Sizes

Gross Properties of Nuclei

Nuclear Spins

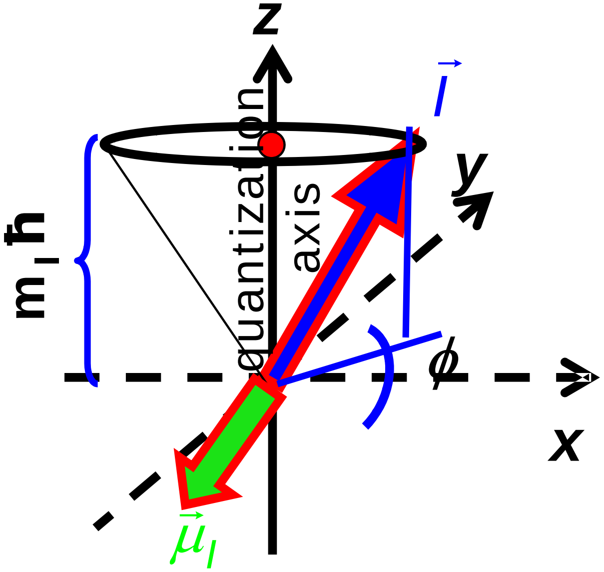
and Magnetic Moments

Intrinsic Nuclear Spin

Nuclei can be deformed → can rotate quantum mech.
→ collective spin and magnetic effects (moving charges)

Intrinsic spin? Nucleons have spin-1/2
Demonstrate via interaction with external $\vec{B}$

"good" quantum numbers: I, m_I



odd-A : I= half-integer multiple of $\hbar$
even-A: I= integer multiple of $\hbar$
even-Z & even-N: I = 0

} expt fact

Quantum mechanical spin:

$$\hat{\vec{I}}^2 \psi_{I,m_I} = I(I+1)\hbar^2 \cdot \psi_{I,m_I} \qquad I = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

$$\left|\hat{\vec{I}}\right| = \sqrt{I(I+1)} > |I|$$

$2I+1$ *different magnetic substates*

$$\hat{I}_z \psi_{I,m_I} = m_I \hbar \cdot \psi_{I,m_I} \qquad -I \le m_I \le +I$$

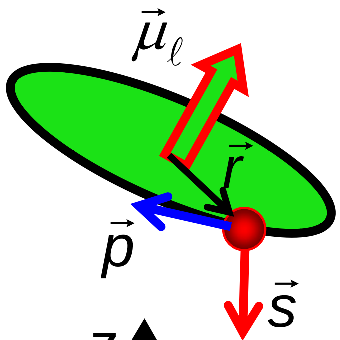
Interactions via magnetic moment $\vec{\mu}_I$

$$W_B = -\vec{\mu}_I \cdot \vec{B} = -\left(g\frac{e\hbar}{2m_p}\right)\vec{I} \cdot \vec{B}$$

Nuclear Magneton $\mu_N = \dfrac{e\hbar}{2m_p}$

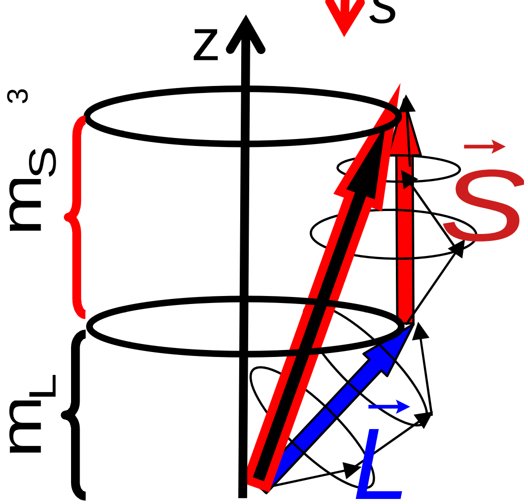
W. Udo Schröder, 2004

Spin Coupling Schemes



Nuclear spin built up of nucleonic angular momenta spin and orbital, $\vec{s}_i, \vec{\ell}_i$

$$\boxed{\vec{I} = \sum_{i=1}^{A}\left(\vec{s}_i + \vec{\ell}_i\right)} \quad \textit{For neutrons } \vec{\ell}_i = 0, \textit{ no moving charge}$$



Extreme coupling schemes of A-body system

$\boxed{LS}: \ \vec{L} = \sum_i \vec{\ell}_i \qquad \vec{S} = \sum_i \vec{s}_i \quad \rightarrow \vec{I} = \vec{L} + \vec{S}$ **L and S good qu. #s**

