
Nuclear Accelerators

Overview

Electrostatic Accelerators

Cascade generator	Van de Graaff Accelerator	V.d.G. Tandem 2-3 stages

→ steady (DC) beam, high quality focusing, energy, currents; but low energies

Electrodynamic Accelerators

Cyclotrons conventional, sector-focusing	Synchrotons	Linacs Wideröe, Alvarez

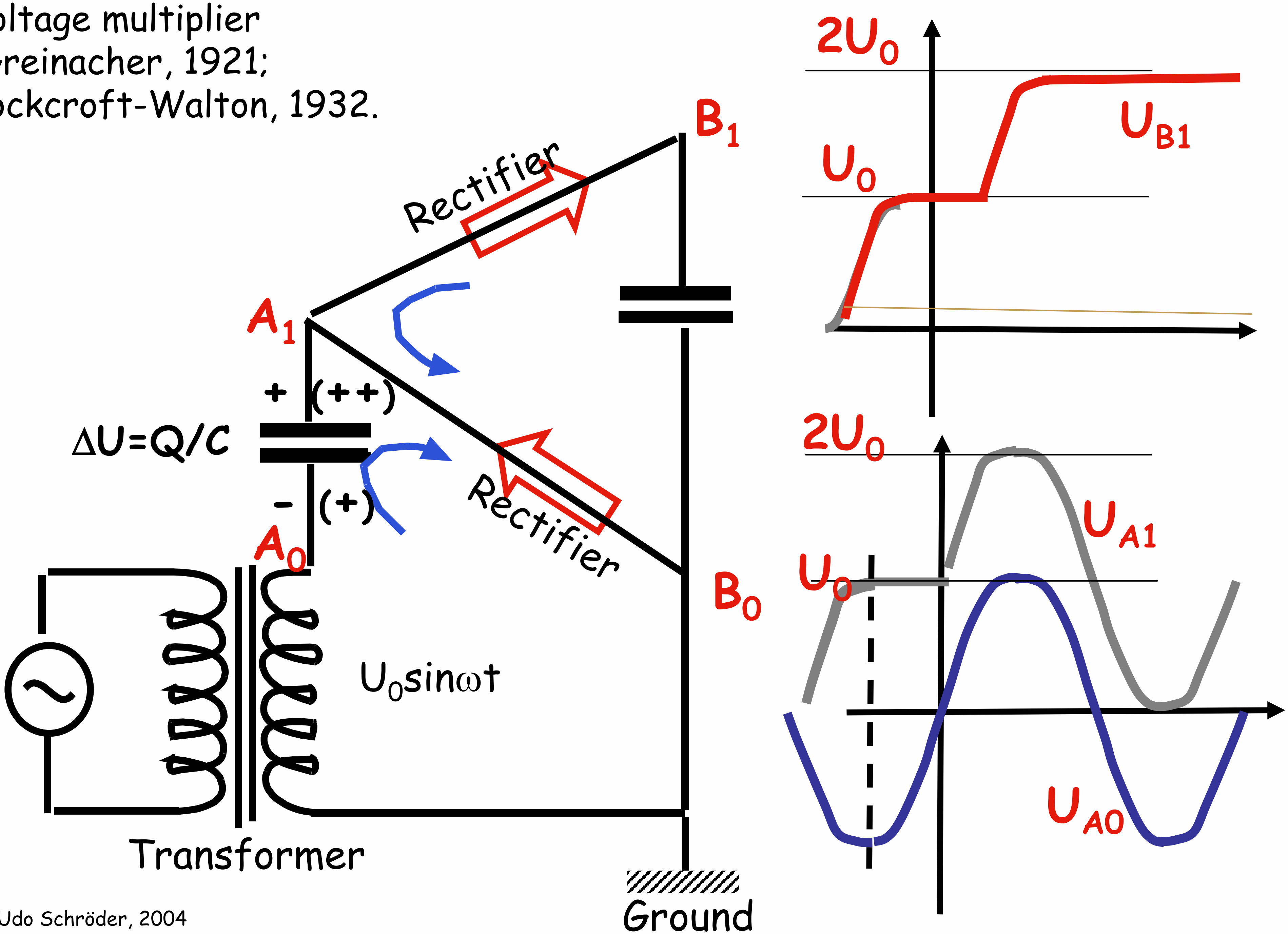
→ pulsed (AC) beam, high energies but lower quality focusing, energy definition, lower currents

Advanced Technology Accelerators

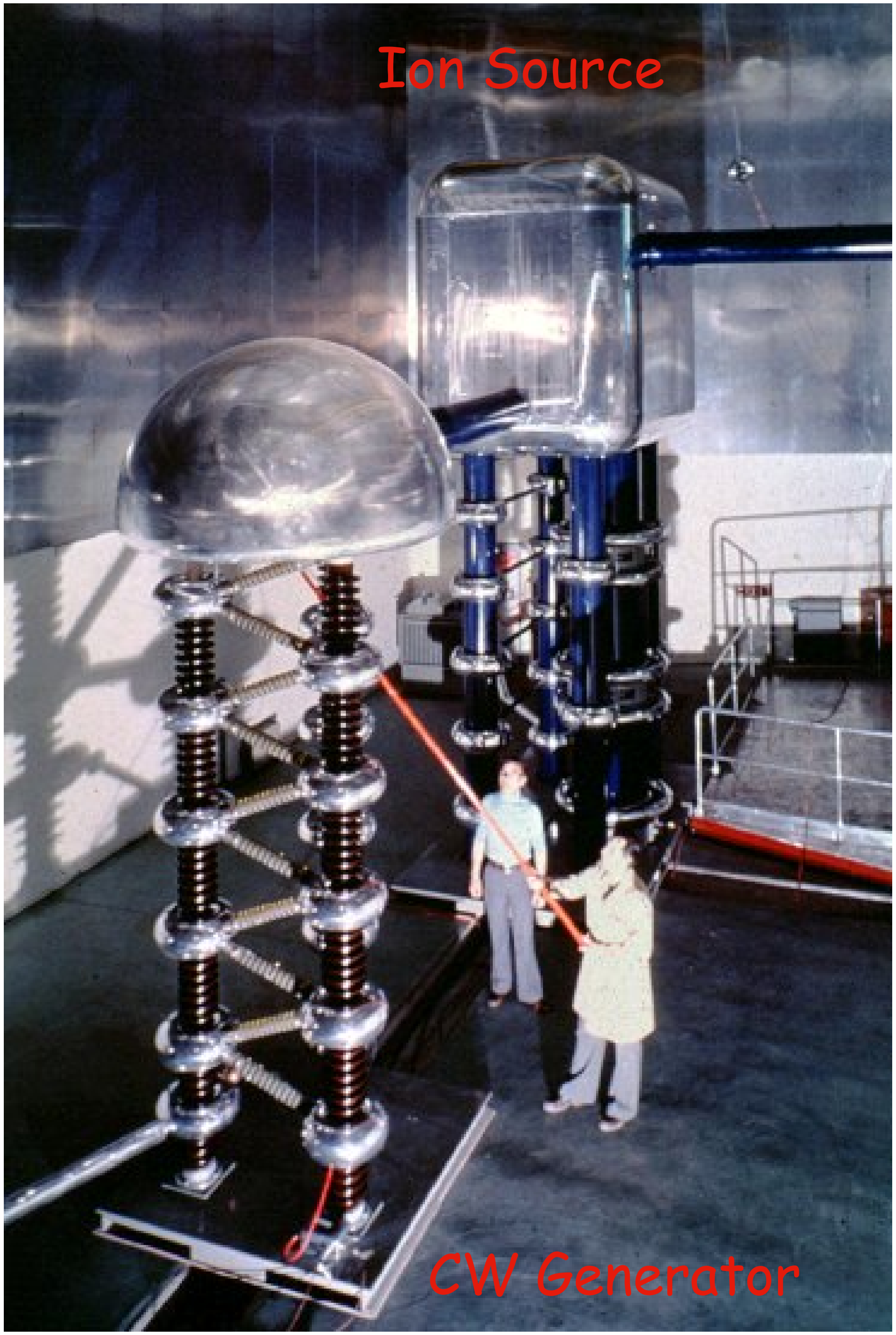
New principles: Collective acceleration, wake-field acceleration conceptual stage

Cockcroft-Walton (Cascade) Generator

Voltage multiplier
(Greinacher, 1921;
Cockcroft-Walton, 1932.



