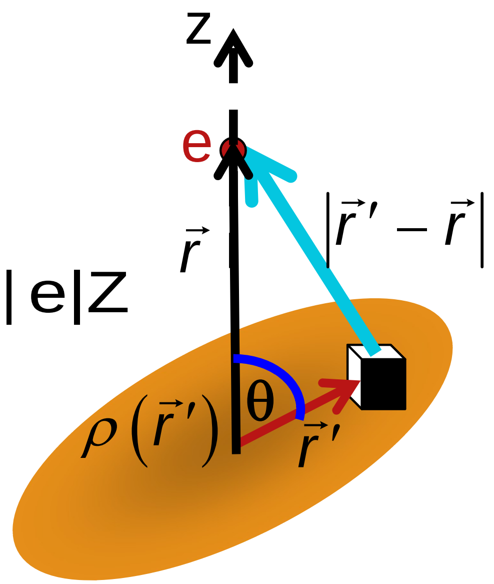
Sizes

Gross Properties of Nuclei

Nuclear Deformations and Electrostatic Moments

Coulomb Fields of Finite Charge Distributions



$\rho(\vec r')$ arbitrary nuclear charge distribution with normalization $\int d^3\vec r' \rho(\vec r') = Z$

Coulomb interaction

$$\boxed{V(\vec r) = e\int d^3\vec r' \frac{e\cdot\rho(\vec r')}{|\vec r' - \vec r|}}$$

Expansion of $|\vec r - \vec r'|^{-1}$ *for* $r \gg r'$

$$|\vec r - \vec r'|^{-1} = \left|(\vec r - \vec r')^2\right|^{-1/2} \approx \left|r^2 + r'^2 - 2\vec r r' \cos\theta\right|^{-1/2}$$

$$= \frac{1}{r}\Big[1 + \underbrace{\frac{r'}{r}\Big(\frac{r'}{r} - 2\cos\theta\Big)}_{\ll 1}\Big]^{-1/2}$$

$$\boxed{[1+x]^{-1/2} = 1 - \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3 + \ldots}$$

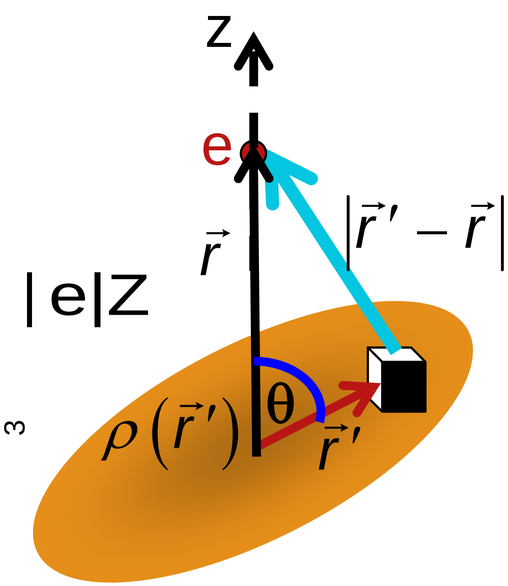
for $|x| \ll 1$:

$$|\vec r - \vec r'|^{-1} = \frac{1}{r}\left\{1 + \left(\frac{r'}{r}\right)\cos\theta + \left(\frac{r'}{r}\right)^2 \frac{1}{2}\left[3\cos^2\theta - 1\right] + \ldots\right\} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta)$$

Legendre Polynomials $P_0 \equiv 1,\ P_1(\cos\theta) = \cos\theta,\ P_1(\cos\theta) = \frac{1}{2}\left(3\cos^2\theta - 1\right)$

W. Udo Schröder, 2004

Multipole Expansion of Coulomb Interaction



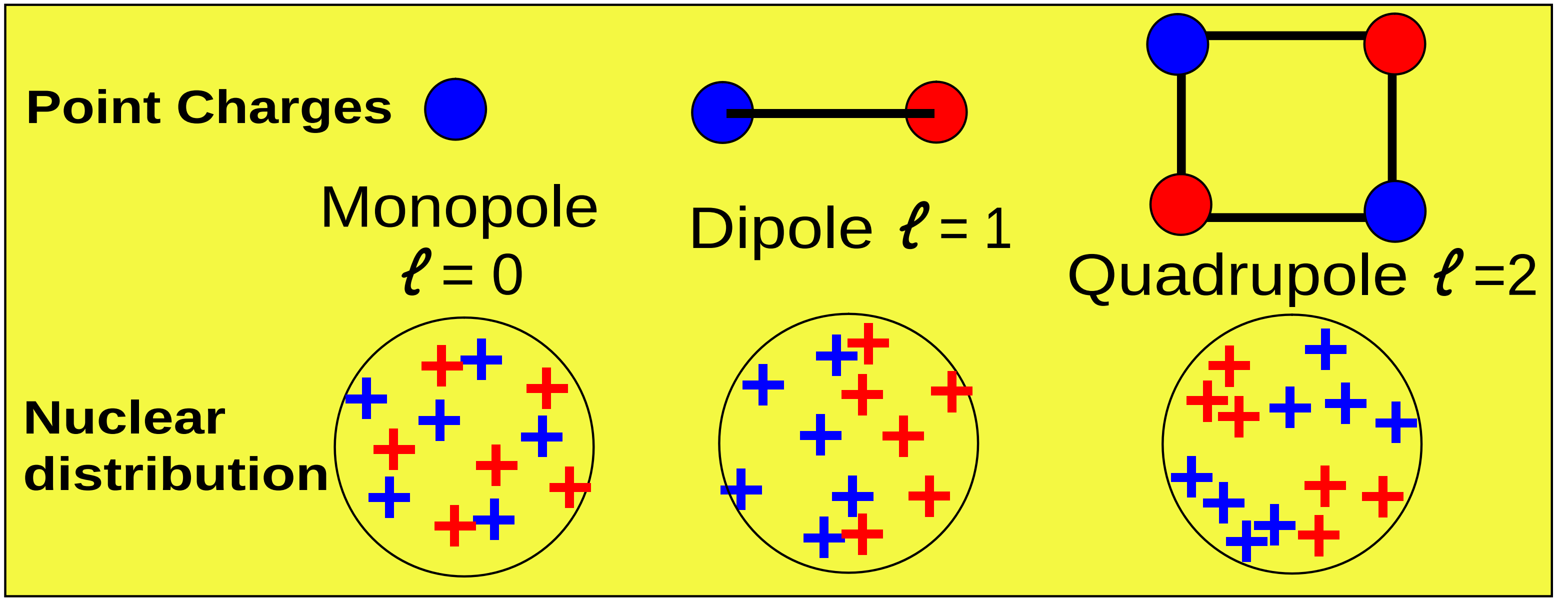
$$V(\vec{r}) = e\int d^3\vec{r}\,' \frac{e\cdot\rho(\vec{r}\,')}{|\vec{r}\,'-\vec{r}|} =$$

$$+\frac{e}{r}\int d^3\vec{r}\,'\rho(\vec{r}\,')\left[e\cdot 1\right] \qquad \leftarrow \frac{e^2 Z}{r}$$

