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Experimental Study of the Nuclear Rotational Motion with Statistical Analysis Methods

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The γ-decay of excited rotating nuclei in the high level density region a few MeV above the yrast line has been studied through a statistical analysis of the fluctuation of counts in γ-γ coincident spectra. In particular, making use of the covariance technique between spectra gated by different intrinsic configurations, one can measure how the cascades feeding into different selected bands are similar. The aim is to learn about the transition between order to chaos in the nuclear many-body system, in terms of the validity of selection rules associated with the quantum numbers of the intrinsic structure. Experimental results on the nucleus ^{164}Yb are presented and compared to model predictions based on cranked shell model calculations including a two-body residual interaction.

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1. Introduction

Detailed γ-ray discrete spectroscopy studies of deformed nuclei can give precious information on the interplay between rotation, superfluidity and quantum numbers of the intrinsic nuclear configurations in the cold excitation energy region just above the yrast line [1].

In recent years it has been possible to obtain information on the properties of the rotational motion at thermal excitation energy even higher than that covered by the resolved bands. This can be done through the study of the unresolved quasi-continuum, forming ridge and valley structures in coincident γ-γ energy spectra [2]. The ridges are formed by transitions from states at low excitation energy obeying rotational energy correlations, while the valley is formed by γ-rays from the region of higher level density above yrast.

A statistical analysis of the fluctuation of counts in the γ-γ spectrum allows to determine the number N_{path} of paths available to the nucleus in the γ-decay through different regions of level density [2], [3], information which can in turn be related to the dynamical properties of the rotational motion. The very existence of a finite number of decay paths

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leads, in fact, to enhanced fluctuations μ_2/μ_1, whose magnitude is fixed by the number of paths through the simple relation $\mu_2/\mu_1 = N_{eve}/N_{path} + 1$. μ_1, μ_2 and N_{eve} are the statistical moments and the number of counts in a $4/\Im \times 4/\Im$ sector, in which each rotational cascade contributes on the average one count, being $\Im$ the dynamic moment of inertia of the nucleus.

Systematic results have been obtained from a number of rare-earth nuclei analyzed [2]. Large fluctuations are observed in the ridge structures, showing that only a rather low number (≈ 40) of discrete rotational bands exists up to excitation energy ≈ 800 keV above yrast. In contrast, very weak fluctuations are found along the central valley, revealing a number of coincidence combinations ($\approx 10^4 - 10^5$), larger than expected assuming that the rotational decay leads to a unique final state. These experimental findings have unambiguously shown that the rotational transition strength gets progressively fragmented with increasing excitation energy above yrast, a manifestation of the damping of the rotational motion [4].

Theoretical and experimental investigation have shown that the mixing among rotational bands, leading to the damping phenomenon, is governed by a residual interaction which includes high multiple terms [5,6] and that in some cases is found to be configuration dependent [7]. It is predicted that the onset of rotational damping sets in gradually over a quite wide interval in internal excitation energy and that scars of discrete rotational bands can be found even in the region of damped rotational states [8]. It is also predicted that the onset of damping is accompanied by an onset of chaotic behaviour in terms of nearest level distributions and distribution of transition strength [9]. This latest finding indicates that the study of the damping of rotational motion as function of nuclear temperature may shed light on the transition which brings the nucleus from the chaotic compound nucleus region [10], described by random combinations of the available configurations, into the zero temperature region, characterized by regular ordered motion, with selection rules and complete sets of quantum numbers for every state [11].

In the present paper we discuss a statistical method based on the covariance technique applied to gated quasi-continuum spectra, which provides a measure of how the cascades feeding into different selected bands are similar. As schematically illustrated in figure 1, while in the compound nucleus region there are no additional constants of motion besides the energy E, the total angular momentum I and the parity π, additional quantum numbers characterizing the single particle excitations of a rotating mean field, can be given to each nuclear state at $\approx$ zero temperature, making the system very close to be integrable [11]. Therefore, by comparing the decay flows which bring the nucleus from the same entry region, reached directly via particles or γ emission from the compound nucleus, down to specific intrinsic configurations characterized by different complete sets of quantum numbers, one may be able to test to which degrees selection rules and complete sets of quantum numbers are conserved by the γ-cascades.