W. Udo Schröder, 2004

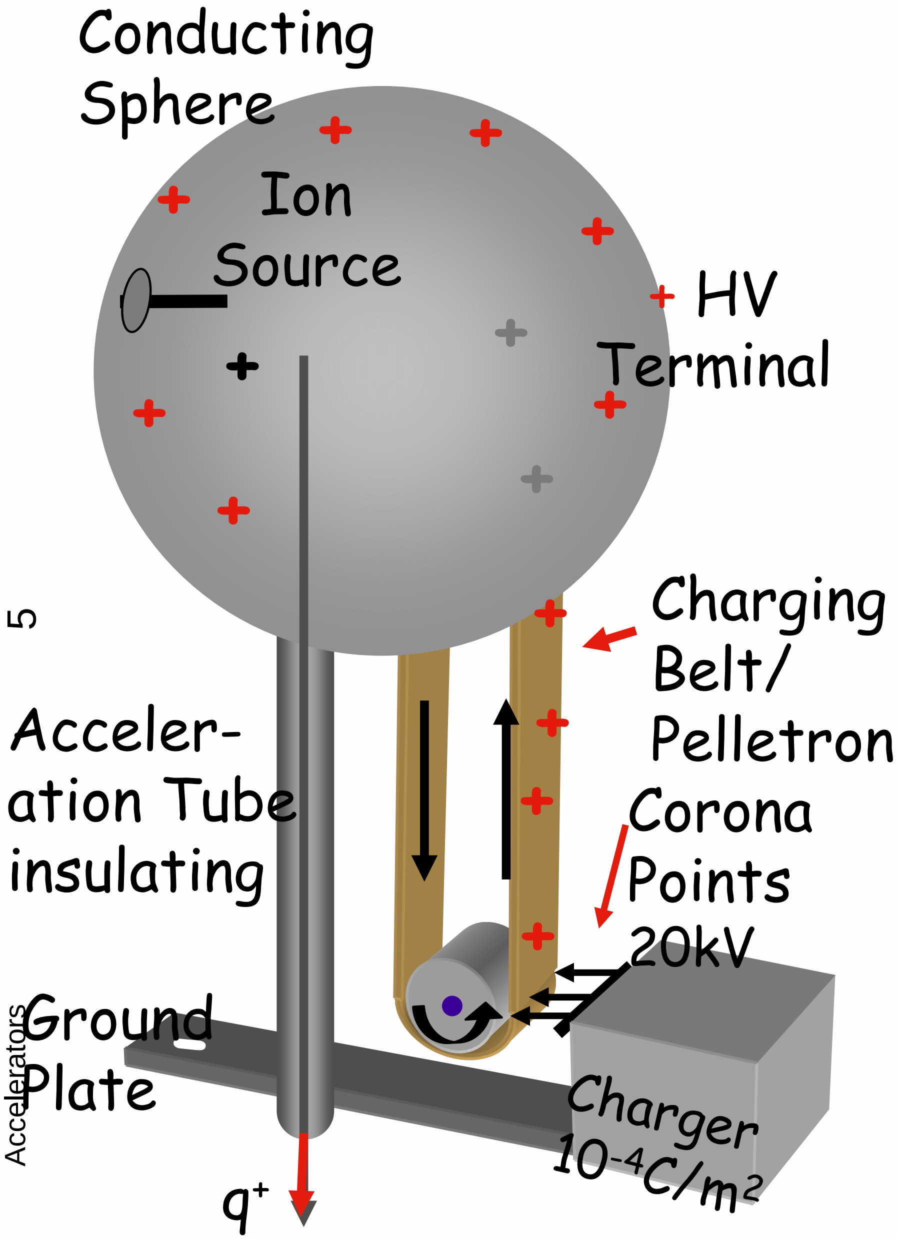


Cockroft-Walton Pre-Accelerator

Voltage doubling for each stage.
Typical range order ~ n·100 kV ~ 10 MV

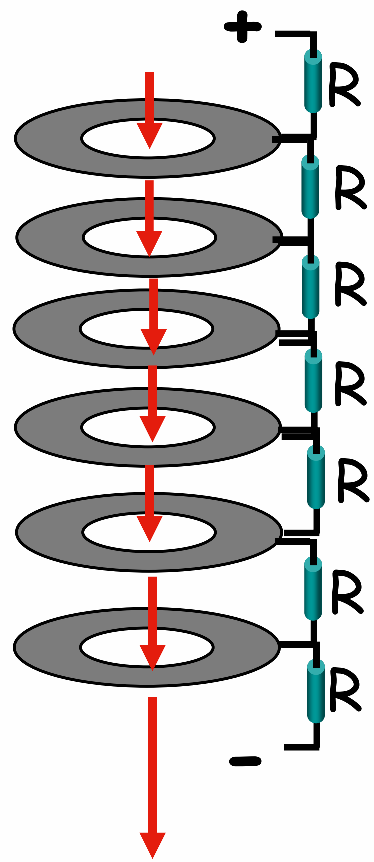
CW used here to charge an enclosed metal terminal containing the ion source. Provides pre-acceleration of ions produced in the source
→ allows magnetic analysis

Van de Graaff Electrostatic Accelerators



Van de Graaff, 1929

Operating limitations: 2 MV terminal voltage in air, 18-20 MV in pressure tank with insulating gas (SF_6 or gas mixture N_2, CO_2)



Acceleration tube has equipotential plates connected by resistor chain (R), ramping field down.

Typical for a CN:

7-8 MV terminal voltage

Early Van de Graaf Accelerators

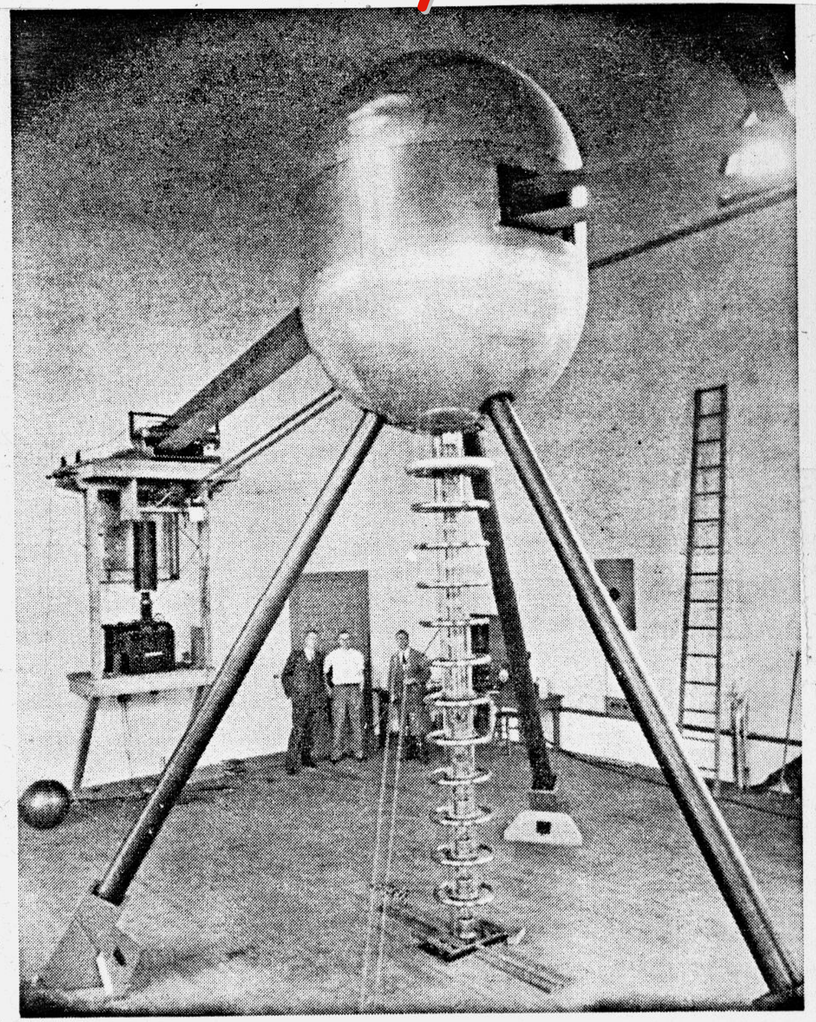


Fig. 15. The 2 m DTM Van de Graaff generator installed in the new DTM Laboratory. This photograph was taken on November 13, 1933. The drive belt as shown passes completely through the terminal from drive motors on the laboratory walls. The spray supply for the charging system is shown attached to the wall at the left. The manually operated shorting rod with a spherical termination is lying on the floor under this power supply. The multielectrode acceleration tube leads into the experimental research room below and the fishline controls for the ion source instrumentation lead downward slightly to the left of the acceleration tube. In the background observing the instrument are from the left M. A. Tuve, L. R. Hafstad, and O. Dahl.