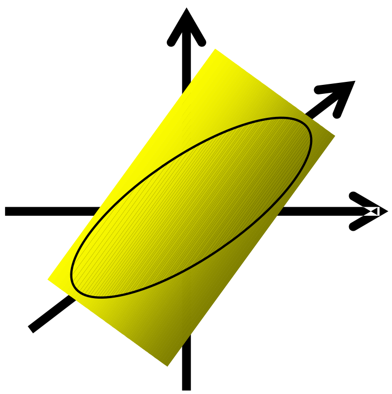
$$+\frac{e}{r^2}\int d^3\vec{r}\,'\rho(\vec{r}\,')\left[e\cdot r'\cos\theta\right] \qquad \leftarrow \frac{e^2 Z}{r^2}d$$

$$+\frac{e}{r^3}\int d^3\vec{r}\,'\rho(\vec{r}\,')\left[e r'^2\cdot\frac{1}{2}\left(3\cos^2\theta-1\right)\right] \qquad \leftarrow \frac{e^2 Z}{r^3}Q$$

$$+\ \ldots\ldots$$



A Quantal Symmetry



symmetric nuclear shape → symmetric $\rho(\vec{r})$

invariance of Hamiltonian against space inversion

$\Pi H(-\vec{r}) = H(\vec{r})$ *Parity Operator* $\vec{r} \xrightarrow{\Pi} -\vec{r}$

$\rightarrow [H, \Pi] = 0 \rightarrow$ *simultaneous eigen functions* ψ_E^π

$$\boxed{H\psi_E^\pi = E \cdot \psi_E^\pi \qquad \Pi \psi_E^\pi = \pi \cdot \psi_E^\pi, \quad \pi = \begin{cases} +1 & even \\ -1 & odd \end{cases}}$$

Electrostatic multipole moments of $\rho(\vec{r})$:

both even or odd

$$M_n^{el} \propto \left\langle r^n P_n(\cos\theta) \right\rangle_\rho \propto \int d^3\vec{r}\, \psi^*(\vec{r}) \left[\underbrace{r^n P_n(\cos\theta)} \right] \psi(\vec{r})$$

n even ➔ $\pi = +1$

n odd ➔ $\pi = -1$

If strong nuclear interactions parity conserving ⇨ $\boxed{M_n^{el} = 0 \;\; for \;\; n = odd}$

Restrictions on Nuclear Field

Expt: No nucleus with non-zero electrostatic dipole moment

Consequences for nuclear Hamiltonian
(assume some average mean field U_i for each nucleon *i*):

$$\hat{H} = \sum_{i=1}^{A} \left\{ \frac{\hat{p}_i^2}{2m_i} + U_i(\vec{r}_1, \ldots, \vec{r}_A; \ldots) \right\} \quad \textit{parity conserving}$$

$$\rightarrow \left[\hat{H}, \hat{\Pi} \right] = 0$$

$$\textit{Since} \;\; \hat{p}_i^2 = -\hbar^2 \frac{\partial^2}{\partial x^2} = -\hbar^2 \frac{\partial^2}{\partial (-x)^2} \rightarrow \left[\hat{p}_i^2, \hat{\Pi} \right] = 0$$

$$\textit{and} \;\; \left[\sum_{i=1}^{A} U_i(\vec{r}_1, \ldots, \vec{r}_A; \ldots), \hat{\Pi} \right] = 0$$

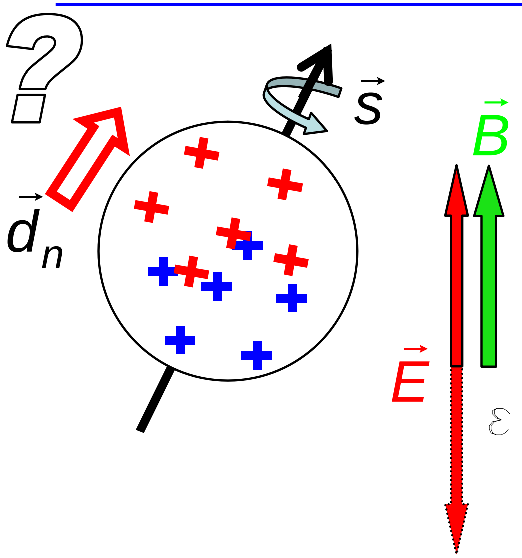
$$\rightarrow\rightarrow \sum_{i=1}^{A} U_i(\vec{r}_1, \ldots, \vec{r}_A; \ldots) = \sum_{i=1}^{A} U_i(-\vec{r}_1, \ldots, -\vec{r}_A; \ldots)$$

$$= \sum_{i=1}^{A} U_i(|\vec{r}_1|, \ldots, |\vec{r}_A|; \ldots)$$

Average mean field for nucleons conserves π
→ inversion invariant, e.g., central potential

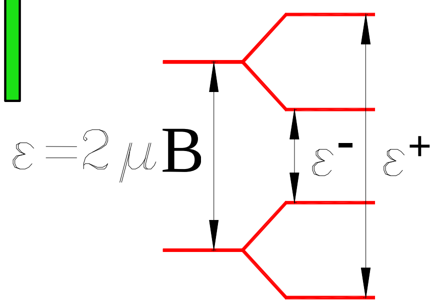
W. Udo Schröder, 2004

Neutron Electric Dipole Moment



$q_n = 0$, possible small $d_n \neq 0$.
CP and P violation → could explain matter/antimatter asymmetry