$\boxed{jj}: \ \vec{j}_i = \vec{\ell}_i + \vec{s}_i \qquad \rightarrow \vec{I} = \sum_i \vec{j}_i$

Correspond to different strength of interactions between nucleons and atomic electrons
→ different symmetry of A-body wave function

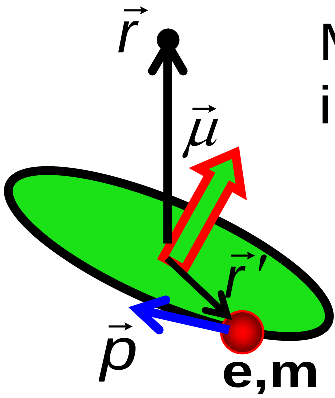
Always conserved: F, no external torque →

$$\boxed{\vec{F} = \vec{I} + \vec{J}} \ (\vec{J}: \textit{total electr. spin}) \ \boxed{\left|\vec{I} - \vec{J}\right| \le \left|\vec{F}\right| \le \left|\vec{I} + \vec{J}\right|}$$

$$\hat{\vec{F}}^2 \psi_{F,m_F} = F(F+1)\hbar^2 \psi_{F,m_F} \qquad -F \le m_F \le +F$$

$$\hat{F}_z \psi_{F,m_F} = m_F \hbar \psi_{F,m_F} \quad \textit{magnetic substates}$$

Magnetic Dipole Moments



$$j = e\rho v$$

$$\boxed{\frac{\mu_0}{4\pi} = 10^{-7}\frac{Vs}{Am}}$$

Moving charge ***e*** → current density ***j*** → vector potential ***A***, influences particles at $\vec{r}$ via magnetic field $\vec{B} = \mu_0 \vec{H} = \vec{\nabla} \times \vec{A}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int d^3\vec{r}\,' \frac{\vec{j}(\vec{r}\,')}{\left|\vec{r}\,' - \vec{r}\right|} \approx \frac{\mu_0}{4\pi}\left[\frac{1}{r}\underbrace{\int d^3\vec{r}\,'\,\vec{j}(\vec{r}\,')}_{=0} + \frac{1}{r^3}\int d^3\vec{r}\,'\left(\vec{r}\cdot\vec{r}\,'\right)\vec{j}(\vec{r}\,') + ..\right]$$

$$\vec{A}(\vec{r}) \approx \frac{\mu_0}{4\pi}\frac{1}{r^3}\int d^3\vec{r}\,'\left(\vec{r}\cdot\vec{r}\,'\right)\vec{j}(\vec{r}\,') + .. = \frac{\mu_0}{4\pi}\frac{1}{r^3}\int d^3\vec{r}\,'\,\vec{r}\times\vec{j}(\vec{r}\,')\times\vec{r}\,' + ..$$

Magnetic dipole moment $\boxed{\mu := \frac{1}{2}\int d^3\vec{r}\,'\left[\vec{r}\,' \times \vec{j}(\vec{r}\,')\right]}$

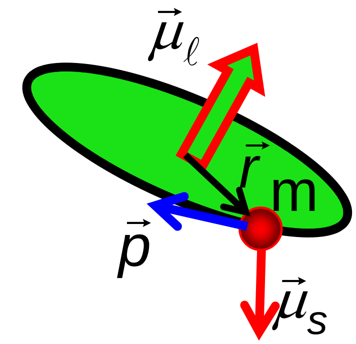
$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{\mu}\times\vec{r}}{r^3}} + \textit{higher multipole moments} \rightarrow B \sim \frac{1}{r^3}$$

$$\vec{r}\times\vec{j}(\vec{r}) = \vec{r}\times\left[\rho(\vec{r})\frac{e}{m}\vec{p}(\vec{r})\right]; \qquad \vec{r}\times\vec{p} = \vec{L}$$

$$\mu = \frac{e}{2m}\int d^3\vec{r}\,'\rho(\vec{r}\,')\vec{L}(\vec{r}\,') \xrightarrow[\rho(\vec{r})=\delta(r-R)]{} = \frac{e}{2m}\vec{L}$$

current loop: $\boxed{\mu_{Loop} = j \times A = \text{current} \times \text{Area}}$

Magnetic Moments: Units and Scaling



Magnetic moments:

$$\boxed{\vec{\mu}_\ell = g_\ell \cdot \left(\frac{e\hbar}{2m_p}\right) \cdot \vec{\ell}} \quad \textit{orbital, Landė } g_\ell \textit{ factor}$$

$$\boxed{\vec{\mu}_s = g_s \cdot \left(\frac{e\hbar}{2m_p}\right) \cdot \vec{s}} \quad \textit{spin, Landė } g_s \textit{ factor}$$