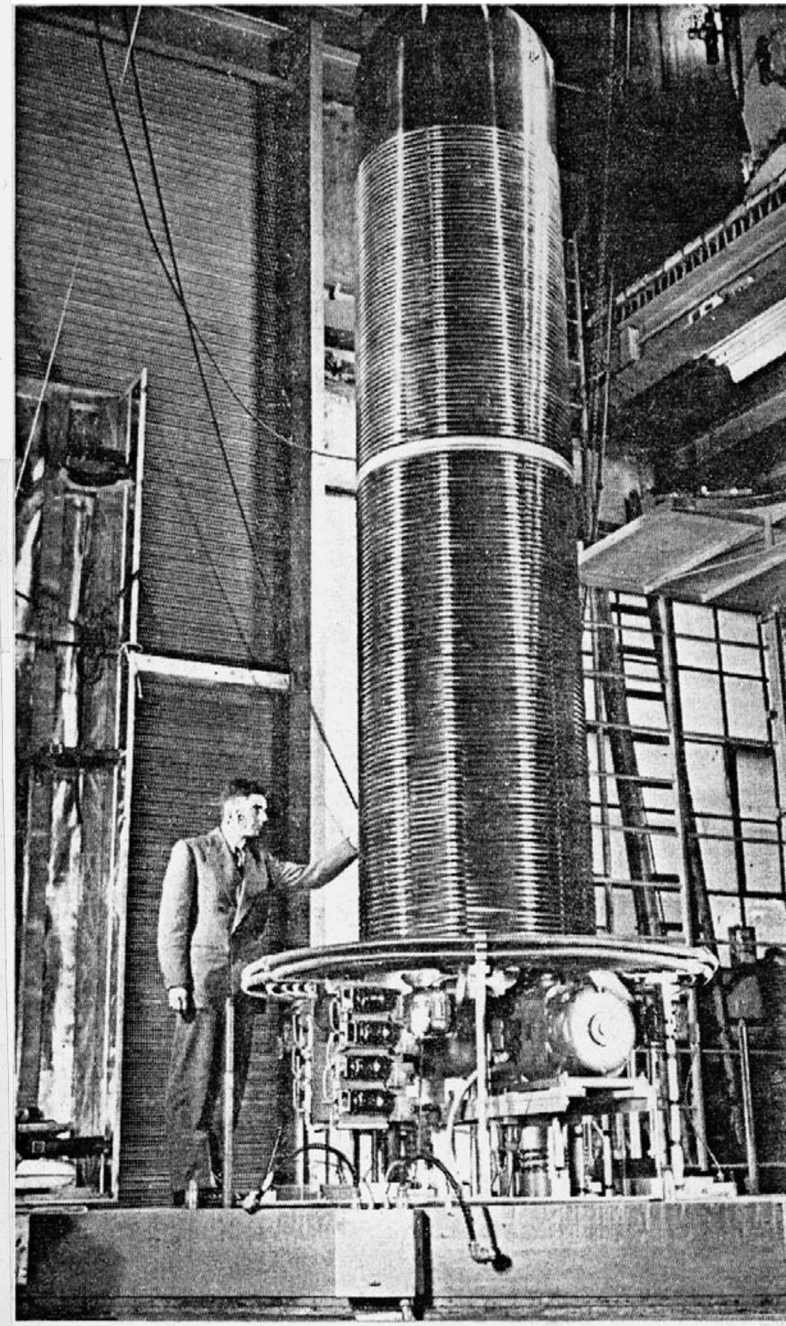
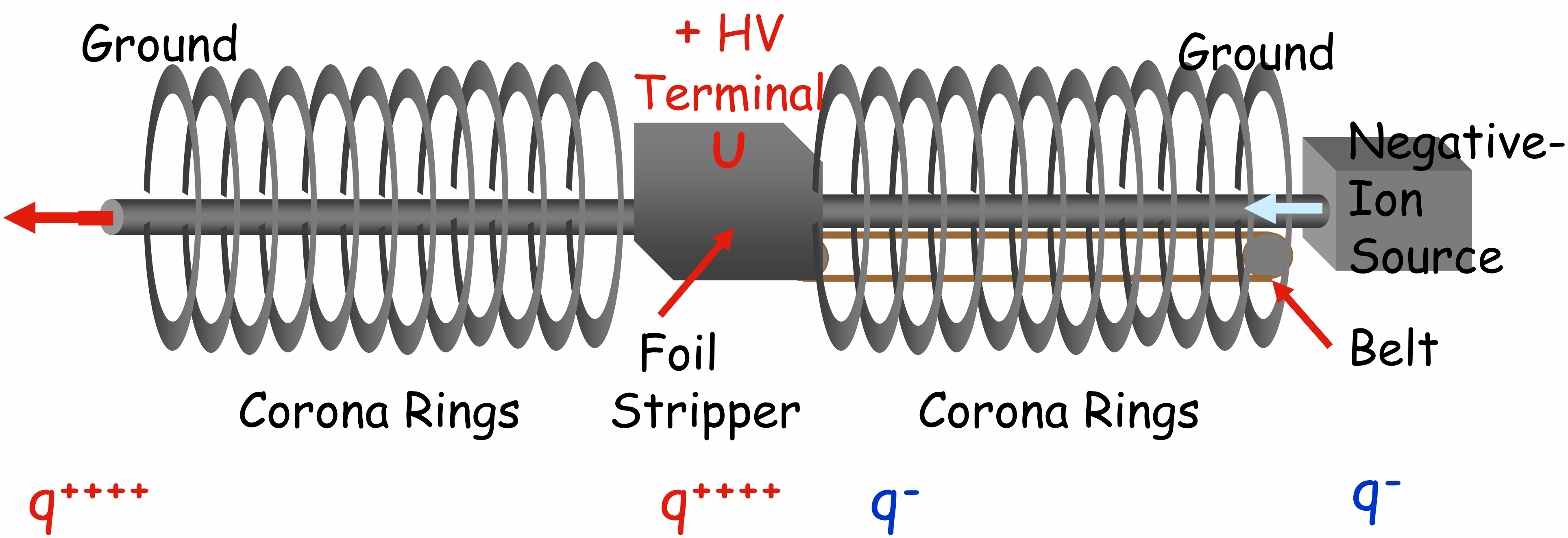


Fig. 27. The column of one of the early HVEC Model CN Van de Graaff accelerators during initial assembly at the HVEC Burlington, Massachusetts factory.

Tandem Van de Graaffs

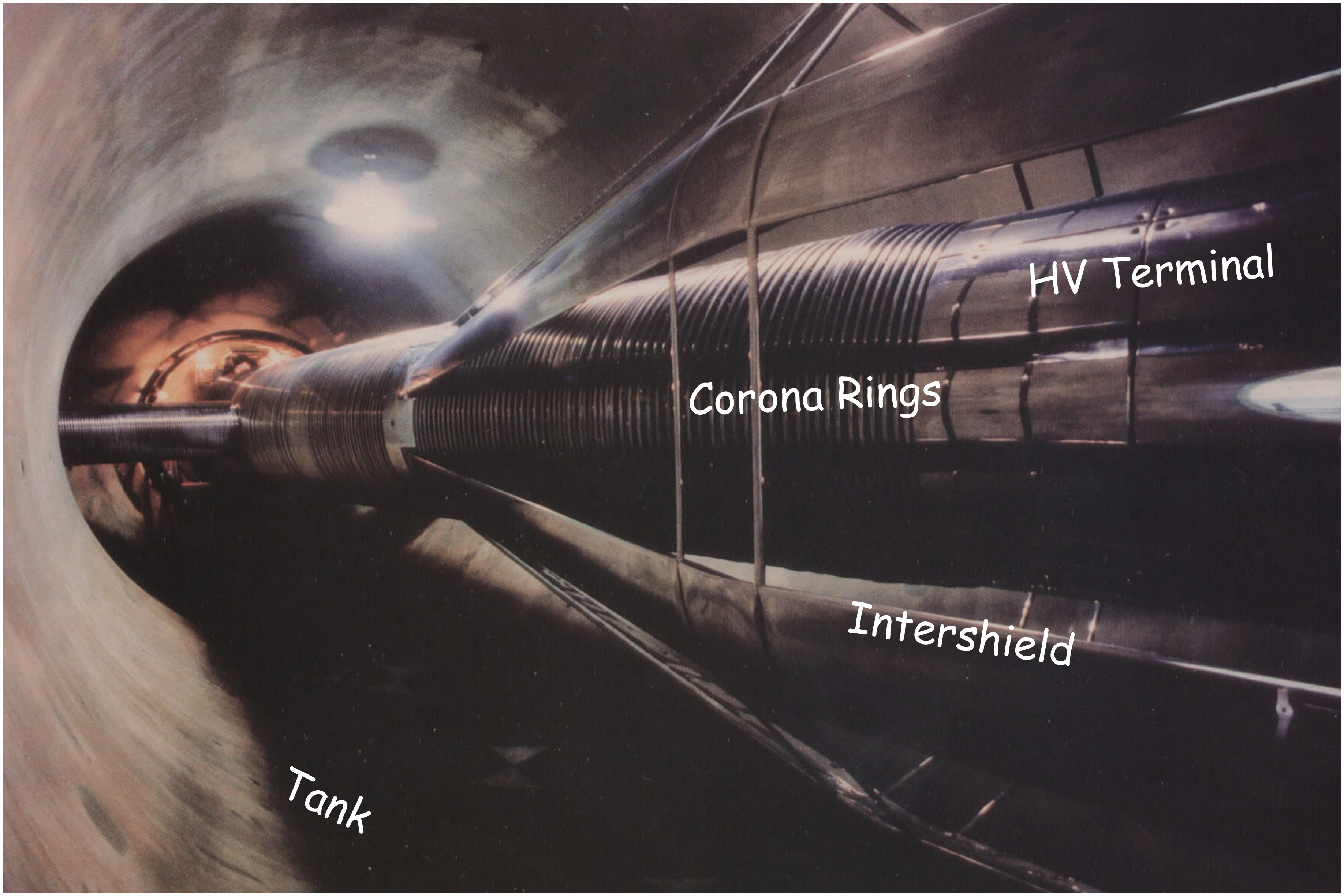
Built in the late 1950s, 2 stage and 3-stage (Brookhaven), to increase beam energy

Accelerator structure enclosed in pressurized tank (SF_6)
Corona rings to homogenize electric field.

