Physics Letters B 276 (1992) 4–10
North-Holland

PHYSICS LETTERS B

Three-dimensional γ-ray correlations and the study of rotational damping in ^{167,168}Yb

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Received 10 October 1991; revised manuscript received 22 December 1991

A new method of analyzing $E_{\gamma1} * E_{\gamma2} * E_{\gamma3}$ correlation data by *rotational plane mapping* is discussed and applied to high spin triple data on ^{167,168}Yb formed in ^{48}Ca + ^{124}Sn reactions. Values for the rotational damping width $\Gamma_{\rm rot} = 124 \pm 20$ keV and 96 ± 30 keV are found for transition energy intervals 820–980 keV and 980–1140 keV corresponding to $I \approx 35\hbar$ and $\approx 45\hbar$, respectively.

The quantum structure and rotational motion of deformed nuclei is well known from the study of discrete rotational bands. For the best studied nuclei, typically 5–15 levels for each spin are known, extending from the yrast line a few hundred keV up in excitation energy.

The γ-decay cascades from high spin states formed in compound reactions mainly flow through the warm region of very high level density ≈ 3–5 MeV above yrast. However, very little specific information has been obtained about the rotational motion in warm nuclei.

It was found a few years ago by the study of $E_{\gamma1} * E_{\gamma2}$ correlations [1,2] that the large collective quadrupole transition strength at higher excitation energies may be distributed over all final states of a given parity within an interval defined as the rotational damping width $\Gamma_{\rm rot}$ [3]. Values for the damping width were extracted indirectly for ^{167,168}Yb from a comparison of spectrum shapes of the $E_{\gamma1} * E_{\gamma2}$ data with similar spectra from simulation calculations [1].

The present letter describes a triple $E_{\gamma1} * E_{\gamma2} * E_{\gamma3}$ correlation measurement on ^{167,168}Yb, and a new method in which analytic approximations are used to more directly extract the damping width in ^{167,168}Yb. The measured values are compared to a microscopic calculation performed for ^{168}Yb [4].

We shall begin by illustrating how $E_{\gamma1} * E_{\gamma2}$ energy correlations can give insight into the rotational decay. Fig. 1 displays in a schematic way how undamped and damped rotational decay generates the ridges and the central valley in $E_{\gamma1} * E_{\gamma2}$ spectra.

For undamped transitions, along a discrete rotational band, the difference in transitional energy between two consecutive γ-ray energies is given by the inverse *second band moment of inertia*

$$\frac{4}{\mathscr{I}^{(2)}_{\rm band}} = 4\,\frac{{\rm d}\omega}{{\rm d}I} = E_\gamma((I+2)\to I) - E_\gamma(I\to(I-2))\ ,$$

as defined in ref. [5]. Combining all transition energies for the decay along a perfect rotational band, a grid pattern is generated in the $E_{\gamma1} * E_{\gamma2}$ spectrum, as illustrated by the grid of large dots in fig. 1b corresponding to the yrast cascade in fig. 1a. In this grid pattern, the points along the diagonal $E_{\gamma1} = E_{\gamma2}$ are missing, since each band transition cannot be in coincidence with itself. For many decays along several of such bands with slightly different $\mathscr{I}^{(2)}_{\rm band}$, grid pat-