Units

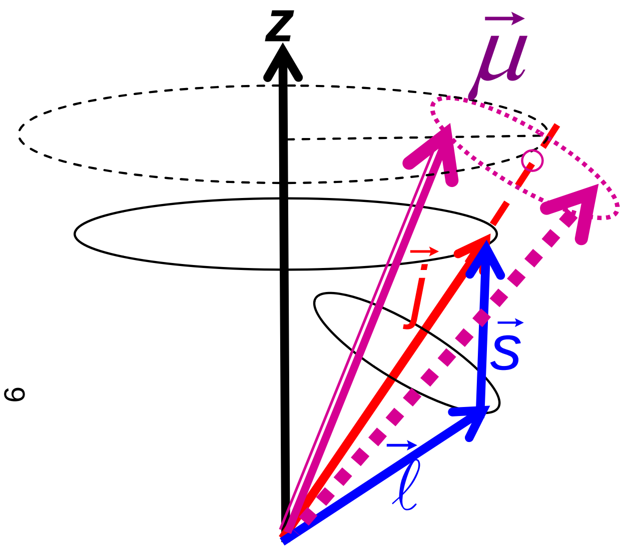
$$\left\{ \begin{array}{l} \textit{electron}: \quad \mu_B = \dfrac{e\hbar}{2m_e} = 5.788 \cdot 10^{-15} MeV/G \;\; \textit{Bohr Magneton} \\ \textit{nucleon}: \quad \mu_N = \dfrac{e\hbar}{2m_p} = 3.152 \cdot 10^{-18} MeV/G \;\; \textit{Nuclear Magneton} \end{array} \right.$$

g factors

$$\left\{ \begin{array}{llll} e^-: & g_\ell = -1 & g_s = -2.00232 & \textit{Free } p, n: \\ p: & g_\ell = +1 & g_s = +5.5855 & \mu_p = 2.79\mu_N \\ n: & g_\ell = 0 & g_s = -3.8256 & \mu_n = -1.91\mu_N \end{array} \right.$$

g<0 → μ ⇅ l

Total Nucleon Magnetic Moment



Superposition of orbital and spin μ:

$$\hat{\vec{\mu}} = \left(g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s} \right) \mu_N \quad \textit{Not} \parallel \textit{to } \vec{j} = \vec{\ell} + \vec{s} \ !$$

below use these single-particle states

$$\hat{\vec{j}}^2 \left| \begin{matrix} \ell & s & j \\ m_\ell & m_s & m_j \end{matrix} \right\rangle = j(j+1)\hbar^2 \left| \begin{matrix} \ell & s & j \\ m_\ell & m_s & m_j \end{matrix} \right\rangle$$

$$\hat{\vec{s}}^2 \left| \ \right\rangle = s(s+1)\hbar^2 \left| \ \right\rangle, \ \hat{s}_z \left| \ \right\rangle = m_s\hbar \left| \ \right\rangle, \ \hat{\vec{\ell}}^2 \left| \ \right\rangle = \ell(\ell+1)\hbar^2 \left| \ \right\rangle, \ \hat{\ell}_z \left| \ \right\rangle = m_\ell\hbar \left| \ \right\rangle$$

Precession of μ around z-axis slaved by precession of $\boldsymbol{j}$ → all μ components perp. to $\boldsymbol{j}$ vanish on average.

$$\frac{\mu_{sp}}{\mu_N} = \frac{\left\langle \ \left| \hat{\vec{\mu}} \cdot \hat{\vec{j}} \right| \ \right\rangle}{\sqrt{j(j+1)}} \cdot \frac{1}{\sqrt{j(j+1)}} \left\langle \begin{matrix} \ell & s & j \\ m_\ell & m_s & j \end{matrix} \right| \hat{j}_z \left| \begin{matrix} \ell & s & j \\ m_\ell & m_s & j \end{matrix} \right\rangle =$$

$$= \frac{1}{j(j+1)} \left\{ g_\ell j(j+1) + (g_s - g_\ell) \left\langle \hat{\vec{s}} \cdot \hat{\vec{j}} \right\rangle \right\}$$

maximum alignment of j

Effective g Factor

$$\frac{\mu_{sp}}{\mu_N} = \frac{1}{j(j+1)}\left\{ g_\ell j(j+1) + (g_s - g_\ell)\left\langle \hat{\vec{s}}\cdot\hat{\vec{j}}\right\rangle \right\} = g_j \cdot j$$

