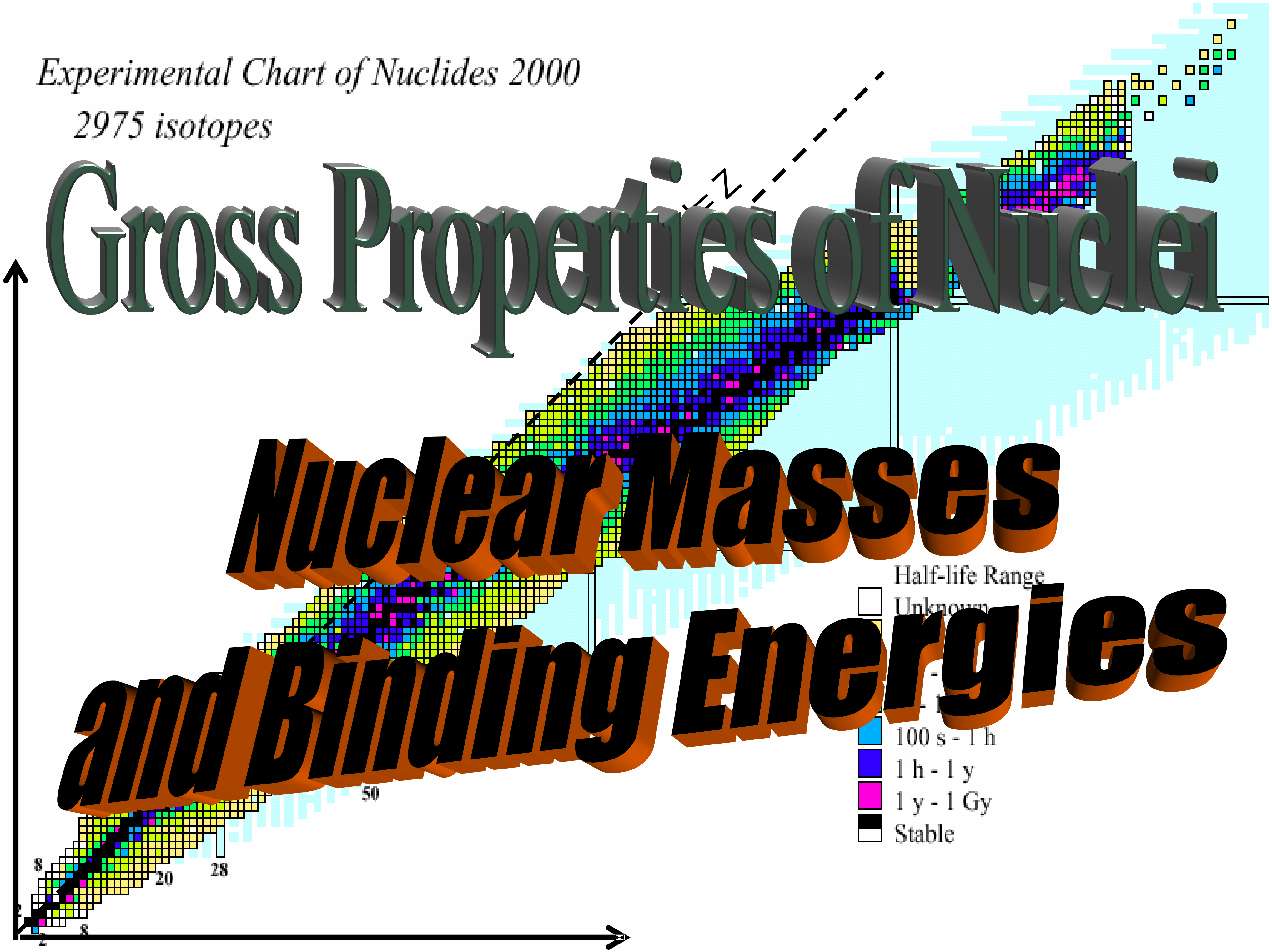
# Gross Properties of Nuclei

## Nuclear Masses and Binding Energies



# Measuring Nuclear Masses

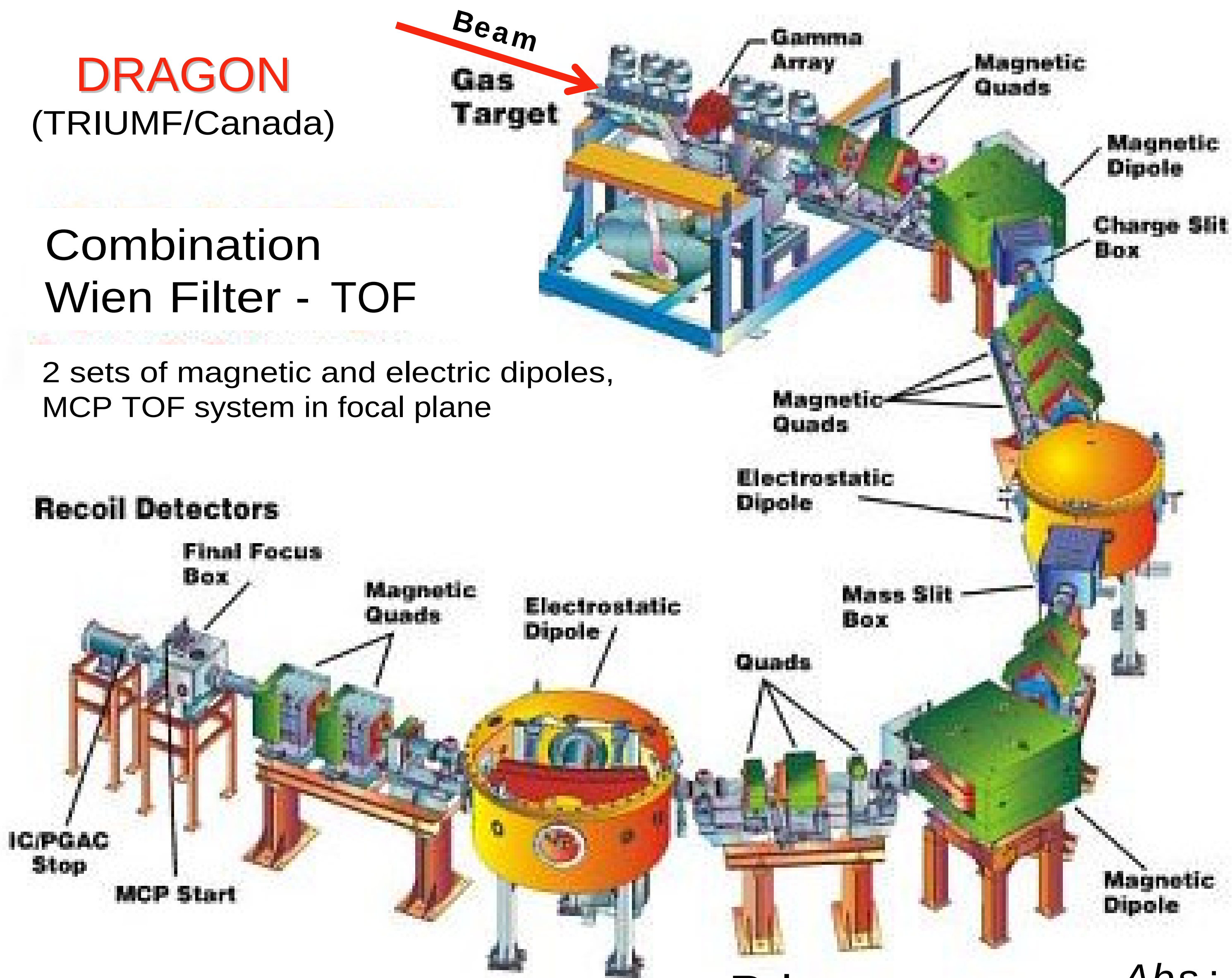
Used with ISOL target to measure exotic reaction product nuclei

new nuclides are produced in high charge states → good mass resolution

DRAGON
(TRIUMF/Canada)

Combination
Wien Filter - TOF

2 sets of magnetic and electric dipoles,
MCP TOF system in focal plane




*Lorentz force*

$$\vec{F} = q\left(\vec{v}\times\vec{B}+\vec{E}\right)$$

$\rightarrow$ *Wien vel. Filter*

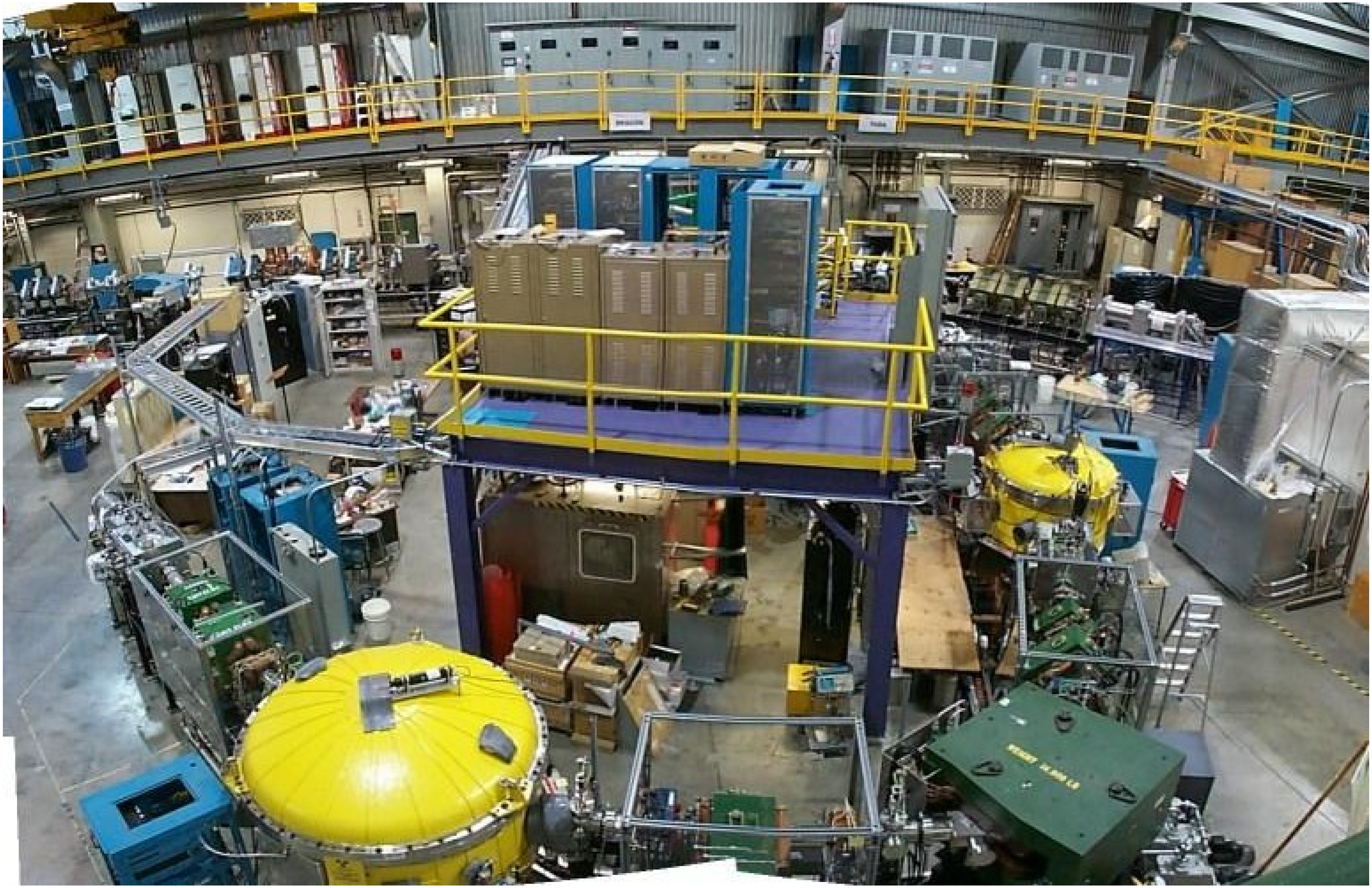
$$\left.\begin{array}{l} B\rho_B \propto \dfrac{mv}{q} \\ E\rho_E \propto \dfrac{E}{q} \end{array}\right\} \rightarrow \frac{m}{q}$$