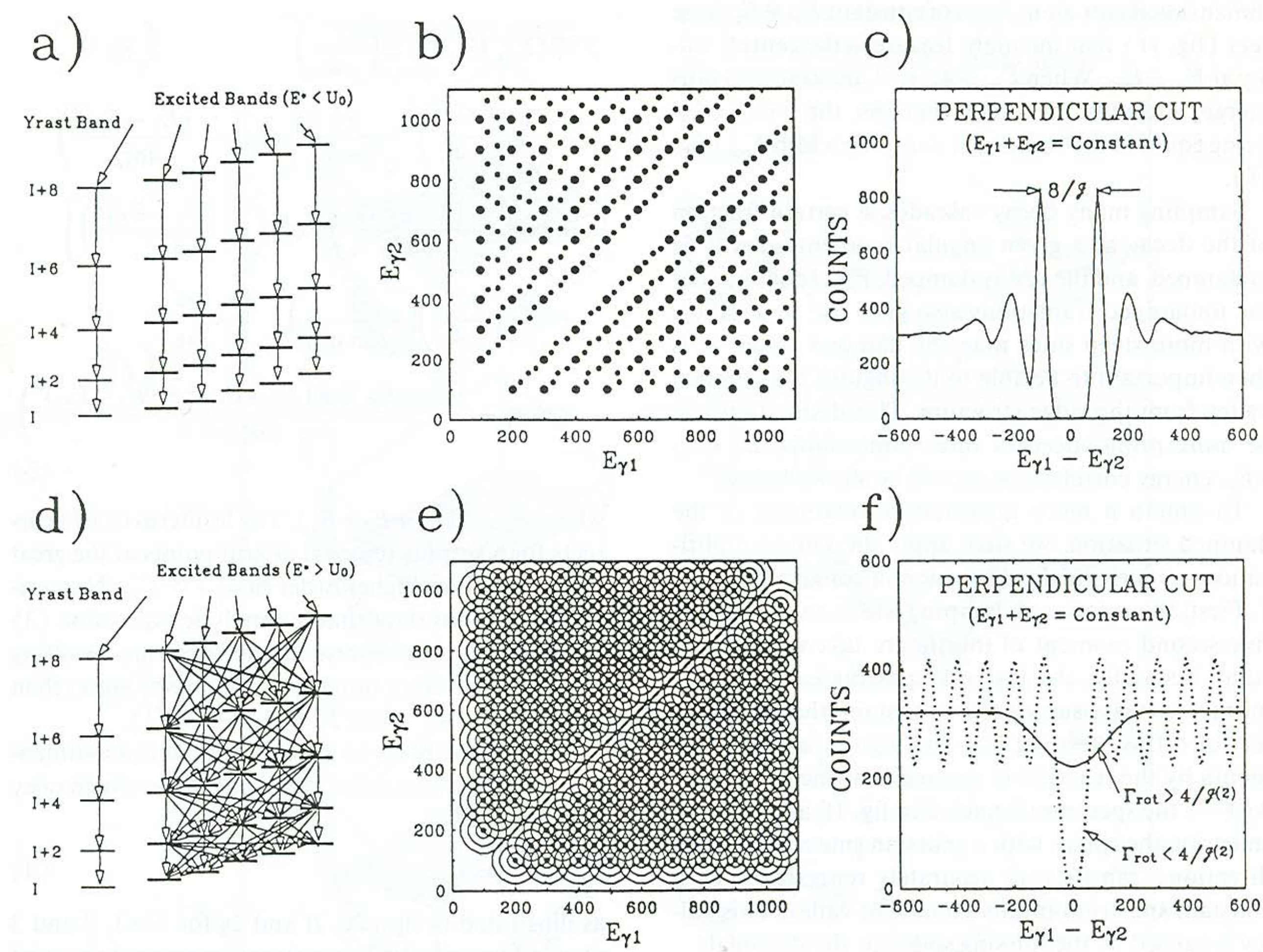

Fig. 1. The schematic figure shows the expected patterns for $E_{\gamma1} * E_{\gamma2}$ spectra in (b) and (e) when the decay paths are regular rotational cascades (a) or when the pathways goes through a region where rotational damping is dominating (d). The spectra (c) and (f) correspond to diagonal cuts for constant $E_{\gamma1}+E_{\gamma2}$ at (b) and (e) respectively.

