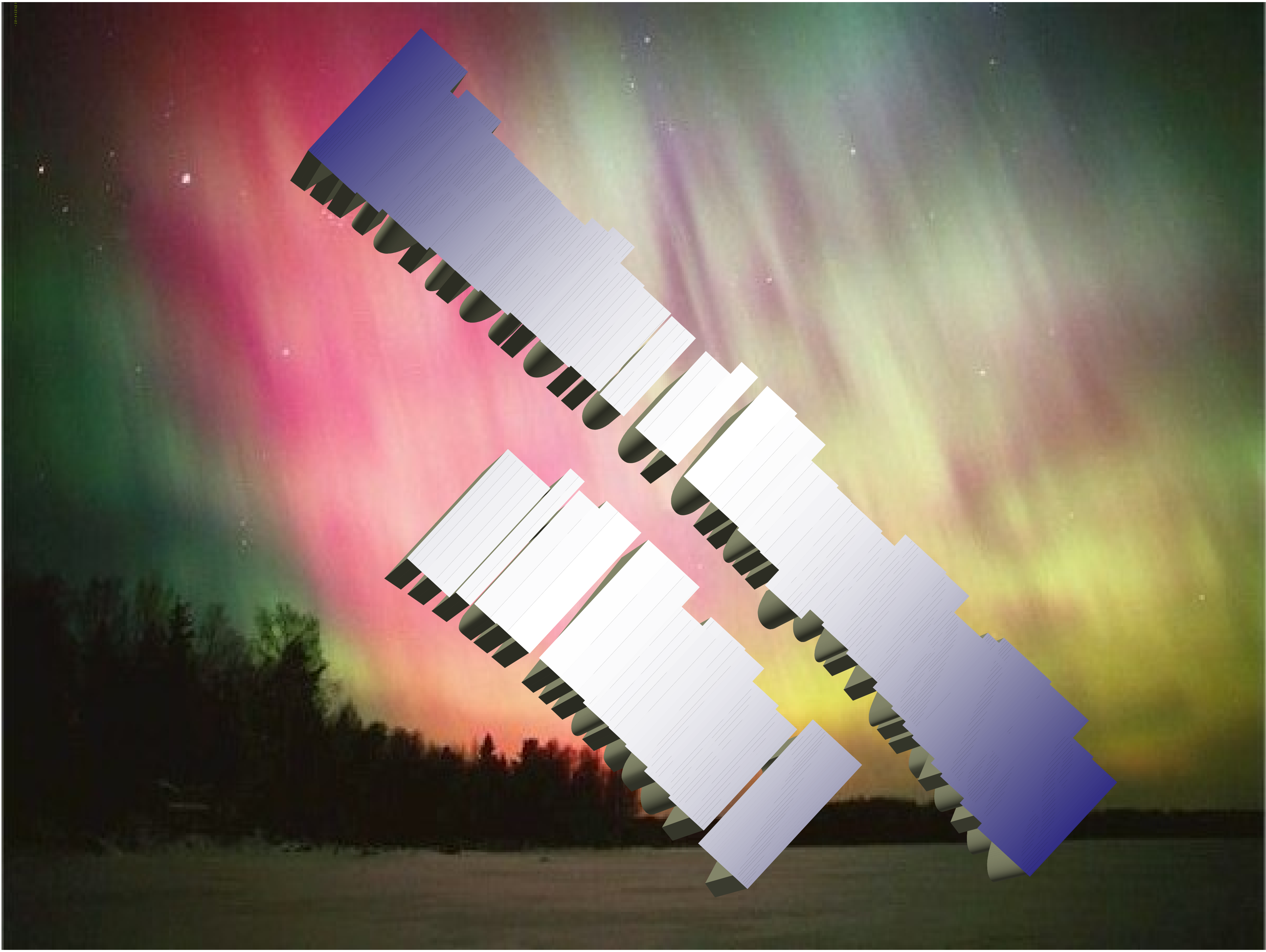