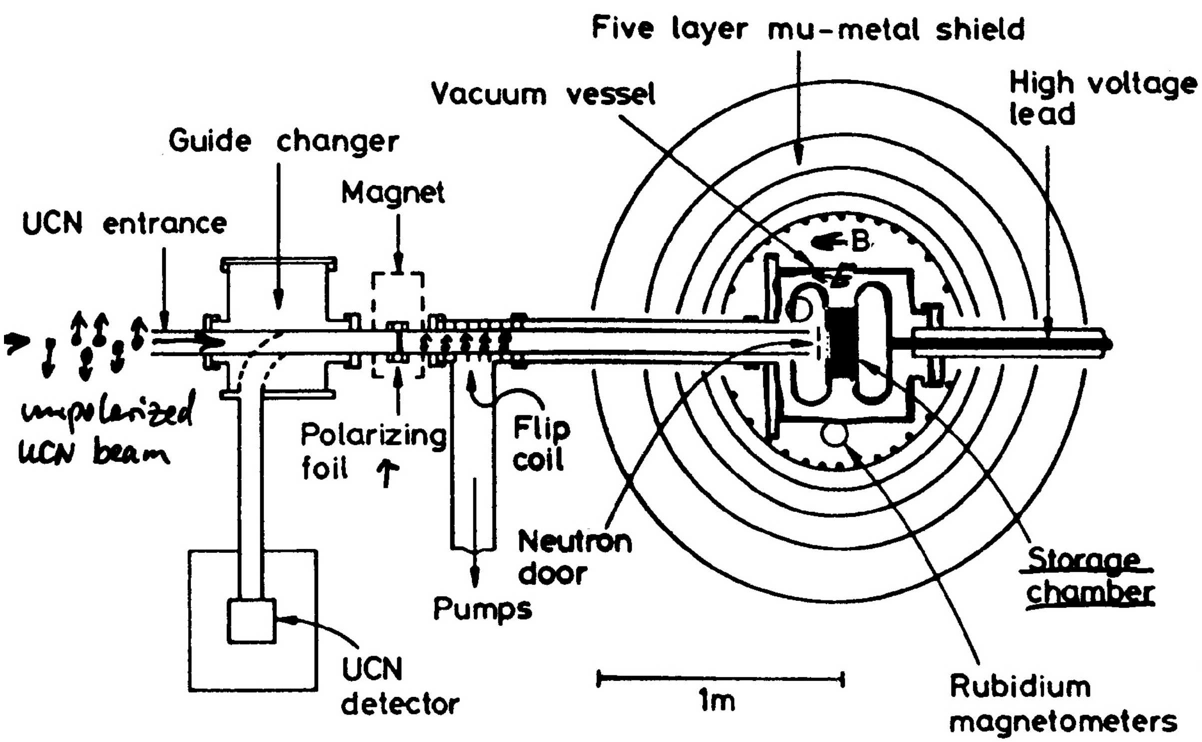
Measure NMR HF splitting for $\vec{E} \uparrow\downarrow \vec{B}$



Interaction $\boxed{\varepsilon = -\mu_n B \mp d_n E}$

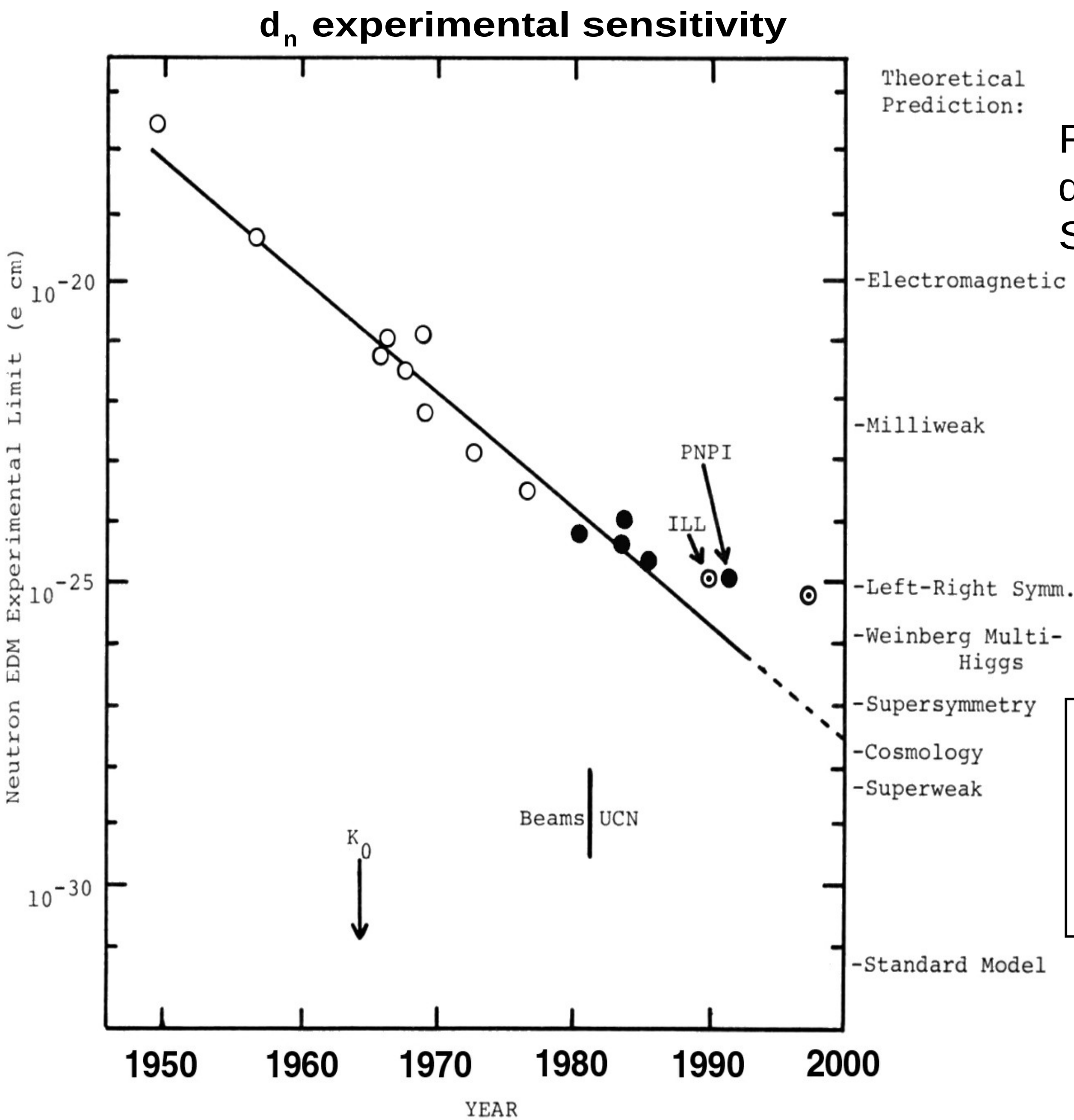
$$\left.\begin{aligned} \varepsilon^+ &= 2\mu_n B + 2d_n E = \hbar\omega^+ \\ \varepsilon^- &= 2\mu_n B - 2d_n E = \hbar\omega^- \end{aligned}\right\}$$

Transition energies
$\Delta\omega = 4 d_n E$



B=0.1mG,
tune with $B_{osc} \perp$ B.
E = 1MV/m
ω= 30Hz spin-flip
of ultra-cold (kT~mK)
$E_{kin}=10^{-7}$eV, λ =670Å
neutrons in mgn.bottle
guided in reflecting Ni tubes

Experimental Results for d_n

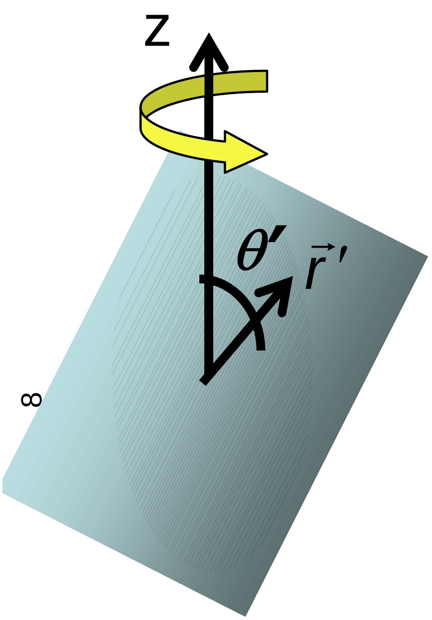


From size of neutron ($r_0 \approx 1.2$fm):
$d_n \lesssim 10^{-15}$ e·m.
So far, only upper limits for d_n