terns corresponding to an ensemble of transition energies are superimposed, forming ridges in the $E_{\gamma1} * E_{\gamma2}$ energy correlation plot (fig. 1b). The width of the ridges is determined by the dispersion in the band moment of inertia, as is best seen from the one-dimensional perpendicular cut in the two-dimensional spectrum with constant $E_{\gamma1}+E_{\gamma2}$, as shown in fig. 1c. The width of the ridges increases for larger energy differences $E_{\gamma1}-E_{\gamma2}$. Thus, the distance between the ridges is determined by *the average second band moment of inertia*, and the width of the ridges is determined by *dispersion in that moment of inertia.*

For damped transitions, the γ transitions from each state branch out to many states according to a distribution of $B(\mathrm{E2})$ values (see fig. 1d). The mean energy of the distribution is twice the average rotational frequency, and the width is the rotational damping width Γ_{rot}. In an $E_{\gamma1} * E_{\gamma2}$ energy correlation plot, the average decay energies at angular momenta $(I+4)$, $(I+2)$, I, etc. form a grid of the type discussed above for the discrete bands, but now with the distance between grid points determined by the *effective second moment of inertia* [6]. The independent selection of damped transition energies at $(I+2) \to I$ and at $I \to (I-2)$, for example, causes the spike at the grid point to be smeared into a round hill having a width Γ_{rot} in both directions. Still, there are no spikes along the diagonal to be smeared out. From making one-dimensional cuts along lines of constant $E_{\gamma1}+E_{\gamma2}$, one sees (fig. 1f) that the main feature is the central valley at $E_{\gamma1}=E_{\gamma2}$. When $\Gamma_{\rm rot}>4/\mathscr{I}^{(2)}$, the smeared hills merge, and only the valley remains, the width of it being equal to the rotational damping width $\Gamma_{\rm rot}$ (fig. 1f).

Sampling many decay cascades, a certain fraction of the decay at a given angular momentum will be undamped, and the rest is damped. Fig. 1c shows that the undamped transitions also give rise to a valley, with more steep sides than the damped valley. It is then important to be able to distinguish the damped valley from the ridge structure. This distinction will be more pronounced in three-dimensional $E_{\gamma1}*E_{\gamma2}*E_{\gamma3}$ energy correlations, as will be shown below.

To obtain a more quantitative treatment of the damped situation, we shall apply the same simplifications underlying the discussion in connecting to fig. 1. First, the rotational damping width and the effective second moment of inertia are taken to be constant. Secondly, the intensity passing each angular momentum is assumed to be constant, that is, the effect of diffuse feeding into the highest angular momenta by the reaction is ignored. For the case $\Gamma_{\rm rot}>4/\mathscr{I}^{(2)}$, the spectrum depicted in fig. 1f, arising from smearing the spikes with a gaussian smearing in both directions, can be very accurately represented by a constant spectrum minus a gaussian valley. This valley is caused by the missing spikes at the diagonal:

$$\mathscr{S}^{(2)}(E_1,E_2)=\left(\frac{\mathscr{I}^{(2)}}{4}\right)^2 - \frac{1}{\sqrt{2}}\frac{\mathscr{I}^{(2)}}{4}\frac{1}{\sqrt{2\pi}\,\sigma_{\rm rot}}\exp\left(-\frac{(E_1-E_2)^2}{4\sigma^2_{\rm rot}}\right), \qquad (1)$$

where $\sigma_{\rm rot}=\Gamma_{\rm rot}/\sqrt{8\ln 2}$. Choosing $E_{\gamma1}-E_{\gamma2}$ as the coordinate perpendicular to the valley, the FWHM of the valley is

$$\mathrm{FWHM}=4\sqrt{\log 2}\,\sigma_{\rm rot}=\sqrt{2}\,\Gamma_{\rm rot}\,. \qquad (2)$$

