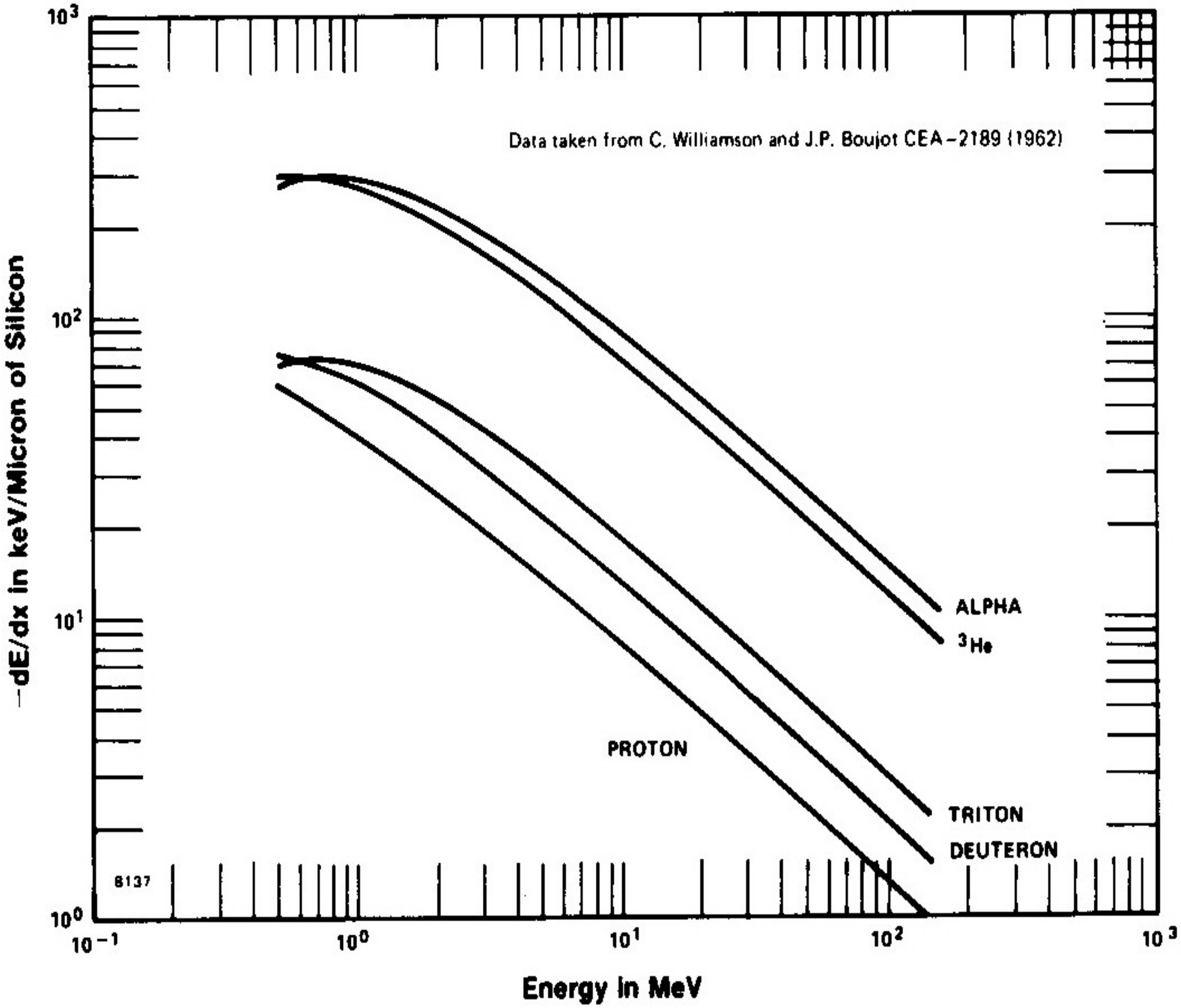
Stopping Power in Silicon

Important for applications in radiation detection



Atomic density = $5 \cdot 0.10^{22}$ atoms/cm^3

Mass density
$\rho = 2.35$g/cm^3

$1\mu m \triangleq 0.235$ mg/cm^2 Si

$\Delta E/ip = 3.62$ eV/ip

Heavier ions have higher stopping power (dE/dx)