$$TOF: \rightarrow v, m, K$$

$$Abs: \frac{\Delta m}{m} \sim 10^{-3}$$

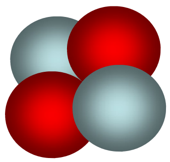
Princ. accuracy $Abs: \dfrac{\Delta m}{m} \sim 10^{-5} \mid Rel :\sim 10^{-8}$

# TRIUMF-Dragon Recoil Mass Spectrometer



# Basic Constituents of Atomic Nuclei

Helium



$${}^{A}_{Z}X_{N} = {}^{4}_{2}He_{2}$$

**A** nucleons: protons ($H^{+}$) plus neutrons

**Z** protons (equals the number of electrons in a neutral atom)

**N=A- Z** neutrons



**Charges**

$e_p = +e = 1.602 \cdot 10^{-19}$C (Coulomb) $e_n = 0$ ($e^2$=1.440MeVfm)

**Masses**

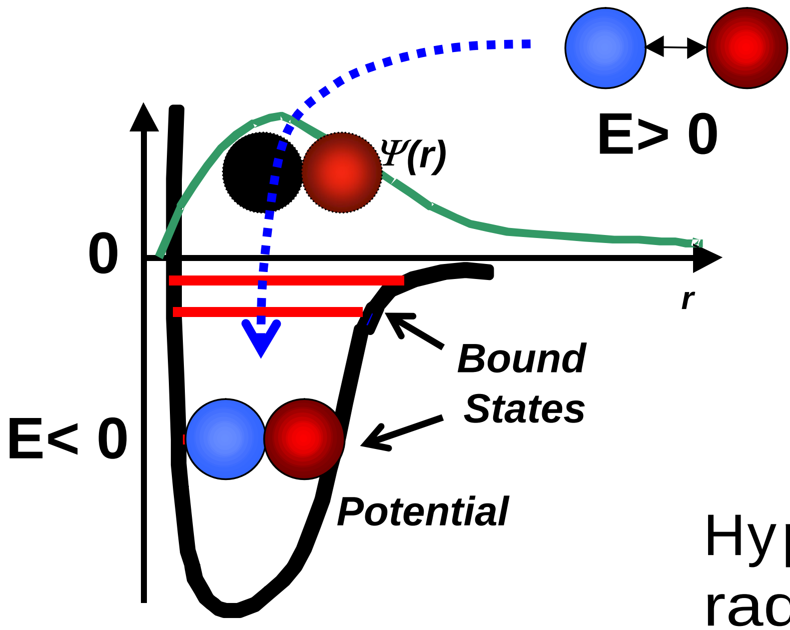
$m_p = 1.673 \cdot 10^{-27}$kg = 938.279 MeV/ $c^2$ = 1.00728 u (mass units)

$m_n = 1.675 \cdot 10^{-27}$kg = 939.573 MeV/ $c^2$= 1.00867 u

1u = m($^{12}$C)/ 12 = $1.6606 \cdot 10^{-27}$kg = 931.502 MeV/ $c^2$

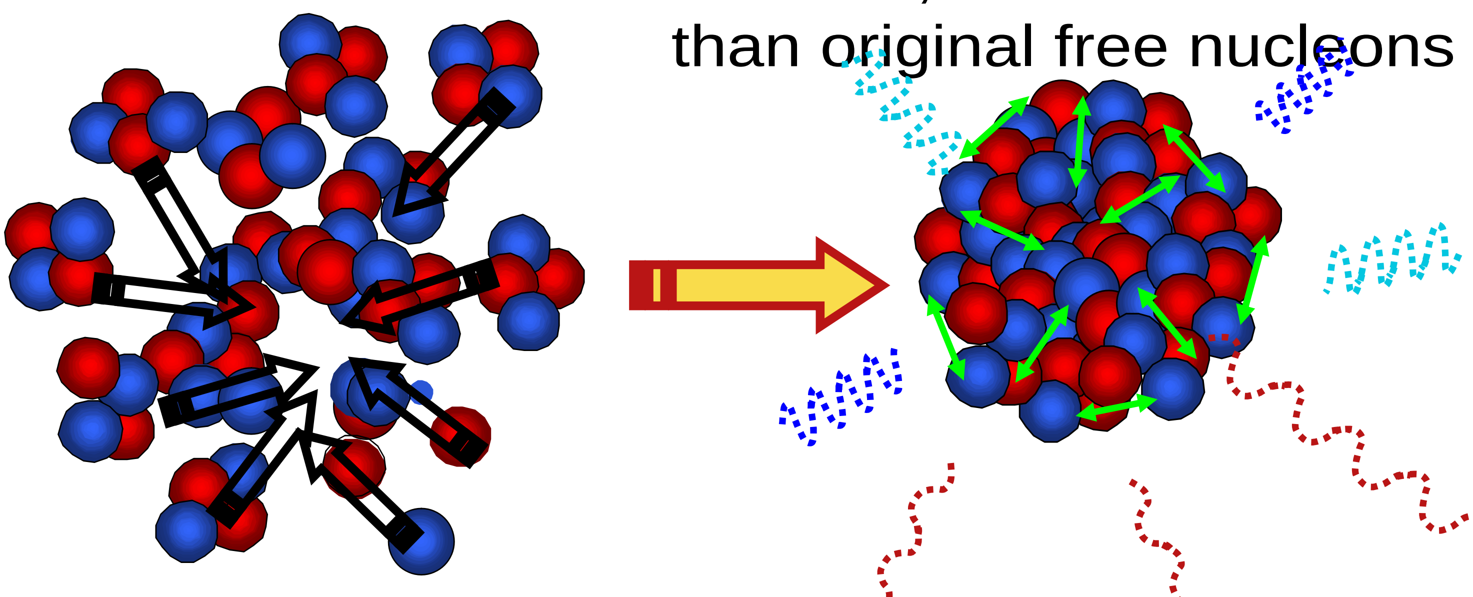
Expect approximately $m\left({}^{A}_{Z}X_{N}\right) \approx Z \cdot m_p + N \cdot m_n + Z \cdot m_e$ ??

# Radiative Capture of Nucleons




Elastic NN scattering ($E \geq 0$) does not lead to a bound ($E < 0$) NN system (nucleus).
**Need to dissipate extra energy → "radiate out" some of the mass.**

Hypothetical nuclear collapse: **γ-rays**, $E_\gamma$ radiated, lost from nucleus → less energy than original free nucleons



*Exotherm*

$\Delta E_{Nucleus} < 0$

Observe radiation emitted in capture of $e^-$, n,...

# Nuclear Masses

$m_H = 1.00782u \neq$
$m_p = 1.00728u$
$m_n = 1.00866u$

*Binding energy*

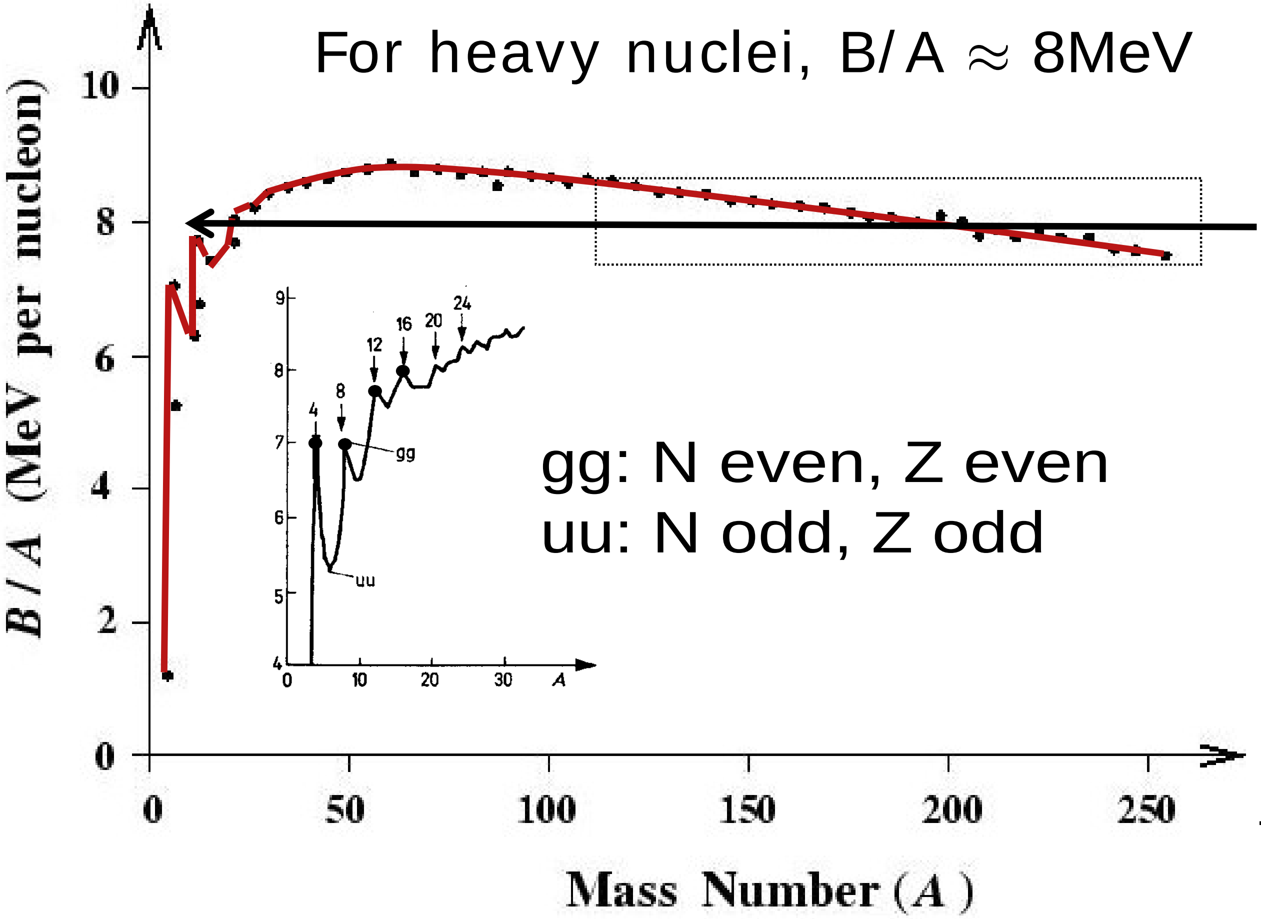
$$B = \left[Z \cdot m_H + N \cdot m_n\right] - m\left({}^A_Z X_N\right) > 0$$