g_j: effective g-factor

$$Use: \left\langle \hat{\vec{s}}\cdot\hat{\vec{j}}\right\rangle = -\frac{1}{2}\left\langle \hat{\vec{j}}^2 + \hat{\vec{s}}^2 - \hat{\vec{\ell}}^2 \right\rangle, \quad etc.$$

$$\frac{\mu_{sp}}{\mu_N} = \begin{cases} \left[g_\ell + (g_s - g_\ell)\dfrac{1}{2\ell+1}\right]\cdot j & for \quad j = \ell + 1/2 \\ \left[g_\ell - (g_s - g_\ell)\dfrac{1}{2\ell+1}\right]\cdot j & for \quad j = \ell - 1/2 \end{cases}$$

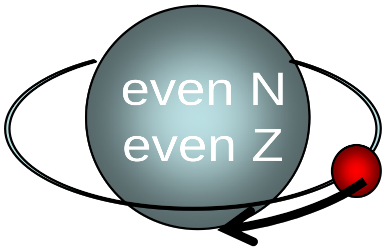
Magnetic moment for entire nucleus: analogous definition for maximum alignment, slaved by nuclear spin I precession

$$\mu = \frac{1}{\sqrt{I(I+1)}}\left\langle \begin{matrix} I \\ m_I = I \end{matrix} \middle| \hat{\vec{\mu}}\cdot\hat{\vec{I}} \middle| \begin{matrix} I \\ m_I = I \end{matrix} \right\rangle$$

Simple s.p. Model: Schmidt-Lines (Odd-A)

odd-A: All but one nucleon paired,. Paired nucleons make spinless, spherical core → central potential for extra nucleon →

$$\vec{I} = \vec{j} \qquad \mu = \mu_{sp}(I)$$



(Z, N) = (odd, even)   unpaired p, $g_\ell = 1$

$$\mu(I) = \begin{cases} I + \dfrac{1}{2}(g_s^p - 1) & for \;\; I = j = \ell + 1/2 \\ I - \dfrac{I}{2(I+1)}(g_s^p - 1) & for \;\; I = j = \ell - 1/2 \end{cases}$$