PNPI (1996):
$d_n < (2.6 \pm 4.0 \pm 1.6)\cdot 10^{-26}$ e·cm

ILL-Sussex-RAL (1999):
$d_n < (-1.0 \pm 3.6)\cdot 10^{-26}$ e·cm

Intrinsic Quadrupole Moment



Consider axially symmetric nuclei (for simplicity), body-fixed system ($'$), $z = z'$ symmetry axis

$$Q_0 = eZ \int d^3\vec{r}\,' \rho(\vec{r}\,') r'^2 \left(3\cos^2\theta' - 1\right)$$

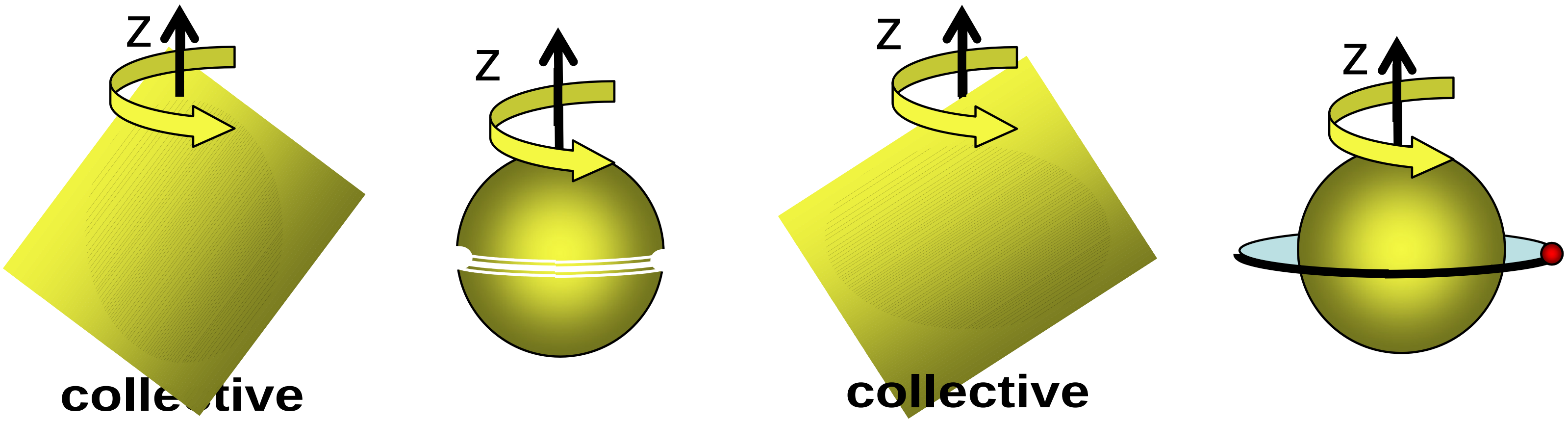
$$z = r' \cdot \cos\theta' \rightarrow \boxed{Q_0 = 3\left\langle z^2 \right\rangle - \left\langle r^2 \right\rangle}$$

Sphere: $$\left\langle r^2 \right\rangle = \left\langle x^2 \right\rangle + \left\langle y^2 \right\rangle + \left\langle z^2 \right\rangle = 3\left\langle z^2 \right\rangle$$

$$\rightarrow \boxed{Q_0^{sph} = 0}$$

→ Q_0 measures deviation from spherical shape.

Collective and s.p. Deformations



collective deformation cigar

single hole around core

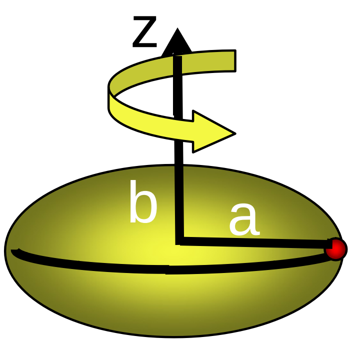
collective deformation disc

single particle around core

$Q_0 > 0$ "prolate"

$Q_0 < 0$ "oblate"

Planar single-particle orbit: $Q_0^{sp} = eZ \cdot \left\{ 3\cancel{\left\langle z^2 \right\rangle} - \left\langle r^2 \right\rangle \right\} = -eZ \cdot \left\langle r^2 \right\rangle$



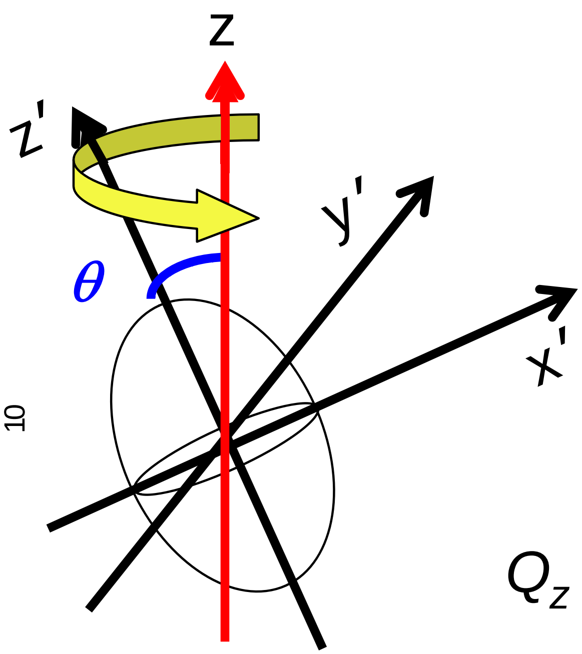
Ellipsoid principal axes *a*, *b*

$$\bar{R} = \frac{(a+b)}{2}; \quad \Delta R = b - a \;\; ; \quad \delta := \frac{\Delta R}{\bar{R}}$$

$$\boxed{Q_0^{coll} = \frac{2}{5} eZ \cdot \left(b^2 - a^2 \right) = \frac{4}{5} eZ \cdot \bar{R}^2 \cdot \delta}$$

Deformation parameter δ

Spectroscopic Quadrupole Moment



body-fixed $\{x', y', z'\}$, Lab $\{x, y, z\}$
Symmetry axis z defined by the experiment

$$Q_0/eZ = 3\left\langle z'^2 \right\rangle - \left\langle r^2 \right\rangle \quad \text{intrinsic}$$

What is measured in Lab system?

$$Q_z/eZ = 3\left\langle z^2 \right\rangle - \left\langle r^2 \right\rangle = \ldots\ldots = \left(Q_0/eZ\right)\frac{1}{2}\left(3\cos^2\theta - 1\right)$$

$$\rightarrow \boxed{Q_z = Q_0 \cdot P_2\left(\cos(\theta)\right)} \qquad \text{finite rotation through } \theta$$

Measured Q depends on orientation of deformed nucleus w/r to Lab symmetry axis. → *define Q_z as the largest Q measurable*.