| Element | | Mass of Nucleons (u) | Nuclear Mass (u) | Binding Energy (MeV) | Binding Energy MeV/Nucleon |
|---|---|---|---|---|---|
| Deuterium | D | 2.01594 | 2.01355 | 2.23 | 1.12 |
| Helium 4 | 4He | 4.03188 | 4.00151 | 28.29 | 7.07 |
| Lithium 7 | 7Li | 7.05649 | 7.01336 | 40.15 | 5.74 |
| Beryllium 9 | 9Be | 9.07243 | 9.00999 | 58.13 | 6.46 |
| Iron 56 | 56Fe | 56.44913 | 55.92069 | 492.24 | 8.79 |
| Silver 107 | 107Ag | 107.86187 | 106.87934 | 915.23 | 8.55 |
| Iodine 127 | 127I | 128.02684 | 126.87544 | 1072.53 | 8.45 |
| Lead 206 | 206Pb | 207.67109 | 205.92952 | 1622.27 | 7.88 |
| Polonium 210 | 210Po | 211.70297 | 209.93683 | 1645.16 | 7.83 |
| Uranium 235 | 235U | 236.90849 | 234.99351 | 1783.8 | 7.59 |
| Uranium 238 | 238U | 239.93448 | 238.00037 | 1801.63 | 7.57 |

Note: hydrogen atomic mass $\boldsymbol{m_H}$ is used in tables, rather than $\boldsymbol{m_p}$.

# Systematics of Experimental Binding Energies

For heavy nuclei, $B/A \approx 8\,\text{MeV}$




gg: N even, Z even
uu: N odd, Z odd

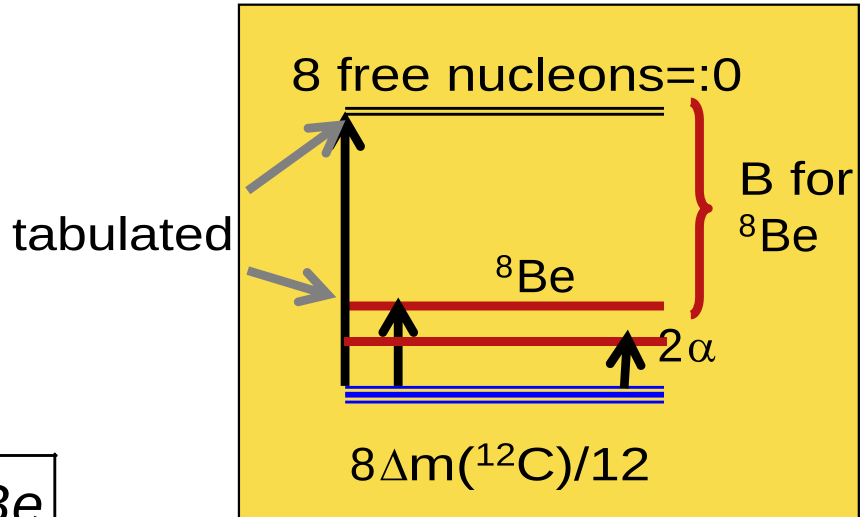
For light nuclei, odd-even staggering
gg nuclei most tightly bound

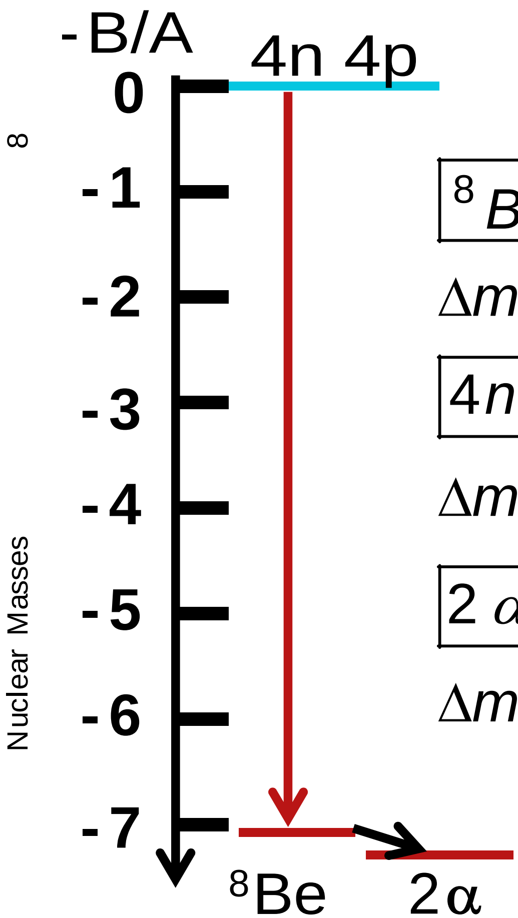
$^{4}_{2}He,\ \cancel{^{8}_{4}Be},\ ^{16}_{8}O,\ ^{20}_{10}Ne, \ldots$

exceptions: $^{8}$Be unstable
$^{8}Be \rightarrow 2\ ^{4}He$

there is no A=5 nucleus

# Energetics of $^{8}$Be




$\boxed{{}^{8}Be}$ $\qquad\qquad\qquad\qquad$ $B$

$$\Delta m = 4.942 MeV \qquad \rightarrow 56.49\, MeV$$

$\boxed{4n + 4p}$

$$\Delta m = 4 \cdot (8.071 + 7.289) MeV = 61.44 MeV \rightarrow "0" \ per\ \ def.$$

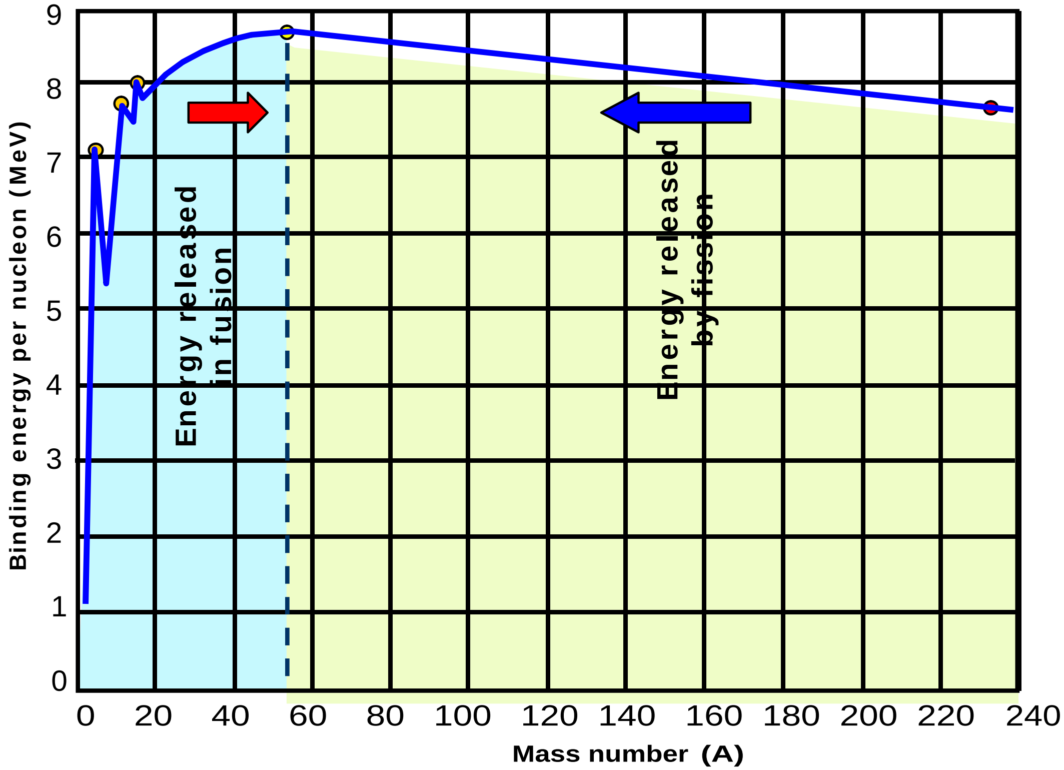
$\boxed{2\ \alpha}$

$$\Delta m = 2 \cdot 2.425 MeV = 4.850 MeV \qquad \rightarrow 56.59 MeV$$

$^{8}$Be is strongly bound but highly unstable ($\Delta E \approx 100$keV) against disintegration into 2 $\alpha$ particles

W. Udo Schröder, 2004

# Energetics of Transmutation




Non-monotonic behavior,

$BE \not\propto A$

Fe/Ni most strongly bound

2 different regions in A, different energetically preferred transmutations: Heavy nuclei are particle unstable, exothermic spontaneous emission of $\alpha$ particles ($B_\alpha$ =28 MeV!) or fission (nuclear power).