In the three-dimensional spectrum, the average transition energies will form grids also, now with the missing points along the diagonals in the three planes obeying the equations $E_{\gamma1}=E_{\gamma2}$, $E_{\gamma1}=E_{\gamma3}$ and $E_{\gamma2}=E_{\gamma3}$. The three planes intersect at the "great valley" [7,8] at $E_{\gamma1}=E_{\gamma2}=E_{\gamma3}$. Smearing these three-dimensional grid points out, the missing points at the three planes each give rise to a valley:

$$\begin{aligned}\mathscr{S}^{(3)}(E_1,E_2,E_3)=&\left(\frac{\mathscr{I}^{(2)}}{4}\right)^3 - \frac{1}{\sqrt{2}}\left(\frac{\mathscr{I}^{(2)}}{4}\right)^2\frac{1}{\sqrt{2\pi}\,\sigma_{\rm rot}}\left[\exp\left(-\frac{(E_1-E_2)^2}{4\sigma^2_{\rm rot}}\right)\right.\\ &\left.+\exp\left(-\frac{(E_1-E_3)^2}{4\sigma^2_{\rm rot}}\right)+\exp\left(-\frac{(E_2-E_3)^2}{4\sigma^2_{\rm rot}}\right)\right]\\ &+\frac{2}{\sqrt{3}}\left(\frac{\mathscr{I}^{(2)}}{4}\right)\left(\frac{1}{\sqrt{2\pi}\,\sigma_{\rm rot}}\right)^2\\ &\times\exp\left(-\frac{(E_1-E_{\rm m})^2+(E_2-E_{\rm m})^2+(E_3-E_{\rm m})^2}{2\sigma^2_{\rm rot}}\right),\end{aligned} \qquad (3)$$

where $E_{\rm m}=\frac{1}{3}(E_1+E_2+E_3)$. The last term of (3) corrects for a surplus removal of grid points at the great valley, and is of higher order in $4/\mathscr{I}^{(2)}\Gamma_{\rm rot}$. Numerical calculations show that the analytic expression (3) for all E_1, E_2, E_3 represents the spectrum resulting from gaussian smoothing of grid points to better than 4% of $(\mathscr{I}^{(2)}/4)^3$ when $\Gamma_{\rm rot}>1.5(4/\mathscr{I}^{(2)})$.

It is an advantage to investigate the three-dimensional spectrum on *tilted rotational planes* which obey the equation

$$E_{\gamma1}-E_{\gamma3}=N(E_{\gamma3}-E_{\gamma2}) \qquad (4)$$

as illustrated in figs. 2e, 2f and 2g for $N=1$, 2 and 3 planes, respectively. These planes intersect at the great valley.

For $N=1$, 2, 3, different ridges corresponding to selected sequences of γ-rays in undamped rotational cascades (figs. 2a–2c) are enhanced as illustrated in figs. 2e, 2f and 2g. It is seen that the ridges are moving away from the diagonal for increasing N. By combining the information from the different planes, it becomes easier to distinguish the undamped part of the spectrum, contributing to the ridges, from the damped part of the spectrum.

The width of the three superimposed damped valleys on the tilted planes can be estimated by inserting eqs. (4) into the spectrum (3). Choosing again $E_{\gamma1}-E_{\gamma2}$ as coordinate, one obtains