How to control or determine orientation of nuclear Q?

→ Nuclear spin ⊥ to symmetry axis, no quantal rotation about z′

Angular Momentum and Q

Q_z =maximum measurable → maximum spin (I) alignment

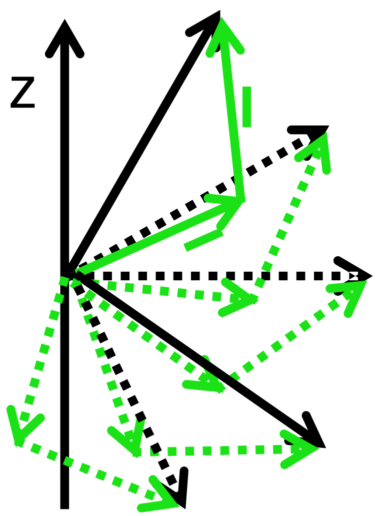
$$Q_z/eZ = \left\langle 3\hat{z}^2 - \hat{r}^2 \right\rangle_{m_I = I} = \left\langle \begin{matrix} I \\ m_I = I \end{matrix} \middle| 3\hat{z}^2 - \hat{r}^2 \middle| \begin{matrix} I \\ m_I = I \end{matrix} \right\rangle = \left\langle \hat{Q} \right\rangle$$

$$\propto \int d^3\vec{r}\,'\,\psi_I^*(\vec{r}\,')P_2(\cos\theta)\psi_I(\vec{r}\,') \qquad \psi_I \text{ nucl. wave funct.}$$

Legendre polyn. complete basis set →

$$\int d\phi\,\psi_I^*(\theta,\phi)\cdot\psi_I(\theta,\phi) = \sum_{\ell=0}^{2I}\alpha_\ell P_\ell(\cos\theta)$$

$$Q_z \propto \int d^3\vec{r}\,'\,\psi_I^*(\vec{r}\,')P_2(\cos\theta)\psi_I(\vec{r}\,') = \sum_{\ell=0}^{2I}\alpha_\ell \underbrace{\int d^3\vec{r}\,P_2(\cos\theta)P_\ell(\cos\theta)}_{\delta_{\ell 2}}$$



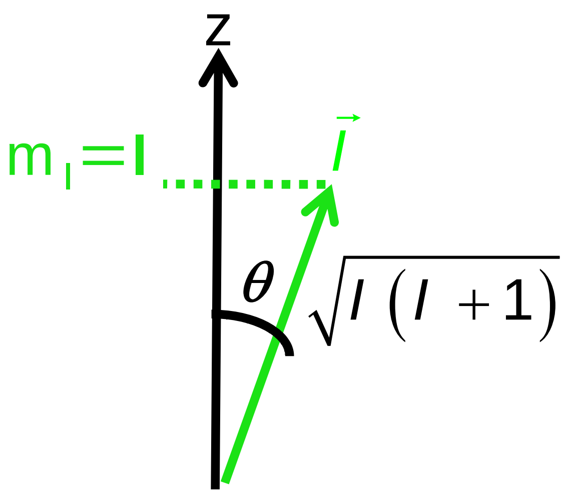
I couples with I to L =2

$$\boxed{Q_z = 0 \quad for \quad 2I < 2 \rightarrow I = 0,\ 1/2}$$

for any Q_0

Spins too small to effect alignment of Q in the lab.

Vector Coupling of Spins



Any orientation

$I \neq 0$:

$$Q_z = \left\langle \begin{matrix} I \\ m_I = I \end{matrix} \middle| \hat{Q} \middle| \begin{matrix} I \\ m_I = I \end{matrix} \right\rangle = \frac{1}{2}\left(3\cos^2\theta - 1\right) Q_0$$

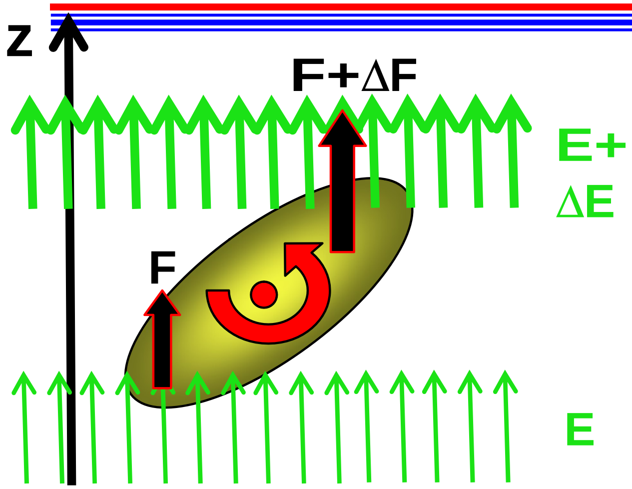
$$m_I = I = \sqrt{I(I+1)}\cos\theta \;\rightarrow\; \cos\theta = \frac{I}{\sqrt{I(I+1)}}$$

$$Q_z(m_I) = \frac{3m_I^2 - I(I+1)}{I(2I-1)} \cdot Q_z(m_I = I)$$

quadratic dependence of Q_z on m_I

"The" quadrupole moment $\boxed{Q_z = \frac{2I-1}{2(I+1)} \cdot Q_0}$ $(= 0 \;\; for \;\; I = 0, 1/2)$

Electric Multipole Interactions



Inhomogeneous external electric field $\vec{E} = \vec{\nabla} U$ exerts a torque on deformed nucleus.
→ orientation-dependent energy W_Q
Examples: crystal lattice,
fly-by of heavy ions

Axial symmetry of field assumed: $\boxed{\left(\frac{\partial^2 U}{\partial x_i \partial x_j}\right) = \left(\frac{\partial^2 U}{\partial x^2}\right)\delta_{ij} \quad \Big| x_i = x, y, z}$

Taylor expansion of scalar potential $\vec{r} = \vec{0}$ *center of nucleus*

$$U(\vec{r}) = U_0 + \vec{r}\bullet\left(\vec{\nabla}U\right)_0 + \frac{1}{2}\left\{x^2\left(\frac{\partial^2 U}{\partial x^2}\right)_0 + y^2\left(\frac{\partial^2 U}{\partial y^2}\right)_0 + z^2\left(\frac{\partial^2 U}{\partial z^2}\right)_0\right\} + \ldots$$

no mixed dervs.

$$W = eZ\int d^3\vec{r}\ \rho(\vec{r})U(\vec{r}) =$$

$$= \underbrace{eZU_0}_{\substack{\textbf{mono pole} \\ W_{IS}}} + \underbrace{eZ\left(\vec{\nabla}U\right)_0\bullet\left\langle\vec{r}\right\rangle}_{\substack{\textbf{dipole} \\ \to 0}} + \underbrace{\frac{eZ}{2}\left\{\left(\frac{\partial^2 U}{\partial x^2}\right)_0\left\langle x^2\right\rangle + \left(\frac{\partial^2 U}{\partial y^2}\right)_0\left\langle y^2\right\rangle + \left(\frac{\partial^2 U}{\partial z^2}\right)_0\left\langle z^2\right\rangle\right\}}_{\textbf{quadrupole}\ W_Q} + \ldots$$