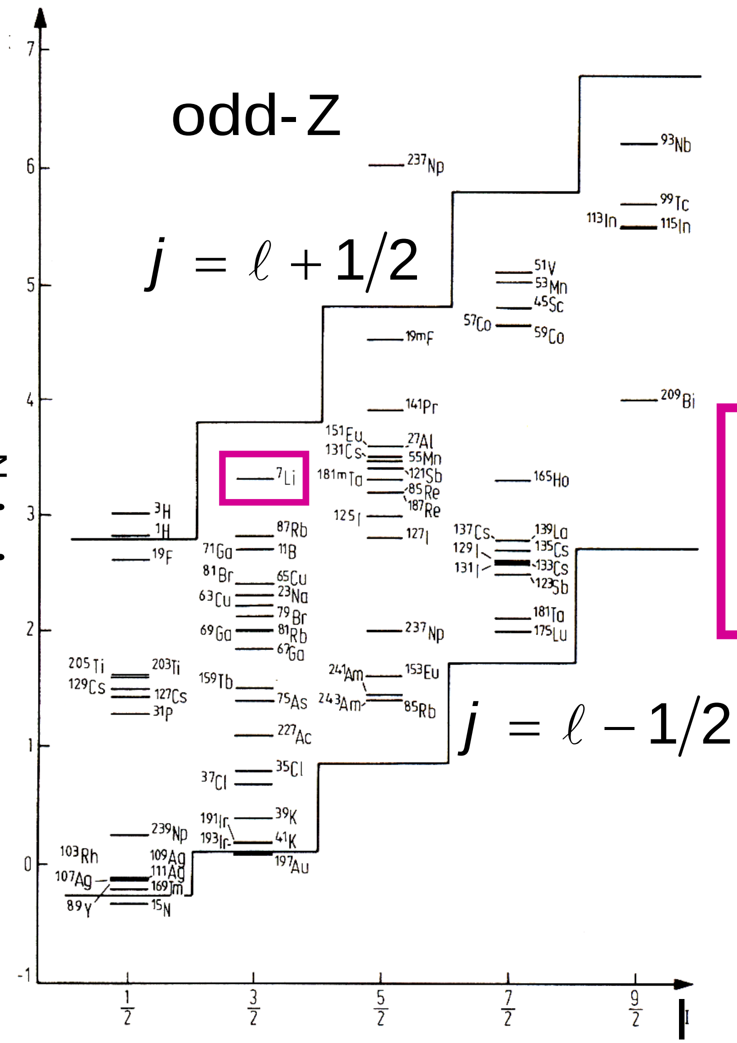
units: μ_N

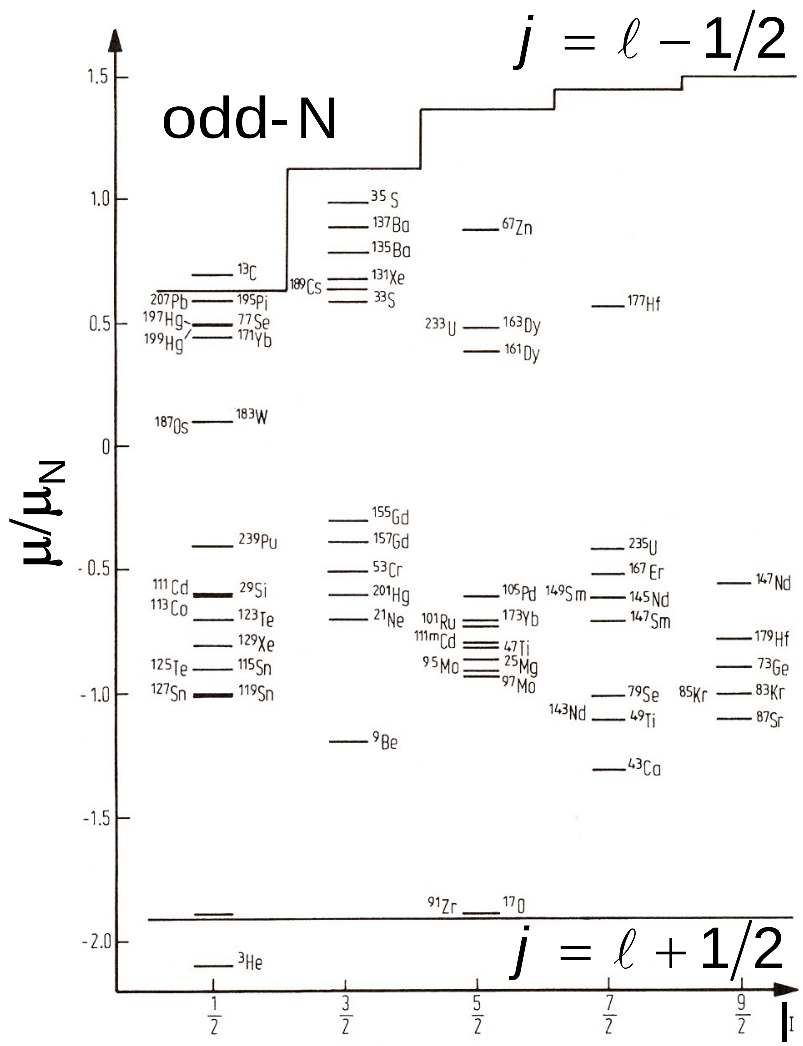
(Z, N) = (even, odd)   unpaired n, $g_\ell = 0$

$$\mu(I) = \begin{cases} \dfrac{1}{2} g_s^n & for \;\; I = j = \ell + 1/2 \\ -\dfrac{I}{2(I+1)} g_s^n & for \;\; I = j = \ell - 1/2 \end{cases}$$

units: μ_N

Experimental μ for Odd-A Nuclei

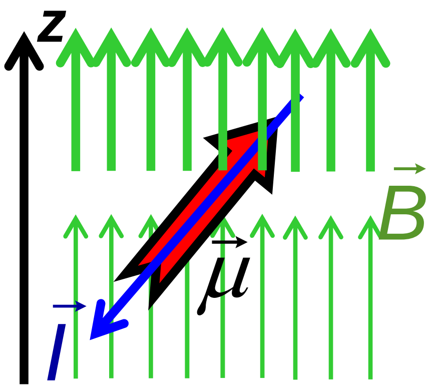




^{7}Li: j=3/2
j ≈ ℓ+ 1/2
→ ℓ = 1

Reproduction of overall trends
Almost all μ lie between Schmidt lines= extreme values for μ.
Quenching of g_s factors due to interactions with other nucleons

