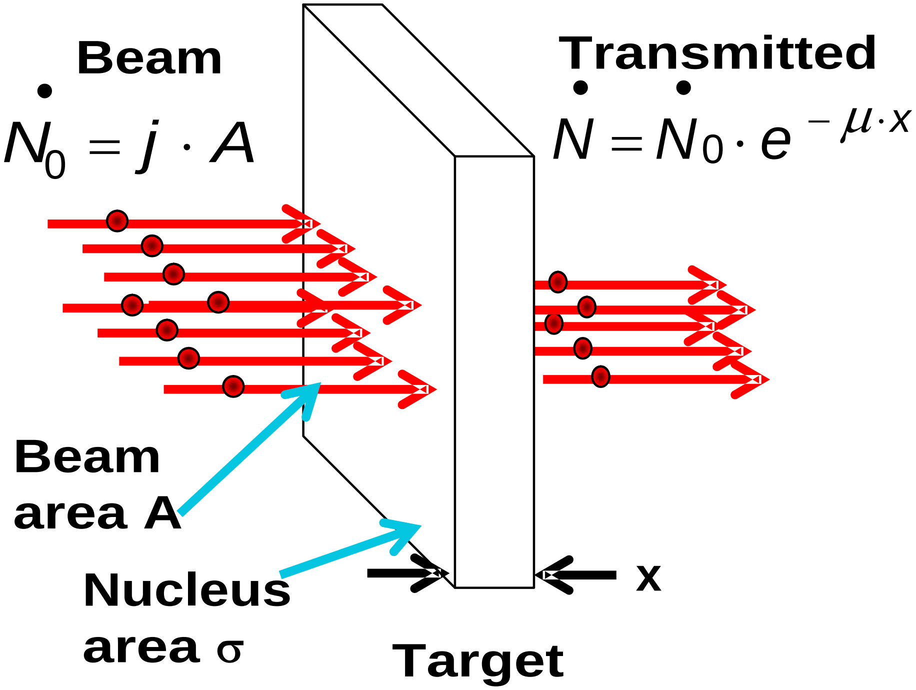
Gross Properties of Nuclei

Nuclear Sizes

Probability and Cross Section



Absorption upon intersection of nuclear cross section area σ

j beam current density
A area illuminated by beam
L= 6.022 10^{23}/mol Loschmidt#
$\boldsymbol{N_T}$ # target nuclei in beam
$\boldsymbol{M_T}$ target molar weight
ρ_T target density
x target thickness
$[\sigma]$=***1barn = 10^{-24}cm²***

Mass absorption coefficient μ

$$\mu x = \left[\begin{matrix} \# \text{ nuclei} \\ \text{in beam} \end{matrix}\right] \times \left(\begin{matrix} P_{absorption} \\ \text{per nucleus} \end{matrix}\right)$$

$$= \left[\left(\frac{L}{M_T}\rho_T\right)\cdot(A \cdot x)\right] \times \left(\frac{\sigma}{A}\right)$$

$$\dot{N}_{abs} = \dot{N}_0 - \dot{N} = \dot{N}_0 \cdot \left(1 - e^{-\mu \cdot x}\right)$$

Thin target

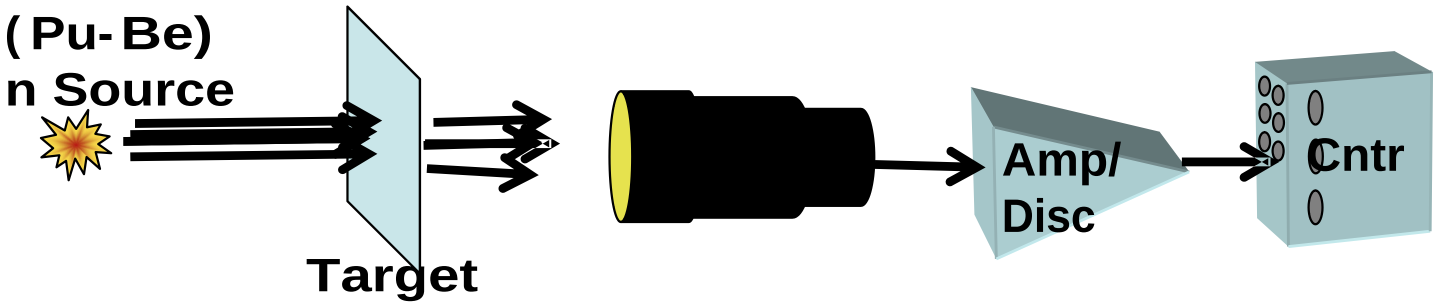
$$\dot{N}_{abs} = \dot{N}_0 \cdot (\mu \cdot x) = \frac{\dot{N}_0}{A} \cdot \left(\frac{L \rho_T A x}{M_T}\right)\sigma$$

$$= N_T \, j \, \sigma \qquad \textit{current density } j$$

$$\boxed{\sigma = \frac{\dot{N}_{abs}}{N_{nucl} \times j}}$$

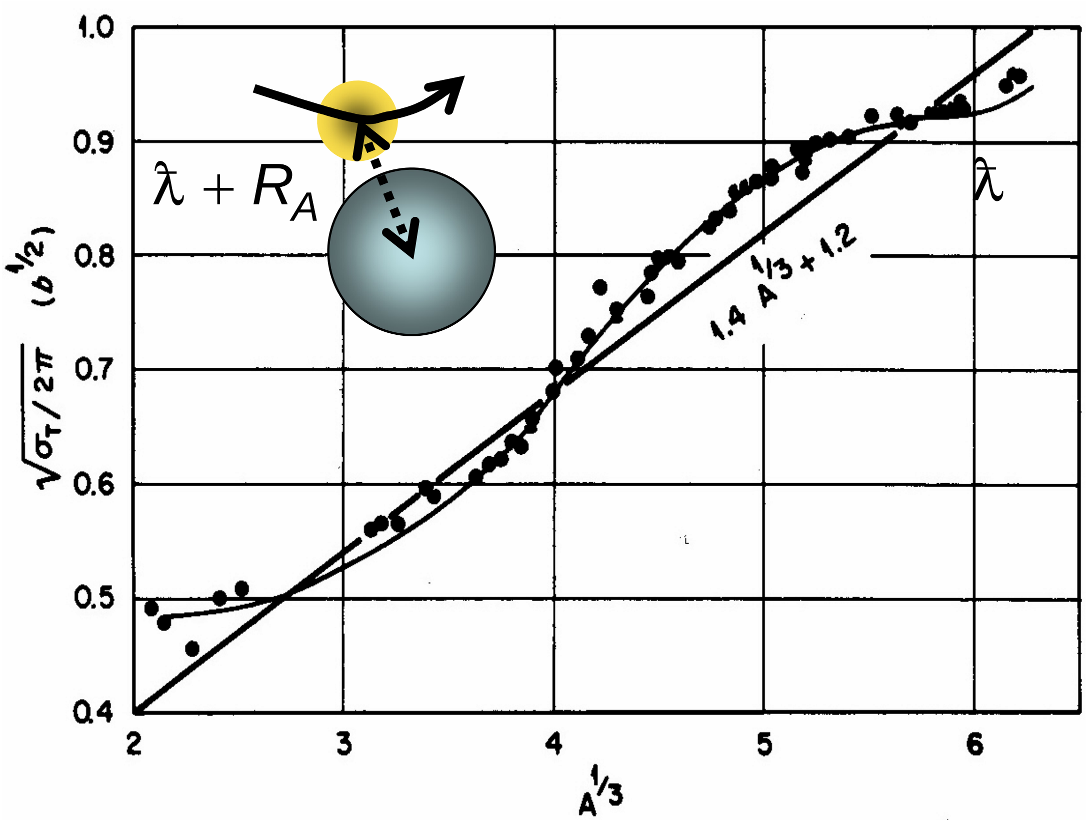
elementary absorption cross section per nucleus

Size Information from Nuclear Scattering



Basic setup with n source

d from small accelerator ($E_d \approx 100$ keV): $T(d,n)^3He \rightarrow E_n \approx 14$ MeV



J.B.A. England, *Techn.Nucl. Str. Meas., Halsted, New York,*1974

dBroglie wave length

$$ƛ = \hbar c \Big/ \sqrt{2mc^2 E} =$$

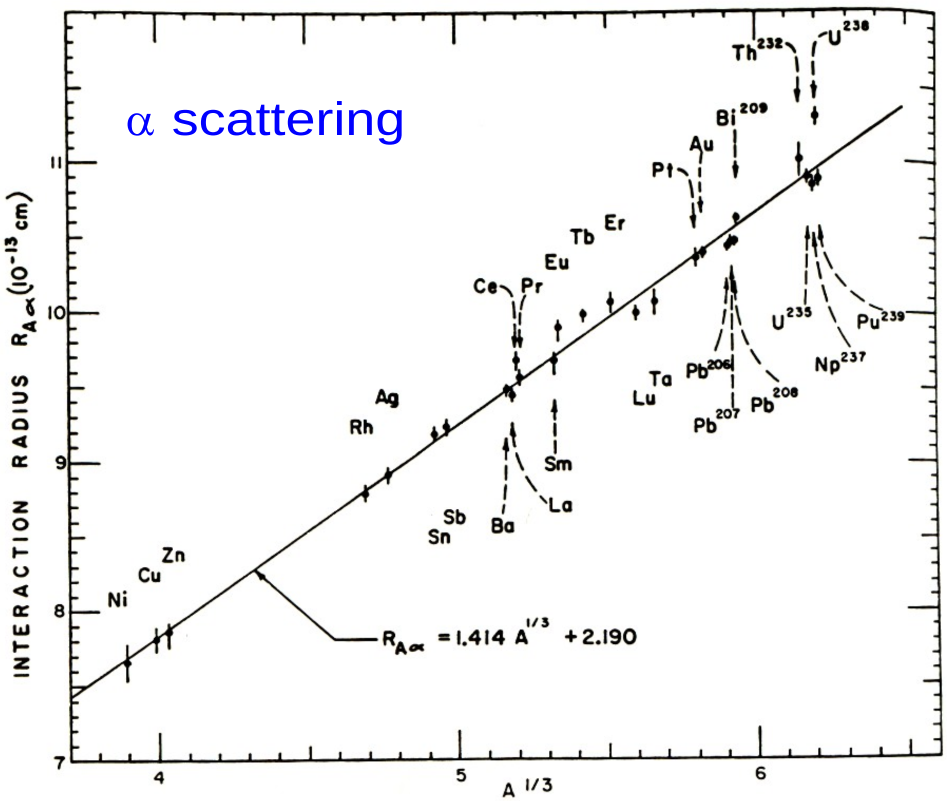
$$\boxed{ƛ = 4.55 \Big/ \sqrt{AE\,/\,MeV}\ fm}$$