In a previous work, the method has been used to study the γ-cascades feeding into low-K and high-K bands in ^{163}Er [7]. It was shown that the statistical method is indeed a sensitive probe of the correlations existing between bands of the same character, and evidence has been found for a possible persistence of the K-selection rules for the unresolved discrete rotational bands forming ridge structure in γ-γ coincident spectra. However, the limited statistics of the experiment did not allow to perform a meaningful analysis of the

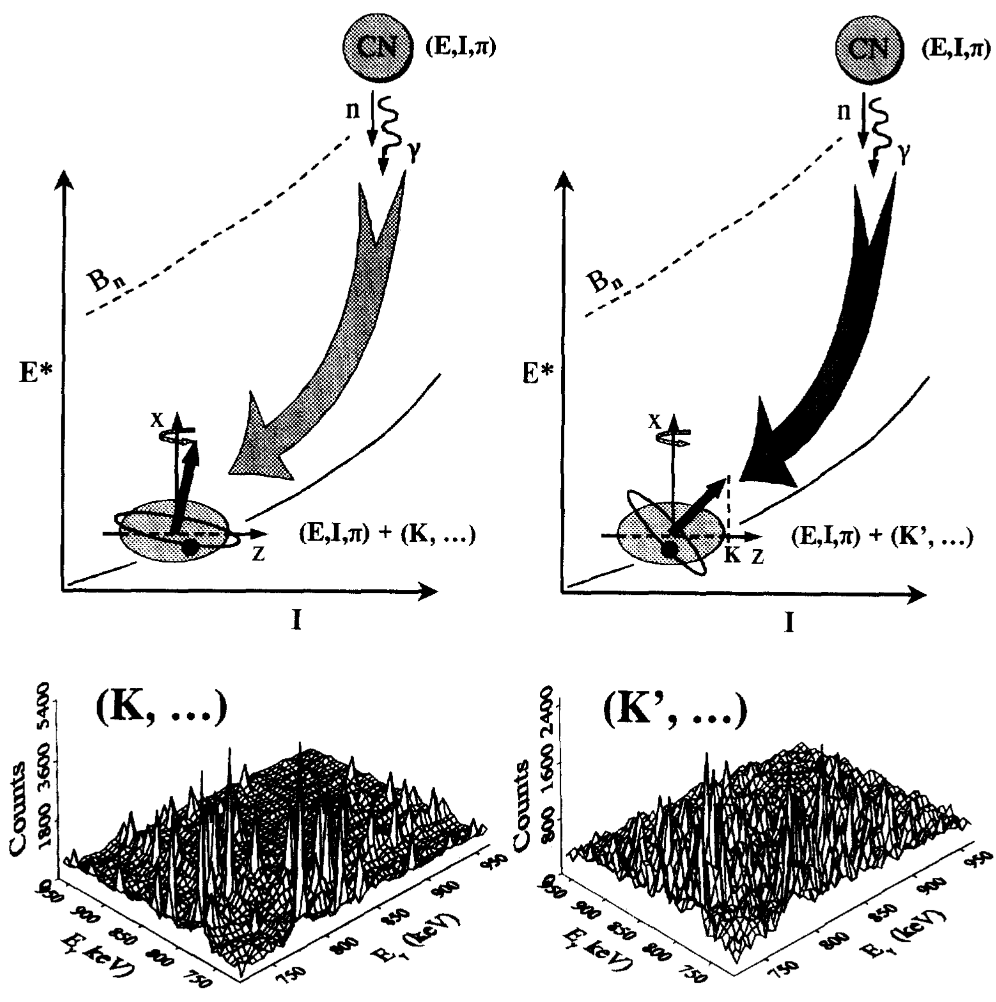

Figure 1. Schematic illustration of the general idea behind the covariance analysis. While only the principal quantum numbers (E,I,π) characterize the compound nucleus system, additional quantum numbers (K, ...) associated to the single particle configuration can be given to the nuclear system at $\approx$0 temperature. Therefore, by comparing the γ-decay from the same compound nucleus region, down into different single particle configurations close to the yrast line, one may learn about the loss of conserved quantum numbers in the nuclear many body system. Such a comparison can be obtained by the use of the covariance technique between pair of γ-γ spectra gated by different single particle configurations. In the bottom part we show, as an example, γ-γ spectra gated by low-K (left) and high-K (right) bands in ^{163}Er.

valley region, mainly populated by damped rotational transitions.

