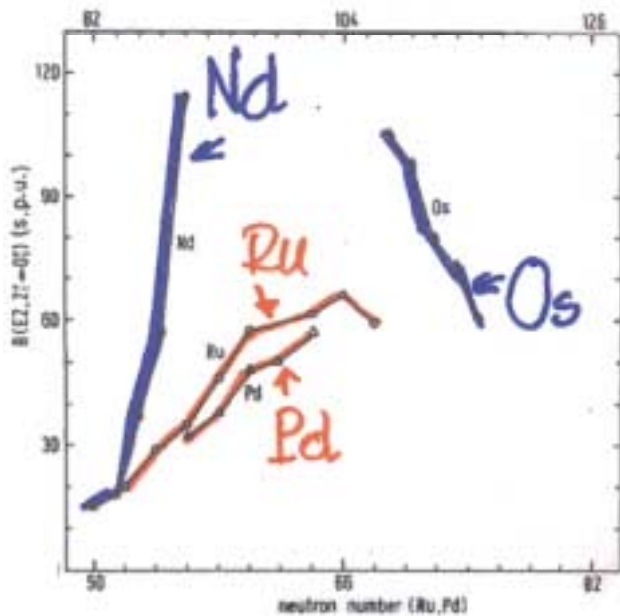
$$E(s, L, n_\gamma, K, M) = E_0 + A \times \frac{L^2}{s, L} + C n_\gamma + D K^2$$



Only states with $n_\gamma = 0$

POSSIBLE EXPERIMENTAL TEST

(a) Ru, Pd REGION : transitional region from vibrational to γ -soft energy spectra



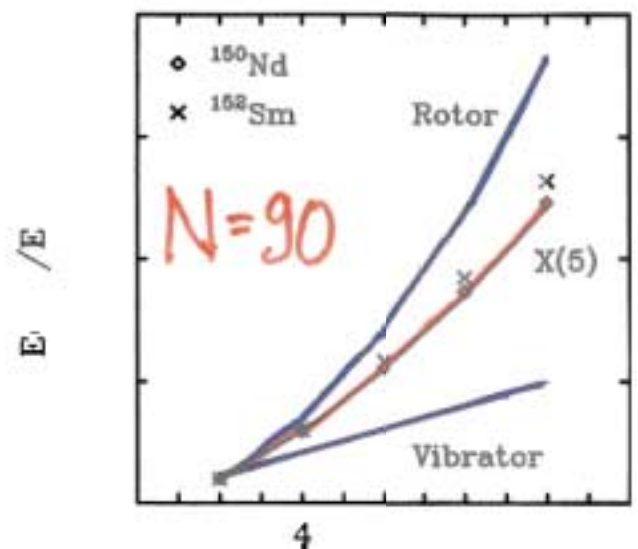
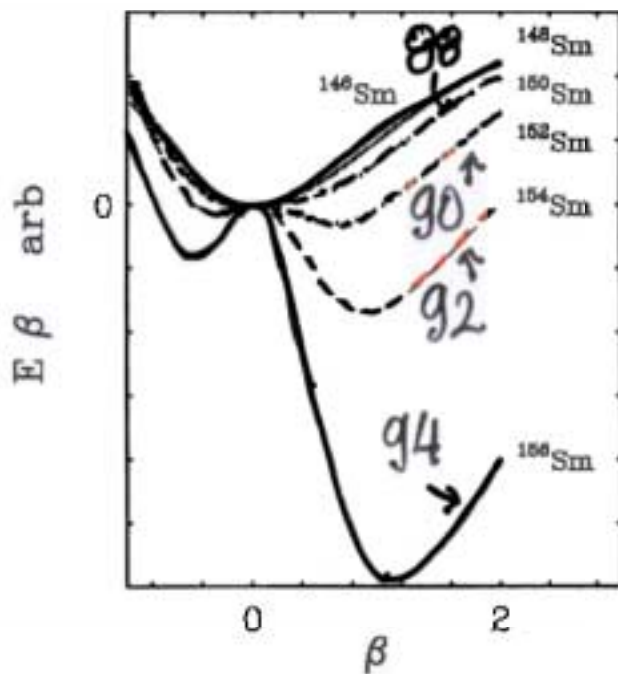
A. Frank et al, PRC 65 (2001), 014301
 V.M Zamfir et al., PRC 65 (2002), 044325