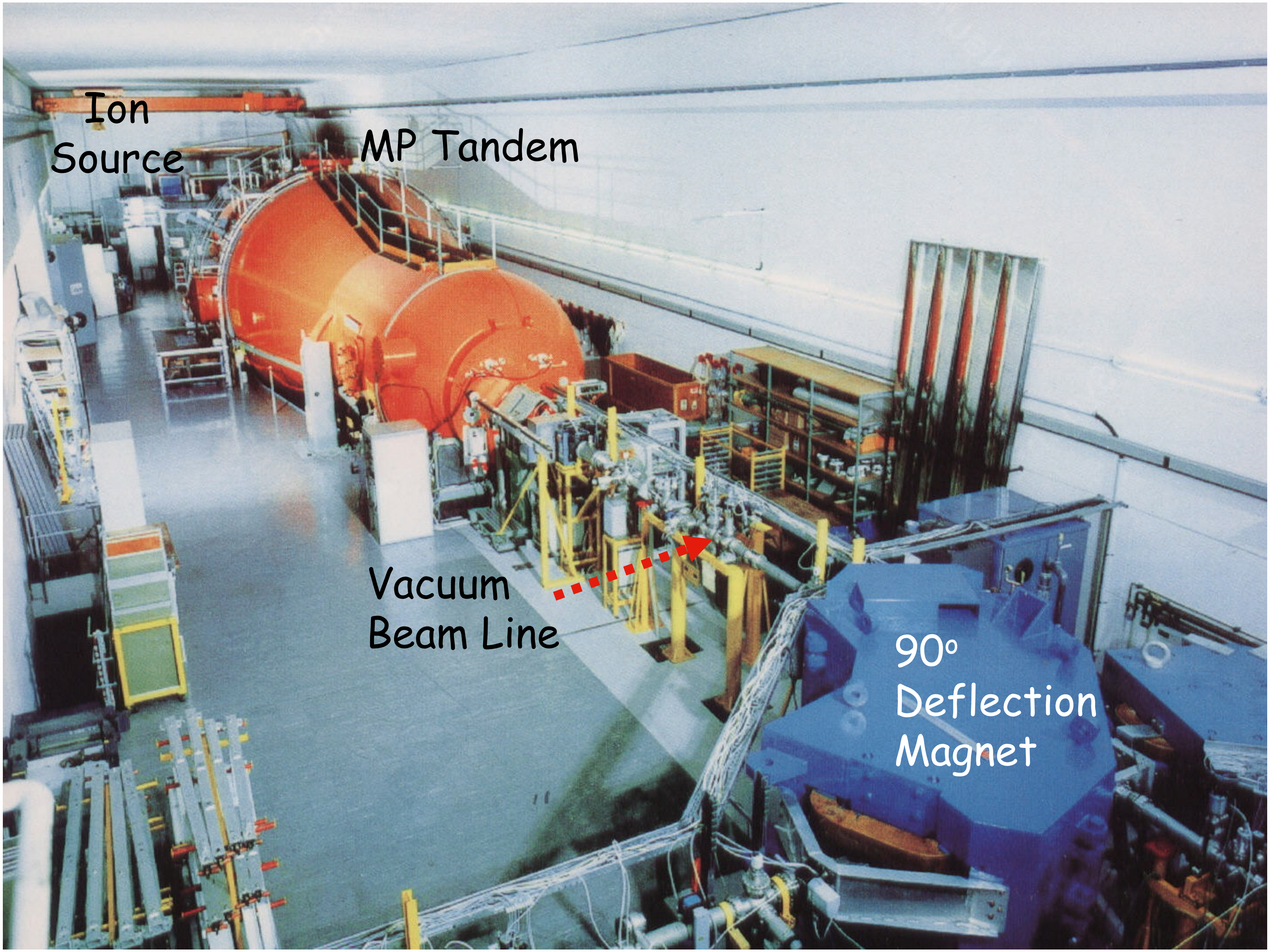


W. Udo Schröder, 2004

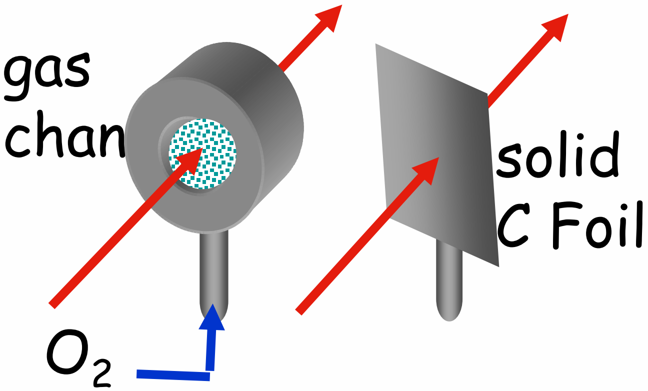
The Yale MP Tandem



Munich University MP Tandem



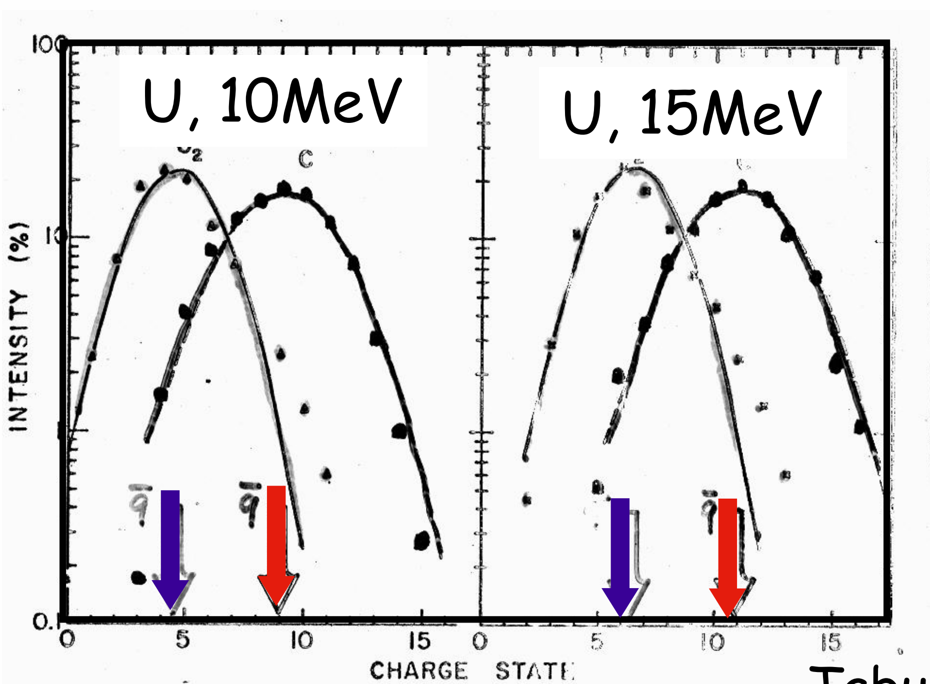
Stripping Probability for Heavy Ions



Charge distribution

$$f(q) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(q-\bar{q})^2}{2\sigma^2} \right\}$$

Average $\bar{q}$, *variance* σ^2

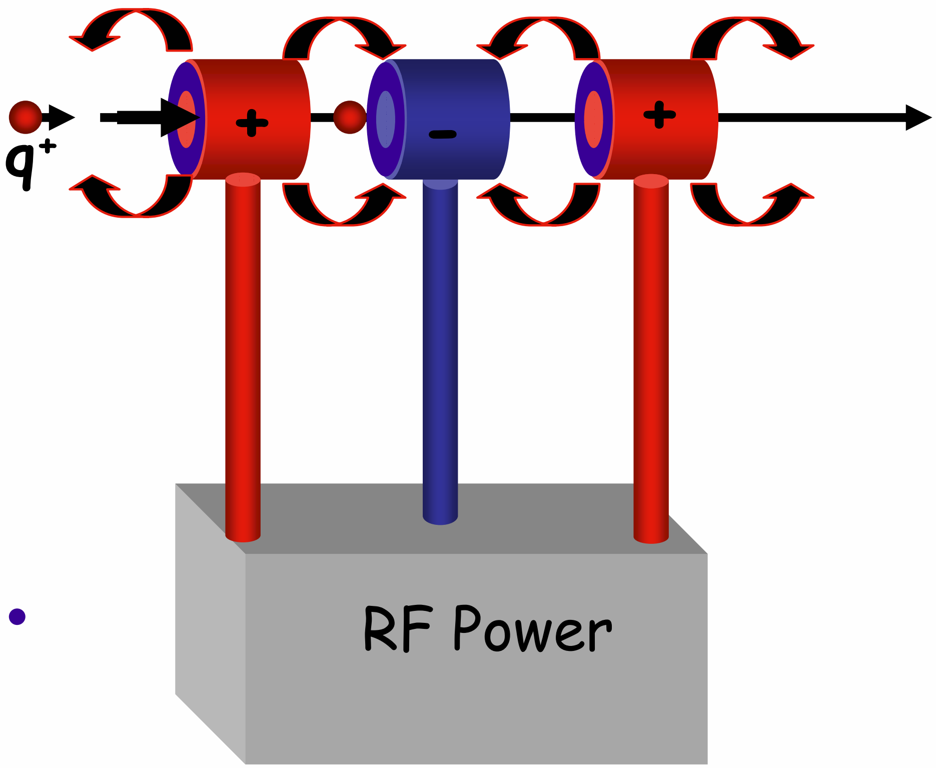


$$\frac{\bar{q}}{Z} = \begin{cases} avZ^{-1/2} \quad for\ \ \bar{q}/Z \le 0.3 \\ \qquad a = 0.16(He), \\ \qquad\quad 0.18(N_2, Ar) \\ \qquad \sigma = (0.3-0.4)Z^{0.4} \\ \ell n\left(v/mZ^{\alpha}\right) / \ell n\left(nZ^{\beta}\right) \\ \qquad for\ 0.3 < \bar{q}/Z \le 0.9 \\ v = 1.38\sqrt{E/AMeV} \quad velocity \end{cases}$$

Tabulation: Chaki&Elmore UR-NSRL-240, 1980

Linear Accelerators

Wideröe 1928, Alvarez 1946



$$L = v \cdot \frac{T}{2} = \beta \cdot \frac{\lambda}{2} \qquad \beta = {}^{v}\!/_{c}$$

$$After\ gap\ n:$$

$$v_n = \sqrt{n \frac{2(qU_0)}{m}} \quad velocity$$

Linear trajectory, no deflection magnet, no radiative losses

Hollow drift tubes, E-field-free interior, contain magn. focusing elements.