14-MeV neutron $ƛ = 1.2 fm$

$$R_A \propto A^{1/3} \rightarrow V_A \approx \frac{4\pi}{3} R_A^3 \propto A$$

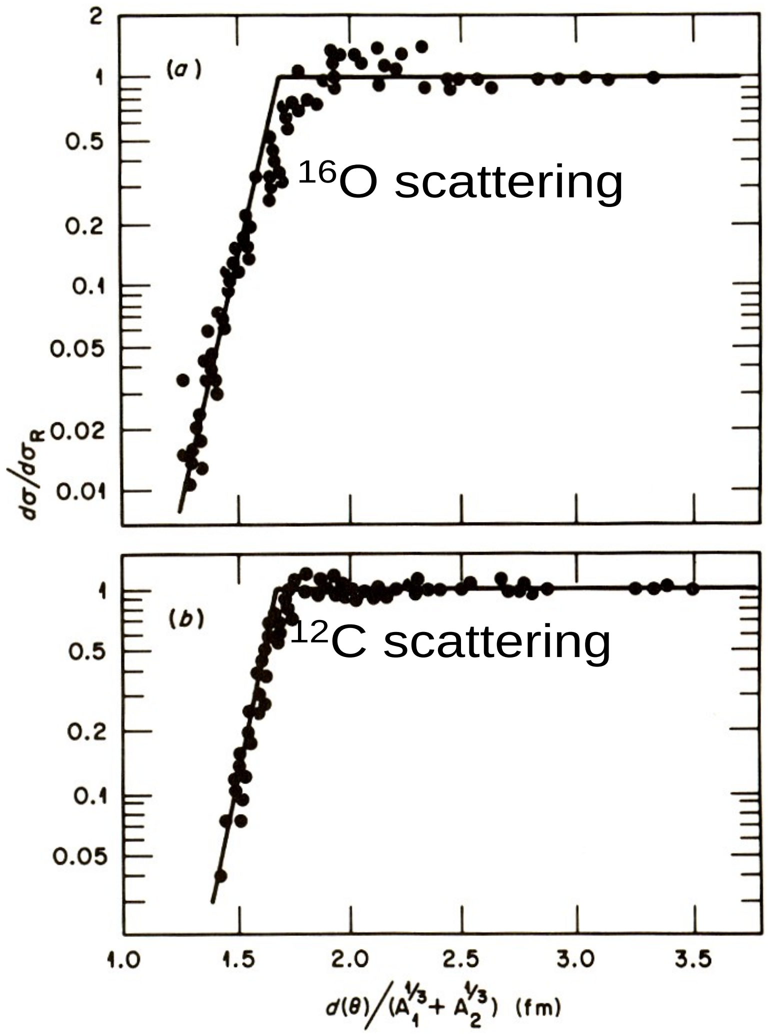
$$\rho_A \approx 0.17 \frac{nucl}{fm^3} \approx const.$$

Interaction Radii

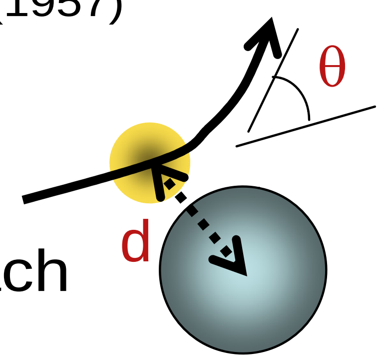


D.D. Kerlee et al., PR 107, 1343 (1957)

$$\frac{d\sigma_{el}(\theta)}{d\Omega} \Bigg/ \frac{d\sigma_{Ruth}(\theta)}{d\Omega}$$



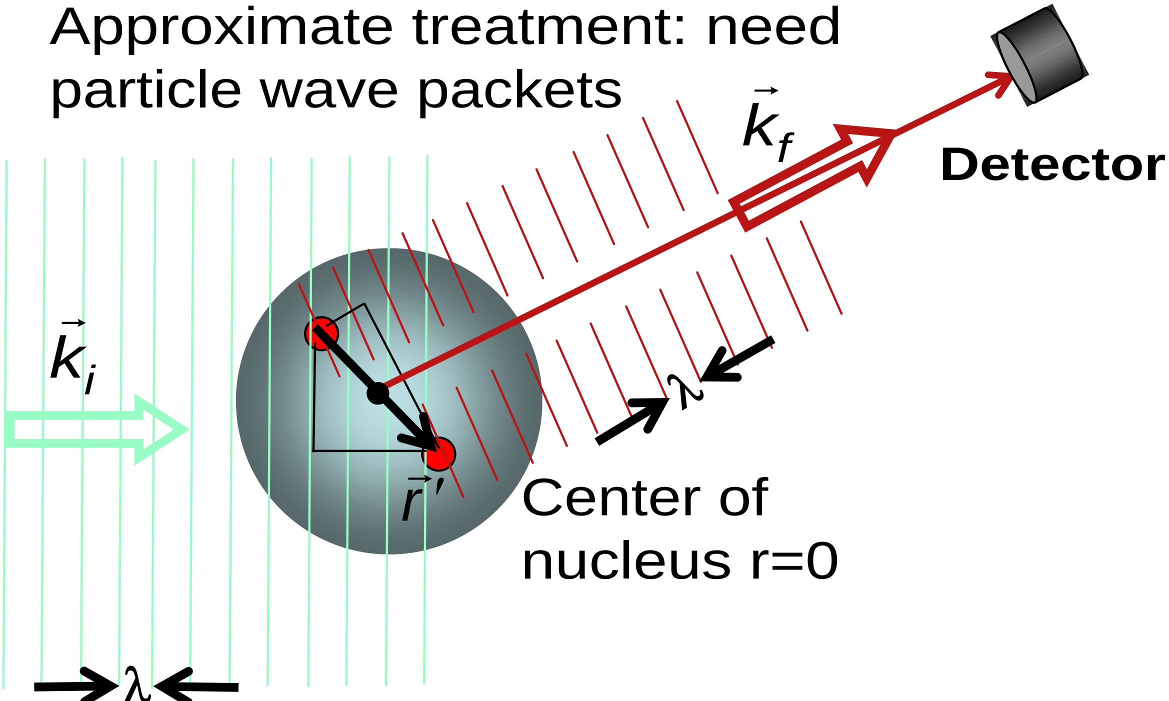
P.R. Christensen et al., NPA207, 33 (1973)

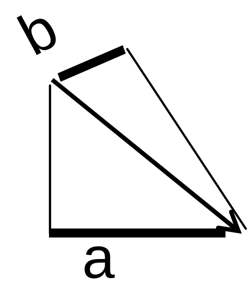


Distance of closest approach
←→ scatter angle θ

Electron Scattering

Approximate treatment: need particle wave packets





phase difference of elementary waves

$$\varphi = \frac{a-b}{\lambda} 2\pi = (a-b)k$$

$$a = \vec{k}_i \bullet \vec{r}\,' / k \qquad b = \vec{k}_f \bullet \vec{r}\,' / k$$

$$\varphi = \left(\vec{k}_i - \vec{k}_f\right) \bullet \vec{r}\,' / k =: \vec{q} \bullet \vec{r}\,' / k$$

Momentum transfer $\hbar\vec{q}$

$$\Psi_i(\vec{r}, t) = (2\pi\hbar)^{-3/2} \exp\left\{\frac{-i}{\hbar}\left(\vec{p}_i \bullet \vec{r} - E_i t\right)\right\}$$

$$= (2\pi\hbar)^{-3/2} \exp\left\{-i\vec{k}_i \bullet \vec{r} + i\varpi t\right\}$$

$$\left|\vec{p}_i\right| = \left|\vec{p}_f\right| = p = \hbar k = \hbar \frac{2\pi}{\lambda}$$

$$\Psi_f(\vec{r}, t) = \sum_n \psi_n(\vec{r}_n, t)$$

Impulse Approximation

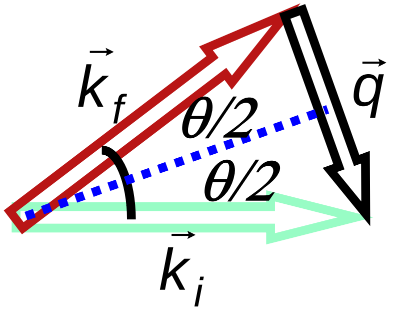
$$\psi_n(\vec{r}_n, t) \propto \psi_{proton_n} \cdot \psi_{electron}$$

$$\propto f(\vec{r}_n\,') \exp\left\{-i\vec{k}_f \bullet \vec{r} + i\varpi t + ik\varphi_n\right\}$$

$$\propto \Psi_f(\vec{r}, t) \cdot f(\vec{r}_n\,') \exp\left\{ik\varphi_n\right\}$$

prob. amplitude for proton n

Momentum Transfer and Scatter Angle in (e,e)