Ba, Xe REGION

R.F. Casten, V.M. Zamfir, PRL 85 (2001), 3584 : ^{134}Ba

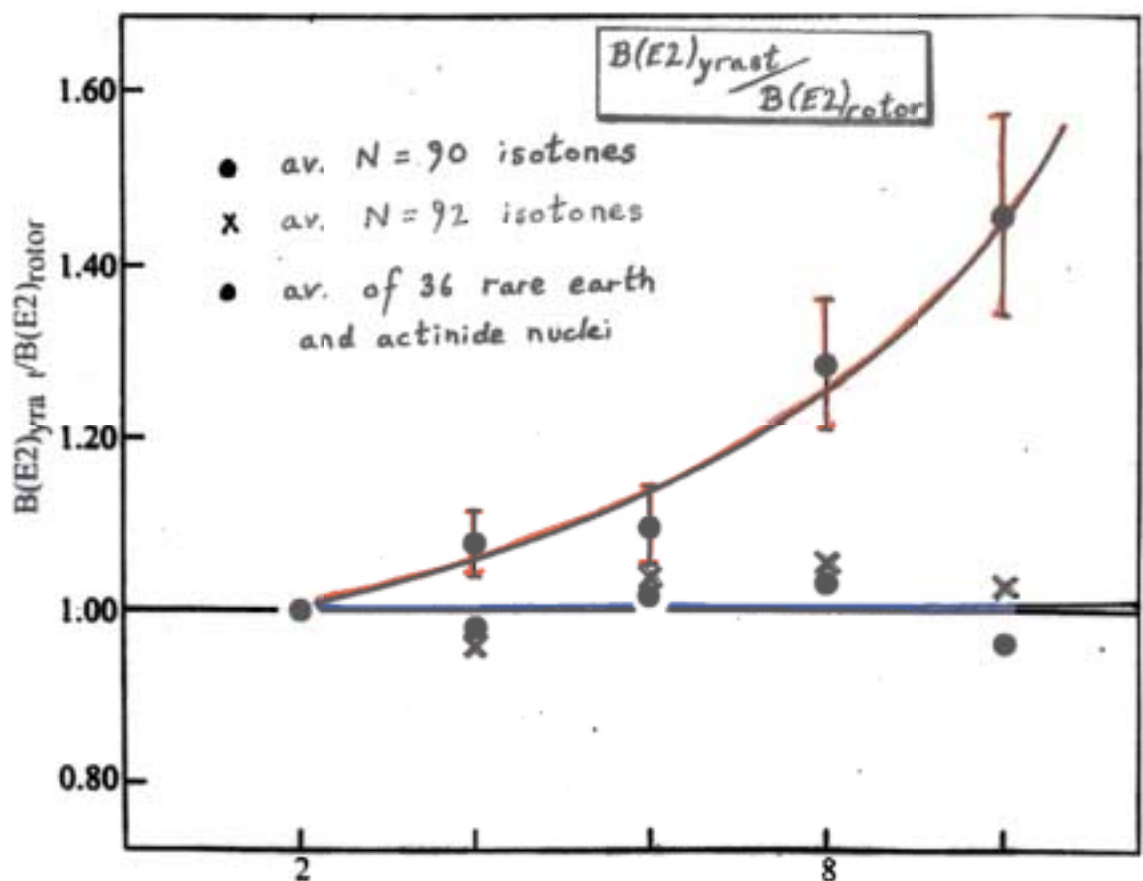