Accelerating gap between 2 drift tubes on different el. potential

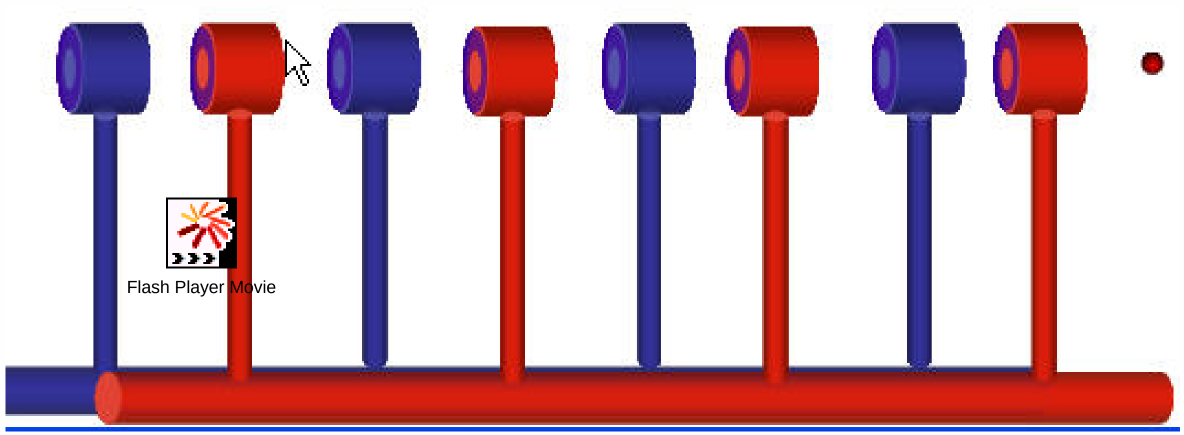
$U(t) = U_0.\sin(\omega t)$ 10MHz-3GHz

on all gaps → alternating E field

switch polarity while particle is hidden.

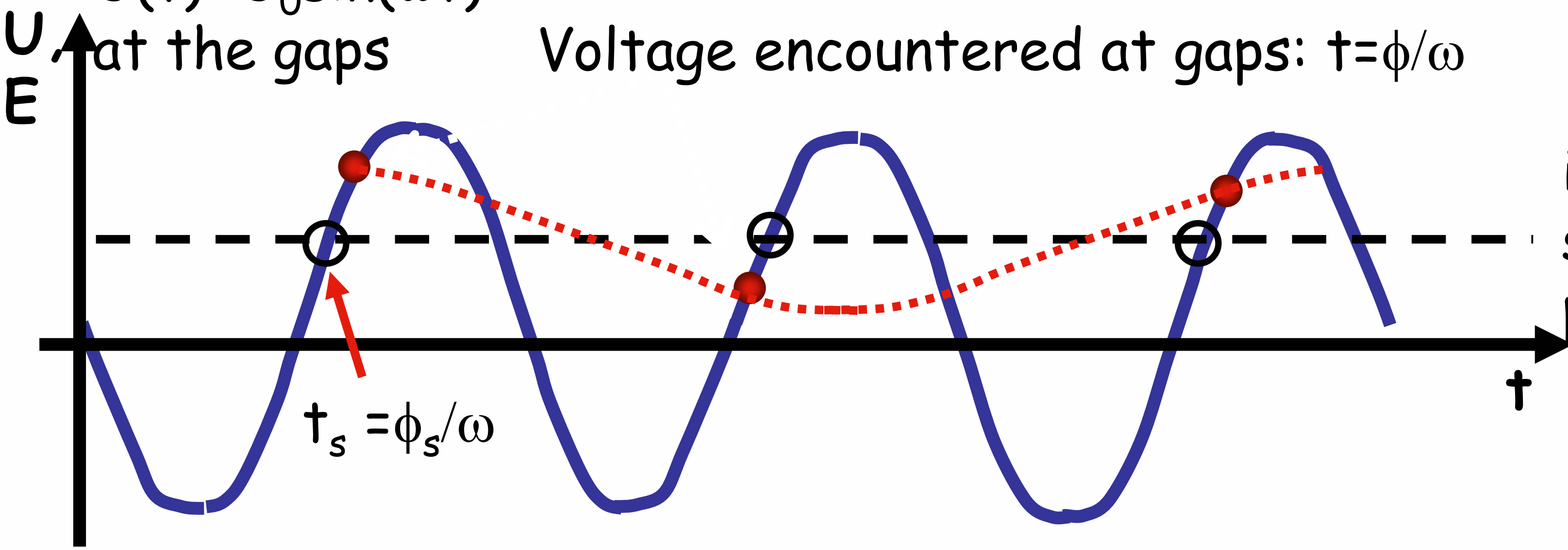
Acceleration with Linac

Alternating directions of the gap fields, change while particle is hidden inside drift tube.



Q: Phase conditions change during acceleration, particle speed. Is continuous operation possible, without loss of particles?

Phase Stability



Phase angle of particle: $\phi = \phi_s + \omega\cdot(t-t_s)$, force $F=qU_0/(\text{gap length } L)$

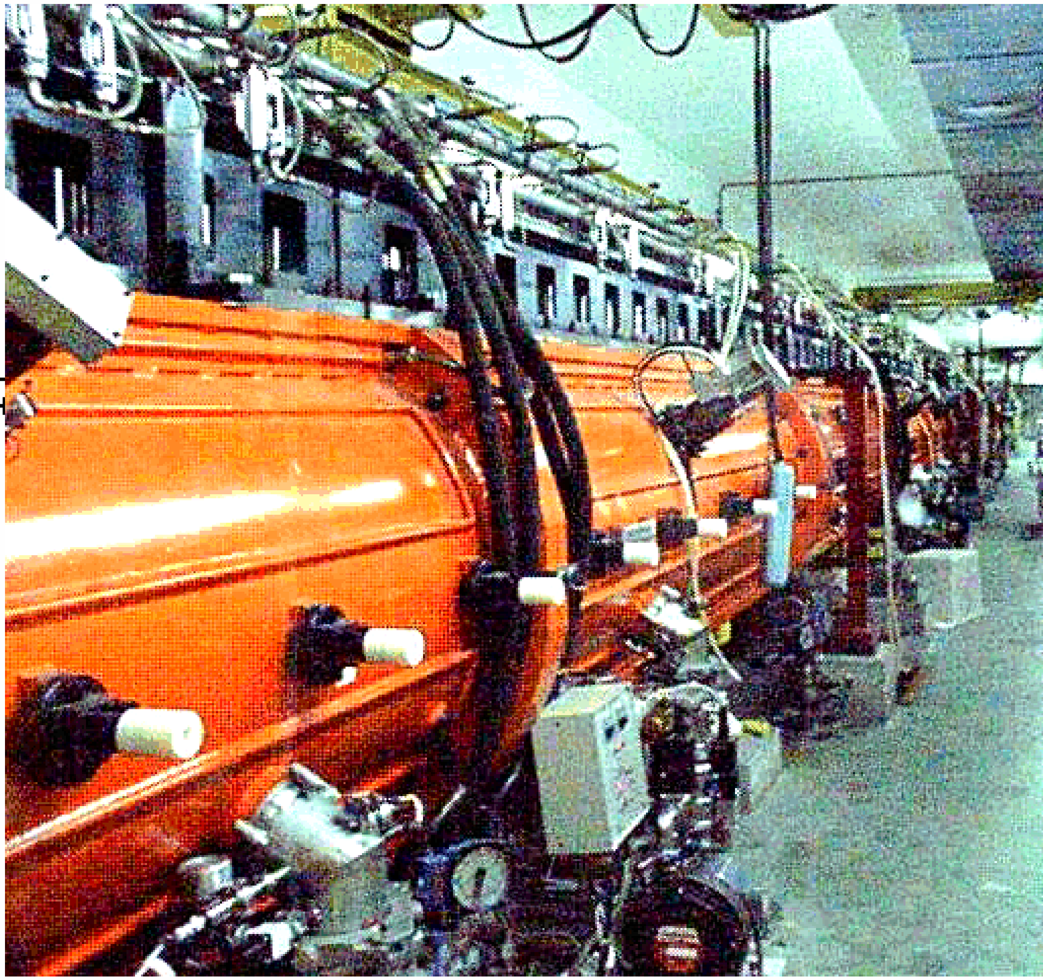
$$\frac{d^2}{dt^2}(\phi-\phi_s)+\frac{\varpi F\cos\phi_s}{p_s}\underbrace{(\phi-\phi_s)}_{:=\tilde{\phi}}=0$$