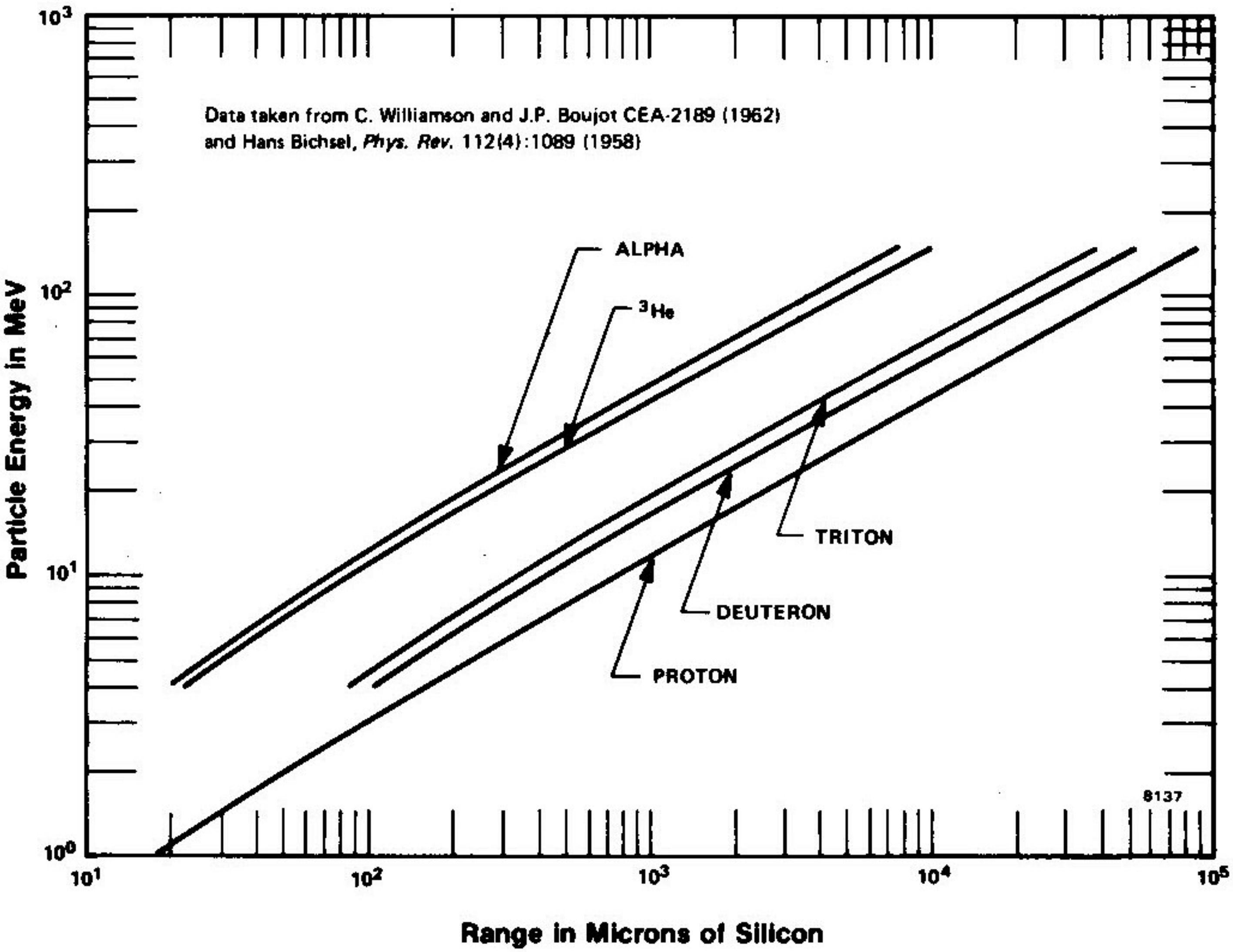
Range in Silicon

Important for applications in radiation detection

Mass density ρ = 2.35g/cm^3

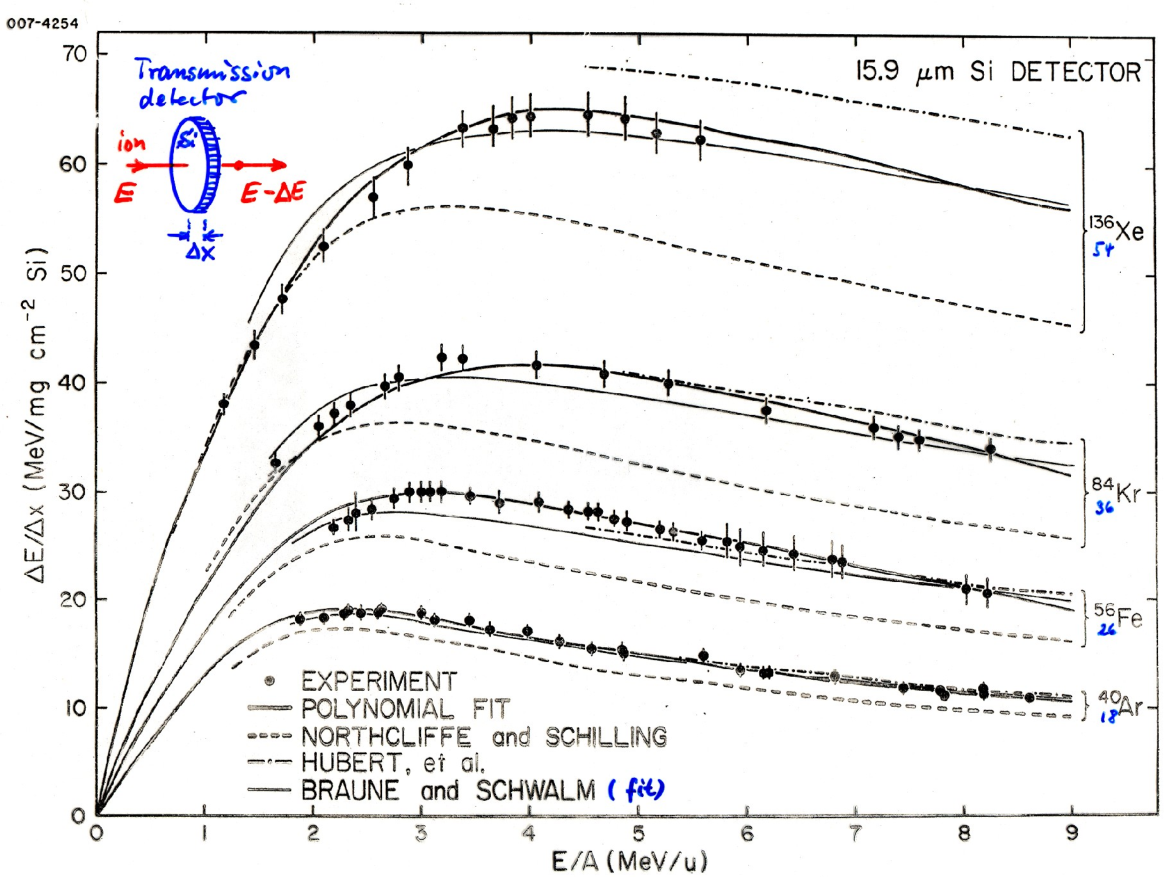
1μm $\triangleq$ 0.235mg/cm^2 Si

$\Delta E/ip$ = 3.62 eV/ip



To reach a certain depth, heavier ions must