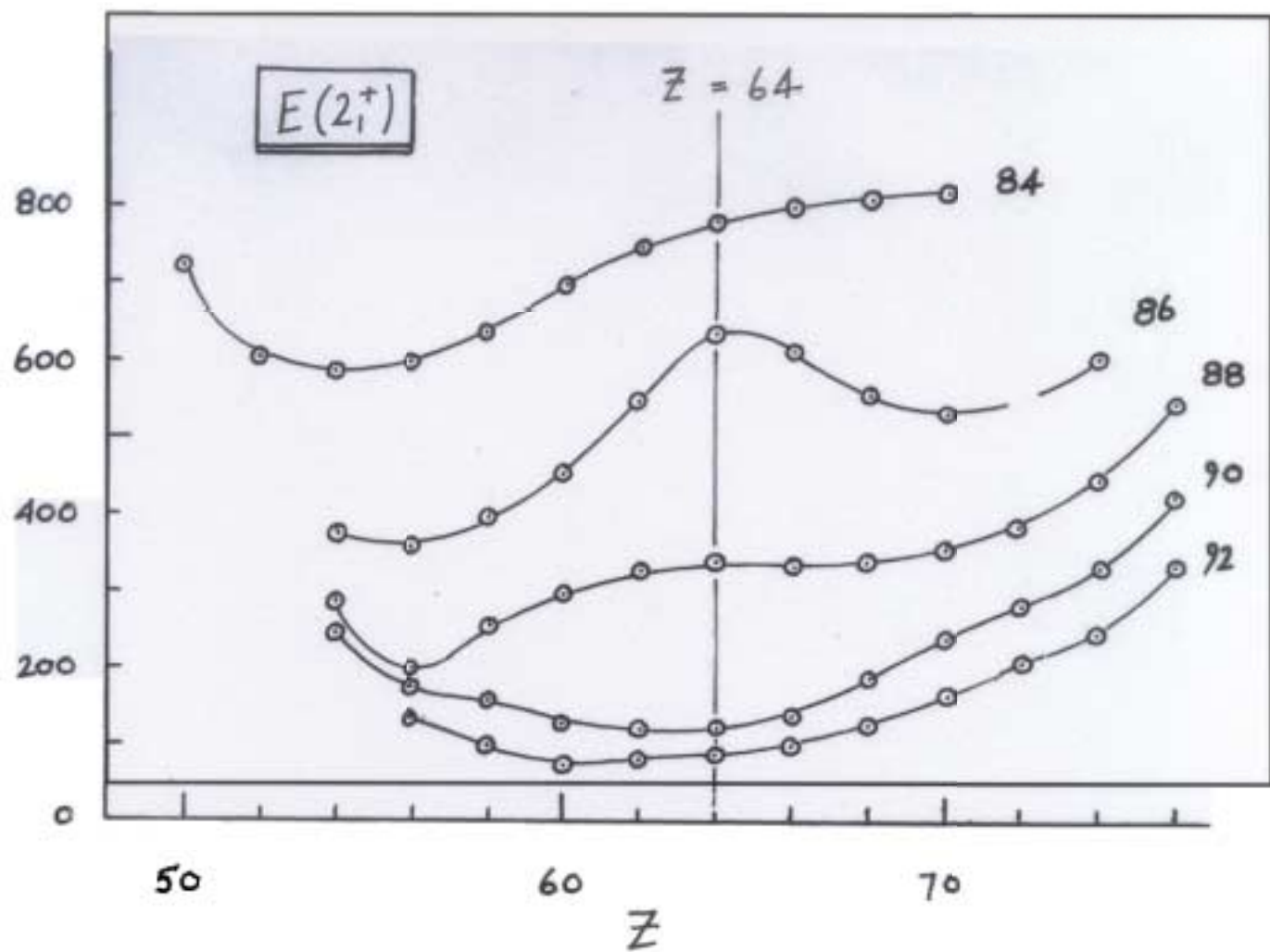
(b) Nd, Sm, Gd REGION NEAR $N = 90$