2 light nuclei can fuse and produce energy (stars, nuclear power)

$^{226}Ra \Rightarrow (^{222}Rn + {}^{4}He)$

(226.0254MeV) (222.0176 + 4.00260)MeV

Mass defect = $m_{Ra} - m_{RnHe}$ = 0.0052 u

→ exothermic, energy released Q=+4.84 MeV transmutation energetically possible



W. Udo Schröder, 2004

# The Semi-Empirical Mass Formula

$$m(Z,N) = \left[Z \cdot m_H + N \cdot m_n\right] - B(Z,N)/c^2; \quad B > 0$$ neglect $e^-$ binding

Original: Weizsacker, Z. Physik 96,431(1935)

$$\boxed{B(Z,N) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} - \delta A^{-1/2}}$$

Wapstra, Handb. Physik, XXXVIII

$a_V = 15.835\,MeV$

$a_S = 18.33\,MeV$

$a_C = 0.714\,MeV$

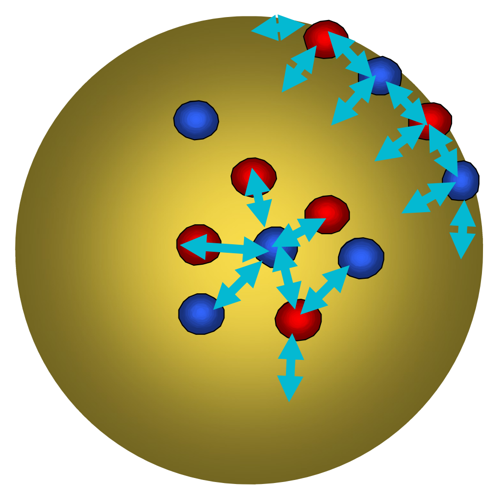
$a_a = 23.20\,MeV$

$$\delta = \begin{cases} +11.2\,MeV \ \text{for} \ o-o \ \text{nuclei} \\ 0\,MeV \ \text{for} \ odd-A\,\text{nuclei} \\ -11.2\,MeV \ \text{for} \ e-e \ \text{nuclei} \end{cases}$$

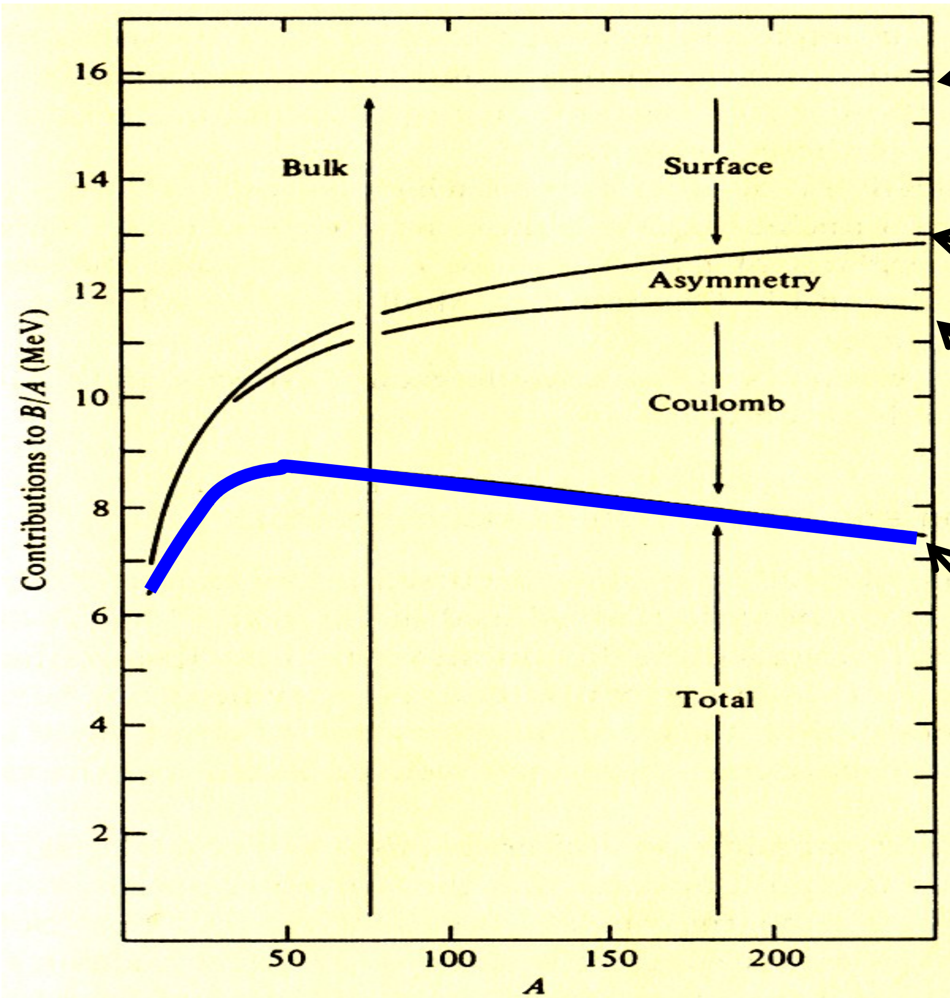
## Implicit assumption: Leptodermous nucleus

$a_V$ = volume, "condensation"
$a_S$ = surface, less binding
$a_C$ = Coulomb repulsion of protons
$a_a$ = asymmetry lessens binding for like particles, compared to unlike
$\delta$ = unpaired particles are less tightly bound

} quantal effects



# Relative Contributions to Nuclear Mass




$R \propto A^{1/3} \rightarrow V_{nucleus} \approx A$

const. contribution from each nucleon→ "saturated" force

fewer interactions on surface → reduce contribution from each surface nucleon $S \propto A^{2/3}$

different interactions between like and unlike nucleons (Fermion statistics, isospin)→ depends on |N-Z|, reduces B

Coulomb self energy becomes large for large Z, heavy nuclei, makes nucleus unstable reduces B

$$E_{Coul} = \frac{e^2 Z^2}{2} \int d^3\vec{r}\, d^3\vec{r}\,' \frac{\rho(\vec{r})\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = \underbrace{\frac{3}{5}\frac{e^2 Z^2}{R_C}}_{\text{hom. sharp sphere}}$$

*Coulomb Radius*

$$R_C = 1.24 A^{1/3} fm$$



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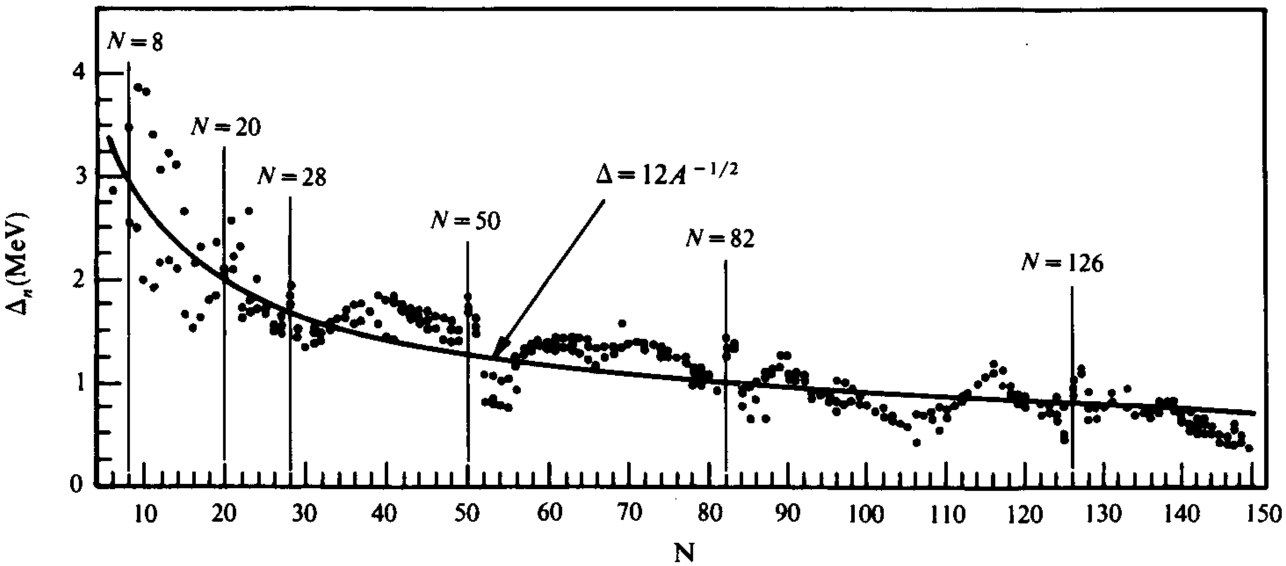
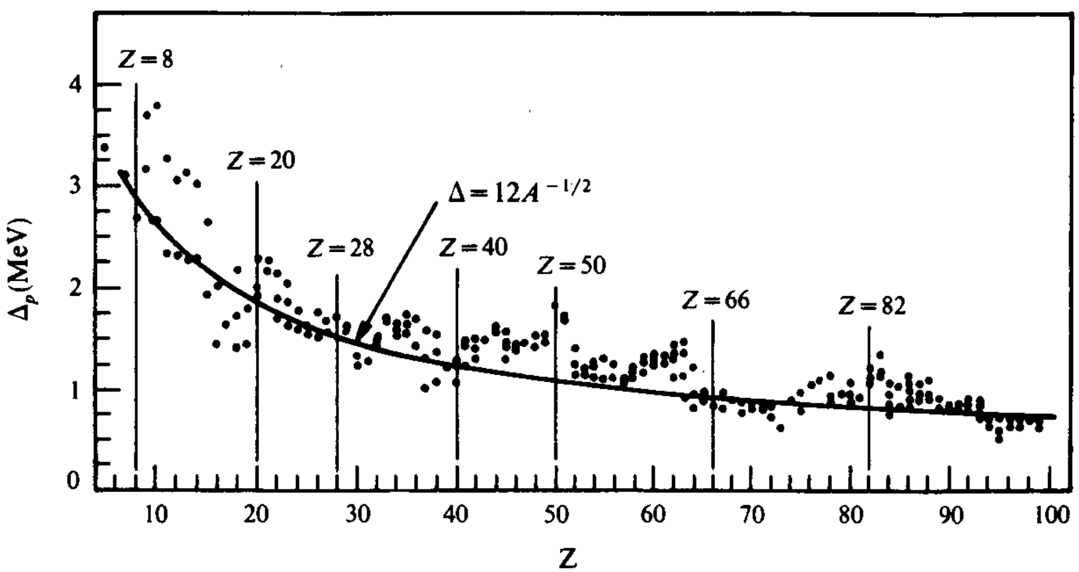
# Pairing Energies




odd-even mass differences, B largest for "paired" nucleons

Average trend:

$$\Delta_n \approx \Delta_p \approx \delta A^{-1/2}$$

Remaining structure and fluctuations: shell and deformation effects.