W_{IS}: isomer shift, W_Q: quadrupole hyper-fine splitting

Electric Quadrupole Interactions

Maxwell equs. → $\Delta U = 4\pi\rho(0) = 0$ No external charge

$$0 = \left(\frac{\partial^2 U}{\partial x^2}\right) + \left(\frac{\partial^2 U}{\partial y^2}\right) + \left(\frac{\partial^2 U}{\partial z^2}\right) \longrightarrow \left(\frac{\partial^2 U}{\partial x^2}\right)_0 = \left(\frac{\partial^2 U}{\partial y^2}\right)_0 = -\frac{1}{2}\underbrace{\left(\frac{\partial^2 U}{\partial z^2}\right)_0}_{=U_{zz}}$$

axial symm

$$W_Q = \frac{eZ}{2}\left\{\left(\frac{\partial^2 U}{\partial x^2}\right)_0 \left[\langle x^2\rangle + \langle y^2\rangle\right] + \left(\frac{\partial^2 U}{\partial z^2}\right)_0 \langle z^2\rangle\right\}$$

$$W_Q = \frac{eZ}{4}\left(\frac{\partial^2 U}{\partial z^2}\right)_0 \left\{-\left[\langle x^2\rangle + \langle y^2\rangle\right] + 2\langle z^2\rangle\right\}$$

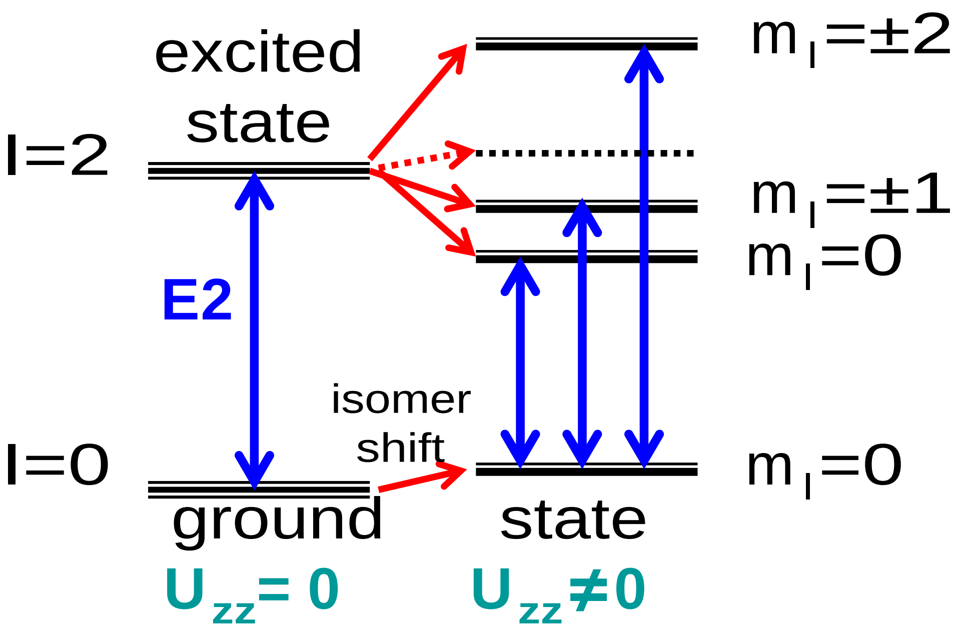
$$= \frac{eZ}{4}\left(\frac{\partial^2 U}{\partial z^2}\right)_0 \left\{-\left[\langle r^2\rangle - \langle z^2\rangle\right] + 2\langle z^2\rangle\right\}$$

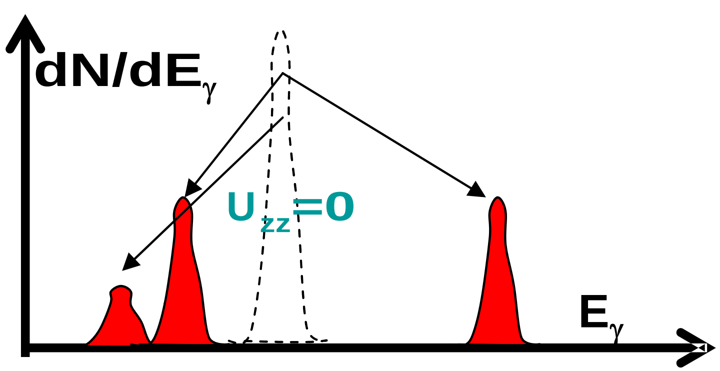
$$\boxed{W_Q = \frac{eZ}{4}\left(\frac{\partial^2 U}{\partial z^2}\right)_0 Q_z(m_I)}$$

Field gradient x spectroscopic quadrupole moment $\propto m_I^2$

Quadrupole Hyper-Fine Splitting

Use external electrostatic field, align Q by aligning nuclear spin **I**,
Measure interaction energies W_Q (**I** >1/2)
Quadrupole hyper-fine splitting of nuclear or atomic energy levels





Slight "hf" splitting of nuclear and atomic levels in $U_{zz}\neq 0$

→splitting of
γ emission/absorption lines

Estimates:
atomic energies ~ eV
atomic size ~ 10^{-8}cm
potential gradient U_z ~ 10^8V/cm
field gradient U_{zz} ~ 10^{16}V/cm^2
Q_0 ~ 10^{-24} e cm^2

→**W_Q ~ 10^{-8} eV**
small !

Experimental Methods for Quadrupole Moments

Small "hf" splitting W_Q of nuclear and atomic levels in **$U_{zz} \neq 0$**

→ splitting of X-ray/ γ emission/absorption lines

Measurable for atomic transitions with laser excitations

nuclear transitions with Mössbauer spectroscopy

muonic atoms:

10^7 times larger hf splittings W_Q with X-ray and γ spectroscopy

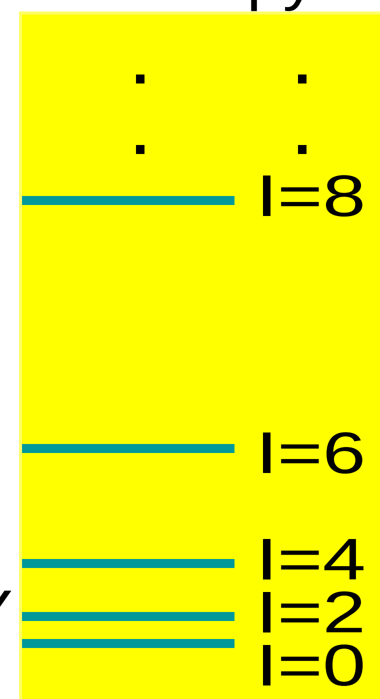
scattering experiments $U_{zz}(t)$

Nuclear spectroscopy of collective rotations
→ model for moment of inertia

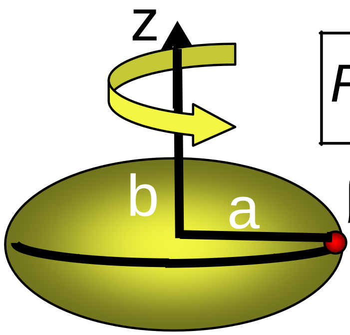
$$E_I = \frac{\hbar^2 I(I+1)}{2\Im}$$

$$\Im \rightarrow Q_0$$

$$\frac{\hbar^2}{2\Im} \sim (10-20)\,keV$$



Collective Rotations



$$R(\theta,\phi) = R_0\left[1+\beta Y_0^2(\theta,\phi)\right]$$

$$\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\,\frac{\Delta R}{\bar{R}}$$