possible transition region from vibrational
towards axial deformed rotor

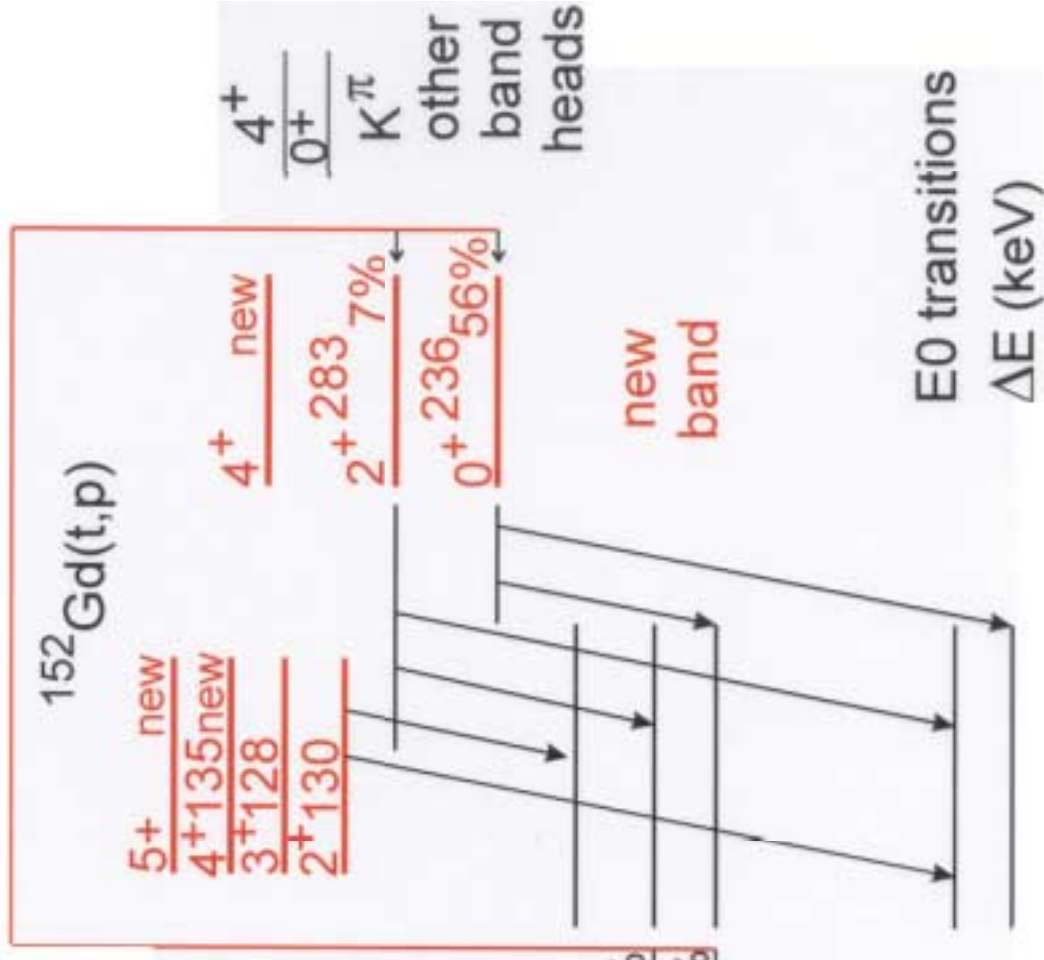