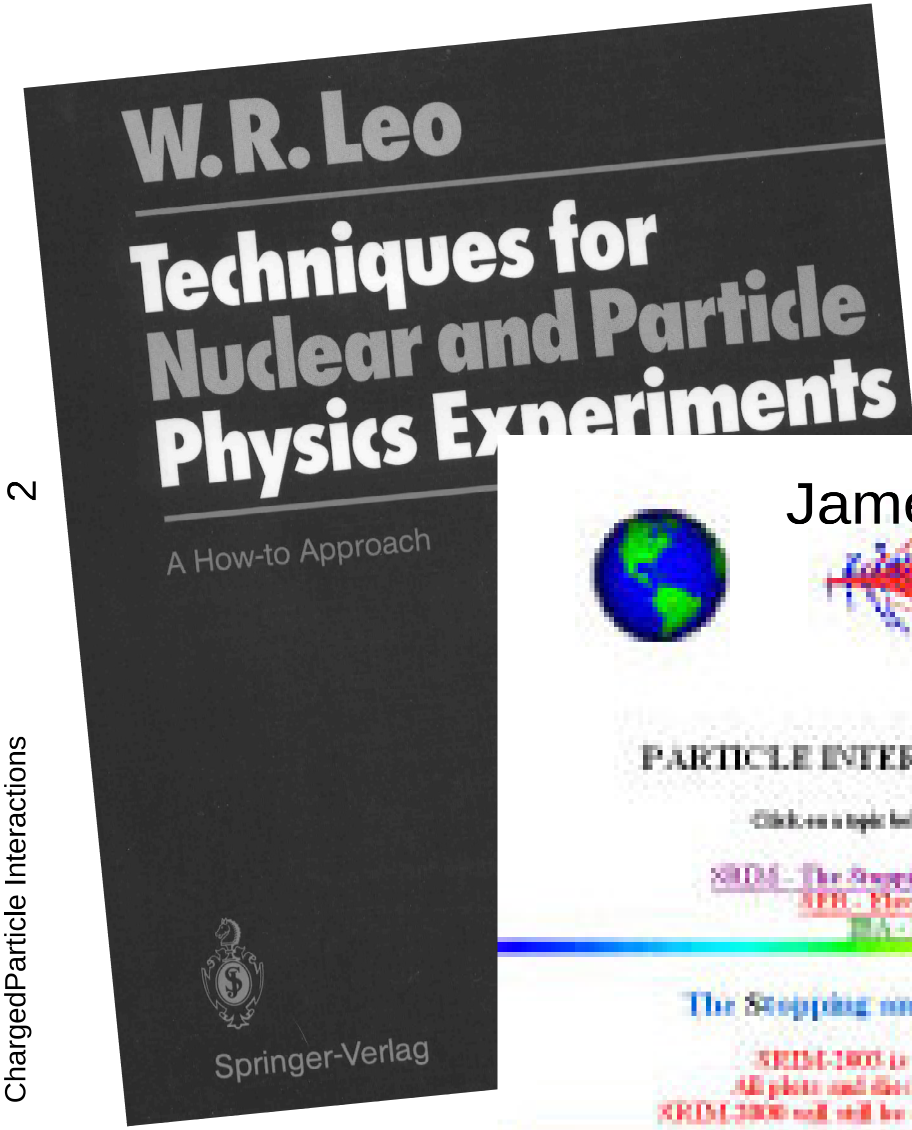
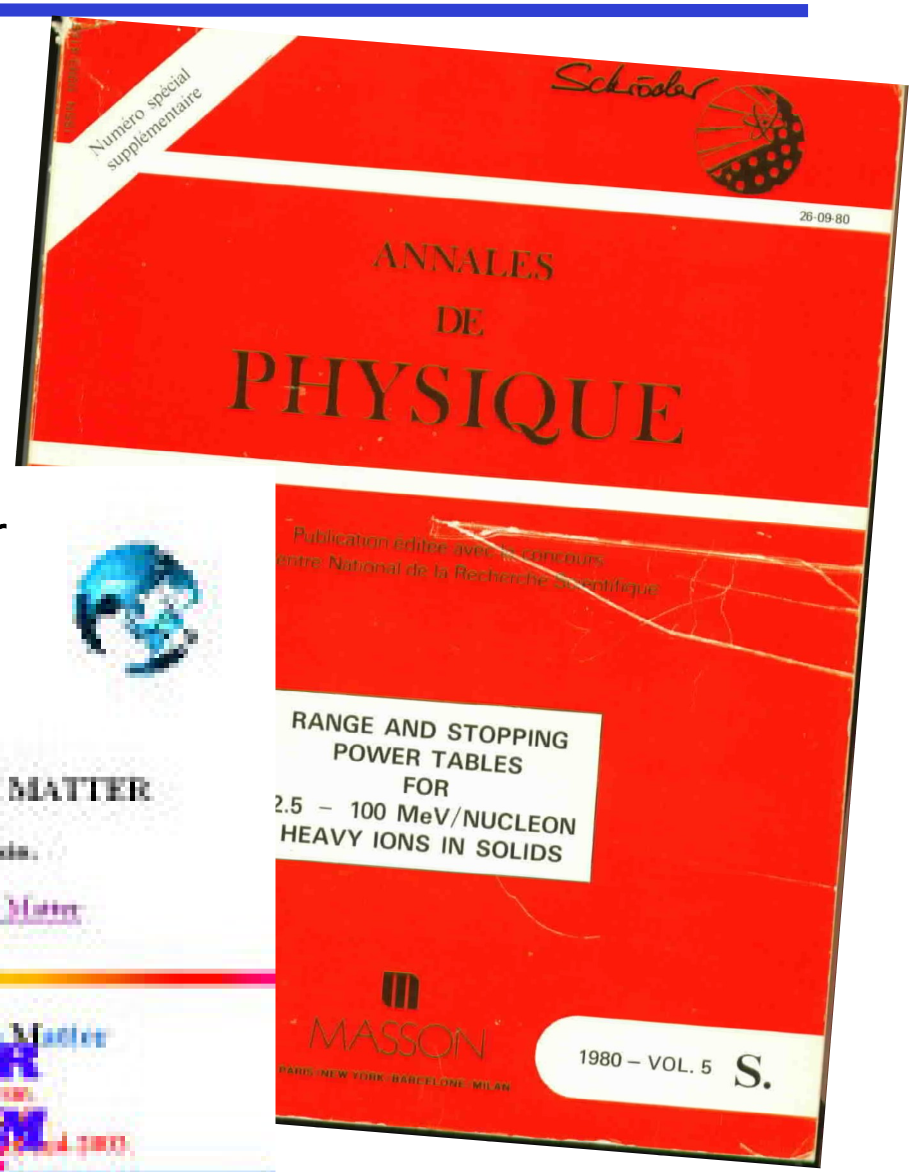