In the present case, we investigate the possible correlations existing between the flows populating low lying bands of different parity and signature in the nucleus ^{164}Yb. This represents the first attempt of covariance analysis in the valley region of quasi-continuum spectra, which could be instrumental to learn about the loss of the selection rules associated with the quantum numbers of the intrinsic structures, at excitation energy high above the yrast line. In fact, in the course of its decay, one expects that the nucleus can change its parity and signature, mainly by emitting E1-gamma rays. If the decay obeys strict selection rules, the cascades populating a low-lying final state of given parity and signature should be essentially independent from those of the other combinations, so that the fluctuations in the total spectrum should be those expected from the superposition of four independent contributions. On the contrary, if there are no strong selection rules besides those associated to parity and signature, one expects that a given state can feed democratically the other configurations, so that the cascades populating given low-lying configurations may have many transitions in common. In this case, the fluctuations in the total spectrum would be strongly affected by the correlations among the different parity-signature configurations.

The experimental results on ^{164}Yb will be also compared with predictions based on band mixing calculations.

2. The experimental analysis

An high statistic, high fold data set on ^{164}Yb has been obtained through the reaction ^{30}Si+^{138}Ba $\rightarrow$ ^{168}Yb, after 4n evaporation. The experiment was carried out at the Institut de Recherches Subatomiques of Strasbourg, using the Vivitron tandem accelerator and the multidetector system EUROGAM II. A stack of two targets, each evaporated on a thin Au foil of $\approx$580 $\mu g/cm^2$, has been used, with a total ^{138}Ba thickness of 450 $\mu g/cm^2$. The initial velocity of the residual nuclei in the middle of the target was calculated to be v/c = 1.8%. A total of 220 M triple and higher-fold events (with an average Ge-fold of 5) was collected. Energy-dependent time gates on the Ge time signal were used to suppress background from neutrons.

The data have been sorted into three γ-γ coincident spectra $G_{\alpha\pi}(E_{\gamma_1},E_{\gamma_2})$ gated by high spin transitions which exclusively select the three basic configurations of signature and parity (α,π) = (0,+), (0,-) and (1,-), as shown in figure 2. A total of $\approx$880 M, 650 M and 550 M events have been collected in the gated matrices respectively, before each spectrum was corrected for the background under the gate-selected peaks by subtracting a properly normalized spectrum $B_{\alpha\pi}(E_{\gamma_1},E_{\gamma_2})$ chosen from narrow gates around the peaks. The Compton and other uncorrelated events are reduced by the standard COR treatment [12]. Due to the low intensity population of the (1,+) rotational band, no γ-γ spectrum gated by this configuration could be obtained.

The fluctuations of counts in each channel of these two dimensional (2D) spectra, expressed as variance and covariance, are evaluated by the program STATFIT [2] and stored into two dimensional spectra. In particular, the correlations in fluctuations between two

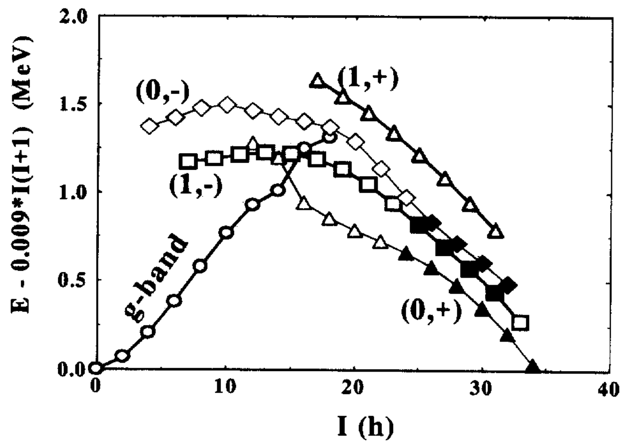

Figure 2. Excitation energy versus spin of the four lowest signature and parity configurations (α,π) = (0,+), (0,-), (1,+), (1,-), relative to a rigid reference. The closed symbols correspond to the high spin transitions used as gates to sort the corresponding γ-γ spectra. The weak population of the configuration (1,+) did not allow to obtain a gated γ-γ spectrum statistically significant.