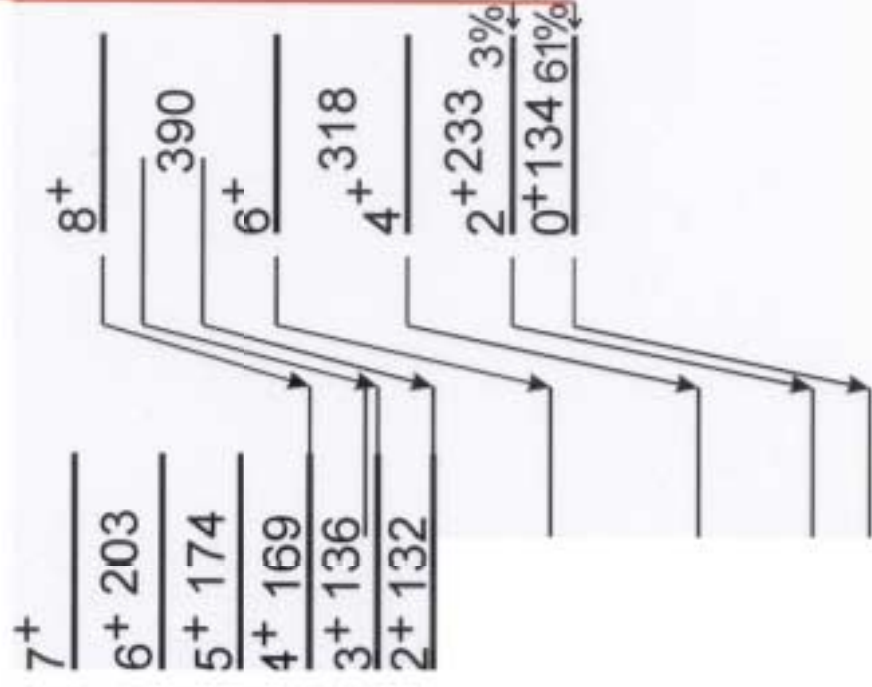
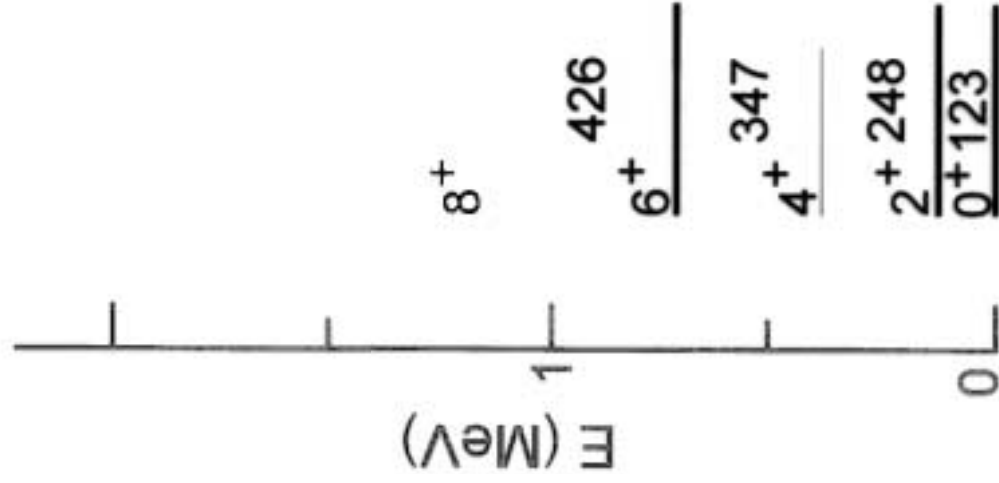