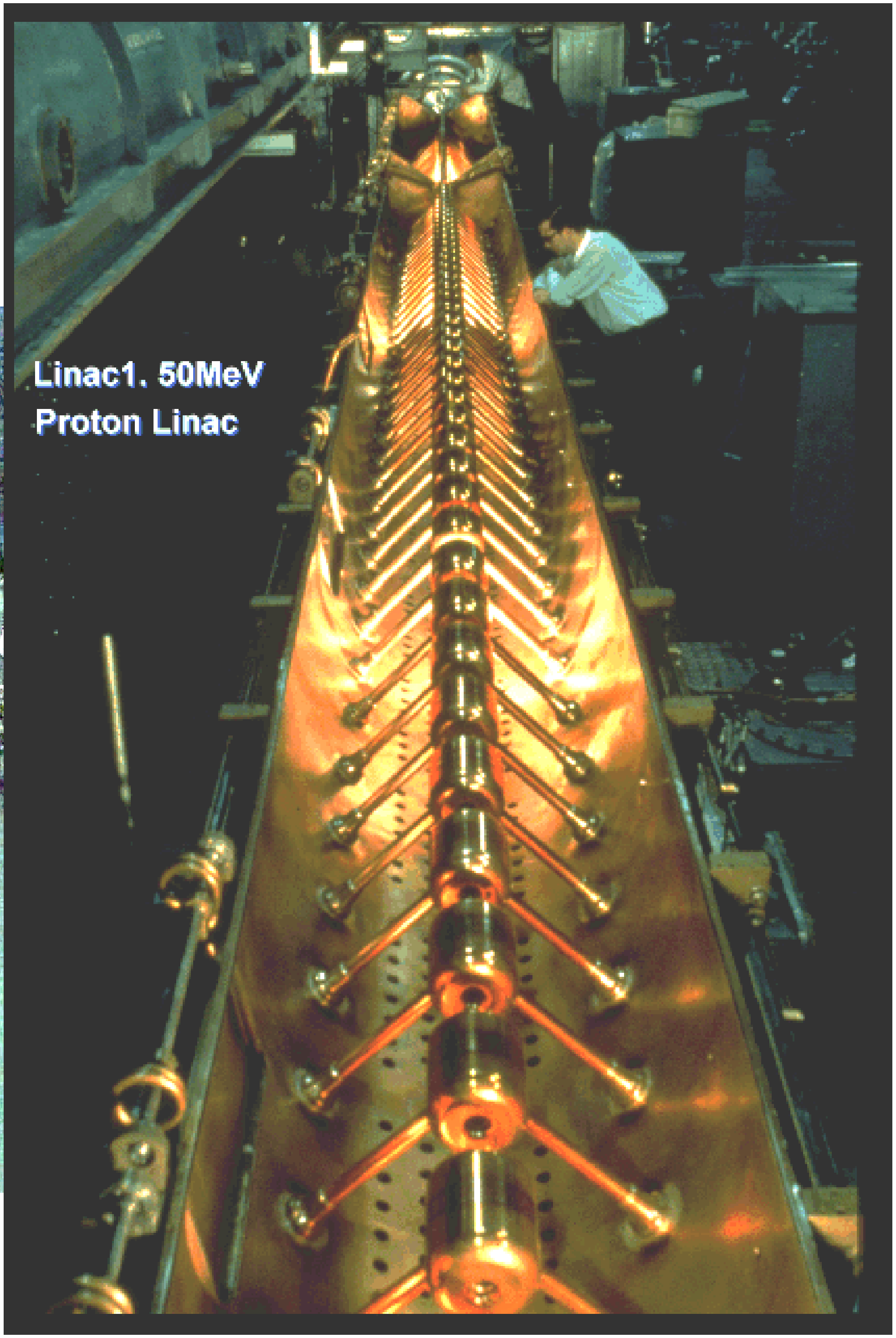
$$\frac{d^2\tilde{\phi}}{dt^2}+\varpi_\phi^2\tilde{\phi}=0 \quad \textit{differential equation}$$

Stability analysis for $\tilde{\phi}\approx 0$
Stable oscillations about synchr. phase for $\cos\phi_s>0$,

→ $\phi_s < 90^0$ inject during first quarter cycle!

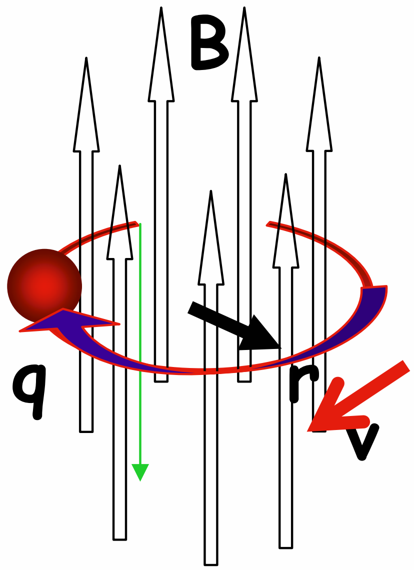
CERN Proton Linac





Cyclotrons

Charged particles in electromagnetic fields, E, B



B: Magnetic guiding field

Lorentz Force

$$F = q\cdot\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

$$E=0: \qquad F=\dot{\vec{p}}=q\cdot\vec{v}\times\vec{B}$$

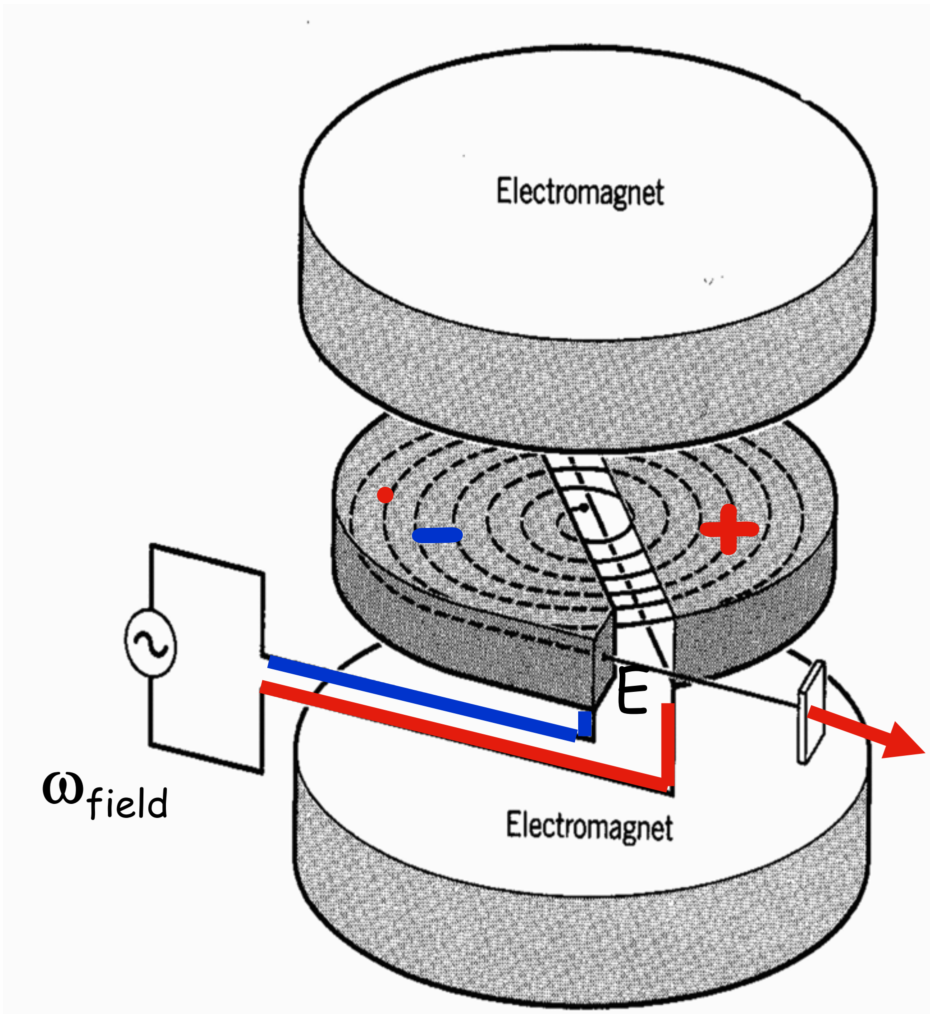
$$\boxed{\vec{p}=q\cdot\vec{r}\times\vec{B}} \quad \textit{orbit radius } r, \left(\vec{r}\perp\vec{B}\right)$$

$$p=q\cdot r\cdot B, \; \textit{equilibrium orbit at } r=\frac{p}{qB}$$

$$\vec{p}=m\vec{v} \;\rightarrow\; \vec{v}=\vec{\varpi}_0\times\vec{r}$$

$$\boxed{\vec{\varpi}_0=-\frac{q}{m}\vec{B}} \quad \textit{Particle Cyclotron Frequency}$$

Cyclotron Max. Energy



Cyclotron Frequency

$$\vec{\varpi}_0 = -\frac{q}{m}\vec{B} \quad \textit{same for all } v$$

Acceleration, if $\omega_{field} = \omega_0$

Equilibrium orbit r: $p = qBr$

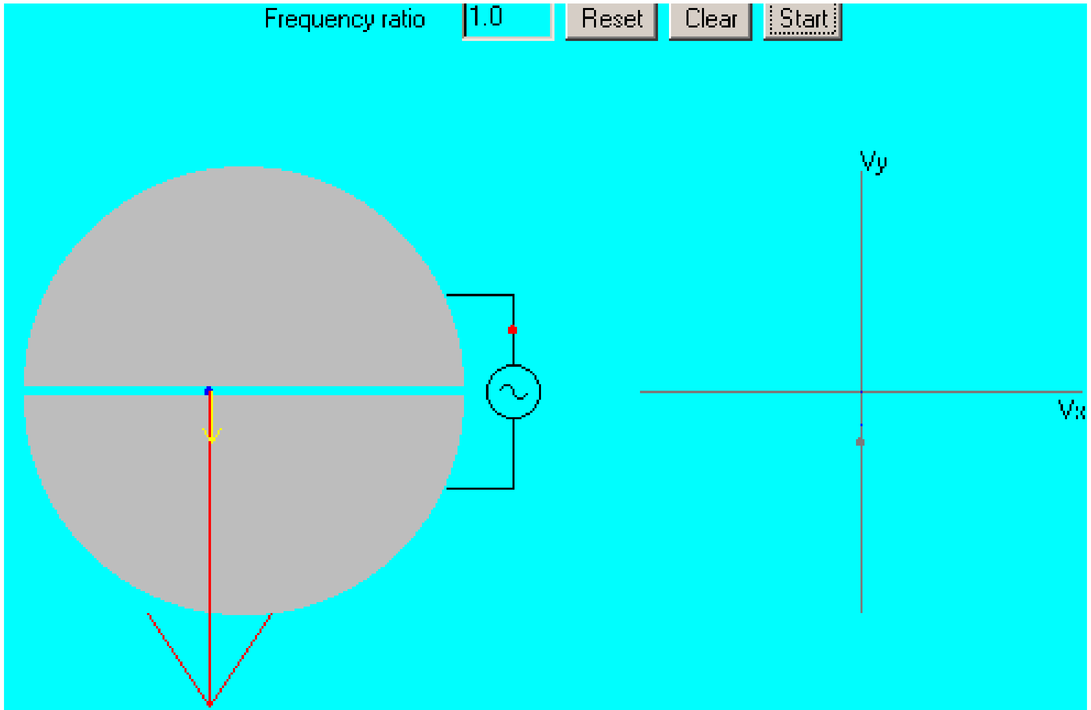
→ maximum $p_{max} = qBR$

Maximum Energy

$$\boxed{\varepsilon_{max} = \frac{(qBR)^2}{2m} = K \cdot \frac{q^2}{A}}$$

Relativistic effects: $m \rightarrow W = \varepsilon + m_o c^2$ shape B field to compensate. Defocusing corrected with sectors and fringe field.

