Microscopic Simulations of γ Cascades in Warm Rotating Nuclei

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The γ decay of the excited rotating ^{168}Yb nucleus has been simulated considering the competition between $E1$ and $E2$ transitions, using microscopically calculated rotational bands. The simulations were made for two different cases: (i) pure rotating mean field bands and (ii) rotational bands mixed by a residual interaction. Simulated γ-γ spectra were analyzed in the typical rotational ridge-valley region. The fluctuation analysis method, sensitive to the effective number of γ-decay paths, was also applied. An overall agreement between experimental and simulated data is found for the case of mixed rotational bands. [S0031-9007(96)00364-X]

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The interplay between rotational motion and intrinsic excitations reveals fundamental properties of atomic nuclei. The rotation is a collective motion of the deformed shape, whereas intrinsic energy is mainly generated by the excitation of individual nucleons. Descriptions of cold rotating nuclei have been mostly based on a rotating mean field, and are able to reproduce the overall systematics of low-lying rotational bands. With increasing internal energy, the bands come closer in energy, and will be mixed by a residual interaction, even of modest strength. Because the bands display a distribution of rotational frequencies at a given spin, this mixing leads to a fragmentation of the rotational decay from each state. For sufficiently high level density it becomes possible to define a rotational strength function and a rotational damping width [1]. Recently, theoretical studies of hot rotating nuclei have addressed the transition between the region of regular rotational bands and the region of rotational damping [2,3]. The studies have focused on the distributions of level distances and transition strengths of the mixed levels, and it has not been possible to go beyond a schematic connection between theory and experiment.

Experimentally, heavy ion fusion reactions are used to produce fast rotating, excited residual nuclei which decay towards the yrast line by emitting a cascade of γ rays, passing through regions of different level densities [4,5]. In the cold region it is possible to resolve γ transitions and to build up the level scheme. This becomes difficult going up in energy from the yrast line. To learn about the major part of the γ-decaying cascades from high spin states, one has rather to investigate the unresolved transitions forming the so-called quasicontinuum spectra and, in particular, the valley and ridge structures observed in coincidence $E_{\gamma_1} \times E_{\gamma_2}$ spectra. The ridges are formed by transitions from states at low excitation energy obeying rotational energy correlations, while the valley is formed by γ rays from the region of damped rotational motion at higher excitation energy.

Information on the rotational damping width has been obtained by analyzing the shape of the central valley [6,7]. In addition, a fluctuation analysis of the counts in the $E_{\gamma_1} \times E_{\gamma_2}$ spectrum has determined the number of paths available to the nucleus in the γ decay [8,9]. The fluctuation analysis of the ridges gives an effective number of paths of $\approx 30-40$ while the valley yields $\approx 10^4-10^5$. The latter, being larger than the value deduced from simple arguments based on level densities, supports the existence of the fragmentation of the rotational transition strength at each decay step.

The present Letter describes the ingredients and the results of a simulation of γ cascades emitted by fast rotating nuclei, with emphasis on the important role of the nuclear residual interaction. The simulations are made for the first time using microscopically calculated nuclear states. Since the simulations are based on a finite number of γ-decay paths, they also allow us to perform a fluctuation analysis on simulated spectra. The aim is to sharpen the comparison between experiment and theory by comparing experimental two-dimensional $E_{\gamma_1} \times E_{\gamma_2}$ spectra to two types of simulated spectra. One is based on regular bands of a rotating mean field, while the second simulation is based on bands mixed by two-body residual interactions, giving rise to a progressive fragmentation of the rotational transitions with increasing excitation energy.

Single-particle levels of the cranked Nilsson Hamiltonian with parameters $\epsilon_2 = 0.255$ and $\epsilon_4 = 0.014$ are first obtained. The energy levels are then followed as a function of rotational frequency, eliminating sharp mean field level crossings, by selecting a diabatic basis. At each