Scattering angle θ determines momentum transfer $\hbar q$

$$m_e \ll m_A \rightarrow k_f \approx k_i = k \quad Lab \approx com$$

$$\boxed{q = 2k \cdot \sin(\theta/2)} \qquad q = q(\theta)\ !$$

$$\frac{d\sigma_{i\to f}}{d\Omega_f} \propto \left|\left\langle \Psi_i(\vec{r},t) \,\middle|\, \Psi_f(\vec{r},t)\right\rangle\right|^2 \propto \left|\left\langle \exp\left\{-i\vec{k}_i \bullet \vec{r} + i\varpi t\right\} \,\middle|\, \sum_n \psi_n(\vec{r}_n,t)\right\rangle\right|^2$$

$$\propto \left|\exp\left\{i\vec{k}_i \bullet \vec{r} - i\varpi t\right\} \exp\left\{-i\vec{k}_f \bullet \vec{r} + i\varpi t\right\}\right|^2 \left|\sum_n f(\vec{r}_n{}') \exp\left\{ik\varphi_n\right\}\right|^2$$

$$\propto \underbrace{\left|\exp\left\{i\vec{q} \bullet \vec{r}\right\}\right|^2}_{=1} \left|\sum_n f(\vec{r}_n{}') \exp\left\{ik\varphi_n\right\}\right|^2 = \underline{\left|\sum_n f(\vec{r}_n{}') \exp\left\{ik\varphi_n\right\}\right|^2}$$

W. Udo Schröder, 2004

Separation of Variables

Point nucleus: a=b, $\varphi_n=0$ →

$$\left.\frac{d\sigma_{i\to f}}{d\Omega_f}\right|_{PN} = \left.\frac{d\sigma_{i\to f}}{d\Omega_f}\right|_{Rutherford} \propto \left|\sum_n f(\vec{r}_n')\right|_{PN}^2 = \left|\sum_n f_0\right|^2 = \left|Zf_0\right|^2$$

$$\frac{d\sigma_{i\to f}}{d\Omega_f} \propto \left|\sum_n f(\vec{r}_n')\exp\{ik\varphi_n\}\right|^2 \to \left|\int_{Nucleus} d^3\vec{r}'\, f_0 \cdot Z\rho(\vec{r}')\, e^{i\vec{q}\bullet\vec{r}'}\right|^2$$

$$\propto \left|Zf_0\right|^2 \left|\int_{Nucleus} d^3\vec{r}'\, \rho(\vec{r}_n')\, e^{i\vec{q}\bullet\vec{r}'}\right|^2$$

$\rho(\vec{r}) =$ *nuclear charge density*

$$\boxed{\frac{d\sigma_{i\to f}}{d\Omega_f} = \left.\frac{d\sigma_{i\to f}}{d\Omega_f}\right|_{PN} \cdot \left|F(\vec{q})\right|^2}$$

Scatter cross section for finite nucleus = cross section for point-nucleus x form factor F of charge distribution

Mott Cross Section for Electron Scattering

In typical nuclear applications, electron kinetic energies K » $m_e c^2$
→ (extreme) relativistic domain

$$E^2 = (pc)^2 + (mc^2)^2$$

$$ƛ_e = \frac{\lambda}{2\pi} = \frac{\hbar c}{\sqrt{K(K + 2m_e c^2)}} = \frac{1.952 \cdot 10^2\, fm}{\sqrt{(K\,/\,MeV)(1 + K/1.022MeV)}}$$

$ƛ_e(100MeV) = 2\,fm$ e^- = good probe for objects on fm scale

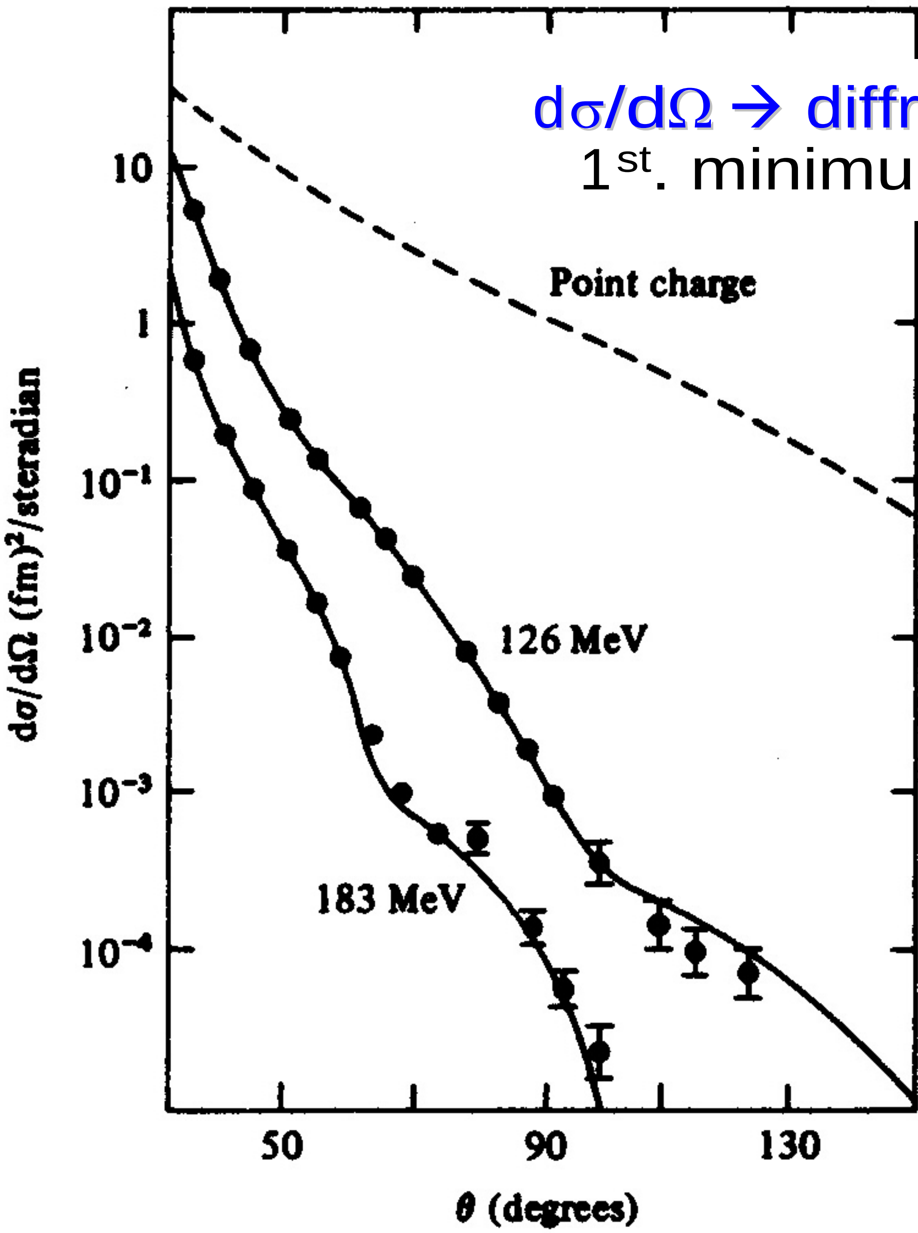
$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \left(1 - \beta^2 \sin^2(\frac{\theta}{2})\right) \xrightarrow[\beta\to 1]{} \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \cos^2(\frac{\theta}{2})$$

Obtained in 1. order qu. m. perturbation theory, neglects nuclear recoil momentum.

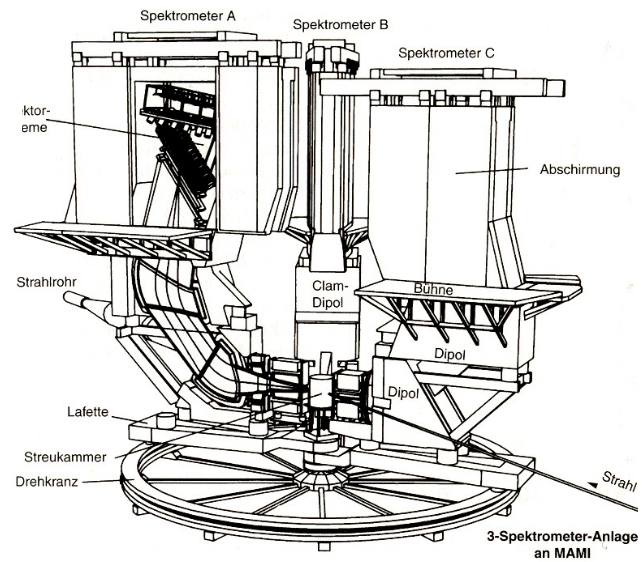
Elastic (e,e) Scattering Data

$d\sigma/d\Omega \rightarrow$ diffraction patterns

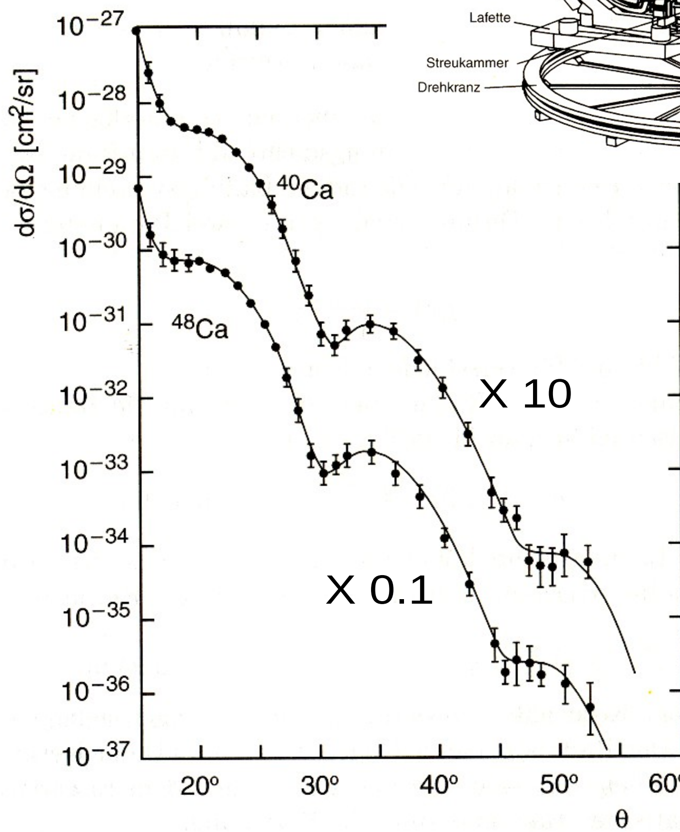
1^{st}. minimum $q(\theta) \approx 4.5/R$