have a higher energy, since they have a higher dE/dx.

Theory and Practice for Very Heavy Ions



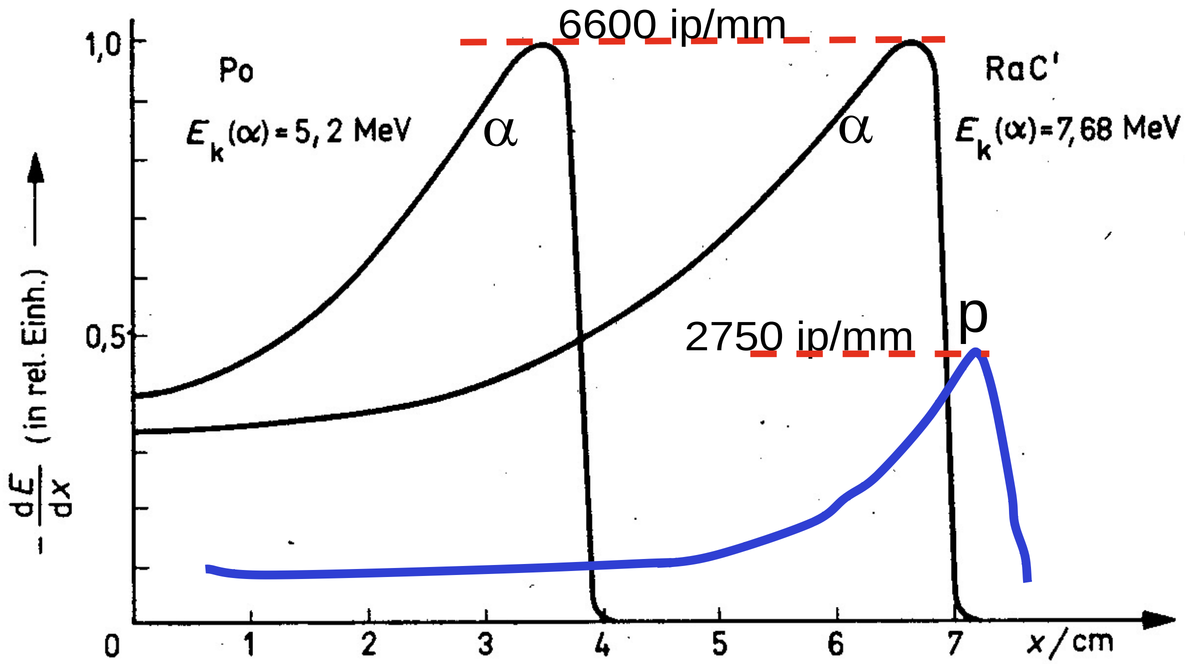
Energy lost by various ions in a 15.9 μm Si transmission detector vs. ion energy per nucleon (mass number A). Curves represent different theories.