Cyclotron Resonance Condition

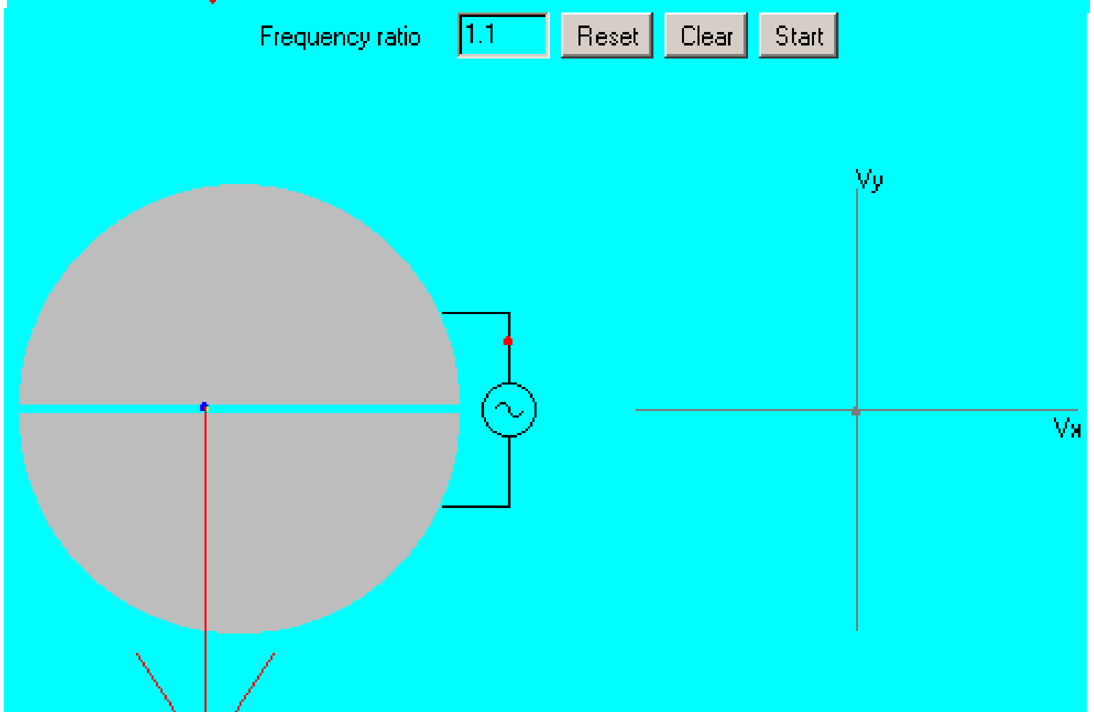


← f=1, matched

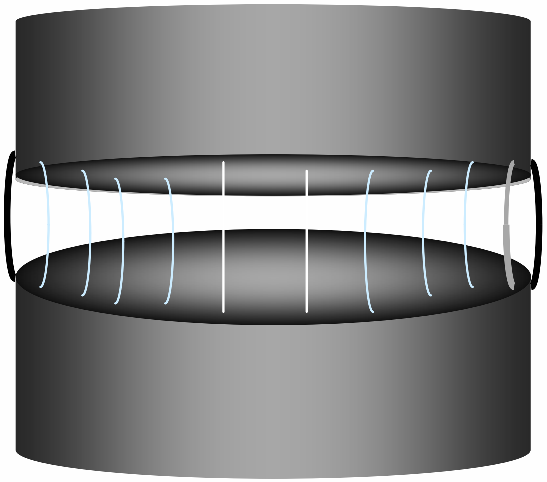
Accelerating rf voltage has to be switched with frequency of gap crossings:

$$\varpi_{rf} = \varpi_{0,particle} = \frac{qB}{m}$$

$$f = \varpi_{rf} \Big/ \varpi_{0,particle} = 1\,!$$



← f=1.1, mismatched

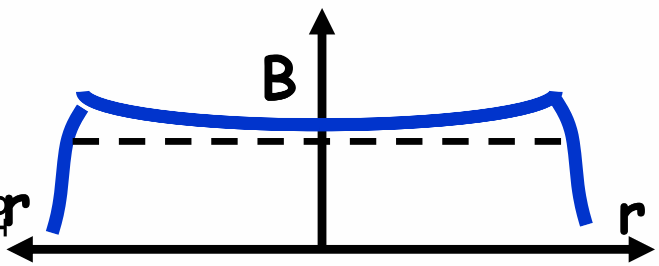


Relativistic Correction

Particle Cyclotron Frequency

$$\varpi_0 = \frac{q}{m} B \quad for\ small\ velocities\ v$$

$$Relativistically: W = mc^2 = m_0 c^2 + \varepsilon_{kin}$$



$$\varpi_0 \to \varpi(W) = \frac{qc^2}{W} B \quad for\ relat.\ velocities\ v$$

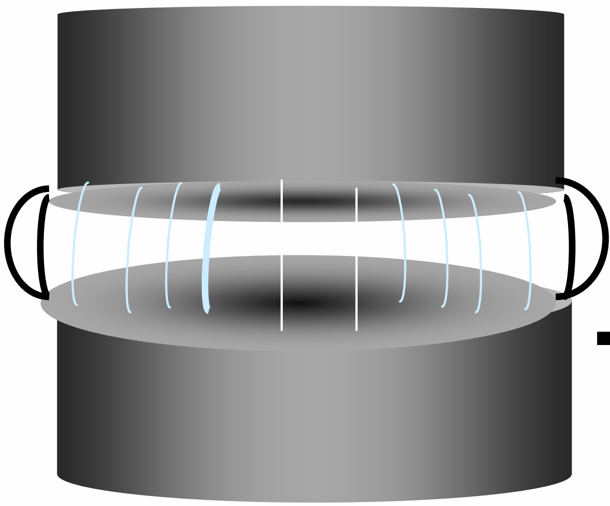
$$\frac{d}{dW}\varpi(W) = -\frac{\varpi(W)}{W} \quad decreases\ with\ incr.\ \varepsilon_{kin} \to \frac{\Delta W}{W} = -\frac{\Delta \varpi}{\varpi}$$

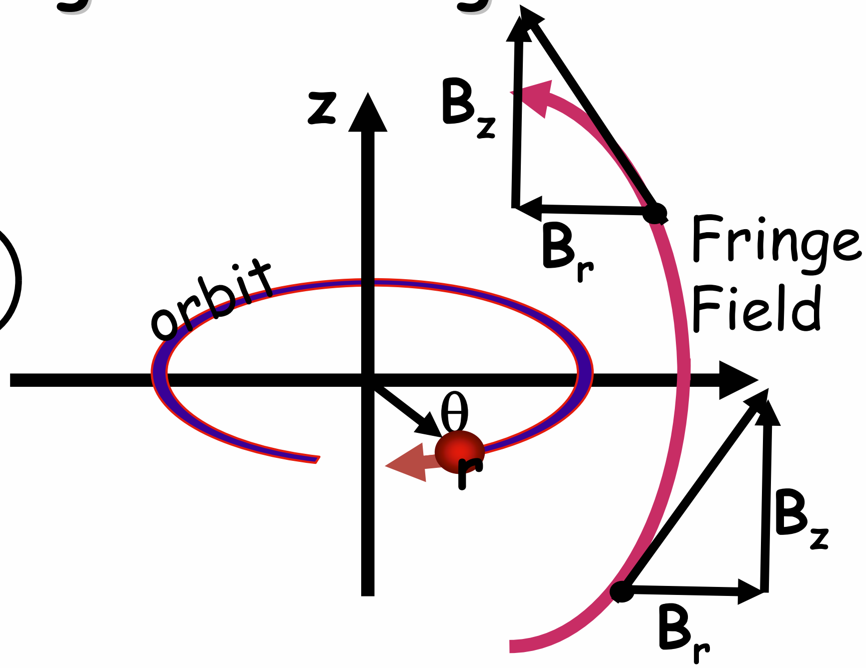
Example: projectile $m_0c^2 = A \cdot 938$ MeV, energy change $\Delta\varepsilon_{kin}$ **= 20·AMeV**

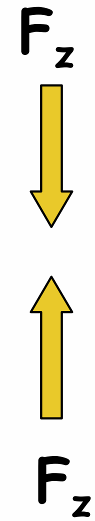
$\Delta\omega/\omega = -\Delta W/W = 20\ \text{MeV}/938\ \text{MeV} = 0.02$ ↷ **$\Delta\phi$ = -7.7^0 per turn**

out of phase within 12 turns! Correction: increase B(r) radially.
Undesirable consequence: defocusing of beam in inhomogeneous B!