R. Hofstadter, Electron Scatt. and Nucl. Struct., Benjamin, 1963

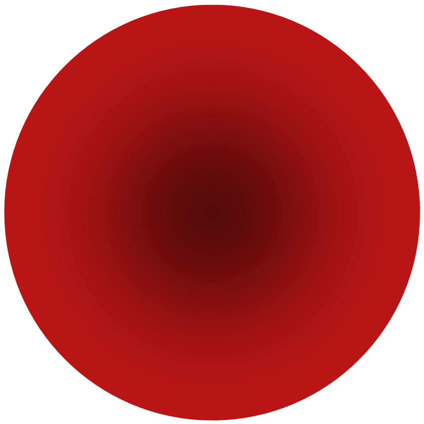


3-arm electron spectrometer (Univ. Mainz)



J.B. Bellicard et al., PRL 19,527 (1967)

Fourier Transform of Charge Distribution



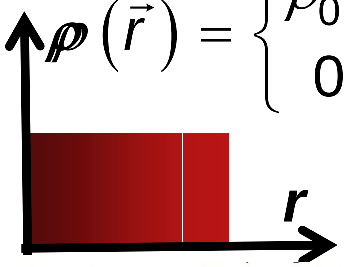
Form factor contains entire information about charge distribution

$$\vec{q}\bullet\vec{r} = qr\cos\theta \qquad \vec{q}\parallel\vec{z}$$

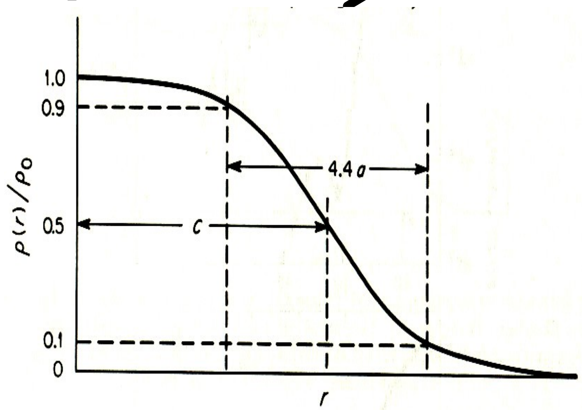
$$F(q) = \int r^2 dr \sin\theta\, d\theta\, d\phi\, \rho(r) e^{iqr\cos\theta}$$

$$= -2\pi \int_0^\infty dr\, r^2 \rho(r) \left[\frac{e^{-iqr} - e^{+iqr}}{i\,qr}\right]$$

$$\rho(\vec{r}) = \begin{cases} \rho_0 & r \le R \\ 0 & r > R \end{cases}$$



$$\boxed{F(q) = \frac{4\pi}{q}\int_0^\infty dr\,[r\rho(r)]\sin(qr)}$$



$$f(r) = \frac{2}{\pi}\int_0^\infty dq\, \tilde{f}(q)\cdot\sin(qr)$$

Fourier transform f

$$\rho(r)\cdot r = \frac{1}{2\pi^2}\int_0^\infty dq\,\left[q\cdot F(q)\right]\sin(qr)$$

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-C)/a}}$$

Fermi distribution, half-density radius C, diffuseness a

Nuclear Charge Form Factor

$$F(\vec{q}) = \int_{Nucleus} d^3\vec{r}\,'\,\rho(\vec{r}_n\,')\,e^{i\vec{q}\bullet\vec{r}\,'}$$

$$\hbar q = momentum\ \ transfer$$

Form factor for Coulomb scattering = Fourier transform of charge distribution.

r-Distribution	Function ρ(r)	Form Factor	θ-Distribution
Point	$\frac{1}{4\pi}\delta(r)$	1	constant
Homogeneous sharp sphere	$\begin{cases}\rho_0 & \text{for } r \le R \\ =0 & \text{for } r > R\end{cases}$	$\frac{3\left(\sin qR - qR\cos(qR)\right)}{(qr)^3}$	oscillatory
Exponential	$\frac{a^3}{8\pi}e^{-a\cdot r}$	$\left[\frac{\hbar^2 a^2}{1+q^2}\right]^2$	exponential
Gaussian	$\left(\frac{a^2}{2\pi}\right)^{3/2} e^{-\frac{a^2r^2}{2}}$	$\exp\left\{-\frac{q^2}{2a^2}\right\}$	Gaussian

W. Udo Schröder, 2004

Model-Independent Analysis of Scattering

Interpretation in terms of radial moments of charge distribution

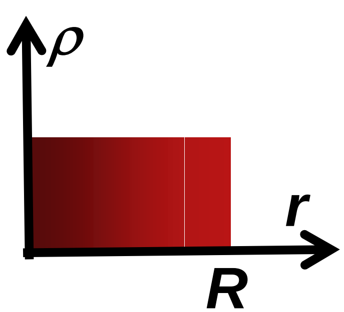
Expansion: $$\sin(qr) = qr - \frac{1}{3!}(qr)^3 + \frac{1}{5!}(qr)^5 - \ldots .$$

$$F(q) = \underbrace{4\pi\int_0^\infty dr\,[r^2\rho(r)]}_{=1} - \frac{4\pi}{6}q^2\int_0^\infty dr\,r^2\,[r^2\rho(r)] + \frac{4\pi}{120}q^4\int_0^\infty dr\,r^4\,[r^2\rho(r)] - \ldots$$

$$\boxed{F(q) = 1 - \frac{1}{6}q^2\left\langle r^2\right\rangle + \frac{1}{120}q^4\left\langle r^4\right\rangle - \ldots}$$

$$\rightarrow\quad \left\langle r^2\right\rangle = -6\left.\frac{dF}{dq^2}\right|_{q=0}$$

$\left\langle r^2\right\rangle$ mean-square radius of charge distribution

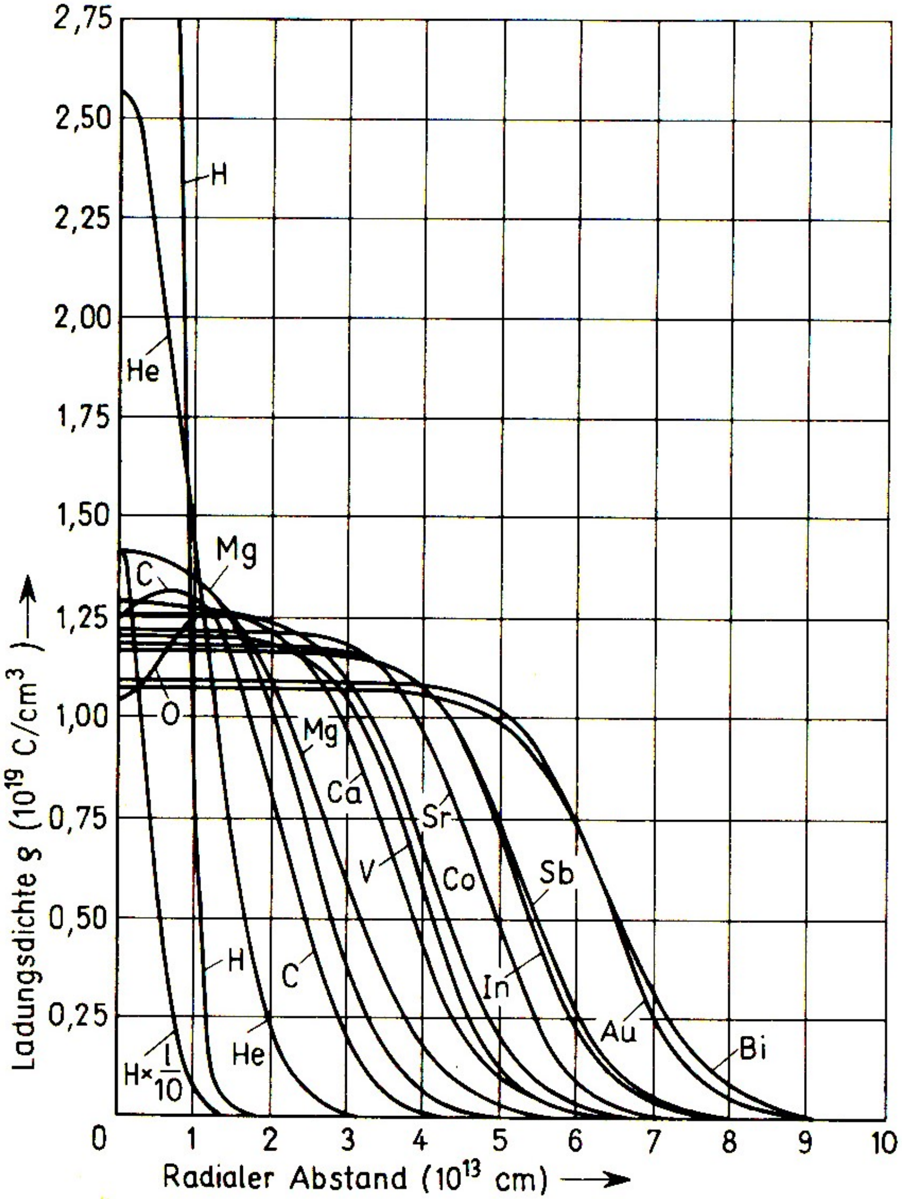


$$\left\langle r^2\right\rangle = \frac{3}{5}R^2$$

Equivalent sharp radius of any $\rho(r)$: $$R_{eq} := \sqrt{\frac{5}{3}\left\langle r^2\right\rangle_\rho}$$