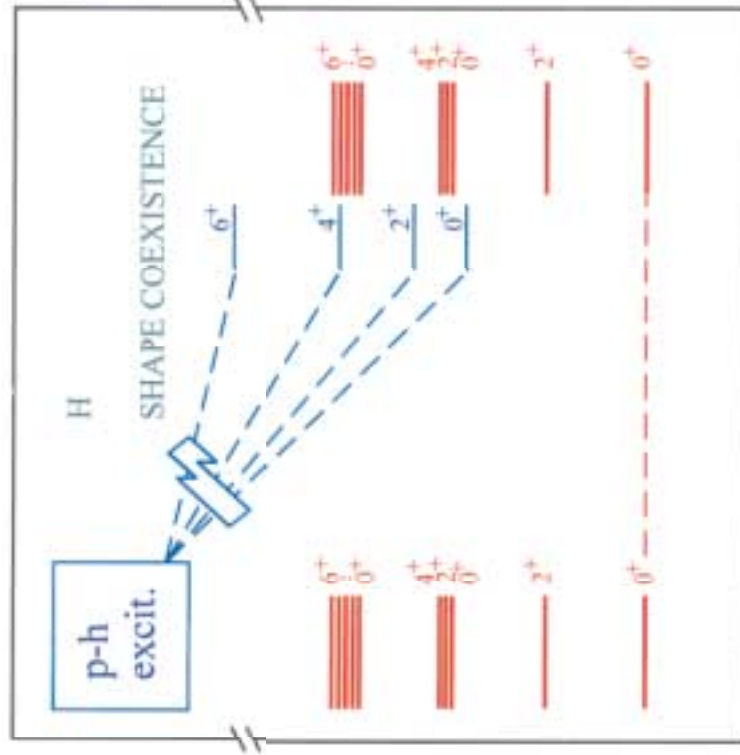
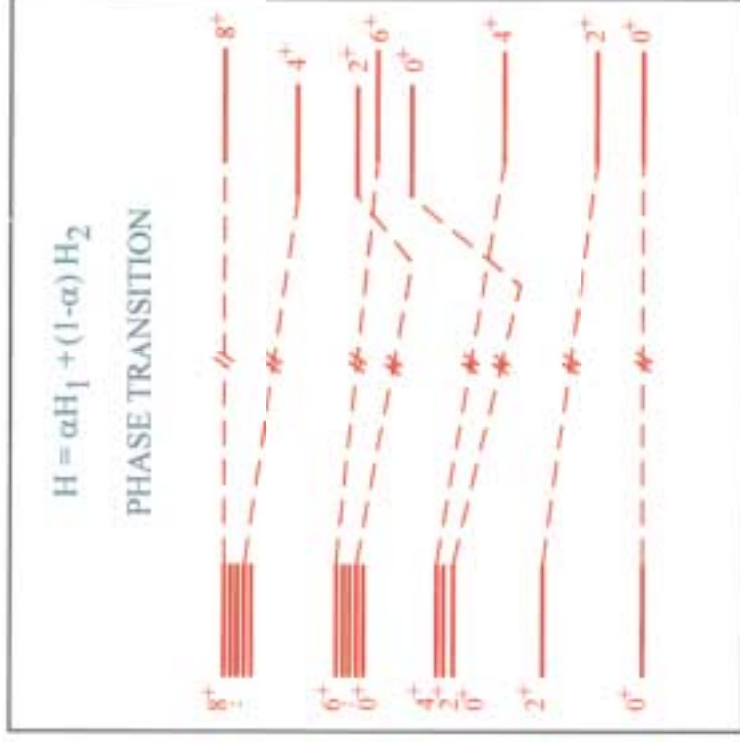
R Krucken et al PRL 88 (2002), 23250 ^{150}Nd
 R V Casten V M Zamfir PRL 87 (2001) 052503 ^{152}Sm
 W D Kulp J L Wood et al PRL 91 (2003) 025011 ^{154}Gd