$$\bar{R} = \frac{a+b}{2};\quad \Delta R = a-b$$

β : deformation parameter

Nuclei with large Q_0 consistent w. collective rotations → lanthanides, actinides

$$Q_0 = \frac{3eZ}{\sqrt{5\pi}} R_0^2 \beta\left[1+0.16\beta\right]$$

Rotational and inversion symmetry → even I

$$E_I = \frac{\hbar^2}{2\Im} I\left(I+1\right)$$

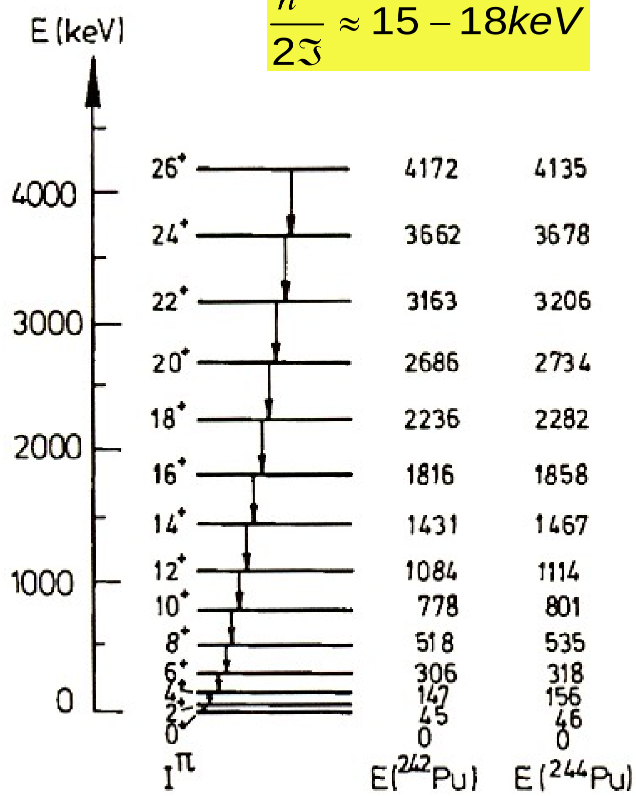
rigid body mom.o.inertia :

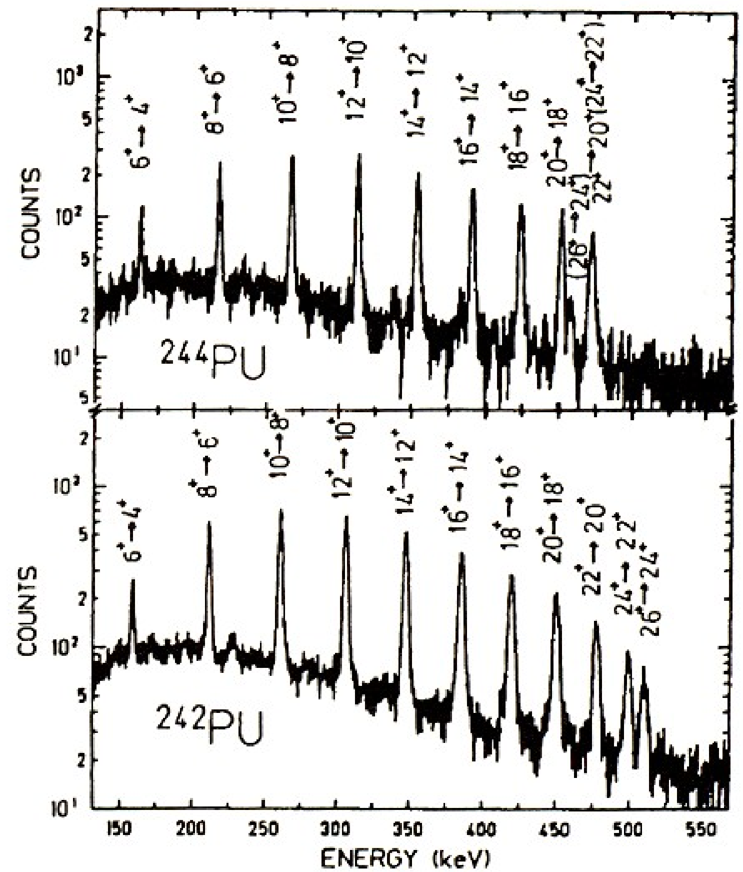
$$\Im_{rig} = \frac{2}{5} M R_0^2\left[1+0.31\beta\right]$$

hydro – dynamical :