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frequency, the lowest 1000 excited n-particle–n-hole (np-nh) configurations $|\mu\rangle$ are generated on the basis of these single-particle states for each of the four parity-signature combinations. The energy is transformed to the laboratory frame. A Strutinsky correction is included to obtain an average moment of inertia similar to the experimental value. In this way, at each spin I, we have obtained a set of unperturbed rotational bands, $|\mu(I)\rangle$. Mixed band eigenstates $|\alpha(I)\rangle = \sum_\mu c_{\alpha\mu}(I)\,|\mu(I)\rangle$ are then obtained by diagonalizing a surface-δ residual interaction in the $|\mu\rangle$ basis. We have adopted the standard interaction strength $V_0 = 27.5$ MeV$/A$ [2]. In accordance with the cranking ansatz, the rotational $E2$ transition operator is taken to be diagonal in the $|\mu\rangle$ basis, $\langle\alpha(I)\,|M(E2)|\alpha'(I-2)\rangle = Q_0 \sum_\mu c_{\alpha\mu}(I) c_{\alpha'\mu}(I-2)$, where a constant value for the quadrupole moment $Q_0 = 7.7\ e\,\mathrm{b}$ is assumed for all the bands. The transition probability for $E2$ decay from a state $|\alpha\rangle$ to a state $|\alpha'\rangle$ is given by

$$T_\alpha(E2) = \frac{12\pi}{225}\,\frac{e^2}{(\hbar c)^5}\,E_\gamma^5 \sum_{\alpha'} |\langle\alpha(I)|M(E2)|\alpha'(I-2)\rangle|^2, \tag{1}$$

where $E_\gamma = E_\alpha(I) - E_{\alpha'}(I-2)$.

The actual calculations have been performed for the nucleus ^{168}Yb. It was found that the first 400 mixed levels of each spin, parity, and signature are almost free of truncation problems with respect to the level density. These states cover a region extending up to an excitation energy above yrast $U = 2.5$ MeV. Although this basis is not large enough to take full account of pairing correlations at low spins, the pairing effects are quenched at high spins and the present simulations are made to cover only $20\hbar < I < 60\hbar$ (corresponding to rotational frequency $300 < \hbar\omega < 900$ keV). We may emphasize that the aim of this work is the understanding of the quasicontinuum, and therefore a very accurate description of the measured low-lying levels is not required.

At higher excitation energies than covered by the selected basis ($U > 2.5$ MeV) it is assumed that the rotational decay proceeds through a continuum of states, governed by a rotational damping strength of Gaussian shape, and a width Γ_{rot} extrapolated from the region of the discrete, microscopically calculated levels. Typical values are at $I = 25\hbar$, $\Gamma_{\mathrm{rot}} = 110$ (150) keV for $U = 1$ (2.2) MeV, and at $I = 60\hbar$, $\Gamma_{\mathrm{rot}} = 230$ (290) keV for $U = 1$ (2.2) MeV. For the calculation with noninteracting bands, it is appropriate to adopt a much smaller damping width, and a value $\Gamma_{\mathrm{rot}} = 10$ keV is used. Also, the level density is extrapolated from the discrete region. This procedure may be adopted because the calculated level density follows rather closely the Fermi gas model expression already for $U > 1$ MeV. A typical value for the level density parameter is $a = 14.1\ \mathrm{MeV}^{-1}$ at spin $I = 50\hbar$.

At each decay step in the (U, I) plane, the rotational $E2$ decay (involving a change in spin of $\Delta I = 2\hbar$ and conserving parity) competes with statistical dipole transitions ($\Delta I = 0, \pm 1\hbar$ and change of parity). The $E1$-transition probability from a state of initial excitation energy U_i is given by

$$T(E1, U_i) = H_{E1} \int_0^{U_i} (U_i - U)^3 \times f_{\mathrm{GDR}}(U_i - U)\,\frac{\rho(U)}{\rho(U_i)}\,dU\,. \tag{2}$$

Here f_{GDR} is given by the tail of the giant dipole resonance (GDR), of Lorentzian shape with centroid $E_0 = 15$ MeV and width $\Gamma_{E1} = 5$ MeV. We have found that one has to reduce substantially the value of f_{GDR}, as compared to the value deduced assuming that the GDR exhausts the $E1$ sum rule, in order to reproduce the average experimental intensity of low-lying bands. This is in agreement with previous studies [10]. The associated hindrance factor is denoted H_{E1} in Eq. (2). In the region of discrete levels, the integral in expression (2) is replaced by a summation.