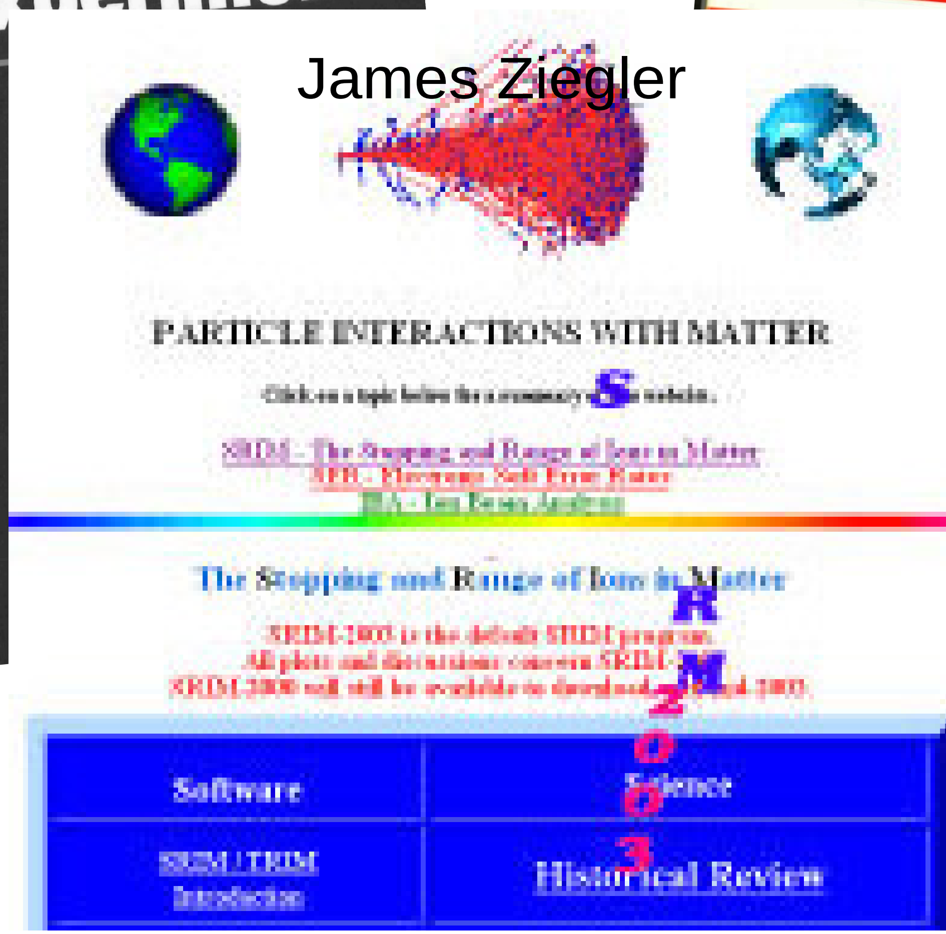