Magnetic e-Nucleus Interactions



$$\vec{\mu} = (g_I \mu_N) \cdot \vec{I}$$

Energy in homogeneous B-field || z axis

$$W = -\vec{\mu} \cdot \vec{B} = -(g_I \mu_N)\, \vec{I} \cdot \vec{B} = -(g_I \mu_N)\, m_I B_z$$

Force in inhomogeneous B-field || z axis

$$F_z = -\frac{\partial W}{\partial z} = \vec{\mu} \cdot \frac{\partial \vec{B}}{\partial z} = +(g_I \mu_N)\, m_I \frac{\partial B_z}{\partial z}$$

Atomic electrons (currents) produce B-field at nucleus, aligned with total electronic spin

$$\vec{J}: \quad \vec{B} = B_0 \frac{\vec{J}}{\left|\vec{J}\right|}$$

$1s_{1/2}$ *electron* $B_0 \sim 10^5 - 10^6\ G$

$2p_{/}$ *electron* $B_0 \sim 10^4 - 10^5\ G$

$1s_{1/2}$ *muon* $B_0 \sim 10^{13} G$

$$\longrightarrow \quad \boxed{W = -\frac{(g_I \mu_N I) B_0}{\left|\vec{I}\right| \cdot \left|\vec{J}\right|}\, \vec{I} \bullet \vec{J}}$$

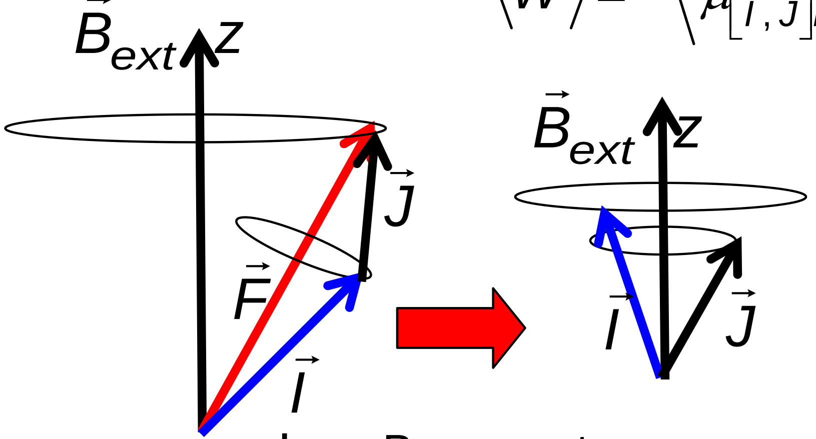
Total spin $\vec{F} = \vec{I} + \vec{J} \;\rightarrow\; \vec{F}^2 = \vec{I}^2 + \vec{J}^2 + 2\vec{I} \bullet \vec{J}$

$$W = -\frac{(g_I \mu_N I) B_0}{2\left|\vec{I}\right| \cdot \left|\vec{J}\right|}\, \vec{I} \bullet \vec{J} = -\frac{(g_I \mu_N I) B_0}{2\left|\vec{I}\right| \cdot \left|\vec{J}\right|} \left[F(F+1) - I(I+1) - J(J+1)\right]$$

Magnetic Hyper-Fine Interactions

HF pattern depends on strength B_{ext}

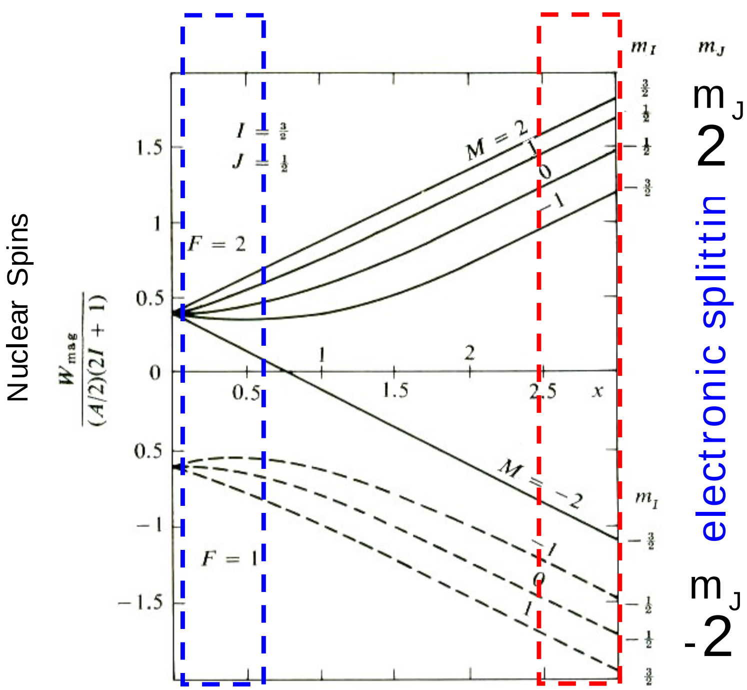
$$\langle W\rangle = -\left\langle \vec{\mu}_{[I,J]F}\cdot\vec{B}_{ext}\right\rangle \approx \begin{cases} \dots - \dfrac{(\vec{\mu}_J\cdot\vec{F})\vec{F}\cdot\vec{B}_{ext}}{F(F+1)} & \textit{weak } B_{ext} \\ \underbrace{-(g_J\mu_B)B_{ext}m_J}_{\text{FS}} + \underbrace{Am_Jm_I}_{\text{HFS}} & \textit{strong } B_{ext} \end{cases}$$