Theoretical energy loss in material of finite thickness is obtained from integration of the Bethe-Bloch formula or equivalent.

Actual data may differ: Calibration required

Range and Specific Ionization

E-loss in Air: 1atm, 15^0C



Stopping power dE/dx (specific energy loss) depends on energy E and therefore on x

→ **Bragg Curve**

Highest E loss close to end of path → Bragg maximum

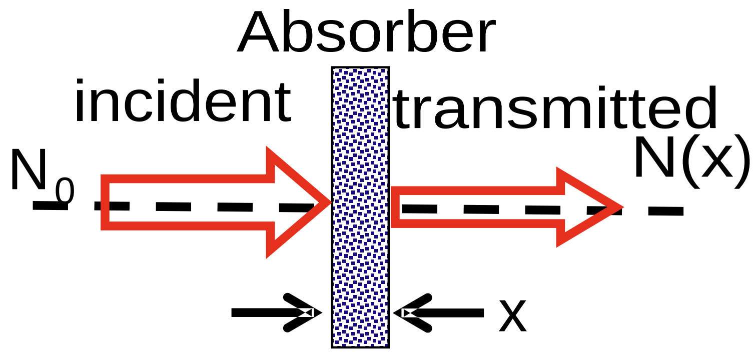
$$\frac{dE}{dx} \triangleq \frac{\#\left(e^- - ion\ pairs\right)}{unit\ \ length}$$

$$\alpha\ particles:$$

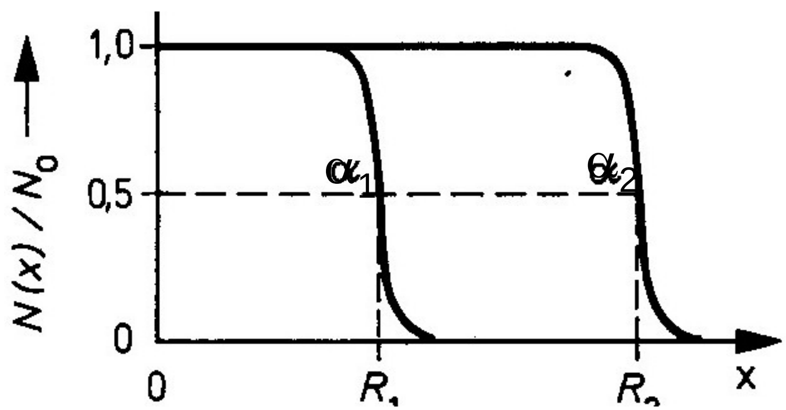
$$\frac{dE}{dx} \le \frac{7 \cdot 10^3\ pairs}{mm}$$

Main E-loss mechanism: ionization, production of δ **electrons**, electron-ion pairs