Evidence for the Z = 64 subshell gap from $E(2_1^+)$ vs Z and



^{154}Gd

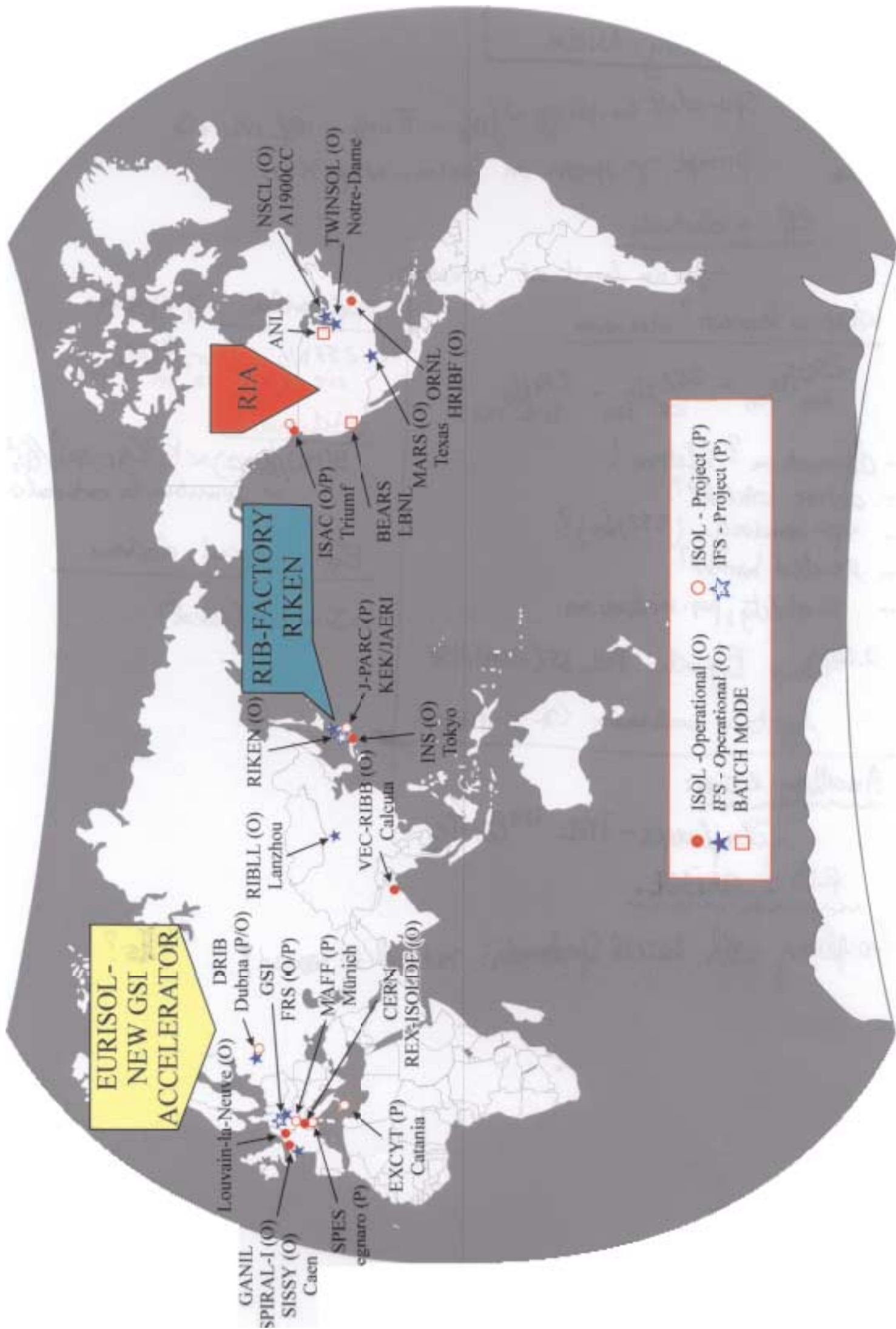




- TRANSITION THROUGH LONG SERIES OF ISOTOPES (ISOTONES)
- TRANSITION POINTS LOCALIZED

- LIGHT $N = Z$ NUCLEI AT SUBSHELL AND SHELL CLOSURE
- CLOSED PROTON SHELL - LARGE OPEN NEUTRON SHELL

np - mh excitations



COLLABORATORS (over the years)

B. DECROIX	Gent *
J. DE BEULE	Gent *
C. DE COSTER	Gent *
R FOSSION	Gent
N. SMIRNOVA	Gent
K. HEYDE	Gent
JOLIE	Köln
H. LEHMANN	Grenoble
P. VAN ISACKER	Ganil
J. WOOD	Georgia Tech., Atlanta

MUCH INPUT FROM EXPERIMENTAL GROUPS

IKS Leuven, Jyväskylä, ISOLDE-CERN, ANU-
Canberra, Oak-Ridge, GSI, Mainz, Ganil, ANL,
Liverpool,...