weak B_{ext} strong

Strong B_{ext} breaks [J,I]F coupling.
F import for weak B_{ext},
independent for strong B_{ext}

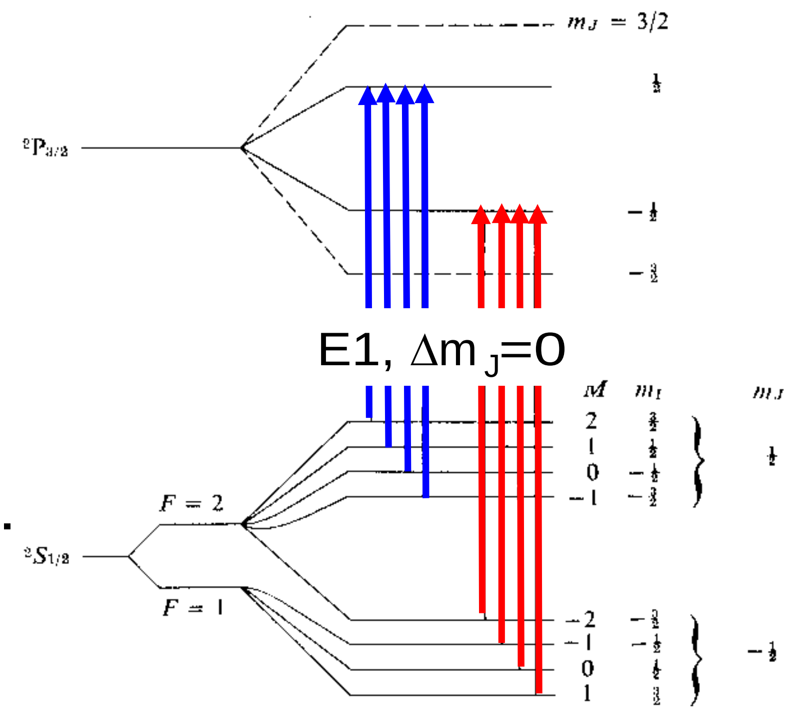
$$\varpi_{Lamor} = \frac{g|e|}{2m\hbar}B_{ext}$$



Na:
$I = 3/2$
$J = 1/2$
$F = 1, 2$

2 separated groups @ 2I+1=4 lines. (F not good qu. #)

1s→2p X-Ray Transition



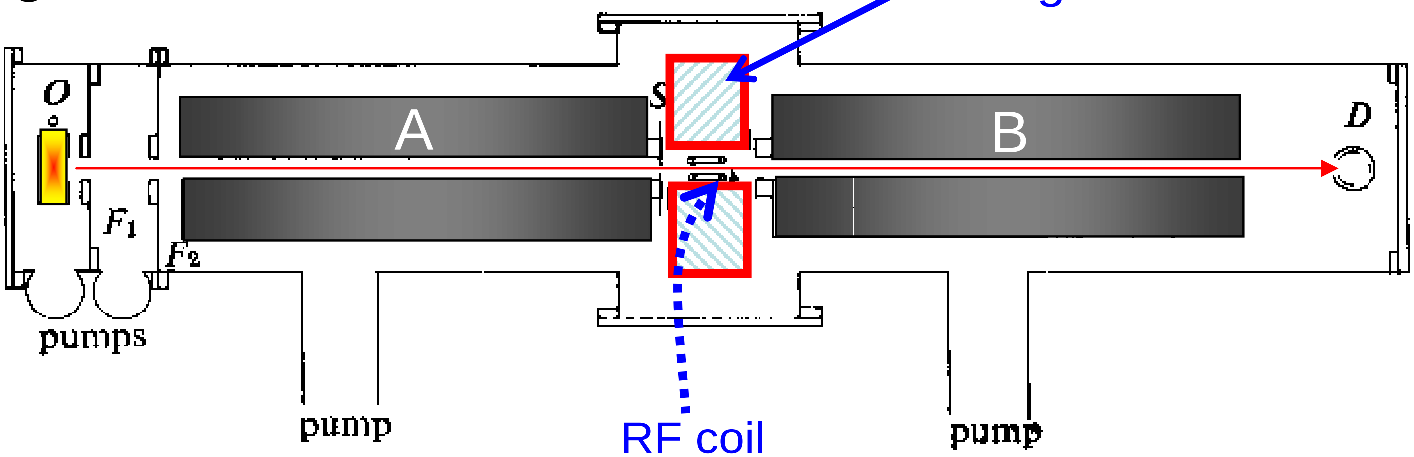
Rabi Atomic/Molecular Beam Experiment (1938)