Literature/Tutorials





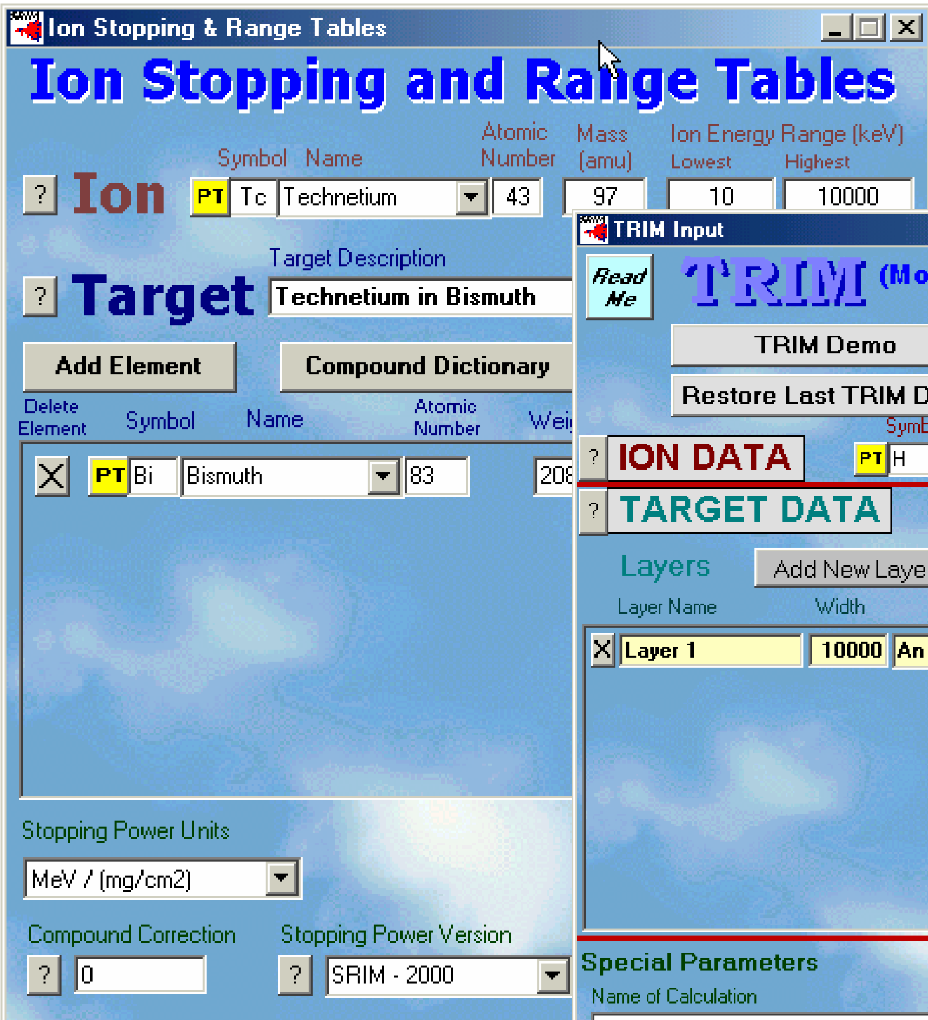
James Ziegler

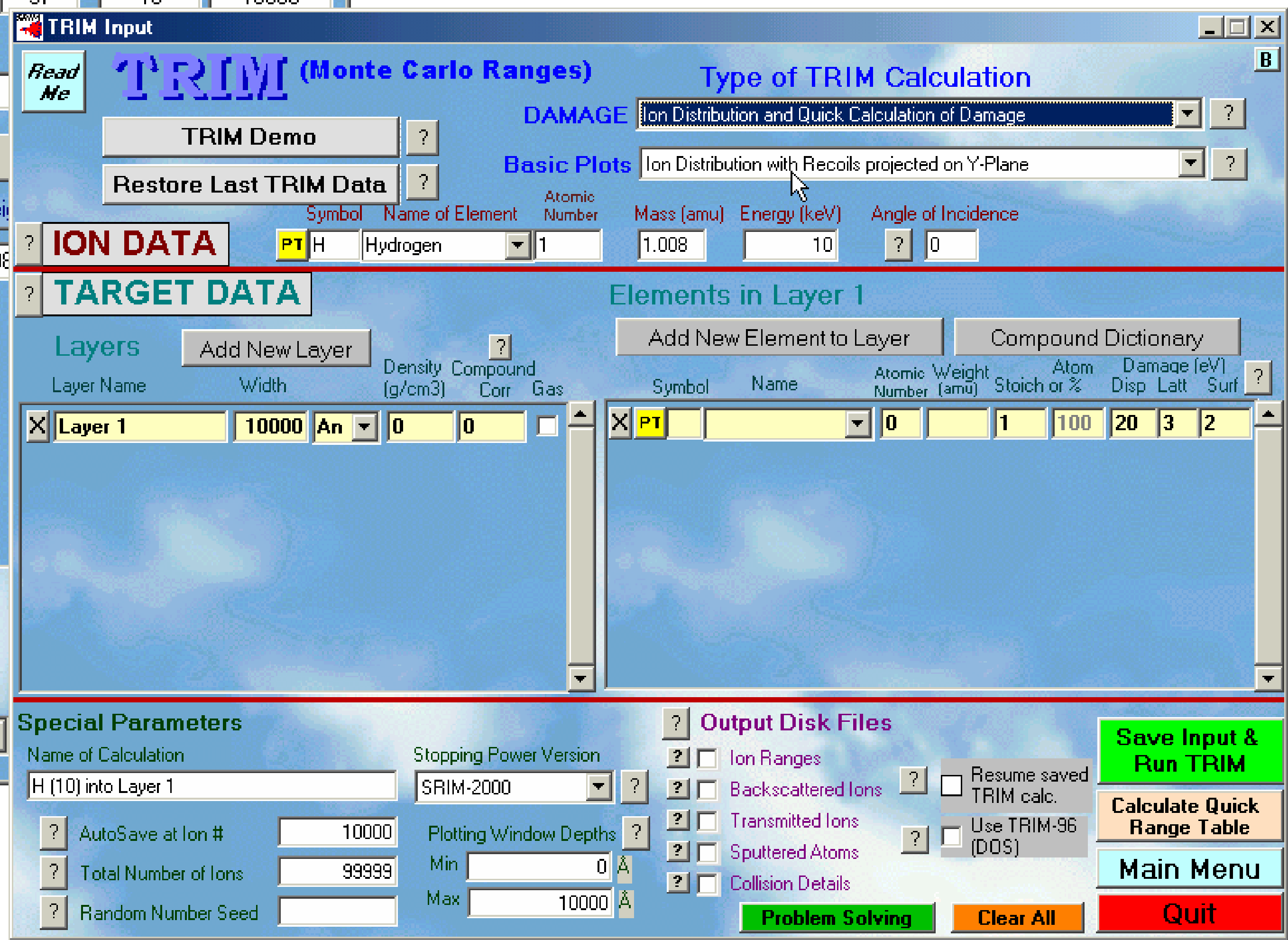


Stopping and Range Tables for Charged Particles

[J.F. Ziegler](#) (IBM&J.P. Biersack (HMI))

http://www.srim.org





Older Literature: Hubert et al., Ann. De Physique Suppl.5, 1 (1980)

Main Interactions of Charged Particles

Dominant type of interaction:
inelastic collisions with electrons $\Rightarrow$ $\left\{\begin{array}{l}\text{Atomic excitation}\\ \text{ionization}\\ \text{fluorescence}\\ \text{phosphorescence}\end{array}\right.$

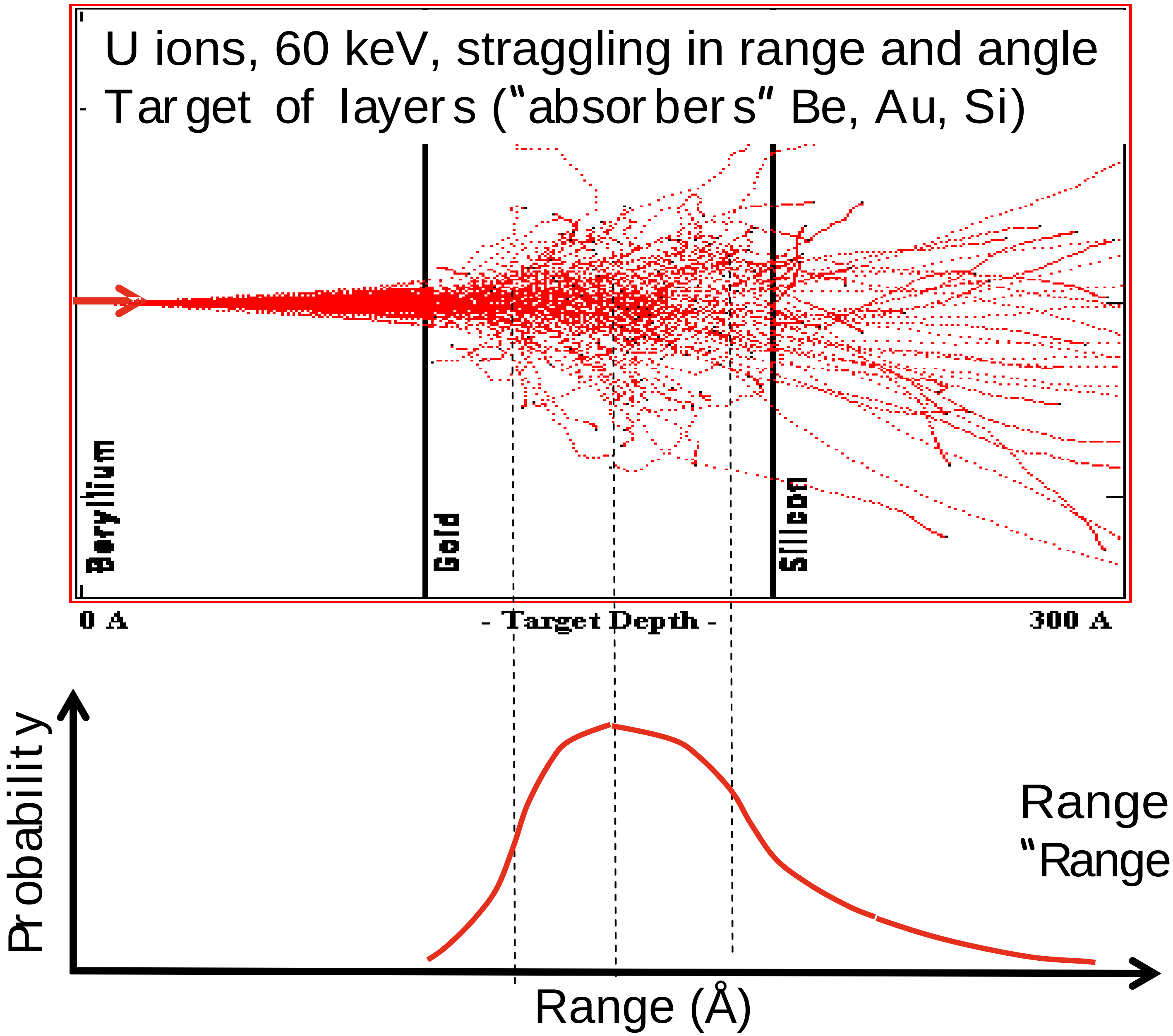
Collisions with nuclei :

$$\frac{\sigma_{nucl}}{\sigma_{atom}} \sim \frac{\pi R_{nucl}^2}{\pi a_Z^2} \sim \frac{Z^2 \cdot 10^{-26} cm^2}{10^{-16} cm^2} \sim Z^2 \cdot 10^{-10} \sim 10^{-6}$$

Most interactions of charged particles with material components occur with atomic electrons.