W. Udo Schröder, 2004

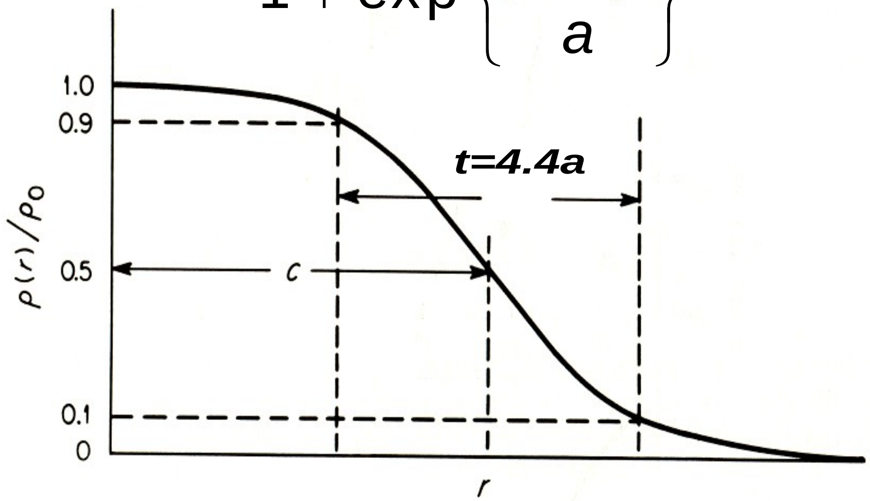
Nuclear Charge Distributions (e,e)



R. Hofstadter, Ann. Rev. Nucl. Sci. 7, 231 (1957)

Fermi Distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp\left\{\frac{r - C}{a}\right\}}$$

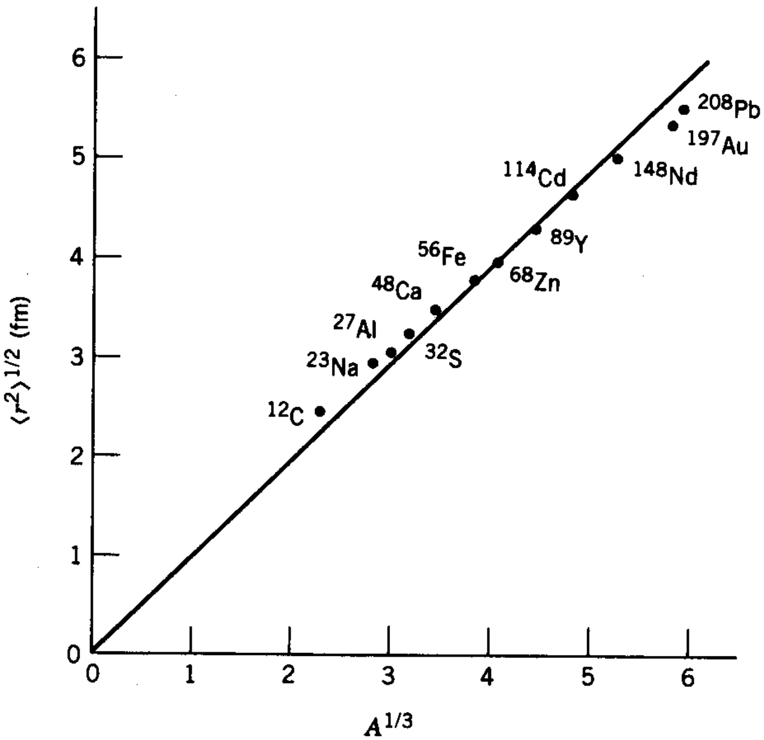


C: Half-density radius
a: Surface diffuseness
t: Surface thickness

Leptodermous: t « C

Holodermous : t ~ C

Charge Radius Systematics



Charge distributions:

$$C(A) = 1.07 \cdot A^{1/3} fm \quad a \approx 0.54\, fm \approx const.$$

$$\sqrt{\langle r^2 \rangle} = r_0 A^{1/3} \quad r_0 = 0.94\, fm$$

Homogeneously charged sphere

$$R_{equ} = \sqrt{\frac{5}{3}\langle r^2 \rangle}$$

$$R(A) = R_{equ}(A) = 1.23 \cdot A^{1/3} fm$$

$a \approx 0.54\, fm \approx const.$ (*small isotopic*

$t \approx 2.4\, fm \approx const.$ *effect*)

ρ_0(charge) decreases for heavy nuclei like Z/A → for all nuclei:

ρ_0(mass) = 0.17 fm^{-3} = const. $\triangleq 10^{14}$ g/cm^3

W. Udo Schröder, 2004

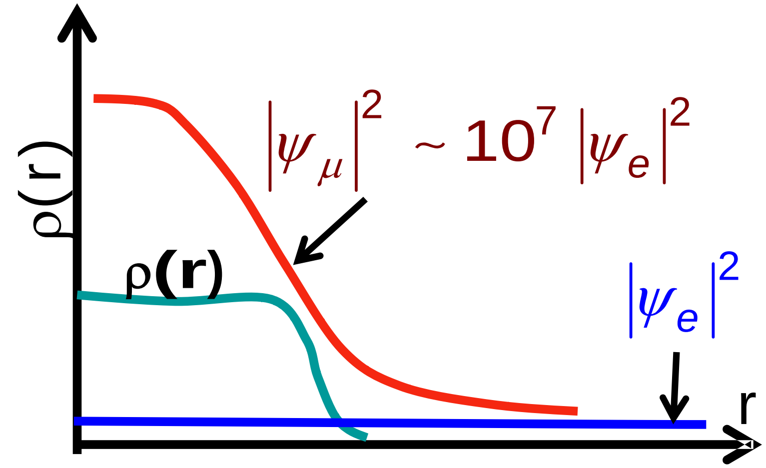
Muonic X-Rays

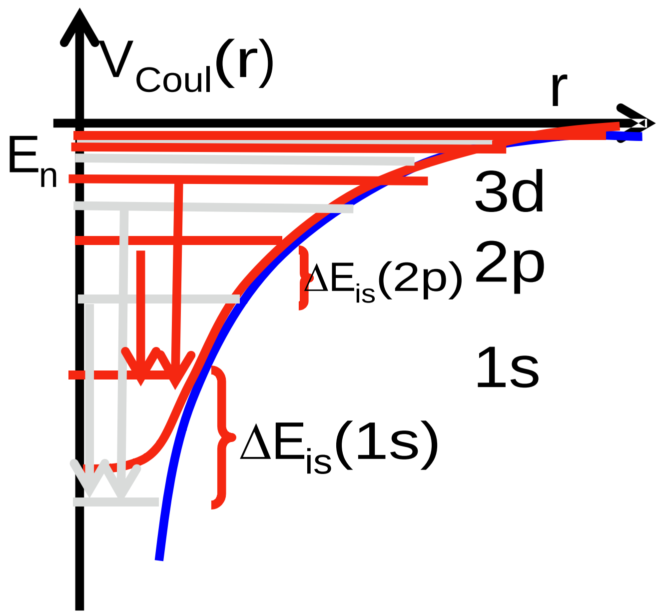
Negative muon:
$\mu^- \approx e^-$ $\quad m_\mu = 207 m_e$

Replace electron by muon → "muonic atom"

Bohr orbits, $a_\mu = a_e/207$

10^7 times stronger fields





1) X-ray energies 100keV−6 MeV

2) Isomeric/isotopic shifts ΔE_{is}

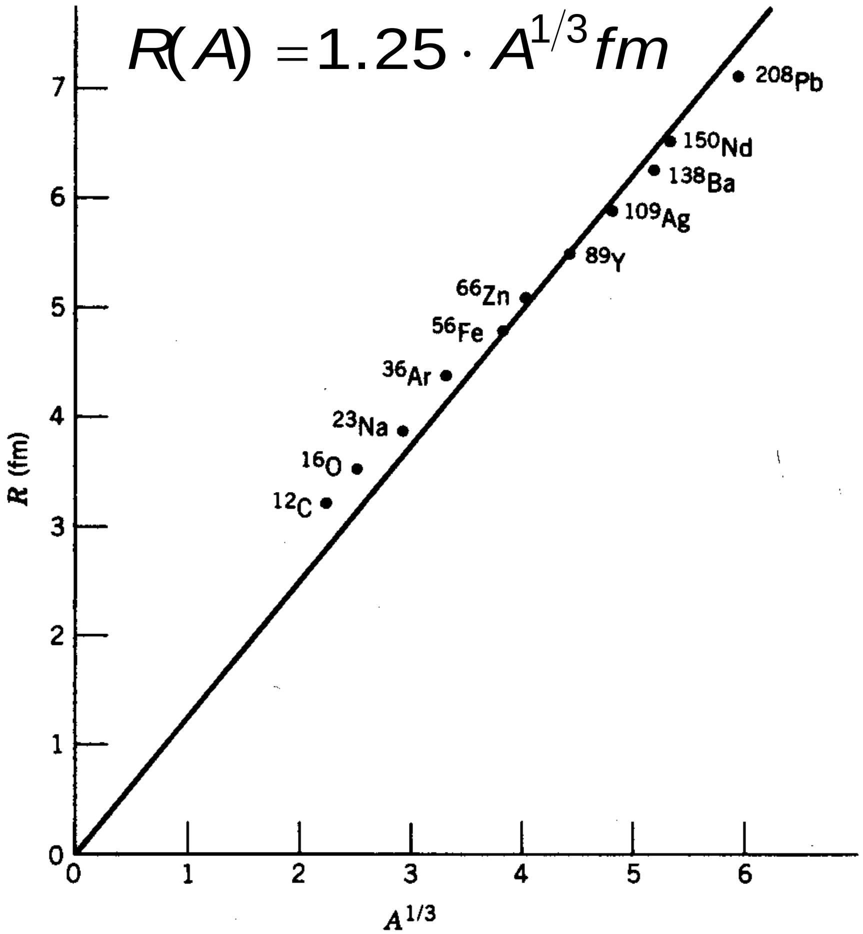
point nucleus

$$\Delta E_{is} = 4\pi e^2 \int_0^\infty dr\, r^2 \left[\delta(r) - \rho(r)\right] |\psi(r)|^2$$