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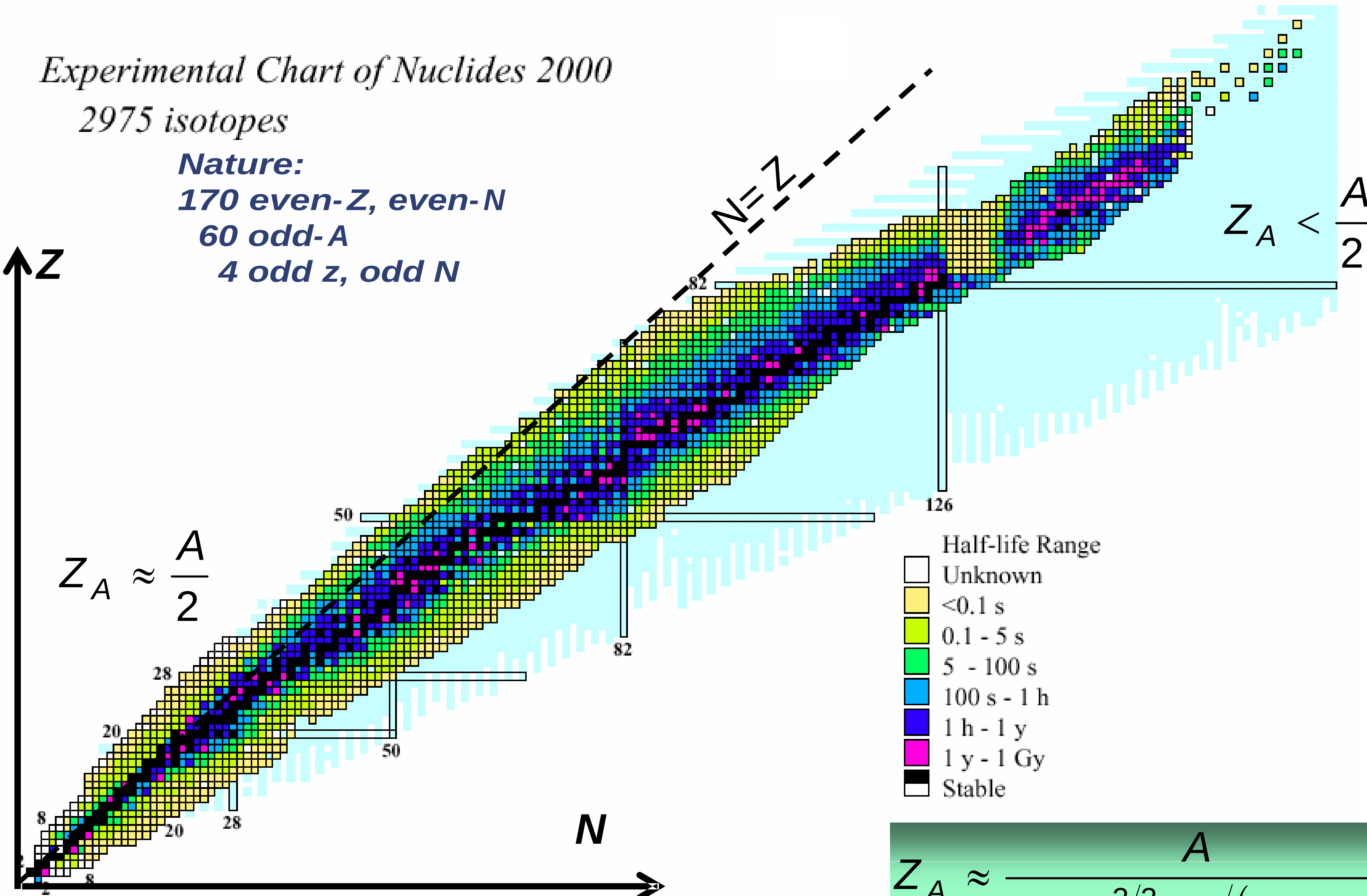
# The β-Stable Valley

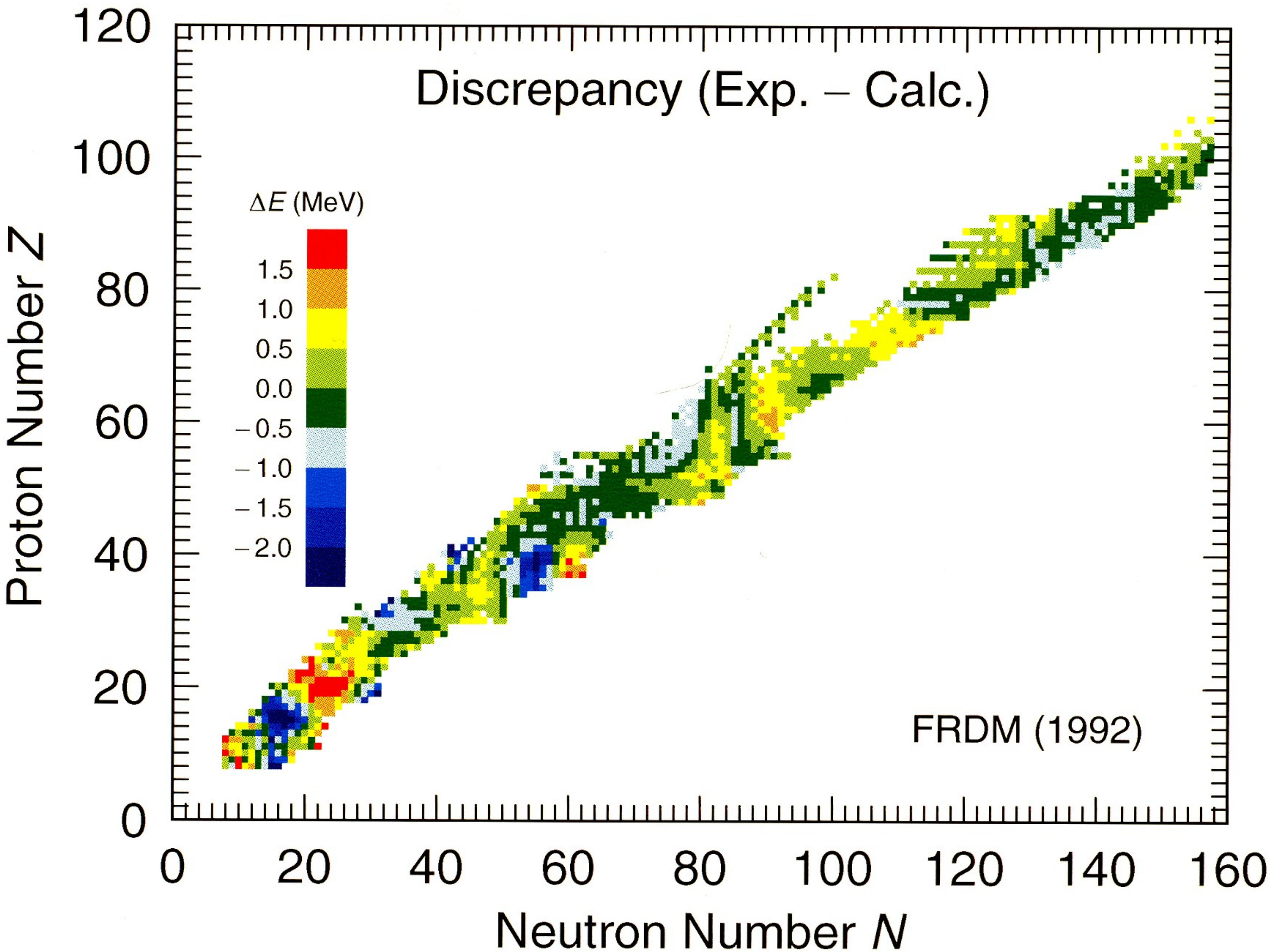



$$Z_A \approx \frac{A}{2}$$

$$Z_A < \frac{A}{2}$$

$$Z_A \approx \frac{A}{2 + A^{2/3} a_C \big/ \left(2 a_{sym}\right)}$$

# Quality of Droplet-Model Mass Fit




Total binding energy of heavy nuclei ~1600 MeV, accuracy of LDM fit: ±0.5 MeV



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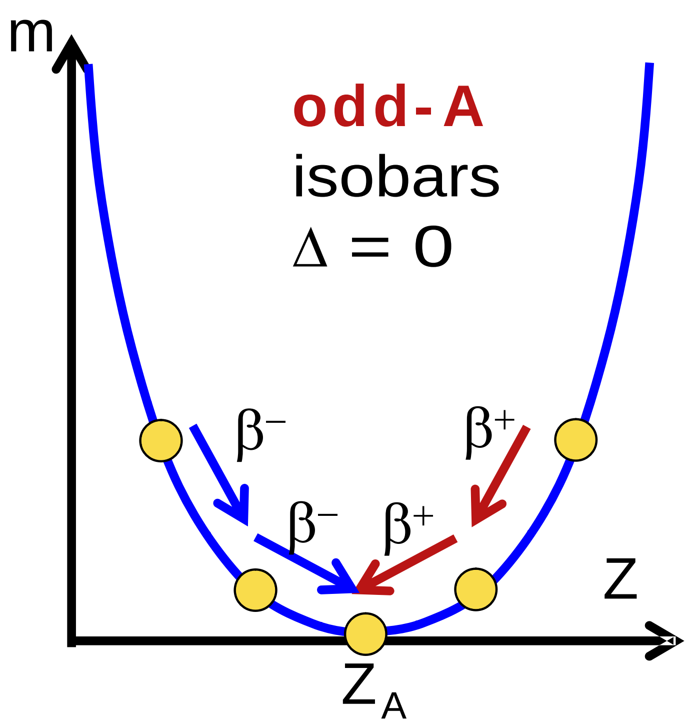
# Beta Decays of Odd-A Nuclei

$$m(A,Z) = \alpha(A) - \beta(A)Z + \gamma(A)Z^2 \pm \Delta$$

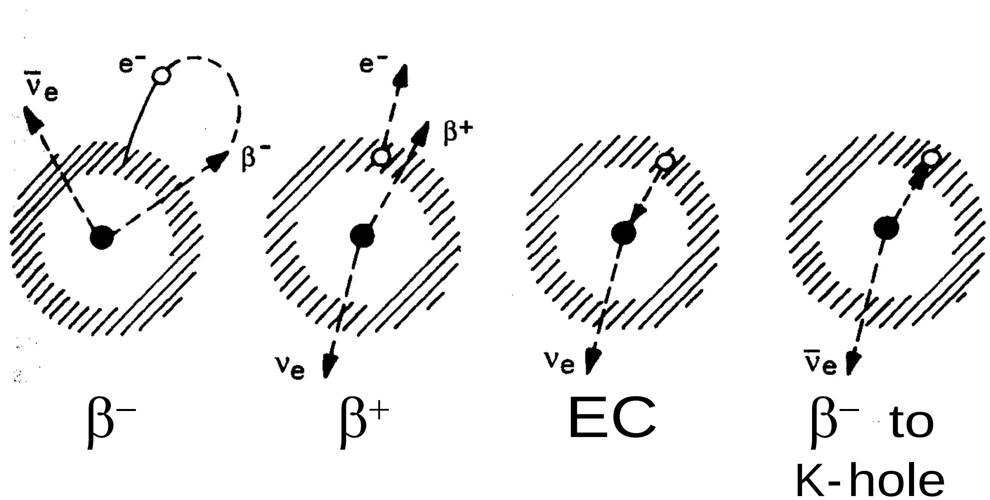
$$m = m_{\min}: \; Z_A = \frac{\beta}{2\gamma} = \frac{\left[4a_s + \left(m_n - m_p - m_e\right)c^2\right]A}{2\left(4a_s + a_C A^{2/3}\right)}$$

bottom of valley

Expand around $Z_A$: Mass parabola $\boxed{m(Z) \approx \left[\tilde{\alpha}(A) \pm \Delta\right] + \tilde{\beta}\left(Z - Z_A\right)^2}$