I. Rabi
1984

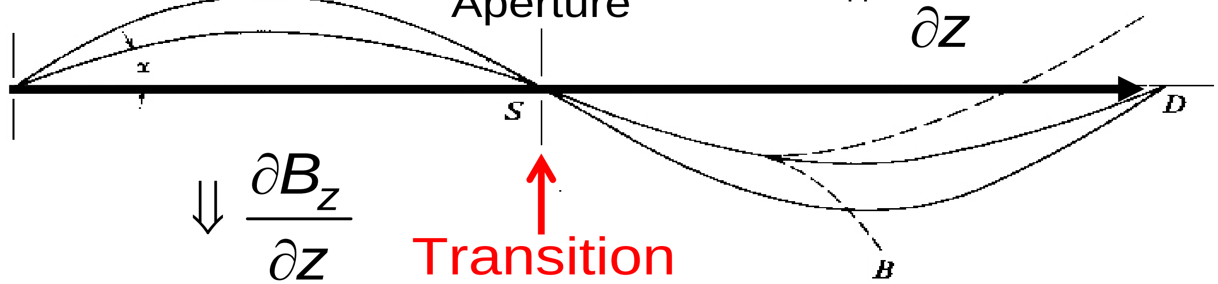
Force on magnetic moment in inhomogeneous B-field ||z axis

$$F_z = -\frac{\partial W}{\partial z} = \mu_I \cdot \frac{\partial B_z}{\partial z} = +(g_I \mu_N)\, m_I \,\frac{\partial B_z}{\partial z}$$

Alternating B gradients



Magnet B compensates for effect of magnet A for a given m_I



W. Udo Schröder, 2004

Summary: Gross Properties of Nuclei

A. <u>Nuclear sizes</u>:

- Finite size, $R = r_0.A^{1/3}$, $r_0 = 1.2\ fm$
  → approximately constant density in interior
  → saturation of nuclear forces, must have repulsive core
- Diffuse surface, *b ~1fm,* weak dependence on A
- Fermi-type charge and mass distributions
- Nuclei with magic N or/and Z numbers slightly smaller than average

B. <u>Nuclear masses and binding energies</u>:

- Approximately constant $B/A \approx 8\ MeV$, weakly dependent of A
  → saturation of nuclear forces, nucleon experiences average interaction with "nearest neighbors"
- Nuclear liquid drop model describes average A-dependence of B/A
  → β stable valley, but: paired nucleons are more tightly bound $\delta B \approx 12^{-1/2}A$ MeV. Structure effects: # of isotopes for odd or even A
- Nuclei with magic N and or Z are more tightly bound than neighbors, ^{64}Ni, ^{56}Fe most tightly bound nuclei

Summary: Gross Properties of Nuclei

C. Nuclear deformations and electrostatic moments:

- Only even electrostatic moments
  → monopole, quadrupole Q most important
  → Spin I=0, ½ nuclei have no measurable Q
- N-Z regions with large Q (Lanthanides, Actinides), domains defined by magic numbers
  → magic N, Z have Q = 0

D. Nuclear spins and magnetic moments:

- Most nuclear spins are small, a few $\hbar$, integer multiple of $\hbar$ for e-A
  → I = 0 for e-e nuclei, half-integer $\hbar$ for o-A
- Nuclear spins = combination of nucleonic orbital and spin angular momenta
- Only odd magnetostatic moments, dipole is first important moment
- Magnetic moments of o-A nuclei related to unpaired nucleon
  → Schmidt Lines (quenching in medium, g factors always smaller than s.p. values)
- Magic nuclei have $I = 0$, $\mu = 0$