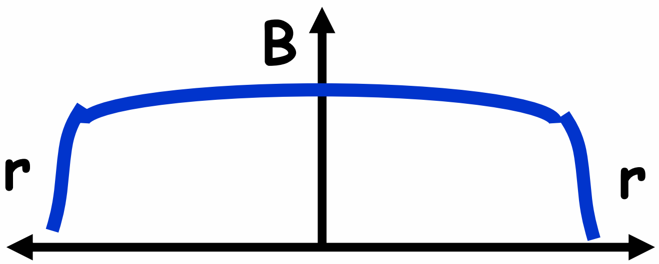
Focusing in Inhomogeneous Fields







Fringe field restoring forces drive particle to median plane



$B_z(r)=B_z(r_0)[r/r_0]^{-n}$

$0<n<1$ field index

$$F_z = \dot{p}_z = q\cdot\left(\vec{v}\times\vec{B}\right)_z = qvB_r$$

$$F_r = \dot{p}_r - mr\dot{\theta}^2 = q\cdot\left(\vec{v}\times\vec{B}\right)_r = -qvB_z$$

Particle prescribes "Betatron Oscillations"

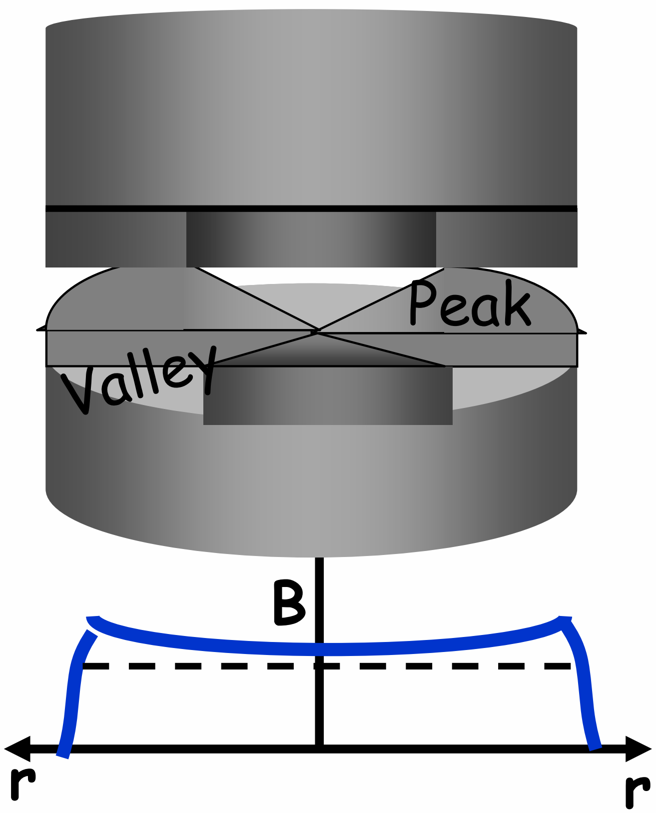
$$m\ddot{\zeta} + n\varpi_0(qB_0)\zeta = 0 \qquad \zeta = z/r_0 \qquad \varpi_r = \sqrt{1-n}\,\varpi_0$$

$$m\ddot{\rho} + m\varpi_0^2[1-n]\rho = 0 \qquad \rho = r/r_0 \qquad \varpi_z = n^{1/2}\varpi_0 \qquad \varpi_0 = v/r_0$$

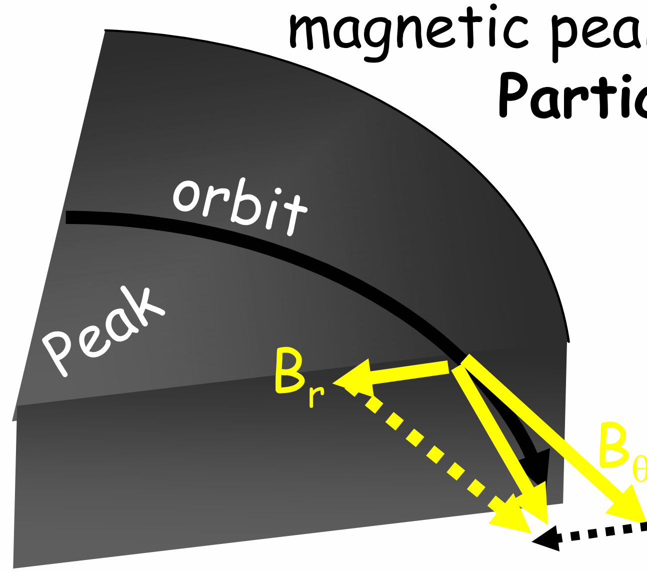
Does not correct for relativistic increase in m!

The Sector-Focused Cyclotron

How to make B both radially concave (to correct for relativistic m) and convex (to have the desired fringe field bulge)?



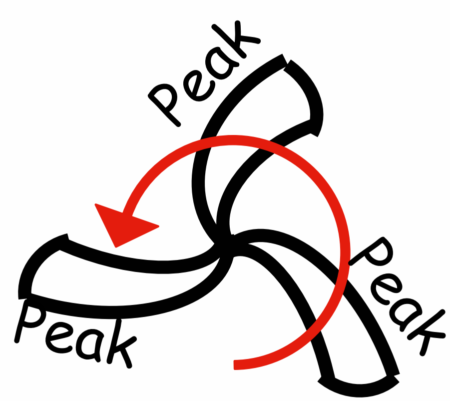
In transition regions between magnetic peaks and valleys, $v_r \neq 0$.
Particle drifts radially



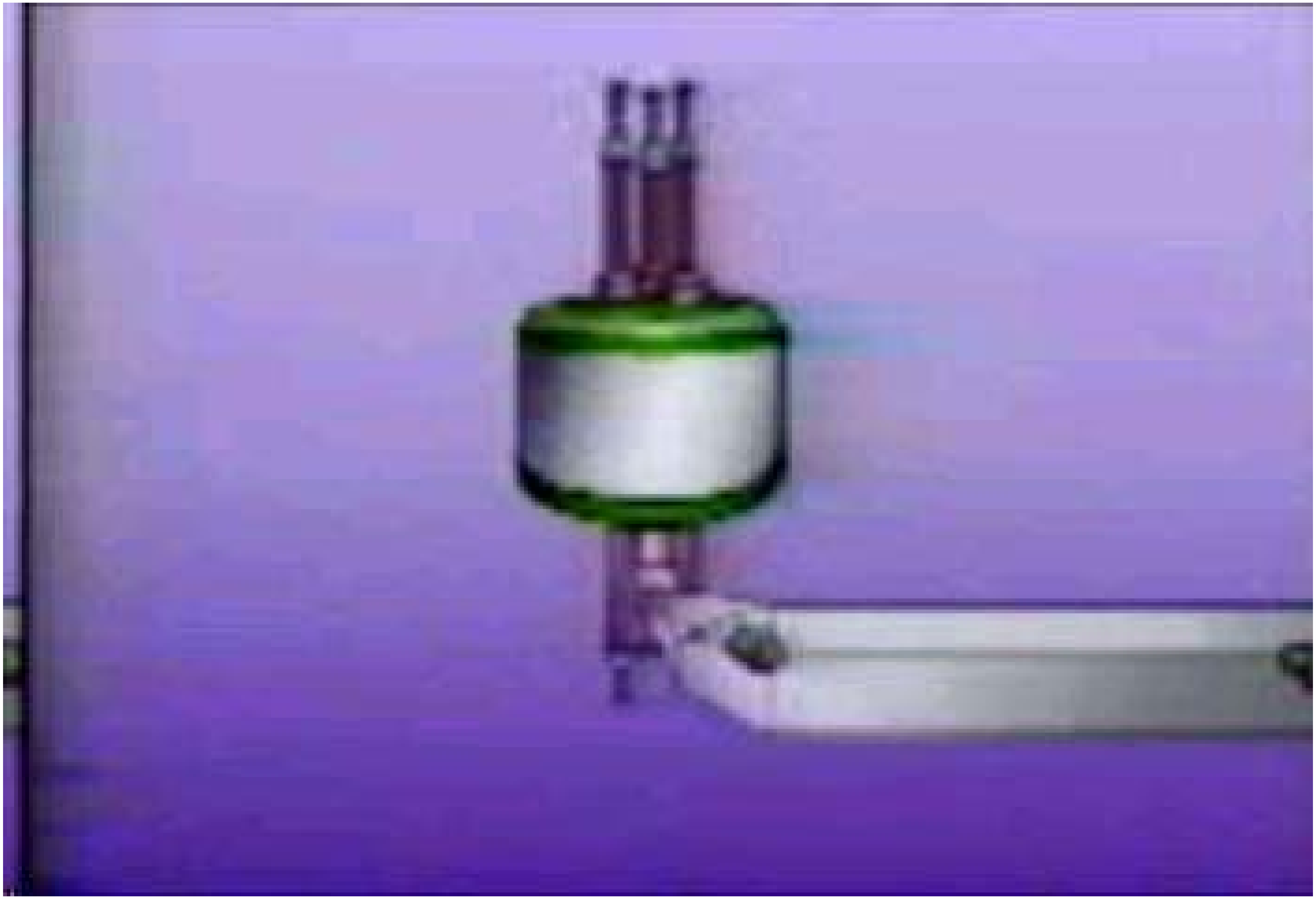
Exit:
Above median plane
$z > 0$:
$F_z = -qv_rB_\theta < 0$ **focus**

below plane, $z < 0$
$F_z = -qv_rB_\theta > 0$ focus
Entrance: defocus

Adjust directions of B_r and B_θ by shaping the pole faces: spiral arms



From Ion Source to Cyclotron



Movie from: NSCL/MSU

The End