Transmission Functions

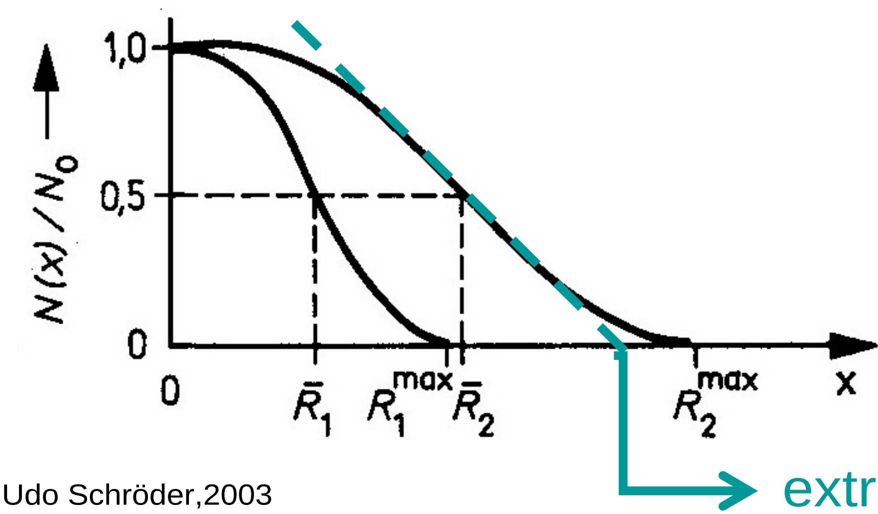


Transmission $T(x) = N(x)/N_0$



Heavy particles (α, p,..HI) have a well defined range, R(E).

Multiple scattering at small angles only, because of $M \gg m_e$



Light particles (electrons) and photons are scattered off original path by large angles

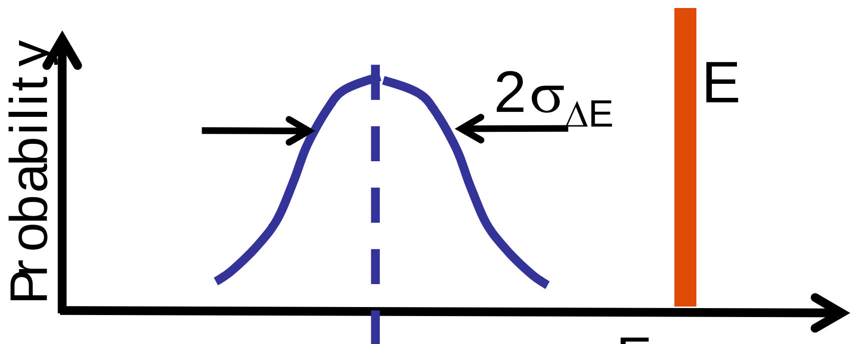
Range is not well defined

extrapolated range

Energy Loss Distributions

Absorber of thickness x; (ρ,Z_T,A_T), many statistical scattering events

→ Central Limit Theorem: Gaussian distribution in energy loss ΔE



$$\Gamma_{FWHM} = 2.35 \cdot \sigma$$

$$P(\Delta E) \propto exp\left\{-\frac{\left(\Delta E - \langle\Delta E\rangle\right)^2}{2\sigma_{\Delta E}}\right\}$$

$$\sigma_{\Delta E} = 4\pi N_A r_e^2 m_e^2 c^4 \rho \frac{Z_T}{A_T} x = 0.1569\, \rho \frac{Z_T}{A_T} x\ MeV^2$$

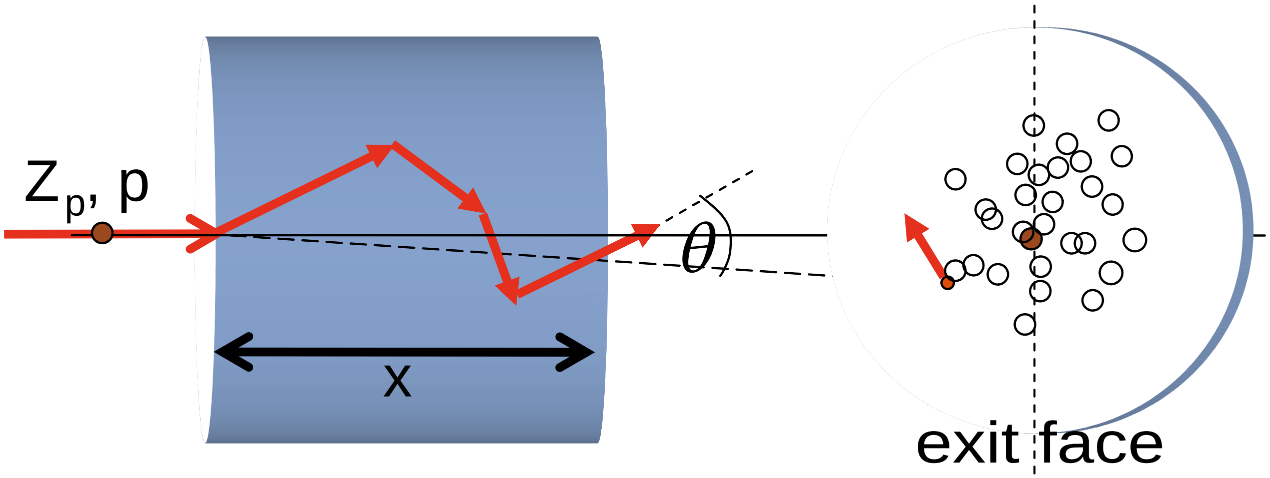
Thin absorber: Asymmetric tail towards higher ΔE