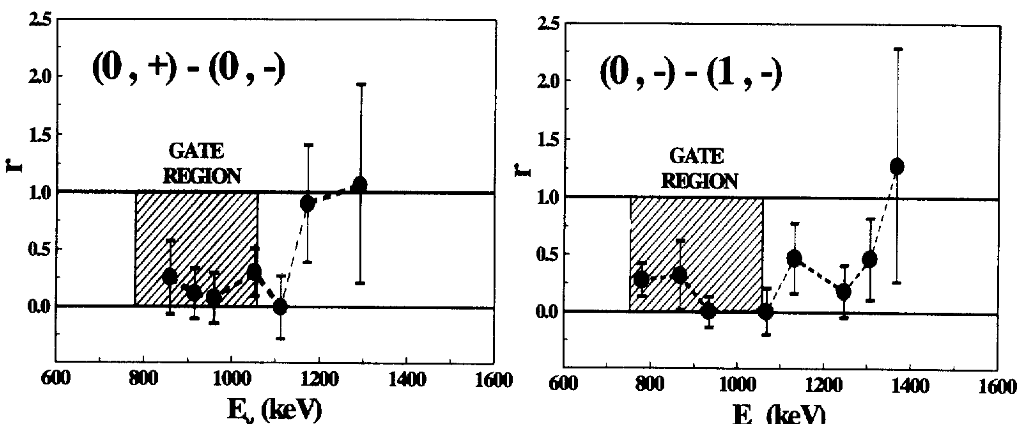

Figure 3. The correlation coefficient r extracted from the experimental analysis of the valley region of ^{164}Yb for the pairs of γ-γ spectra (0,+)-(0,-) and (0,-)-(1,-). The shaded area indicate transition energy regions covered by the gate-selected peaks. In the case of the combination (0,+)-(1,-), no statistically significant values could be obtained outside the gate-selected region.

spectra are given by the covariance of counts, defined as

$$\mu_{2,cov}(a,b) = \frac{1}{N_{ch}} \sum_j [M_j(a) - \tilde{M}_j(a)] \times [M_j(b) - \tilde{M}_j(b)] \quad (1)$$

where M(a) and M(b) refer to spectra gated by transitions from two different (α,π) configurations, denoted by a and b. The sum is over a region spanning N_{ch} channels in a two-dimensional sector, and $\tilde{M}$ denotes an average spectrum (which in our case is found by the routine STATFIT as a numerical smoothed third order approximation of the 2D spectrum). To normalize the covariance and thereby determine the degree of correlation between the two spectra, the correlation coefficient r(a,b) is calculated:

$$r(a,b) = \frac{\mu_{2,cov}(a,b)}{\sqrt{[\mu_2(a) - \mu_1'(a)][\mu_2(b) - \mu_1'(b)]}} \quad (2)$$

While the variance and covariance terms appearing in equation (2) are calculated on the final COR background subtracted spectra, the first moments μ_1' are calculated as

$$\mu_1'(\alpha\pi) = \mu_1(G_{\alpha\pi}) + g^2_{\alpha\pi}\mu_1(B_{\alpha\pi}) \quad (3)$$

Here the μ_1 terms are the averages of M over the region N_{ch} of the 2D gated and background spectra, $G_{\alpha\pi}$ and $B_{\alpha\pi}$, respectively. The label $\alpha\pi$ refers to a given signature and parity configuration, while $g_{\alpha\pi}$ is the background scaling factor, which is typically of the order of 0.5. The subtraction of the first moments μ_1' from the denominator of (2) corrects for the direct contribution to μ_2 from counting statistics, which is linear in the number of counts. In fact, the contribution to μ_2 given by the most interesting fluctuations, related to the nature of the finite number of transitions available to each decay cascade, is quadratic in the number of events.