Beta decay and K-electron capture (EC)




Bethge, Kernphysik

# Energetics of β Decay

Beta decay and K-capture

$$\boxed{(Z,A) \to (Z+1,A) + e^- + \bar{\nu}_e \mid \beta^-}$$

$$m(Z,A) > m(Z+1,A)$$

$$\boxed{(Z,A) \to (Z-1,A) + e^+ + \nu_e \mid \beta^+}$$

$$m(Z,A) > m(Z-1,A) + \underbrace{2m_e}$$

1 extra e⁺
1 extra e⁻

$$\boxed{(Z,A) + e^- \to (Z-1,A) + \bar{\nu}_e \mid EC}$$

$$m(Z,A) > m(Z-1,A)$$

$$Example: \left[{}^{11}_{6}C + 6e^-\right] \xrightarrow[\beta^+]{} \left[{}^{11}_{5}Be + 6e^-\right] + e^+ + \nu_e + Q_\beta$$

$Q_\beta > 0$
exotherm.

5 1

*Mass balance:*

$$m({}^{11}C)c^2 \xrightarrow[\beta^+]{} \left[m({}^{11}Be)c^2 + m_e c^2\right] + m_e c^2 + Q_\beta$$

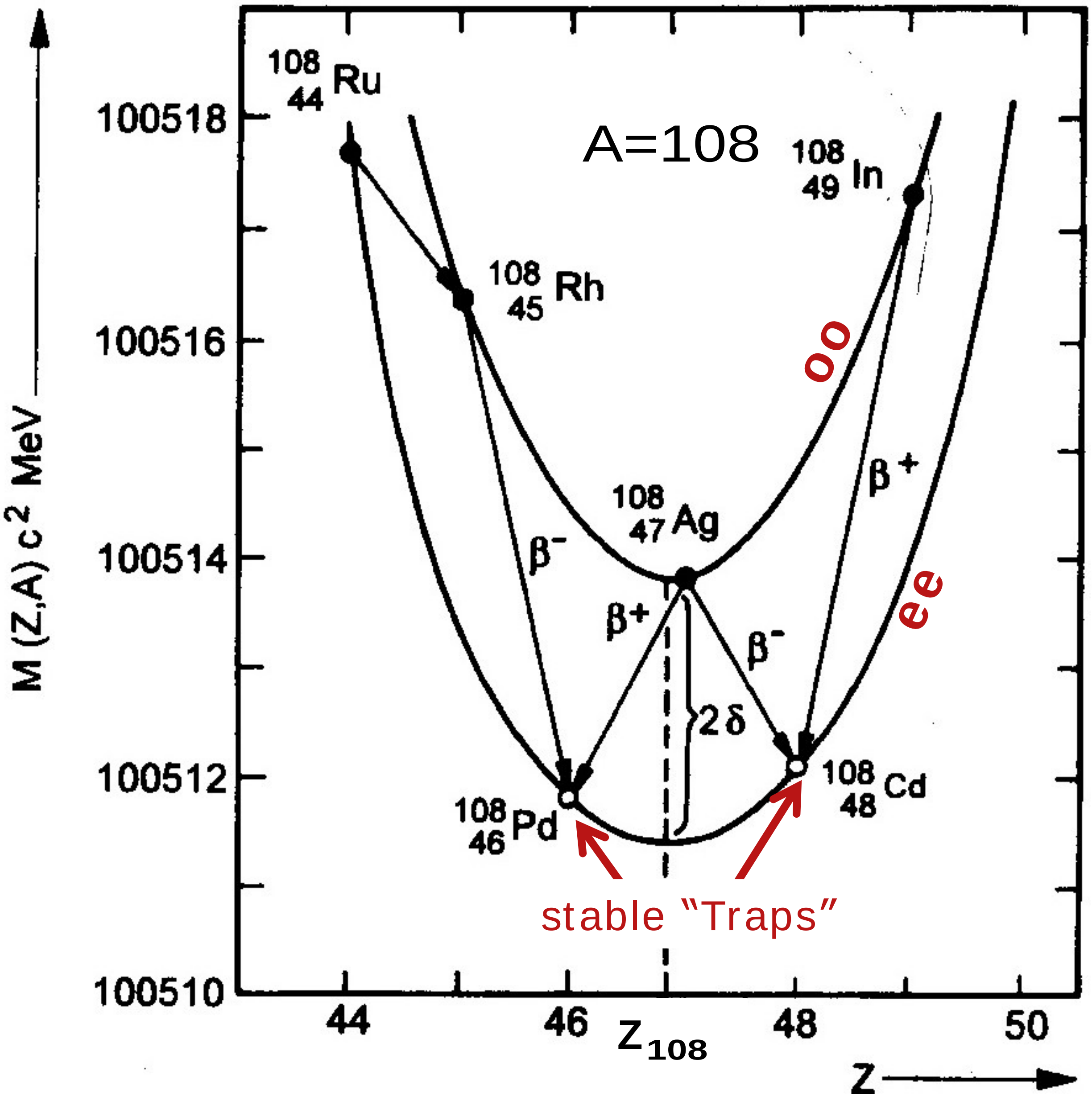
$$\boxed{Q_\beta = \left\{m({}^{11}C)c^2 - m({}^{11}Be)c^2\right\} - 2m_e c^2}$$

Decay Q-value smaller by $2m_ec^2$ for $\beta^+$ decay than for $\beta^-$



W. Udo Schröder, 2004

# β Decay of an Even-A Nucleus




$$even\ A = \begin{cases} o - o \\ e - e \end{cases}$$

→o-o weakly bound

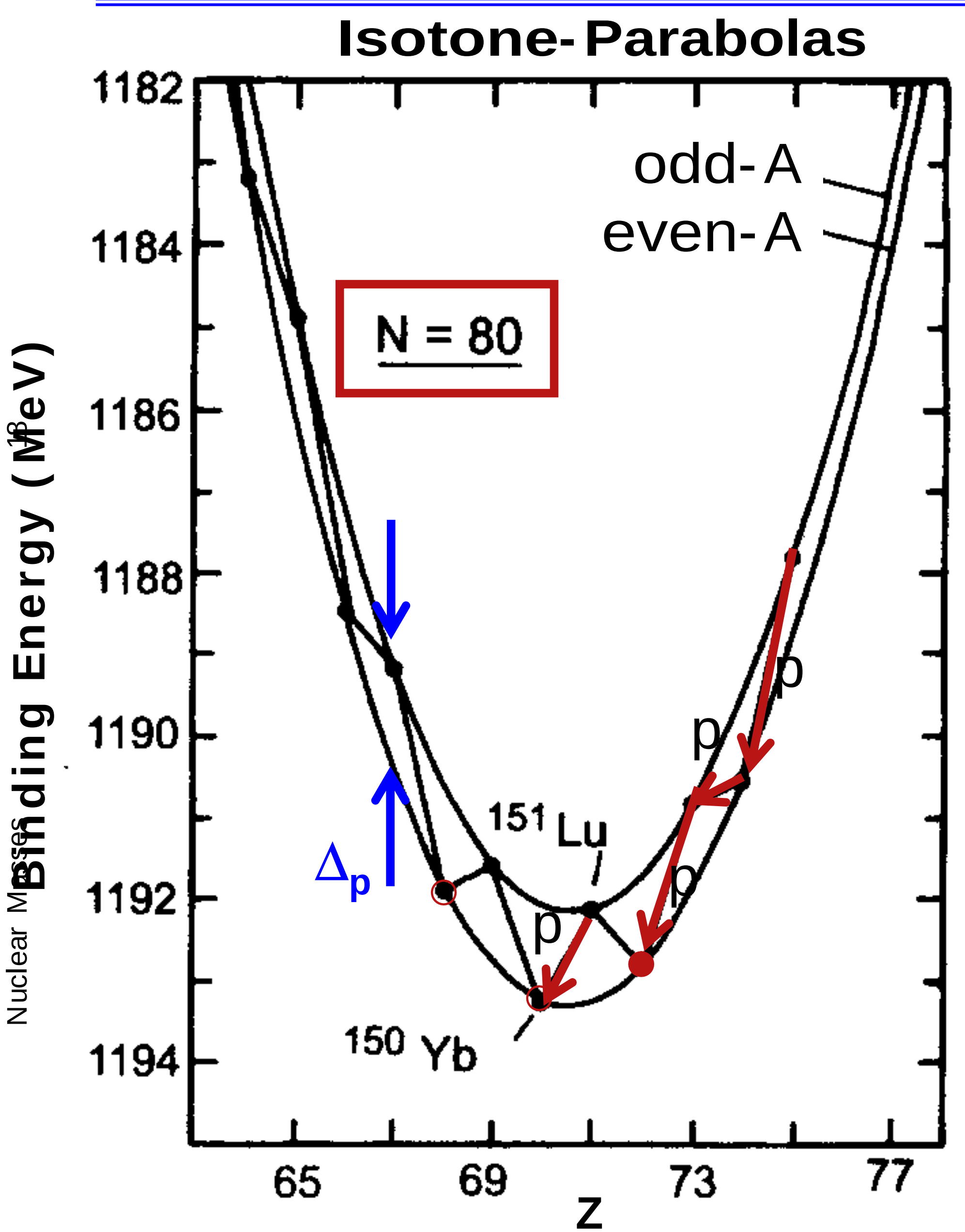
→e-e strongley bound

$$\delta = \begin{cases} +11.2\,MeV\ \ for\ \ o-o\ nuclei \\ 0\,MeV\ \ for\ \ odd-A\,nuclei \\ -11.2\,MeV\ \ for\ \ e-e\ nuclei \end{cases}$$

→ for even A, 2 distinct mass parabolas separated by $2\delta = 22.4.A^{-2}\ MeV$

$Z_{108} \approx 47$ is relative minimum for oo A=108 nuclei, but not for ee.