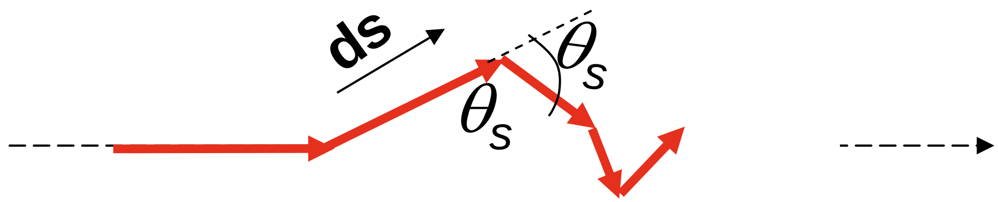
Nuclear collisions are noticeable only at low kinetic energies.

Multiple Scattering and Straggling



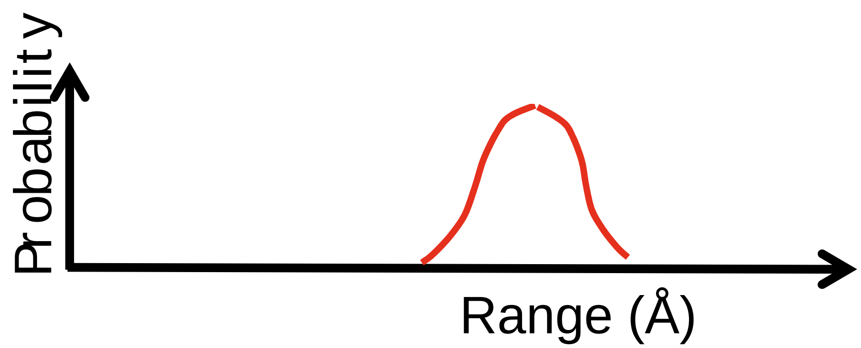
Range is diffuse:
"Range straggling"

Range and Stopping Power



Scattering angle θ_s path variable s

Stochastic multiple scattering process produces straggling in range, energy loss, angle



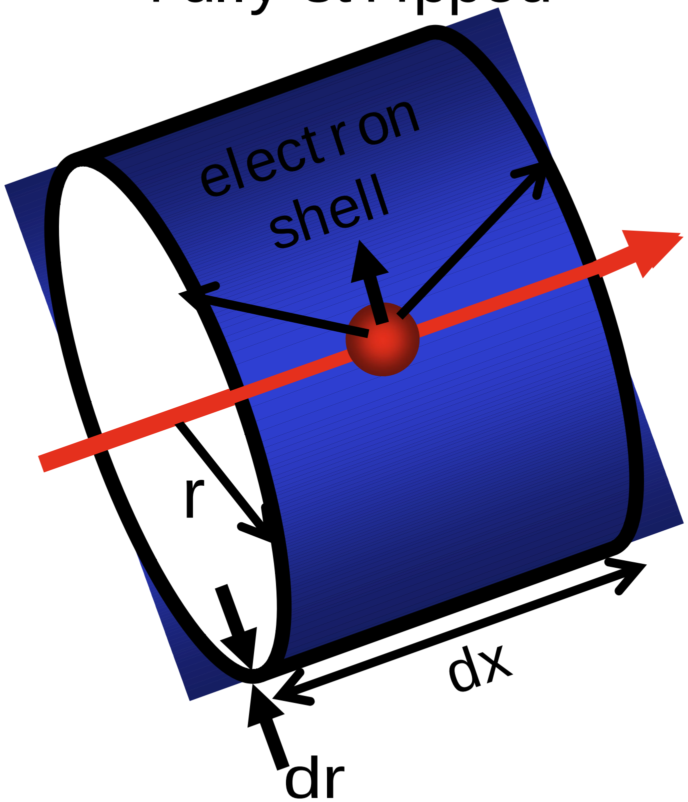
$$R(E) = \int ds \left\langle \cos\theta_s \right\rangle = \int_0^E dE' \left[\frac{dE'}{ds}\right]^{-1} \cdot \left\langle \cos\theta_s \right\rangle \quad \text{Range}$$

$$\left[\frac{dE}{ds}\right] \quad \text{Stopping power}$$

$$S(E) = \int ds \geq R(E) \quad \text{Path length of trajectory}$$

Phenomenological Model Of Energy Loss in Matter

Bethe et al. (1930-1953), Lindhardt's electron theory
Describes energy loss through ionization, incoming ions are fully stripped