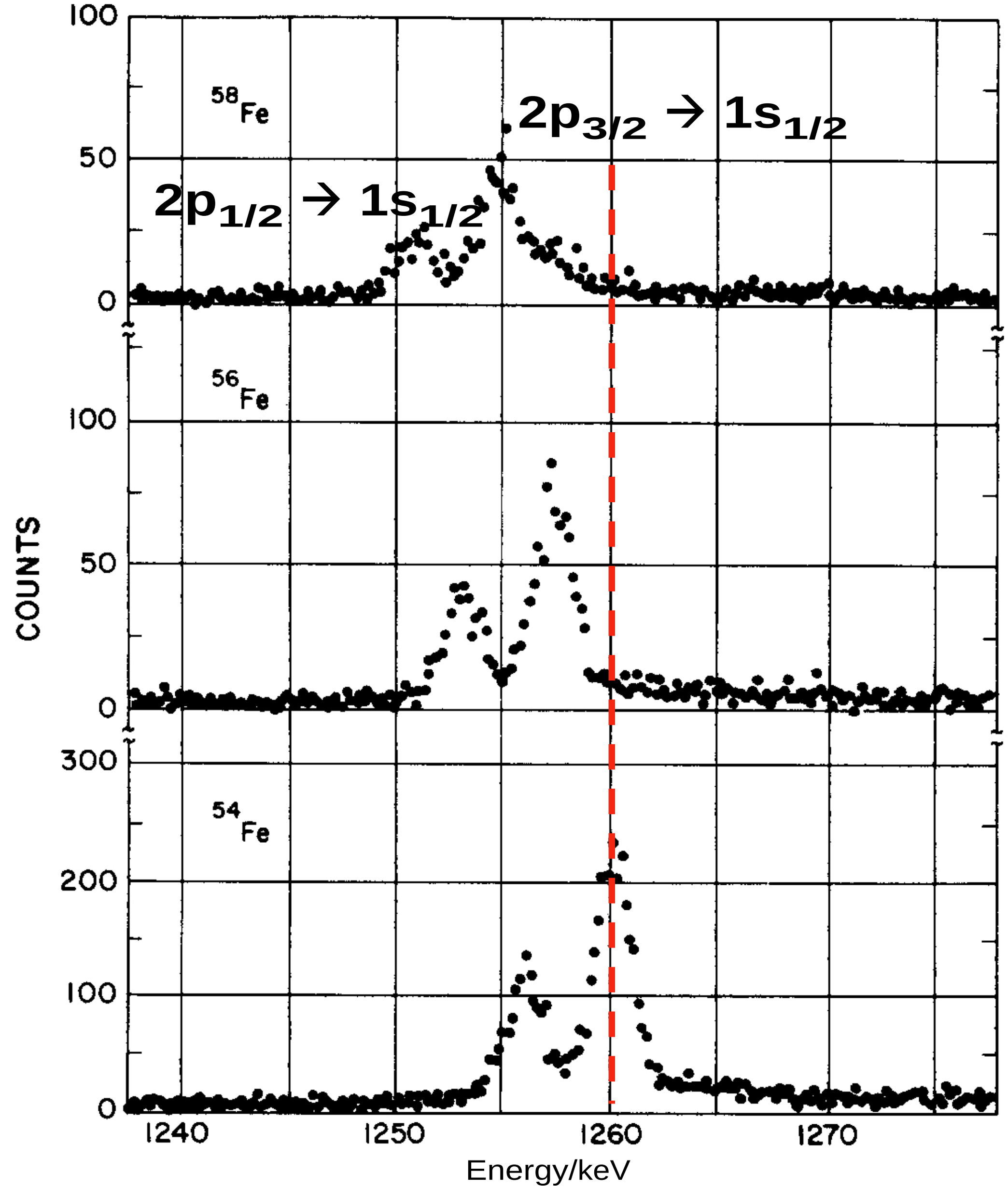
$$\Delta E_{is} = 4\pi e^2 \int_0^\infty dr\, r^2 \left[\rho_{gs}(r) - \rho_{ex}(r)\right] |\psi(r)|^2$$

ground excited
nuclear state

Charge Radii from Muonic Atoms



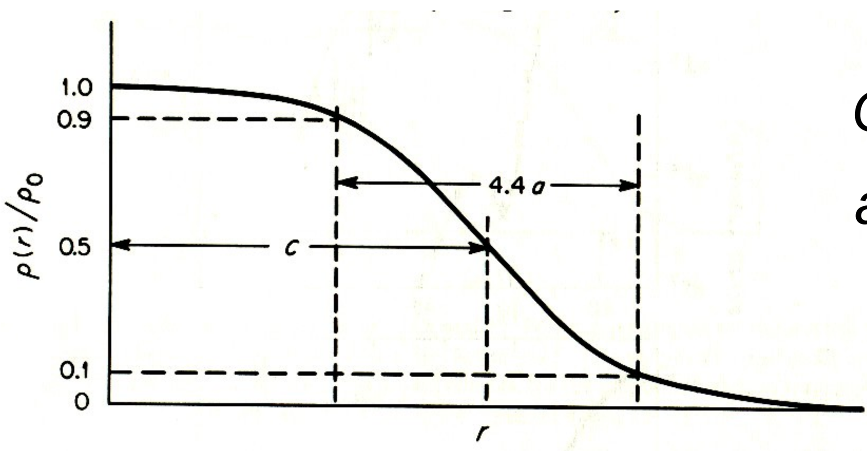
Engfer et al., Atomic Nucl. Data Tables 14, 509 (1974)



E.B. Shera et al., PRC14, 731 (1976)

Sensitive to isotopic, isomeric, chemical effects

Mass and Charge Distributions



$$C(A) = 1.07 \cdot A^{1/3} fm$$
$$a \approx 0.54\, fm$$
$$\sqrt{\langle r^2 \rangle} = r_0 \cdot A^{1/3}; \quad r_0 = 0.94\, fm$$
$$R(A) = 1.21 \cdot A^{1/3} fm$$
$$t = 2a \ln 9 = 2.40\, fm$$

Charge density:

$$C_Z(A) = 1.1 \cdot A^{1/3} fm \quad R_Z(A) = 1.23 \cdot A^{1/3} fm$$
$$a_Z \approx 0.55\, fm \quad a_Z \approx 0.55\, fm$$

Mass density distribution:

$$\boxed{\rho_A(r) \approx \frac{A}{Z} \cdot \rho_Z(r)}$$

except for *small* surface increase in n density ("neutron skin")

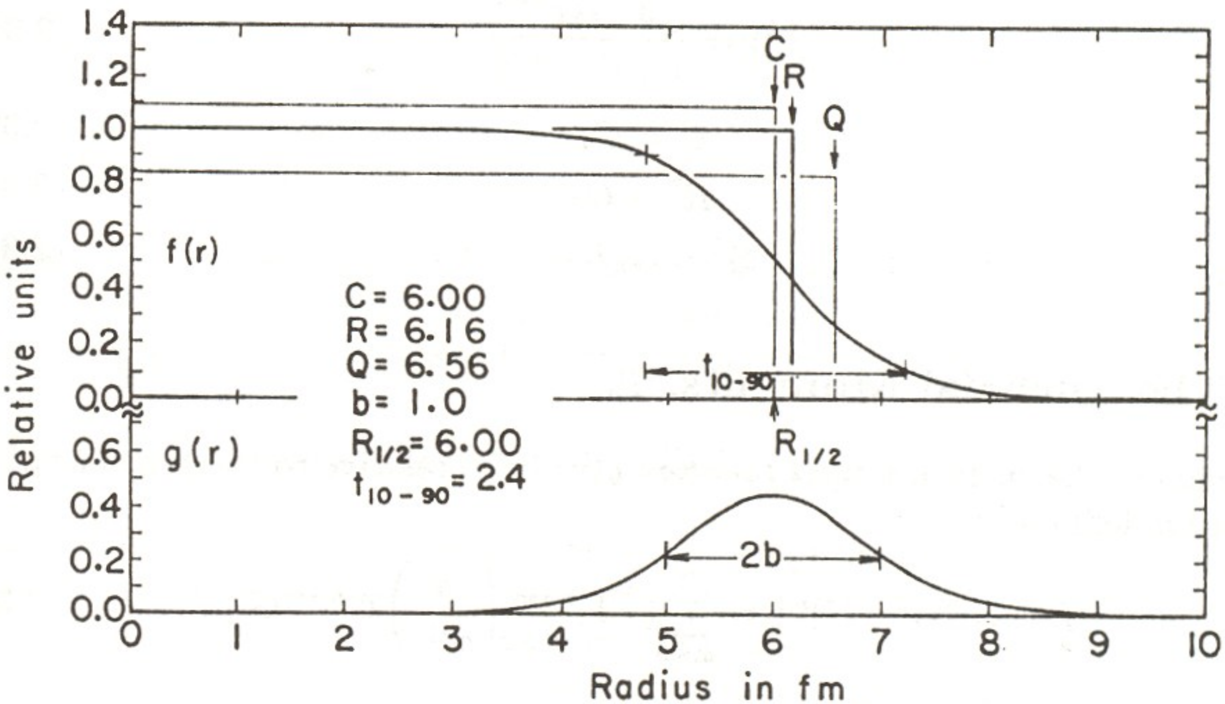
Constant central density for all nuclides, except very light (Li, Be, C,..)