Making use of Monte Carlo techniques, the γ decay can be followed along a path in the (U, I) plane. Each cascade starts from initial values of energy and spin, randomly chosen from a two-dimensional entry distribution of Gaussian form, with centroids and FWHM taken to be $I_{\mathrm{av}} = 55\hbar$, $\Gamma_I = 10\hbar$, $U_{\mathrm{av}} = 5.5$ MeV, $\Gamma_U = 2$ MeV. The simulated spectra in the ridge-valley region for $E_\gamma \leq 1100$ keV were found to be insensitive to the parameters of the entry distribution. In fact, the properties of the γ decay, flow, and, in particular, the average intrinsic excitation energy $\langle U(I)\rangle$ approach rather rapidly a general asymptotic behavior, determined by the level density and by the deformation and the moment of inertia [11]. The average gamma multiplicity of the simulated cascades is 18 and the statistical transitions represent about 10% of the total, in good agreement with experiments. The spectra have been obtained by updating all possible pairs of gamma transitions of 10^6 cascades.

The most direct comparison between simulated and experimental spectra is made by examining the $E_{\gamma_1} \times E_{\gamma_2}$ spectrum itself. Figure 1 shows a perpendicular cut at $\frac{1}{2}(E_{\gamma_1} + E_{\gamma_2}) = 920$ keV projected onto an axis $E_{\gamma_1} - E_{\gamma_2}$. Such a cut shows the ridges flanking the valley in the $E_{\gamma_1} \times E_{\gamma_2}$ spectrum. The strongest transitions were subtracted from the right-hand side of the spectra. The transitions populating the first order ridges must follow the rotational relation $E_{\gamma_1} - E_{\gamma_2} = \pm 4/\mathcal{F}$, where $\mathcal{F}$ is the band moment of inertia. The simulation based on mixed rotational bands is fairly successful in reproducing the experimental shape of the ridge, and especially its width, related to the dispersion in the moment of inertia.

The intensity of the lowest states of the four (σ, π) combinations and that of the ridges are displayed in Figs. 2(a) and 2(b), respectively. These quantities are affected by the competition between statistical and rotational transitions, which in turn also determine the asymptotic behavior of

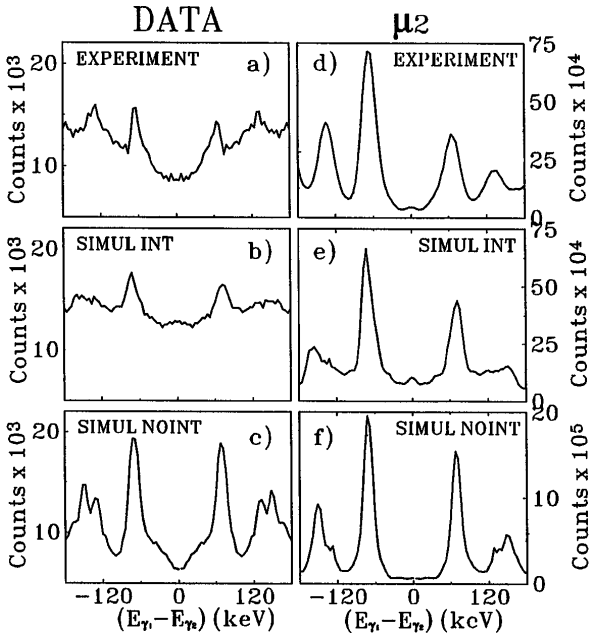

FIG. 1. Comparison between experimental data of ^{168}Yb (top row) and the simulated spectra for the same nucleus based on cranking plus band-mixing calculation (middle row) and with noninteracting bands (bottom row). The associated μ_2 spectra are also shown. In both the experimental and simulated spectra the separation of the first ridges is $\approx$120 keV $= 8/\mathcal{F}_{\rm eff}$. The most intense discrete transitions were subtracted from the right-hand part of the spectra before applying the fluctuation analysis.