$$\mathrm{FWHM}_{12}=\sqrt{2}\,\Gamma_{\rm rot}\,,$$

$$\mathrm{FWHM}_{13}=\sqrt{2}\,\frac{N+1}{N}\,\Gamma_{\rm rot}\,, \qquad (5)$$

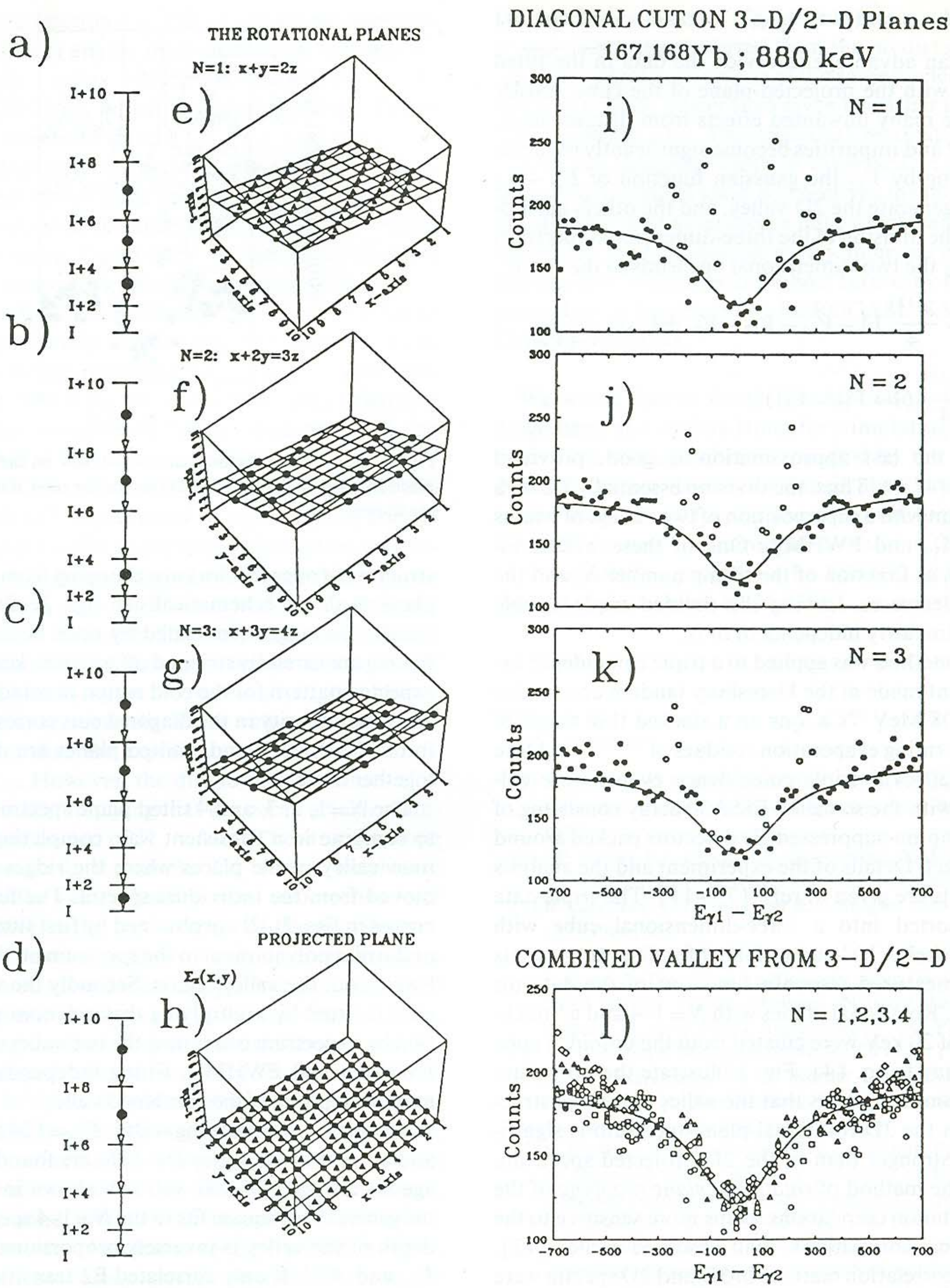

Fig. 2. Spectra of diagonal cuts on rotational planes $N=1, 2, 3$ divided by the plane of total projection for $(E_{\gamma1}+E_{\gamma2})/2=820$–$980$ keV are shown in sections (i), (j), and (k) with the corresponding planes shown schematically in (e), (f), and (g) as they would appear for the selected transitions indicated by filled circles in sections (a), (b) and (c), respectively. The (l) inset shows the result after the data from $N=1, 2, 3, 4$ rotational planes have been combined as described in the text. The data points taken to be ridge structures in the analysis are indicated by open circles. The curve shown to guide the eye is obtained by a higher order polynomial fit.

$$\mathrm{FWHM}_{23} = \sqrt{2}(N+1)\Gamma_{\mathrm{rot}} . \qquad (5\text{ cont'd})$$