Estimate of trends/Order of magnitude

E=particle kinetic energy, e^- ≈ at rest

$$\Delta p_e = F_{Coul} \cdot \Delta t_{coll} \approx \left(\frac{e^2 Z_p}{r^2}\right) \cdot \left(\frac{2r}{v}\right)$$

$$-dE(r,x) \sim [2\pi r\, dr\, dx\, N_e] \cdot \left[\frac{(\Delta p_e)^2}{2m_e}\right]$$

$$-\frac{dE(r,x)}{dr\, dx} \sim \left[\frac{4\pi e^4 Z_p^2}{m_e v^2} N_e\right] \cdot \frac{1}{r}$$

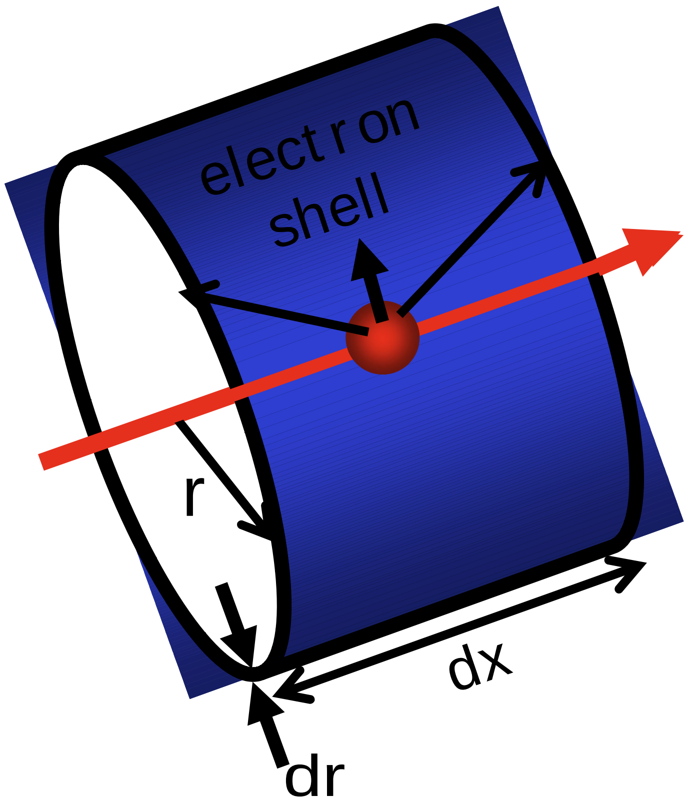
W. Udo Schröder,2003

Phenomenological Model Of Energy Loss in Matter

Integrate over radial coordinate:

$$\frac{dE(x)}{dx} = \int_{r_{min}}^{r_{max}} dr \frac{dE(r,x)}{dr\,dx}$$

$$-\frac{dE(x)}{dx} \sim \left[\frac{4\pi e^4 Z_p^2}{m_e v^2} N_e\right] \cdot \ln\left(\frac{r_{max}}{r_{min}}\right)$$



Estimate of radial limits: N_e = e^- density

λ_e = electronic wave length,

IE=ionization energy

$$r_{min} \geq \lambda_e = h/p_e = h\sqrt{1-\beta^2}/m_e v$$

$$non-adiabatic\ motion$$

$$r_{max}/v \leq \Delta t_{coll} = T_e/\sqrt{1-\beta^2} = 1/\left[\sqrt{1-\beta^2}\langle\nu_e\rangle\right]$$

$$\langle\nu_e\rangle = IE/h = average\ e^-\ orbital\ frequency$$

Phenomenological Model Of Energy Loss in Matter

Further: $$-\frac{dE}{dx} \sim \left[\frac{4\pi e^4 Z_p^2}{m_e v^2} N_e\right] \cdot \ln\left(\frac{2m_e v^2}{h\langle \nu_e\rangle\left(1-\beta^2\right)}\right)$$

Insert: ρ = atomic density, Z_T = atomic number of target

$N_e = Z_T \cdot \rho$

$$-\frac{1}{\rho}\frac{dE}{dx} \approx 4\pi \frac{e^4 Z_p^2}{m_e v^2} Z_T \ln\left(\frac{2m_e v^2}{IE\left(1-\beta^2\right)}\right)$$

$$IE = h\langle \nu_e\rangle \approx \begin{cases} (12\cdot Z_T + 7)\,eV & \text{for } Z_T < 13 \\ (9.76\cdot Z_T + 58.8\cdot Z_T^{-0.19})\,eV & \text{for } Z_T \geq 13 \end{cases}$$

Bethe-Bloch Equation

Quantum mechanical calculation (for heavy particles M»m_e):

$$-\frac{1}{\rho}\frac{dE}{dx} \approx 0.1535\,\frac{Z_p^2}{\beta^2}\frac{Z_T}{A_T}\left[\ln\left(\frac{2m_e c^2}{IE\left(1-\beta^2\right)}\right)^2 - 2\beta^2\right]\frac{MeV}{g/cm^2}$$