# Energetics of Proton Decay




S. Hofmann, *Handbook of Nuclear Decay Modes*, CRC Press 1993

p decay energetically possible if

$Q_p$ = -[B(Z,A) – B(Z-1,A-1)] >0

equivalent to

$$Q_p = m(Z,A)c^2 - \\ -\left[m(Z-1,A-1)+m_p+m_e\right]c^2 > 0$$

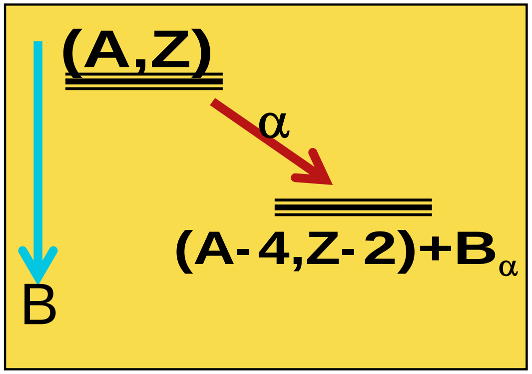
| Proton Emitter | Decay Q Value | Daughter | Half Life |
|---|---|---|---|
| $^{53m}$Co | 1.560 MeV | $^{52}$Fe | 247 ±12 ms |
| $^{109}$I | 0.813 MeV | $^{108}$Te | 103 ±5 μs |
| $^{113}$Cs | 0.959 MeV | $^{112}$Xe | 28 ±7 μs |
| $^{146}$Tm | 1.189 MeV | $^{145}$Er | 72 ±23 ms |
| $^{147}$Tm | 1.051 MeV | $^{146}$Er | 560 ±40 ms |
| $^{150}$Lu | 1.263 MeV | $^{149}$Yb | 35 ±10 ms |
| $^{151}$Lu | 1.233 MeV | $^{150}$Yb | 85 ±10 ms |
| $^{156}$Ta | 1.022 MeV | $^{155}$Hf | 165 $\pm^{165}_{55}$ ms |
| $^{160}$Re | 1.261 MeV | $^{159}$W | 790 ±160 μs |

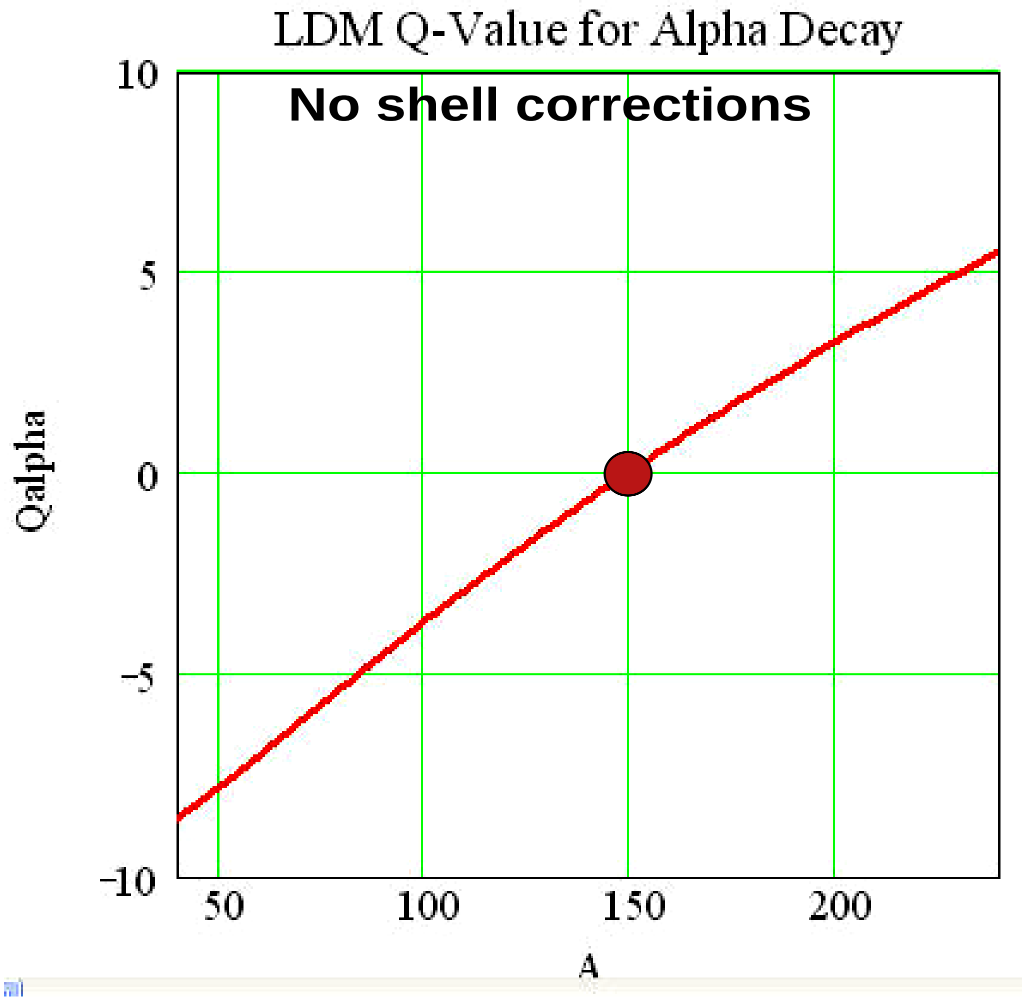
Search for location of p-Drip Line continues

# Energetics of $\alpha$ Decay

One of the most frequent decays of heavy nucleus because $B_\alpha$ = 28.3 MeV is relatively large.

$$Q_\alpha = \left[ B(A-4, Z-2) + 28.3 MeV \right] - B(A, Z) > 0$$



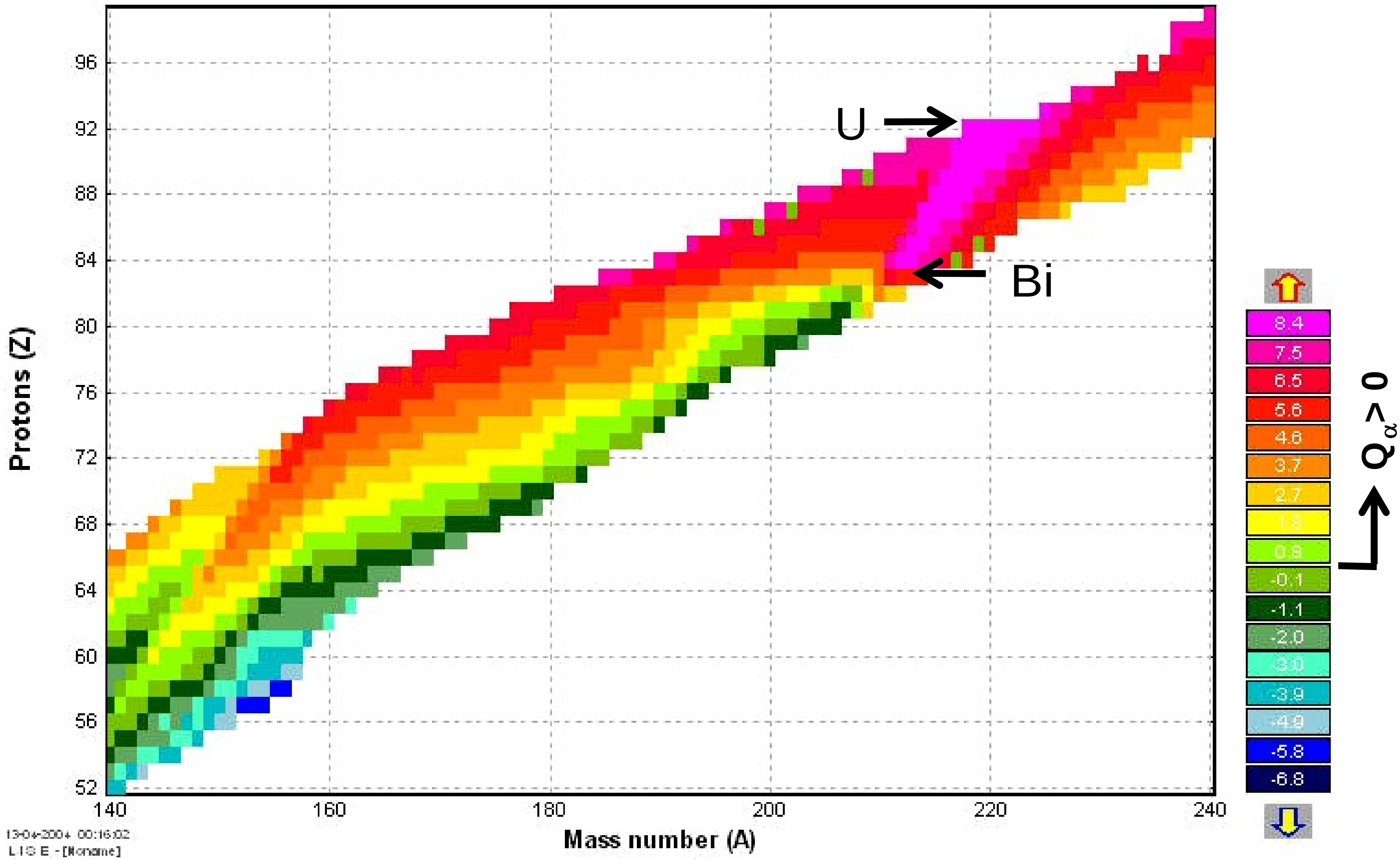




→ energetically allowed (along $\beta$-stable valley) for A > 150, e.g., in the *rare earth region* of elements (Gd).