It is an advantage to divide the data in the tilted planes with the projected plane of the cube, mostly because many unwanted effects from discrete lines, feeding and impurities become significantly reduced. Denoting by V_{12} the gaussian function of $E_{\gamma 1}-E_{\gamma 2}$ which generate the 2D valley, and the others accordingly, the division of the three-dimensional spectrum $\mathscr{S}^{(3)}$ by the two-dimensional one leads to the result

$$\frac{\mathscr{S}^{(3)}}{\mathscr{S}^{(2)}} \sim \frac{\mathscr{I}^{(2)}}{4}(1-V_{12}-V_{13}-V_{23}+V_{123})\frac{1}{1-V_{12}}$$

$$\approx \frac{\mathscr{I}^{(2)}}{4}(1-V_{13}-V_{23}) ,$$

where the last approximation is good, provided $\Gamma_{\mathrm{rot}} > 2(4/\mathscr{I})$. Thus, the division essentially leaves a spectrum with a superposition of two valleys of widths FWHM_{13} and FWHM_{23}. One of these widths increases, as function of the tilting number N, and the other decreases, leaving the divided triple/double spectrum fairly independent of N.

The method was applied to a triple coincidence experiment made at the Daresbury tandem accelerator with 208 MeV ^{48}Ca ions on a stacked thin target of ^{124}Sn forming evaporation residues of ^{167,168}Yb. More than 130×10^6 triple coincidence events were collected with the so-called ESSA30 array consisting of 30 Compton-suppressed Ge detectors packed around the target. Details of the experiment and the analysis methods are given in refs. [7,9–11]. The triple data were sorted into a three-dimensional cube with $4*4*4(\mathrm{keV/ch})^3$ and unfolded for Compton events using measured response functions of the detector system. Rotational planes with $N=1$–4 and a "thickness" of 20 keV were created from the unfolded cube according to eq. (4). Fig. 3 illustrate the procedure of division. One sees that the valley and ridge structures in the 3D rotational-plane-spectrum is significantly stronger than in the 2D projected spectrum. Thus the method of *rotational plane mapping* of the three-photon correlations seems more sensitive to the rotational correlations, than observed earlier [12], where correlation features in 3D and 2D spectra were found to be very small. Diagonal cuts on the $N=1$, 2 and 3 planes for an energy interval of $\frac{1}{2}(E_{\gamma 1}+E_{\gamma 2}) = 820$–980 keV are shown in figs. 2i, 2j, and 2k, respectively. It is clearly seen how the ridge

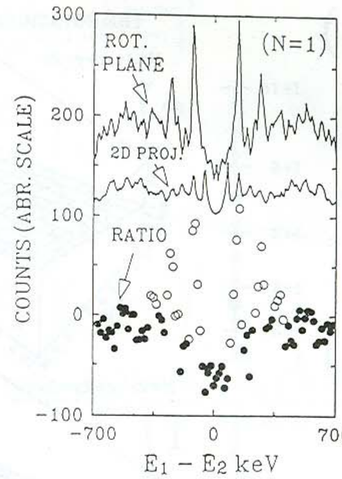

Fig. 3. Spectra from diagonal cuts at 840 keV on the rotational plane ($N=1$), on the projected 2D and on the ratio 3D/2D planes are shown.

structures (open circles) are changing from plane to plane as shown schematically in figs. 2e, 2f and 2g. Finally the ridges (indicated by open circles in the figure) are carefully stripped off by hand, keeping the expected pattern for the cold region in mind. The remaining intensity in the diagonal cuts corresponding to the $N=1$, 2, 3, and 4 tilted planes are displayed together in fig. 2l.

The $N=1$, 2, 3, and 4 tilted plane spectra are seen to combine in a consistent way, completing a common valley in the places where the ridges were removed from the individual spectra. The full drawn curves in figs. 2i–2l are obtained by first fitting a second-order polynominal to the spectrum on the wings, leaving out the valley region. Secondly the valley region is fitted by multiplying this polynominal function by a spectrum containing the two valleys of width FWHM_{13} and FWHM_{23}, fitting independently Γ_{rot} and the depth D of the combined valley.