$$\Im_{irr} = \frac{9}{8\pi} M R_0^2 \beta$$

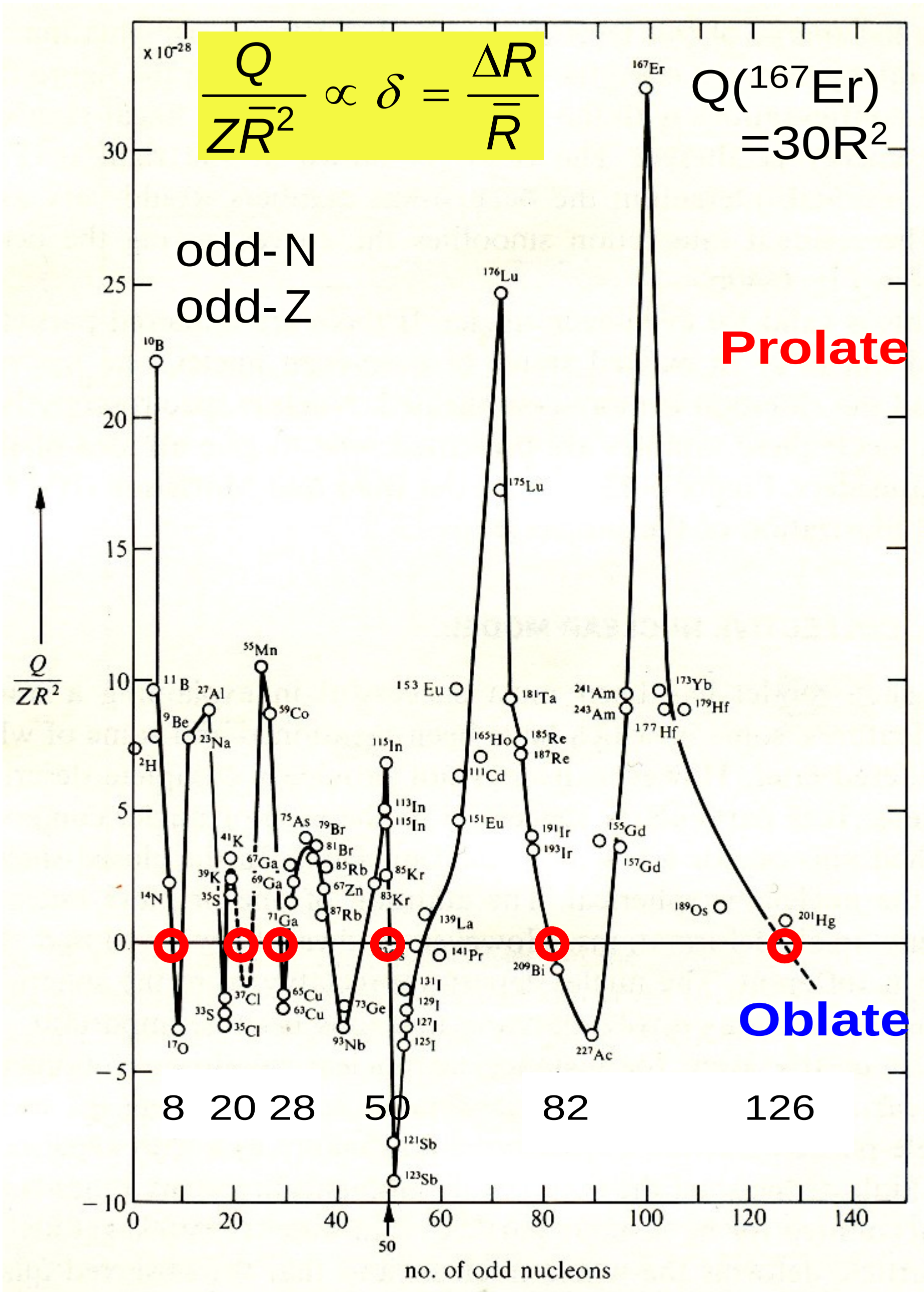
$$\frac{\hbar^2}{2\Im} \approx 15-18\,keV$$





Wood et al.,Heyde

Systematics of Electric Quadrupole Moments



Nuclear Deform

Mostly prolate (Q>0) heavy nuclei

$$^{27}_{13}Al,\ ^{55}_{25}Mn,\ ^{115}_{49}In,\ ^{176}_{71}Lu,\ ^{167}_{68}Er$$

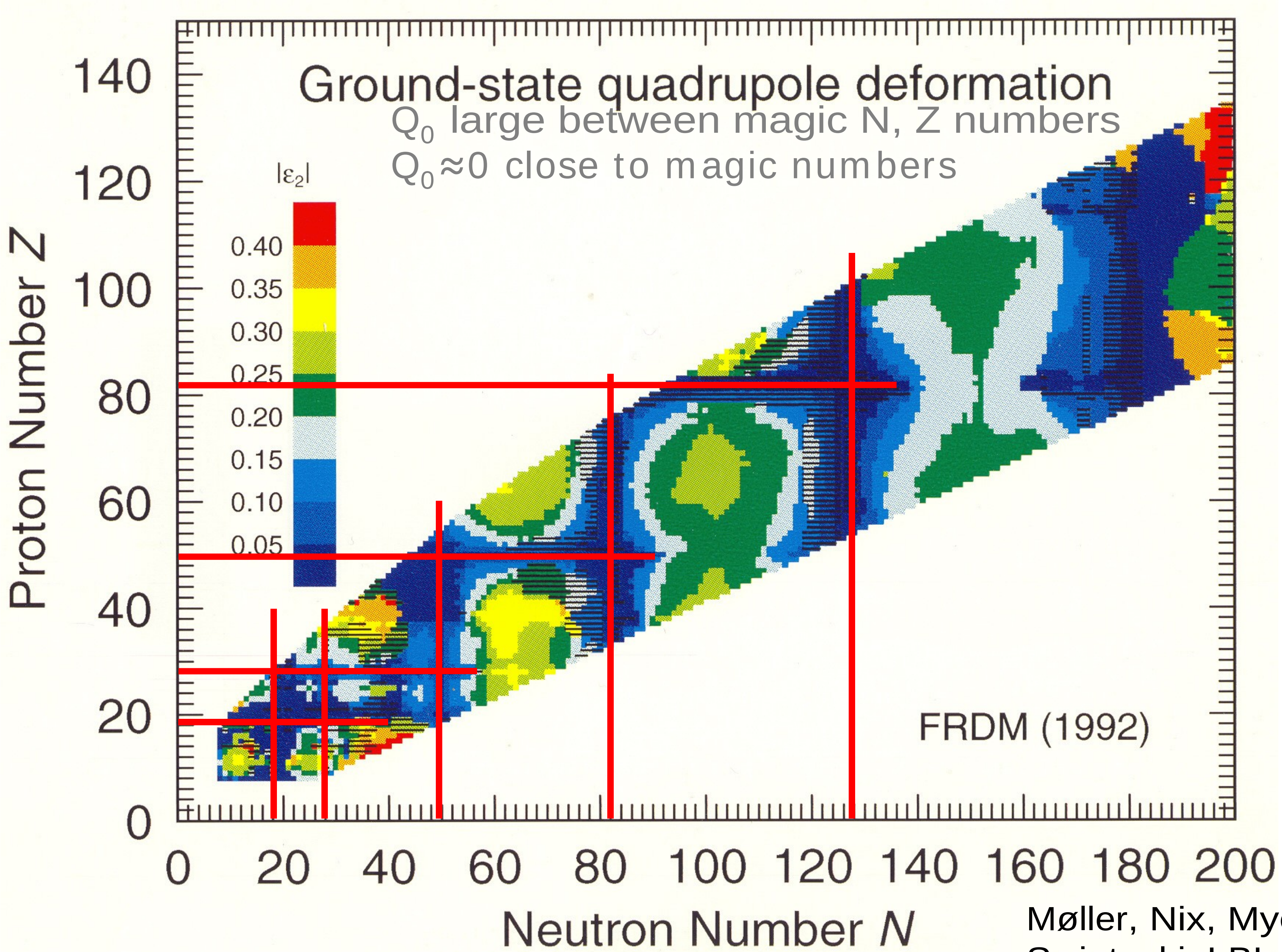
Q>0 : e.g., hole in spherical core → pattern not obvious. If such nuclei exist, weak effect of hole for Q

Tightly bound nuclei are spherical: "Magic" N or Z = 8, 20, 28, 50, 82, 126, ...

Q<0 : e.g., extra particle around spherical core. pattern recognizable

$$^{17}_{8}O,\ ^{63}_{29}Cu,\ ^{123}_{51}Sb,\ ^{209}_{83}Bi$$

Q_0 Systematics



Møller, Nix, Myers, Swiatecki, LBL 1993

No more deformations!