W. Udo Schröder,2003

Angular Straggling

Theory by Moliere, see W.T. Scott, Rev. Mod. Phys. 35, 231 (1963)

HI : M. Wong et al., Med. Phys. 17, 163 (1990)



Non-trivial geometry !

$$\frac{d\sigma_{MS}(\theta)}{d\Omega} = \frac{1}{2\pi\theta_0^2} exp\left\{-\frac{\theta^2}{2\theta_0^2}\right\}$$ *cross section*

Assume multiple Coulomb scattering

$$\theta_0 \approx \frac{13.6MeV}{\beta pc} Z_p \sqrt{\frac{x}{L}}\left[1 + 0.038\, ln\left(\frac{x}{L}\right)\right]$$

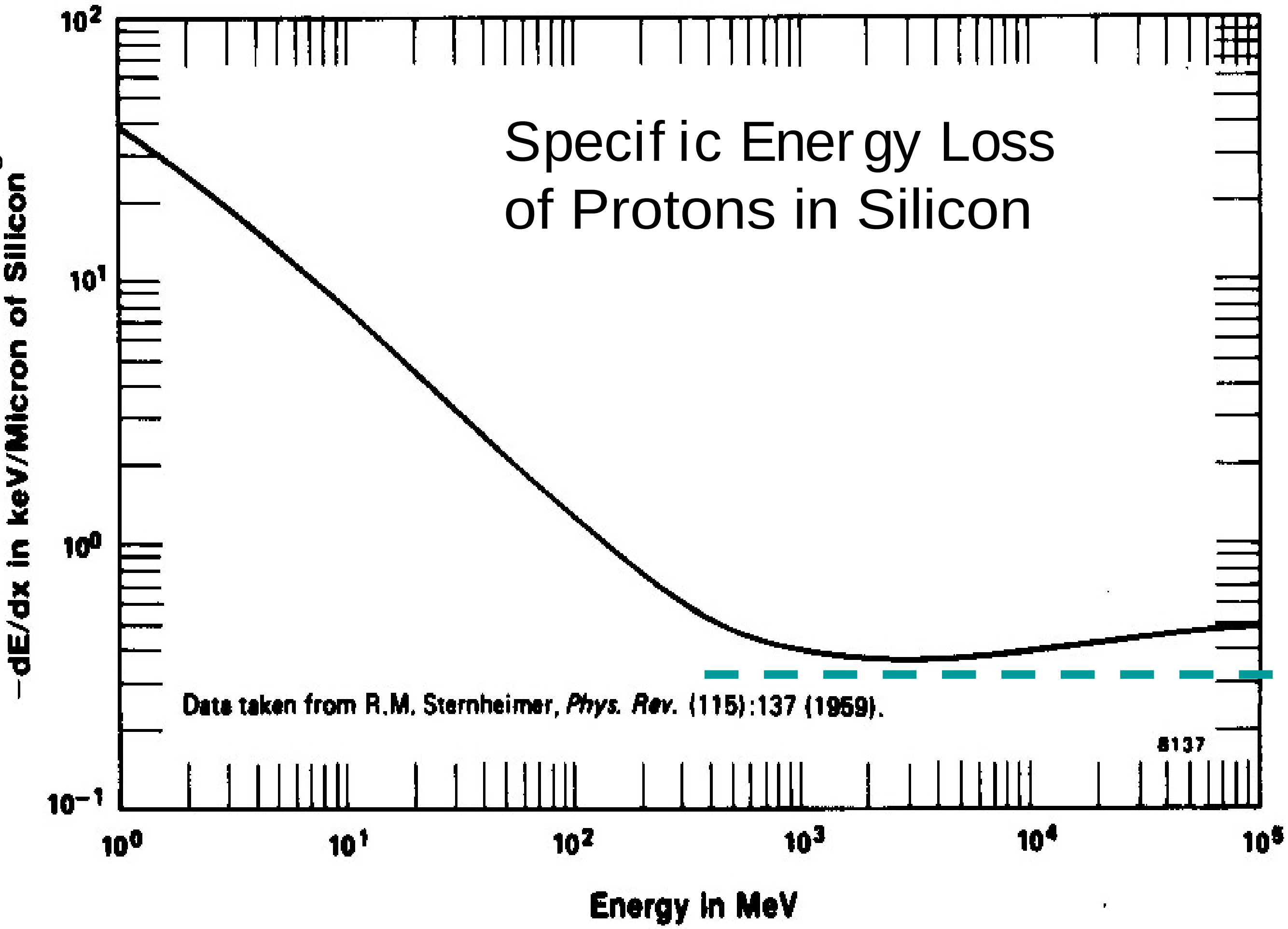
Gaussian angular distribution

$$L \approx \frac{716.4\, A_T}{Z_T\,(Z_T+1)\,ln\left(287/\sqrt{Z_T}\right)}\frac{g}{cm^2}$$ *radiation length*