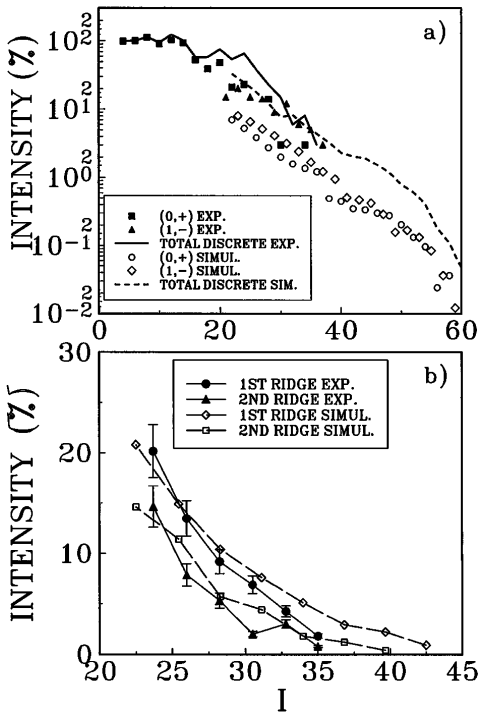

FIG. 2. Comparison of the measured and calculated intensities of discrete γ transitions of ^{168}Yb as a function of spin relative to the strongest yrast transition (top part). In the bottom part, the fraction of the total intensity at a given spin populating the first and second ridges is shown for the measured and simulated spectra.

$\langle U(I)\rangle$. In the present simulation this competition is controlled by the hindrance factor H_{E1}, adjusted to have an overall agreement between the measured and simulated intensities. For the mixed band calculation, $H_{E1} = 0.06$. In the case of noninteracting bands a slightly different value $H_{E1} = 0.09$ was used in order to have the same asymptotic behavior of $\langle U(I)\rangle$ for the two simulations and, consequently, the same intensities for the low-lying bands. It is remarkable how well the ridge-valley structure reflects the different nature of the used rotational levels and transition strengths. In fact, as shown in Fig. 1, the simulation based on pure mean field bands displays ridges which are very different from the data and much stronger than those obtained with interacting bands.

Both simulated (with mixed bands) and experimental intensities decrease exponentially with increasing angular momentum, as expected from the overall behavior of the decay flow [11]. These quantities depend strongly on the level density, and the fact that the simulated intensities decay more weakly than the data may be caused by the low level density produced by the cranking calculation. In the rather wide region of excitation energy where the onset of damping takes place, rotational sequences only survive a few steps. The average length of rotational bands is revealed by the ratio between the intensity of the first and second ridges. There is also good agreement between simulations and data on this quantity, as shown in Fig. 2(b).

In order to estimate the number of bands contributing to the ridges, we analyze count fluctuations in the $(E_{\gamma_1} \times E_{\gamma_2})$ spectra [8,9]. In the course of the decay, the nucleus can select among a finite pair of γ transitions $(E_{\gamma_1}, E_{\gamma_2})$ leading to a given sector of the plane $(E_{\gamma_1} \times E_{\gamma_2})$. This finiteness leads to fluctuations superposed on ordinary statistical fluctuations in the number of counts. If all the decay paths consisting of pairs of γ transitions were equiprobable, their number would be given by $N^{(2)}_{\rm path} = N_{\rm eve}(\mu_2/\mu_1 - 1)^{-1}$, where μ_1 and μ_2 are the first and second moment of the number of counts and $N_{\rm eve}$ is the number of events in the selected sector (chosen to be a square of $4/\mathcal{F}_{\rm eff} \times 4/\mathcal{F}_{\rm eff}$ keV^2) along the valley or along the ridges. The values of μ_1 and μ_2 were obtained from the experimental and simulated spectra using the program STATFIT [9]. In practice, the decay paths have different probabilities $W_{\rm path}$, and the effective number of paths is defined by $N^{(2)}_{\rm path} = (\sum_{\rm path} W^2_{\rm path})^{-1}$, emphasizing the discrete bands which have escaped rotational damping; they lie mostly at $U < 1.5$ MeV and are populated rather uniformly, with the exception of the yrast transitions, removed before performing the ridge analysis. In Fig. 3(a) we report the

number of two consecutive transitions with $n_{\text{branch}} \leq 2$. This quantity gives a measure of the fragmentation of the $E2$ strength defined as $n_{\text{branch}}(i) = (\sum_j S_{i,j}^2)^{-1}$, where $S_{i,j}$ is the normalized strength from level i at spin I to level j at spin $I-2$. One can see that this number is very close to the value of $N_{\text{path}}^{(2)}$ extracted from the simulated ridge. This result demonstrates the validity of the heuristic interpretation, previously given, that the application of the fluctuation analysis to the first ridge yields the number of regular rotational bands.