$$\boxed{\rho_N = 0.17\, fm^{-3}} \quad nucleons$$

W. Udo Schröder, 2004

Leptodermous Distributions

R.W. Hasse & W.D. Myers, *Geometrical relationships of macroscopic nuclear physics*, Springer V., New York, 1988



Fermi Distribution ($a \ll C$)

$$f(r) = \frac{f_0}{1 + \exp\left\{\dfrac{r - C}{a}\right\}}$$

$$g(r) = df(r)/dr$$

C = Central radius
R = Equivalent sharp radius
Q = Equivalent rms radius
b = Surface width

Coulomb self energy

$$E^0_{Coul} = \frac{3}{5}\frac{(eZ)^2}{R} \quad \textit{sharp sphere}$$

$$E_{Coul} = E^0_{Coul}\left(1 - \frac{5}{2}\left(\frac{b}{R}\right)^2 + ..\right)$$

$$C = \int_0^\infty dr\, g(r)\cdot r = R\cdot\left(1 - \left(\frac{b}{R}\right)^2 + \ldots\right)$$

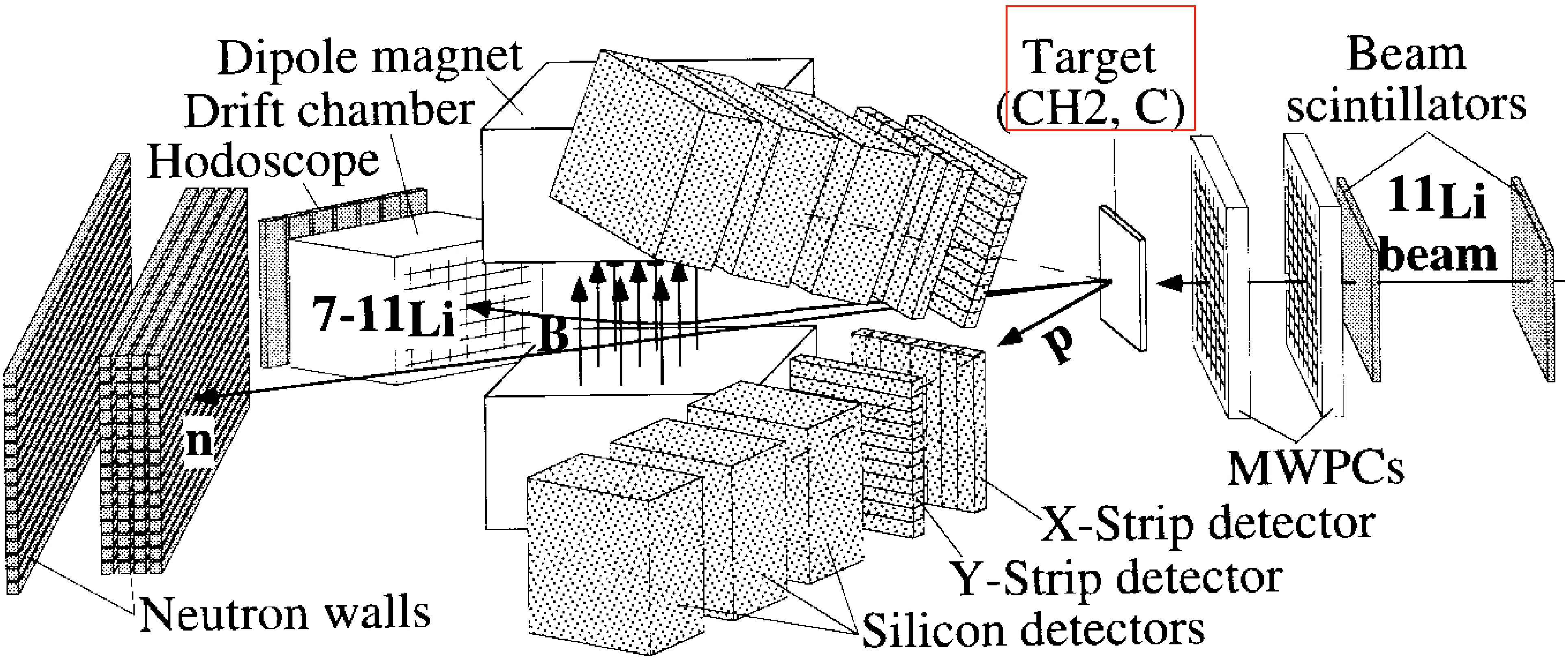
$$b^2 = \int_0^\infty dr\, g(r)\cdot(r - C)^2 = R\cdot\left(1 + \frac{5}{2}\left(\frac{b}{R}\right)^2 - \ldots\right)$$

$$b \approx \frac{\pi}{\sqrt{3}}a \quad (a \ll C)$$

leptodermous

Studies with Secondary Beams

Produce a secondary beam of projectiles from interactions of intense primary beam with "production" target → projectiles rare/unstable isotopes, induce scattering and reactions in "p" target



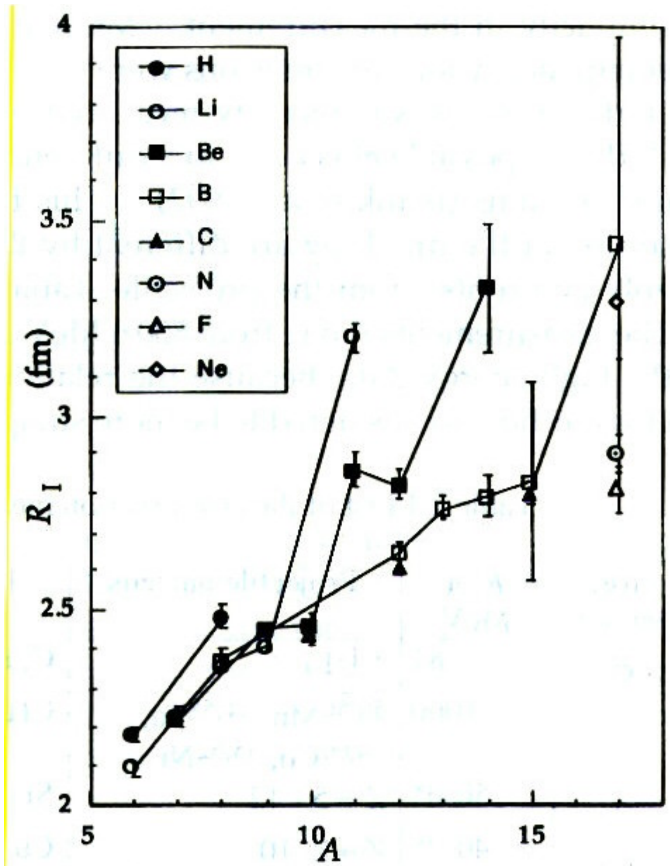
Tanihata et al., RIKEN-AF-NP-233 (1996)