Minimum Ionizing Particles

Bethe-Bloch Formula

$$\frac{-1}{\rho}\frac{dE}{dx} \approx 0.1535\,\frac{Z_p^2}{\;}\,\frac{Z_T}{A_T}\left[\frac{2}{\beta^2}\ln\left(\frac{2m_ec^2}{IE}\right) - \frac{2}{\beta^2}\ln\left(1-\beta^2\right) - 2\beta^2\right]\frac{MeV}{g/cm^2}$$



At high (relativistic energies, the b terms become dominant.

In addition, radiation losses (bremsstrahlung) and ρ-dependent plasma effects become important.

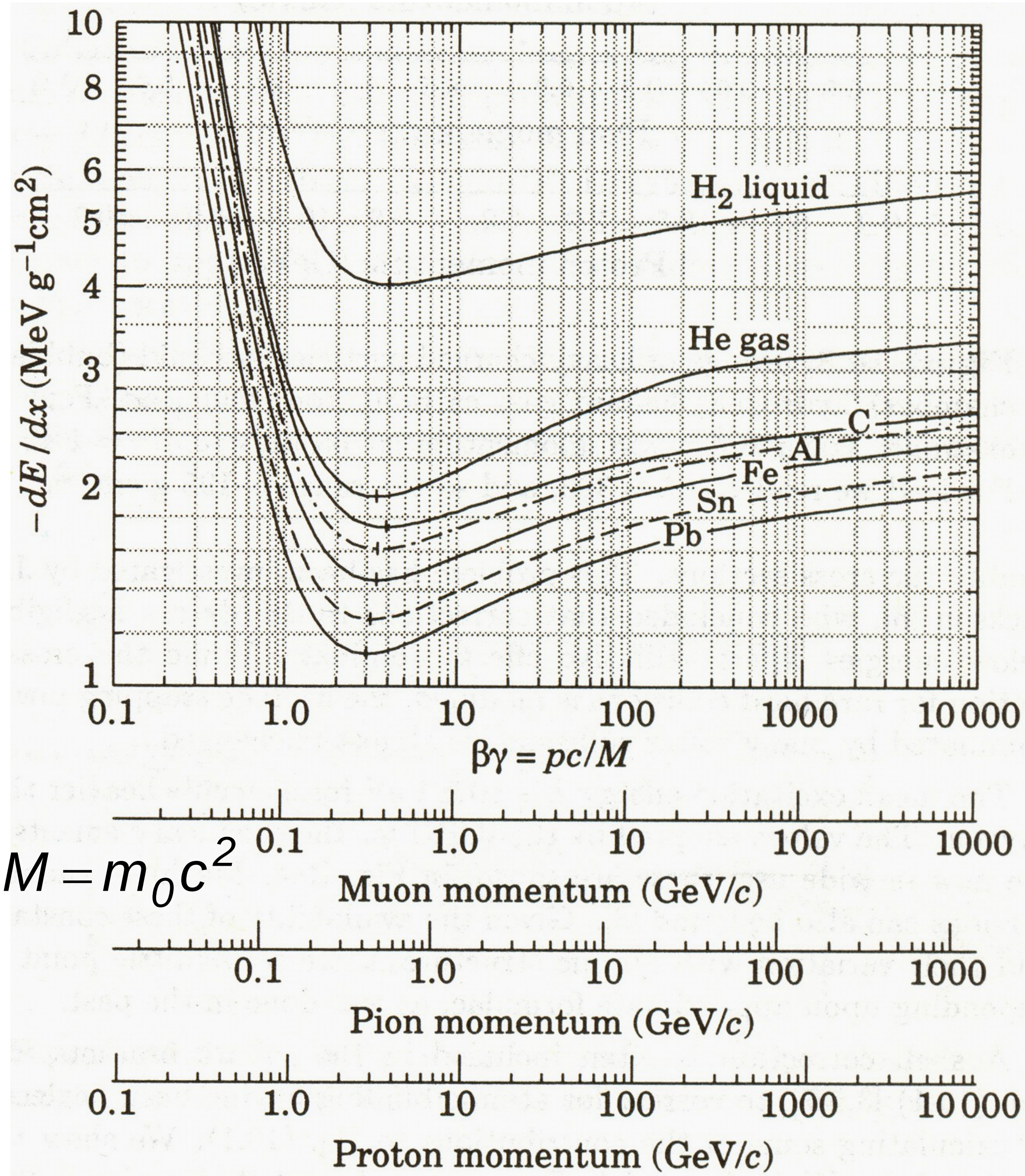
→dE/dx(E) has minimum

→"minimum-ionizing particles (mip)"