The number of paths deduced from the simulated ridges using mixed band calculations also agrees quite well with the experimental findings. For nonmixed rotational bands, $N_{\text{path}}^{(2)}$ is much larger and disagrees strongly with the experiment. In addition, preliminary investigations of the $N_{\text{path}}^{(2)}$ dependence on the interaction strength V_0 show that, if V_0 goes from 27.5 to 15 MeV/A, $N_{\text{path}}^{(2)}$ increases by $\approx 50\%$.

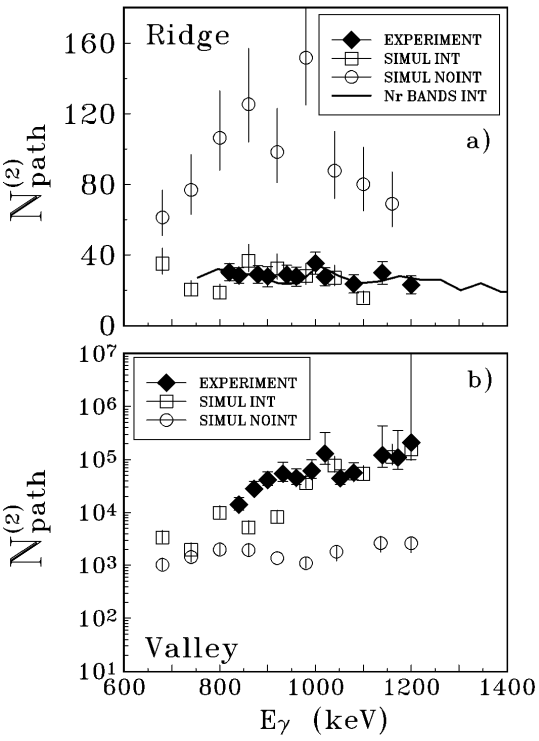

FIG. 3. Top part, the effective number of paths $N_{\text{path}}^{(2)}$ extracted from the first ridge analysis of the measured and calculated spectra is shown. Filled diamonds correspond to the data, empty squares to simulations based on mixed bands, and empty circles to simulations based on noninteracting bands. The full drawn line is the number of discrete bands calculated by the condition $n_{\text{branch}} \leq 2$. Bottom part, the quantity $N_{\text{path}}^{(2)}$ extracted from the valley analysis of the measured and calculated spectra is plotted with the same symbols used in the top part.

The valley region collects mostly damped transitions, and the value of $N_{\text{path}}^{(2)}$ is much larger and depends on the damping properties as well as on the average properties of the flow, which determines which regions are mostly populated. Again, we find good agreement with the experiment only for mixed bands, confirming the presence of the rotational damping.

In conclusion, the present work shows that the residual interaction mixing rotational bands are indeed needed to obtain a satisfactory and consistent description of the quasicontinuum in ^{168}Yb, both with respect to the ridge and valley structures. This was observed from both the intensity and fluctuation spectra associated with the most sensitive perpendicular cuts across the $E_{\gamma_1} \times E_{\gamma_2}$ spectra. So far, agreement is found within the sensitivity that can be obtained by studying two-dimensional spectra. The new generation of γ-detector arrays, Gammasphere and Euroball III, allowing for higher fold coincidences will soon make it possible to test these concepts in greater detail. All together, the new simulation model gives a more solid base for the study of the quasicontinuum of rotational nuclei, a good testing ground for non mean field effects.

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[1] B. Lauritzen, T. Døssing, and R. A. Broglia, Nucl. Phys. **A457**, 61 (1986).
[2] M. Matsuo *et al.*, Phys. Rev. Lett. **70**, 2694 (1993).
[3] S. Åberg, Phys. Rev. Lett. **64**, 3119 (1990); Nucl. Phys. **A477**, 18 (1988).
[4] J. Bacelar *et al.*, Phys. Rev. Lett. **35**, 1858 (1985).
[5] F. Stephens *et al.*, Phys. Rev. Lett. **35**, 1858 (1985).
[6] S. Leoni *et al.*, Nucl. Phys. **A587**, 513 (1995).
[7] B. Herskind *et al.*, Phys. Lett. B **276**, 4 (1992).
[8] B. Herskind *et al.*, Phys. Rev. Lett. **68**, 3008 (1992).
[9] T. Døssing *et al.* (to be published).
[10] G. Leander, Phys. Rev. C **38**, 728 (1988).
[11] T. Dossing and E. Vigezzi, Nucl. Phys. **A587**, 13 (1995).