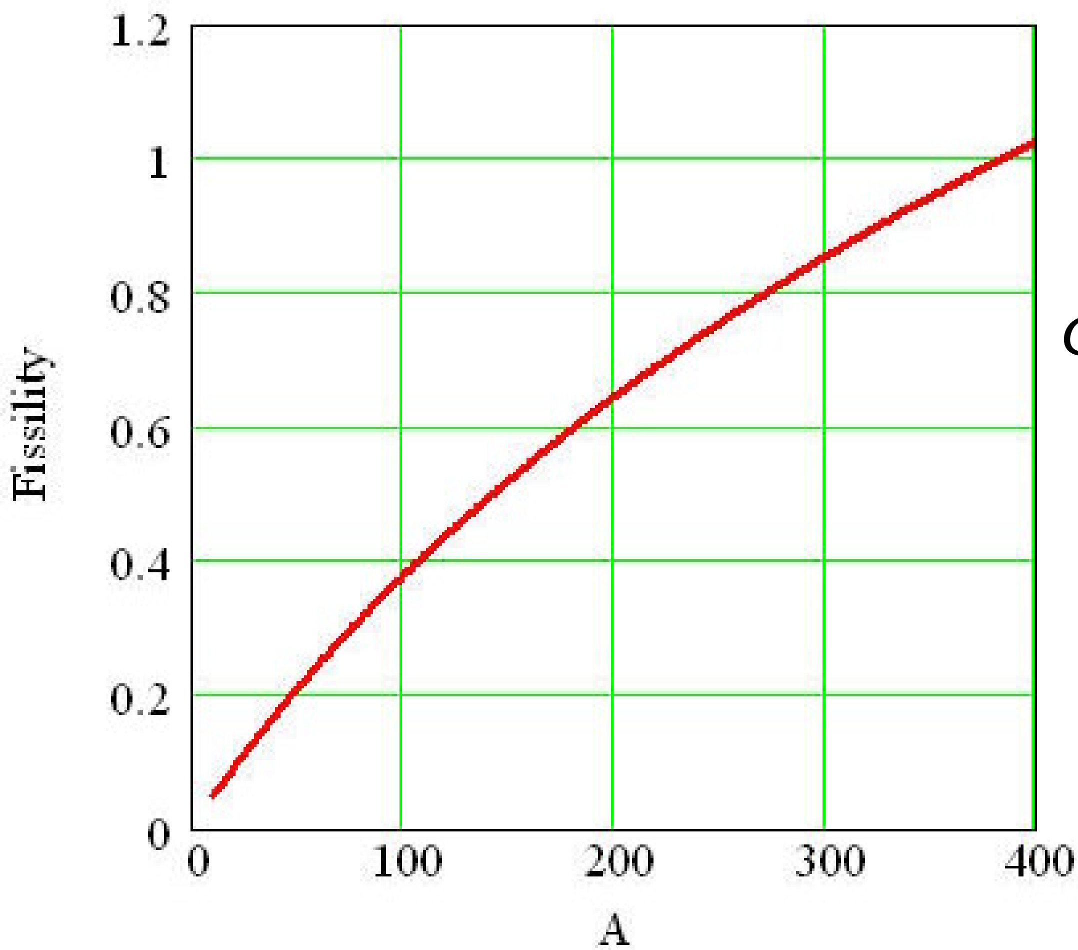
$\alpha$ decay *from nuclear ground state* is mostly slower than $\beta$ decay, except for very heavy nuclei > Pb, actinides, e.g., Po, Am, Cf,...

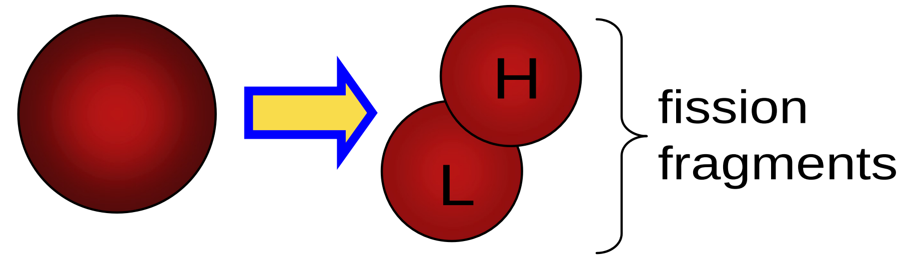
# Alpha Q-Values

## Lise calculation



# Fissility of Nuclei




$$Q_f = B(A_L, Z_L) + B(A_H, Z_H) - B(A, Z)$$
$$= \Delta B_s + \Delta B_{Coul}$$

Volume and symmetry terms cancel → Balance of Coulomb vs. surface energy
→ fissility parameter x

$$\boxed{x = \frac{E_C^0}{2E_s^0} = \frac{3e^2/(5r_0)}{2a_s}\frac{Z^2}{A} \approx \frac{Z^2/A}{50}}$$

$^{239}$U: $E_s = 650\text{MeV}$ $E_C = 950\text{MeV}$

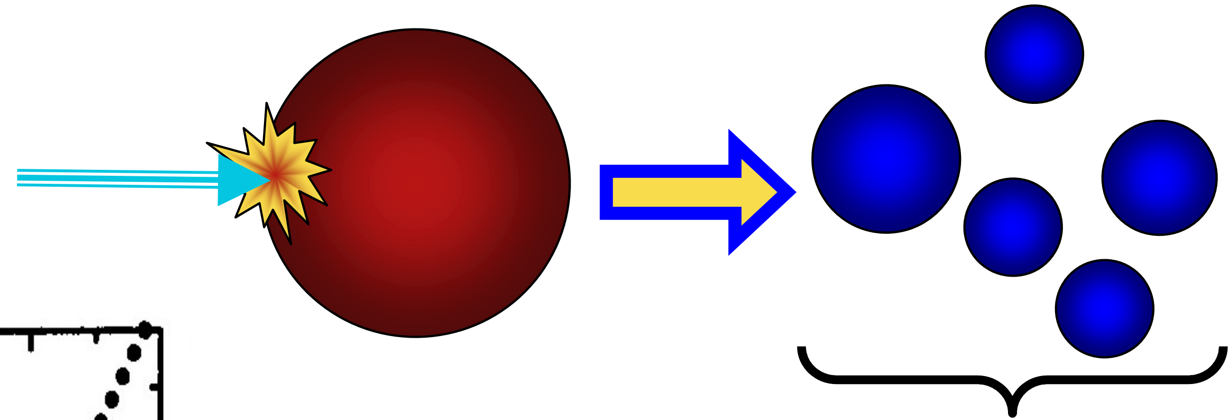
<u>Fiss.frgm. : $E_s = 813\text{MeV}$ $E_C = 607\text{MeV}$</u>

Balance: $\Delta E_s = -163\text{MeV}$ $\Delta E_C = 343\text{MeV}$

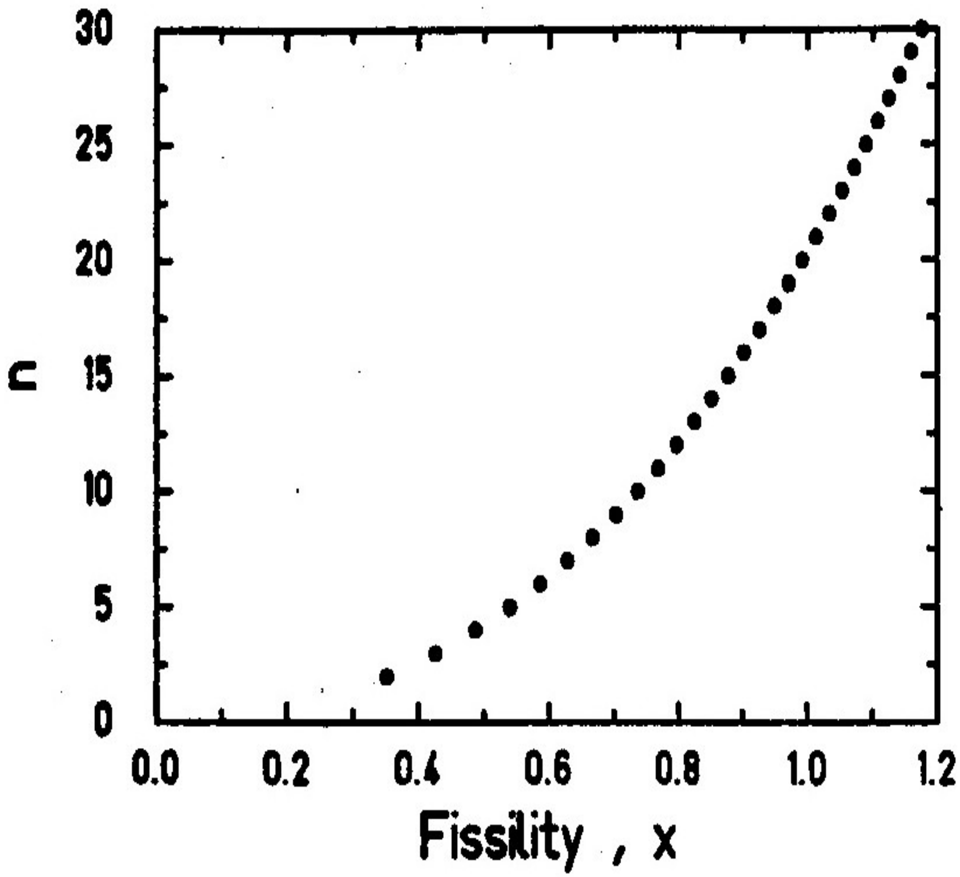
$\Bigg\}$ $Q_f =$ 180 MeV

# Energetics of Multifragment Decay

Important for spallation and applications: n fragments at infinity $Q_n > 0$??



n fragments




*Threshold Fissility for* $Q_n > 0$

$$x = \frac{E_C}{2E_s} = \frac{Z^2}{A}\frac{3e^2/5r_0}{2a_s}$$

x ≈ 1, n ≤ 20 (many)

However: competition with β decay for every emission.

Hasse & Myers, *Geometrical Relationships Of Macroscopic Nuclear Physics,* Springer, New York, 1988

# Droplet Model

W.D. Myers & W. J. Swiatecki, Ann. Phys. 55, 395(1969); 84, 186 (1974)

Extension of LDM → 1 higher order in $A^{-1/3}$, $I=(N-Z)/A$

finite compressibility, deformed shapes, $\rho_n$ and $\rho_p$ different surfaces.
Most accurate: **Finite-Range Droplet Model**



$$\boxed{\begin{aligned} B(N,Z;shape) &= \left[-a_1 + J\bar{\delta}^2 - \frac{1}{2}K\bar{\varepsilon}^2 + \frac{1}{2}M\bar{\delta}^4\right]A + \left[a_2 + \frac{9}{4}\frac{J^2}{Q}\bar{\delta}^2\right]A^{2/3}B_{surf} \\ &+a_3A^{1/3}B_{curv} + c_1Z^2A^{-1/3}B_C - c_2Z^2A^{1/3}B_{red} - c_5Z^2B_w - c_3Z^2A^{-1} - c_4Z^{4/3}A^{-1/3} \end{aligned}}$$

$$\bar{\delta} = \frac{I + (3c_1/16Q)ZA^{-2/3}B_V}{1+(9J/4Q)A^{-1/3}B_{surf}}; \qquad \bar{\varepsilon} = \left[-2a_2A^{-1/3}B_{surf} + L\bar{\delta}^2 + c_1Z^2A^{-4/3}B_C\right]/K$$

bulk asymmetry

deviation from aver density

$$c_1 = 3e^2\Big/(5r_0); \quad c_2 = \frac{c_1^2}{336}\left(\frac{1}{J}+\frac{18}{K}\right); \quad c_3 = \frac{5c_1}{2}\left(\frac{b}{r_0}\right)^2;$$

$$c_4 = \frac{5c_1}{4}\left(\frac{3}{2\pi}\right)^{2/3}; \quad c_5 = \frac{1}{64}\frac{c_1^2}{Q}$$

Functions B describe contributions of shape/spherical