examples: $e^\pm$, $\mu^\pm$

Stopping Power of Relativistic Particles

Review of Particle Properties, Phys. Rev. D50, 1173 (1994)



$$M = m_0 c^2$$

$$E^2 = (pc)^2 + \left(m_0 c^2\right)^2$$

$$m_0 = \text{particle rest mass}$$

$$v = \text{particle velocity}$$

$$c = 2.9979 \cdot 10^8 \, m s^{-1}$$

$$\beta = v/c$$

$$\gamma = \left(\sqrt{1-\beta^2}\right)^{-1}$$

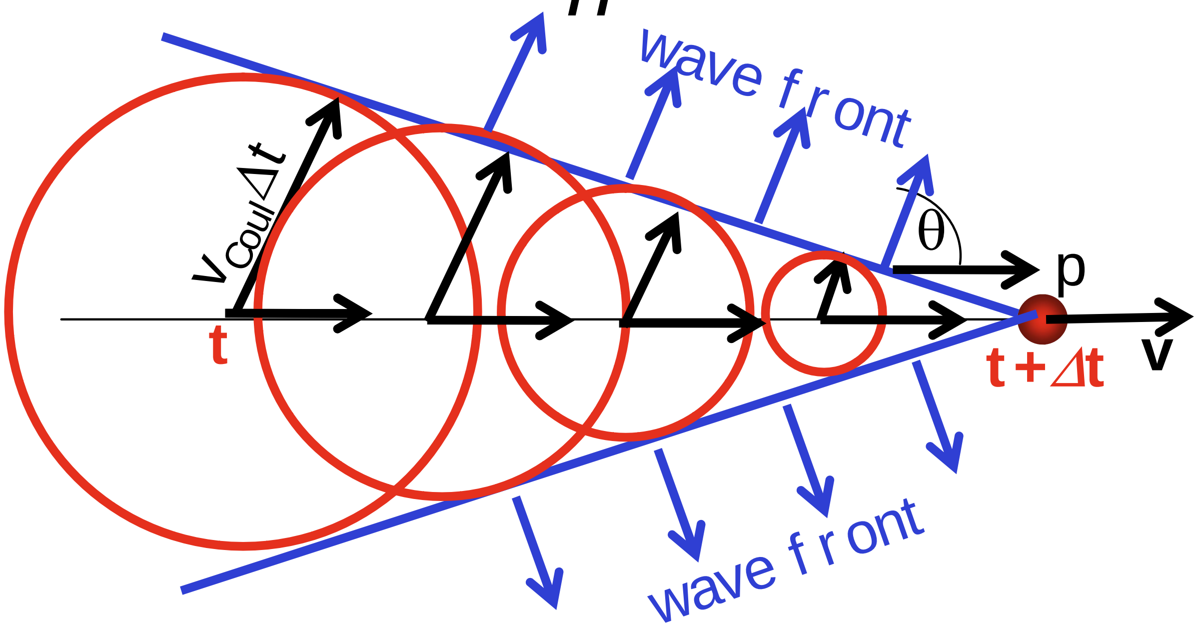
$$pc = (\gamma m_0) vc = \gamma\beta m_0 c^2$$

Note: here $\rho dE/dx \rightarrow dE/dx$

Čerenkov Effect

Similar to supersonic boom in acoustic medium. Occurs for particle velocity

$$v > c_{medium} = \frac{c_{vacuum}}{n} \qquad n = index\ of\ refraction$$



Atoms become electrically polarized by Coulomb field (field velocity v_{Coul}=c/n) of fast particle, atoms radiate coherently

$$\cos\theta = \frac{c}{nv} = \frac{1}{\beta n} \qquad Emission\ of\ \breve{C}\ light$$

$$Light\ \ spectrum: \frac{dN(\nu)}{d\nu} = \frac{4\pi^2}{hc^2}\left(eZ_p\right)^2\left[1 - \frac{1}{(\beta n)^2}\right]$$

End of Charged-Particle Interactions