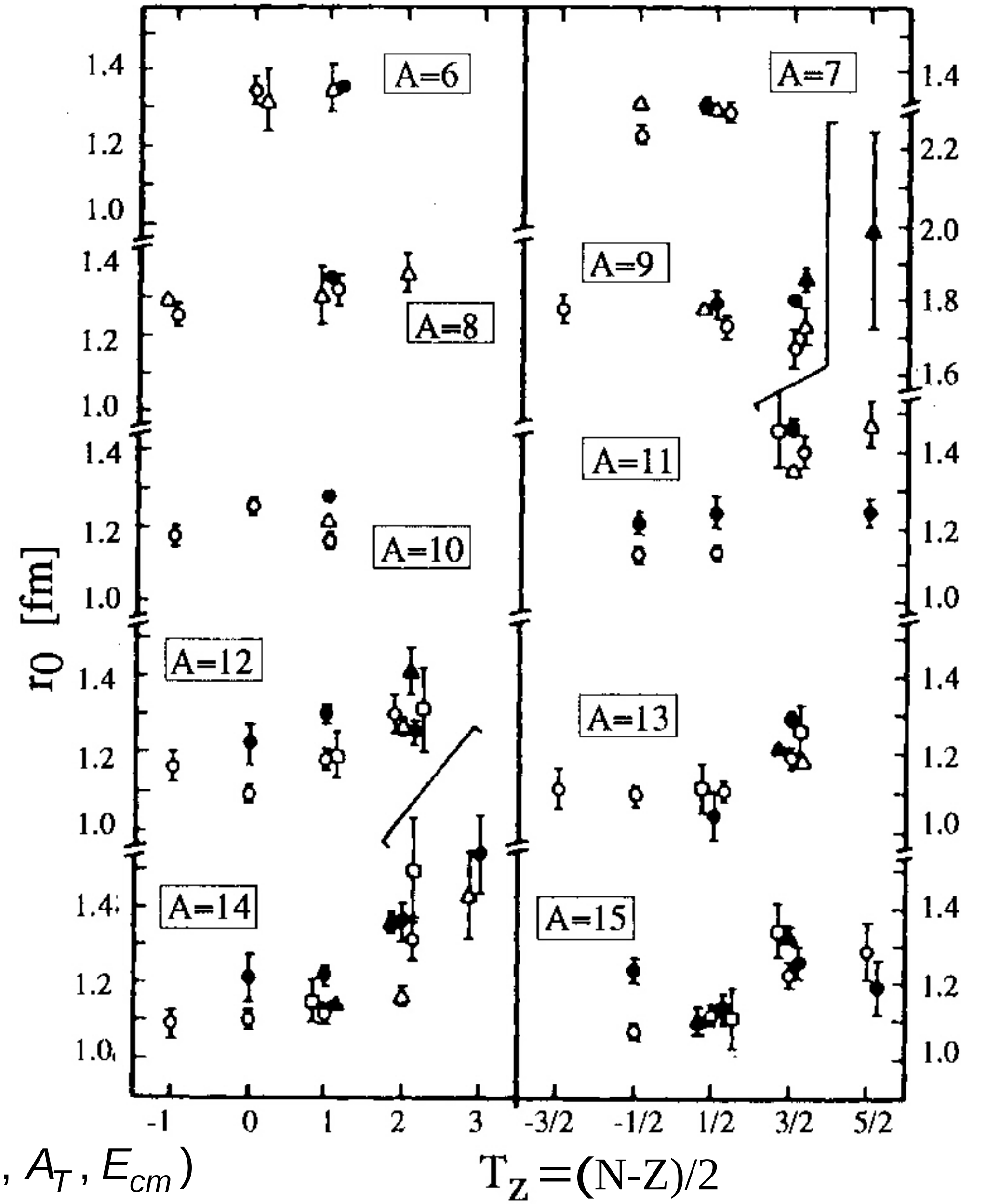
W. Udo Schröder, 2004

"Interaction Radii for Exotic Nuclei

Derive $\sigma_R = \sigma_T - \sigma_{el}$,

$\sigma_R =: \pi[R_I(p) + R_I(T)]^2$



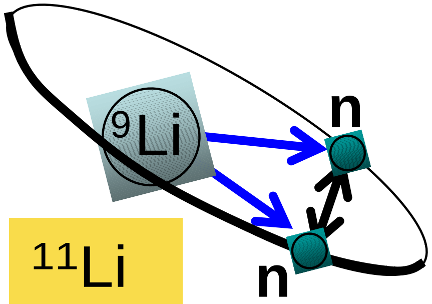


Kox Parameterization:

$$R_{int} = R_{vol}(A_p, A_T) + R_{surf}(A_p, A_T, E_{cm})$$
$$= r_0 \cdot f(A_P^{1/3}, A_T^{1/3})$$

Tanihata et al., RIKEN-AF-NP-168 (1995)

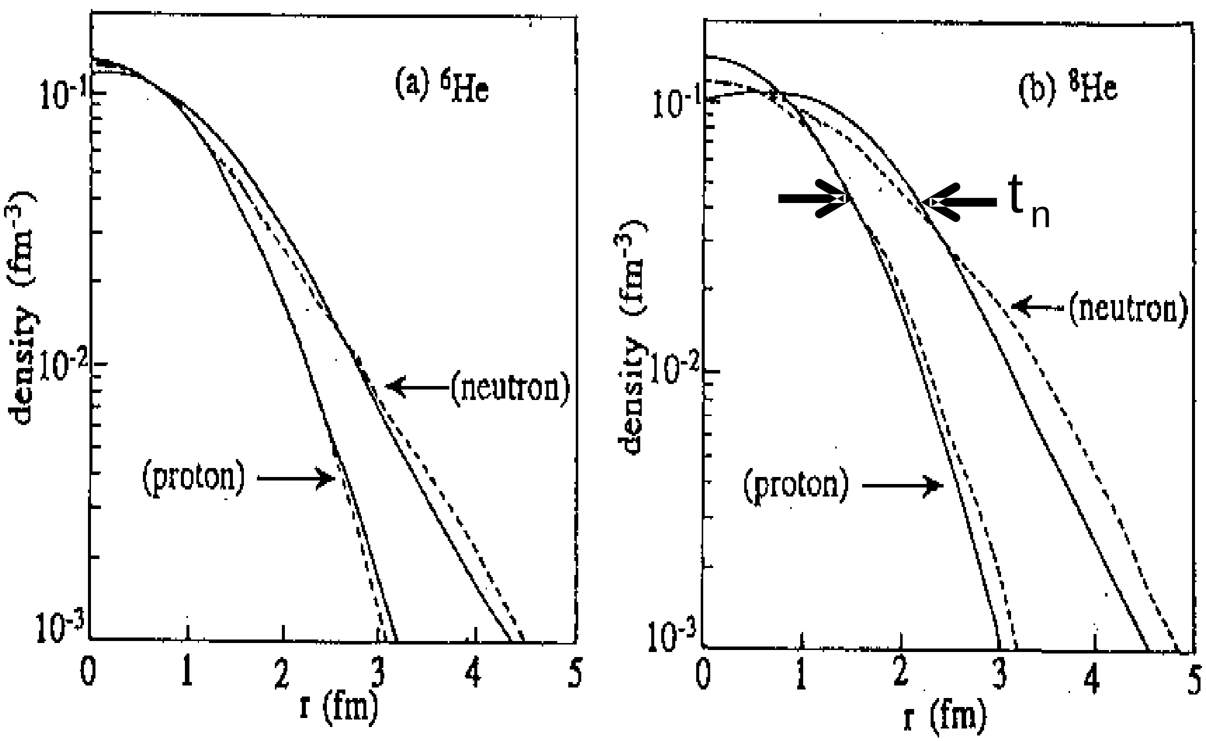
"Halo" Nuclei



From p scattering on ^{11}Li → extended mass distribution ("halo"). Valence-neutron correlations in ^{11}Li: $r_1 = r_2 = 5$fm, $r_{12} = 7$ fm

$$\rho_i(r) = \frac{N_{ci}}{\pi^{3/2}a^3}\exp\left(-\frac{r^2}{a^2}\right) + \frac{2N_{ni}}{3\pi^{3/2}b^5}\exp\left(-\frac{r^2}{b^2}\right)\left[Ar^2 + B\left(r^2 - \frac{3}{2}b^2\right)^2\right]$$

$i = n, p.$



$^{11}Li: \; N_{cp} = 3, N_{cn} = 6,$

$N_{np} = 0, N_{nn} = 2,$

$a = 1.89fm, b = 3.68fm$

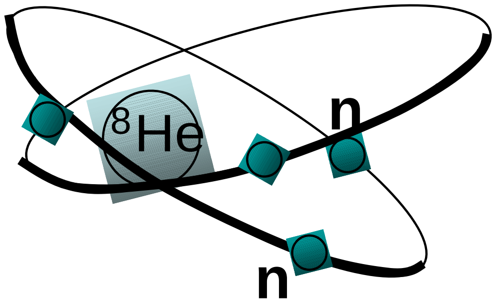
$A = 0.81, B = 0.19$

^{6}He - ^{8}He mass density distributions

Experiment: dashed, Theory:solid

Korshenninikov et al., RIKEN-AF-NP-233, 1996

Neutron Skin of Exotic (n-Rich) Nuclei



Q_{rms} (^{4}He) = (1.57 ± 0.05) fm

Q_{rms} (^{6}He) = (2.48 ± 0.03) fm

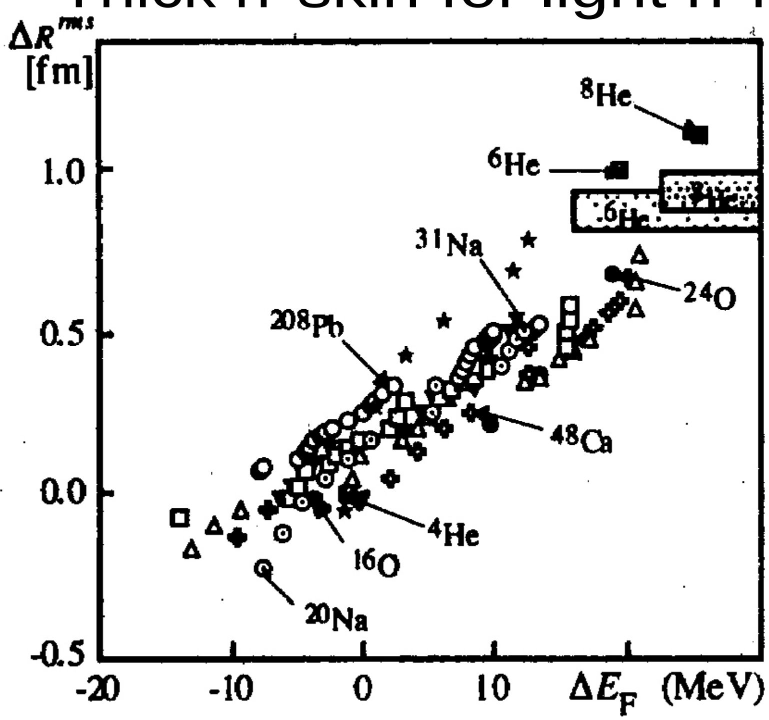
Q_{rms} (^{8}He) = (2.52 ± 0.03) fm

V(^{8}He) = 4.1 x V(^{4}He) !

matter radii

Tanihata et al., PLB 289,261 (1992)

Thick n-skin for light n-rich nuclei: $t_n \approx 0.9$ *fm* (*^{6}He, ^{8}He*)



D.H. Hirata et al., PRC 44, 1467(1991)

Relativistic mean field calculations:

$\boldsymbol{t_n} \leftarrow\rightarrow \varepsilon_F$

^{133}Cs$_{78}$ stable, insignificant n-skin, t_n ~0.1fm

^{181}Cs$_{126}$ unstable, significant n-skin, t_n ~ 2fm

Can one make ^{181}Cs ??

p-halos ? Coulomb barrier keeps p together, expansion could reduce it

End of Nuclear Sizes