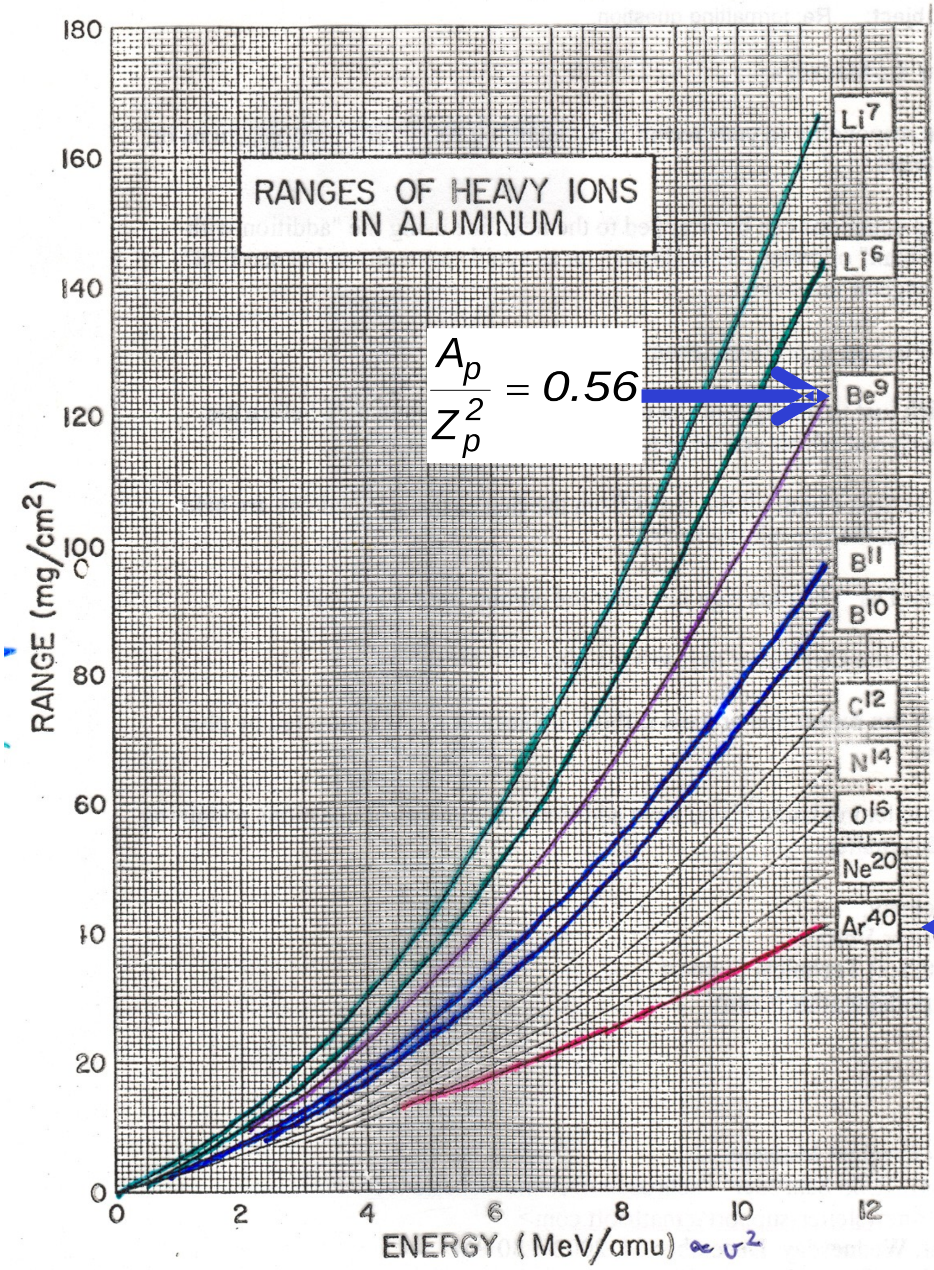
ρ = atomic density

Z_T = atomic number of target

A_T = mass number of target

$$IE = h\langle\nu_e\rangle \approx \begin{cases} (12\cdot Z_T + 7)\,eV & \text{for } Z_T < 13 \\ (9.76\cdot Z_T + 58.8\cdot Z_T^{-0.19})\,eV & \text{for } Z_T \geq 13 \end{cases}$$

Stopping Power Isotopic Scaling Laws



$$\frac{dE}{dx} = \frac{d\left(m_p v^2 / 2\right)}{dx} = Z_p \cdot f\left(\frac{E}{A_p}\right)$$

$$\frac{dv}{dx} = \left(\frac{Z_p^2}{A_p}\right) \cdot g(v) \rightarrow$$

$$R(v) \approx \int dv \left(\frac{dv}{dx}\right)^{-1} \approx \left(\frac{A_p}{Z_p^2}\right) \cdot h(v)$$

Describes well the difference of R for different isotopes of a given element, but:

$$\frac{A_p}{Z_p^2} = 0.12$$

R(Be)/R(Ar)=2.97 expt 4.67 theo

$Z_{eff} \neq Z_p$ effective charge

End of Part 1