A value for the damping width, $\Gamma_{\mathrm{rot}} = 124 \pm 20$ keV, and a depth of the valley $D = 35\%$, are found in average for the interval 800–960 keV shown in fig. 2 by the general least square fits to the $N=1$–4 spectra. The depth of the valley is inversely proportional to both Γ_{rot} and $\mathscr{I}^{(2)}$. If only correlated E2 transitions were present, a valley depth of 54% in the divided spectra is expected, assuming $\mathscr{I}^{(2)} \approx 75$ MeV^{-1} taken from ref. [13]. However, the value of the depth is diluted by the background of statistical transitions which comprise a fraction $f_s \approx 20\%$ of the single spectrum for the present energy interval. Taking this into account the valley depth is expected to be $54(1-f_s)^2 = 35\%$, which is in fortuitous agreement with the measured value.

The statistics of the present data requires an averaging over rather large energy intervals. Therefore only one other set of values for $\Gamma_{rot} = 96 \pm 30$ keV and $D = 15\%$, using $\mathscr{I} \approx 85$ MeV^{-1}, was extracted from the data in the high transition energy region of 980–1160 keV. In this case, the depth of the valley is found to be significantly smaller than the expected depth $D = 60\%$ for the measured narrow valley. This indicates that 50% of the intensity in the valley originates from other components, out of which only 20% can be ascribed to statistical transitions.

The two regions in transition energy around 880 and 1060 keV correspond to spin regions around $I = 35\hbar$ and $I = 45\hbar$ respectively. Simulation calculations predict average excitation energies above yrast for the decay flow of $U = 2.2$ MeV at $I = 35\hbar$ and $U = 2.8$ MeV at $I = 45\hbar$. The cranking model calculations of ref. [4] for these two sets of values of I, U yield Γ_{rot}(cranking) = 175 keV and Γ_{rot}(cranking) = 200 keV, respectively. These values are considerably larger than the experimentally determined values of Γ_{rot}. However, the distribution of values of U is rather wide, implying that the decay actually samples a distribution of Γ_{rot} values. The stronger slope of the spectra for the smaller Γ_{rot} tends to emphasize these smaller values, leading to an underestimate of Γ_{rot} relative to the true average of the distribution. We do not known yet how strongly the method causes such a bias towards lower Γ_{rot}, but the calculated cranking model values of Γ_{rot} for the range of U are all larger than the values determined here, except for the predicted abrupt onset of damping around $U \approx 700$ keV. For E_γ around 880 keV, we may tentatively point to either the region of onset of damping, at low U, or to a much stronger motional narrowing at moderate to high U, as causing the discrepancy. For E_γ around 1060 keV, the significant component of the spectrum, which is not accounted for, makes it difficult to make any specific interpretation, on the basis of the present analysis.

In conclusion, values for the damping width have been extracted for the continuum in ^{167,168}Yb by *rotational plane mapping* of three-dimensional γ-ray energy correlation data. The deduced values are 1.4–2 times lower than inferred from the prediction by a cranking calculation on one of the same nuclei ^{168}Yb, but significantly higher than given by earlier indirect measurements [1]. The analytical method for deducing the damping width is expressed in general terms and promises good perspectives for the analysis of higher fold coincidence data which can be obtained for small transition energy intervals and different entry conditions in the near future, by the next generation of Ge arrays like EUROGAM, GASP and GAMMASPHERE.

We would like to thank Ben Mottelson, Gudrun Hagemann and Anders Holm for stimulating discussions. The work has been supported by the Danish Natural Science Research Council, the Commemorative Association for the Japan World Exposition Foundation and the European Economic Community Stimulation program (the ESSA30 Collaboration, contract number ST2J-0205).

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