The error bars on the correlation coefficient r can also be estimated in terms of variance and covariance of the γ-γ spectra. Assuming Porter-Thomas fluctuations in the E2 decay probability, as expected in the rotational damping region [2], one finds the expression

$$\mu_2(r(a,b)) = \frac{r^2(a,b)}{N_{ch}}\left[\frac{\mu_1(a)\mu_1(b)}{(\mu_{2,cov}(a,b))^2} + \frac{(\mu_1(a))^2}{2(\mu_2(a) - \mu_1'(a))^2} + \frac{(\mu_1(b))^2}{2(\mu_2(b) - \mu_1'(b))^2}\right] + \frac{1}{4n_{ch}}(1-r^2)^2 + \frac{3}{2N_{ch}}(1-r^2)^2 \quad (4)$$

Here n_{ch} denotes the number of channels in one dimension, which is typically 15, and N_{ch} the number of channels in two dimensions, typically 225.

Correlation coefficients (2) have been extracted from the analysis of the valley regions for combinations of the selected configurations, and the results are shown in figure 3. As one can see, the correlation coefficient r, which measures directly the similarities between the M(a) and M(b) spectra, is close to 0 in the transition energy region covered by the gate selected γ-rays (shaded area in the figure). This shows that the transitions feeding into the band must obey selection rules. The transitions in question here feed the low lying bands directly from the excitation region around 1 MeV above yrast, which is probed by valley fluctuations. At higher transition energies r starts to deviate from 0, suggesting a stronger sharing of decay paths among cascades which finally feed into different intrinsic configurations. Stated in general terms, this can be taken as an indication of a compound behavior of the part of the decay cascades which lead from a heat energy $\geq$1 MeV and rotational frequency around 650 keV, down to temperature close to zero and rotational frequency around 500 keV. This segment of a whole cascade contains about five stretched rotational E2 transitions, which conserve (α, π), and few feeding transitions, which change (α, π), and whose nature may be E1, M1, or unstretched E2's. In the case these E1, M1 and unstreched E2's transitions connecting the different configurations have statistical nature, one would expect to observe a compound behaviour, implying a covariance coefficient $r \rightarrow 1$. However, the large error bars on r at the highest transition energies, related to the poor counting statistics in the valley region of the gated γ-γ matrices, tell that such a conclusion cannot be firm.

3. Comparison with model prediction

In order to better assess the significance of the experimental results, values of the correlation coefficient r have been extracted from a similar analysis on calculated γ-γ spectra, obtained through a simulation of the γ-decay of the excited rotating nucleus ^{168}Yb. The model is based on microscopic nuclear levels, obtained by cranked shell model calculations, and on a Montecarlo treatment of the E1-E2 competition [6], in which no selection rules for the E1 decay are imposed, besides those associated to parity and signature.

It has been shown previously that such a simulation is able to reproduce well several features observed in experiments, and in particular the fluctuations. The E2 transition probabilities are calculated microscopically in the region covered by the calculated levels, which extends up to about 2.5 MeV above yrast [5]. It is assumed that at higher energy the rotational decay proceeds through a continuum of states governed by a rotational damping strength of Gaussian shape, and a width extrapolated from the region of discrete, microscopically calculated levels.

In the simulated decay the progressive fragmentation of the E2 rotational strength takes place only within a given (α,π) configuration, and the only correlations between configurations of different (α,π) are those associated to E1-decay, treated in a purely statistical way. During the decay, only E1 transitions, which can be stretched ($|\Delta I| = 1$) or unstretched ($|\Delta I| = 0$), can change the parity and possibly also the signature.

Simulated 2D spectra of ^{168}Yb, gated by high spin transitions of the lowest (α,π) rotational bands have been obtained from 10^6 cascades. Figure (4) shows the values of r obtained from the covariance analysis of all possible pairs of simulated spectra. The shaded area in the figures indicate the region covered by the gate-selected transitions, corresponding to the spin range 22 to 35 $\hbar$. This is comparable to the experimental gating conditions (see figure (2)).

As shown in figure (4), the values of r from simulated spectra do not seem to display any particular trend, and appear to be consistent with 0 even outside the region of the gates, at variance from the experimental results. The result is in contrast with our assumption of purely statistical E1 decay, which mixes different (α, π) configurations and should give

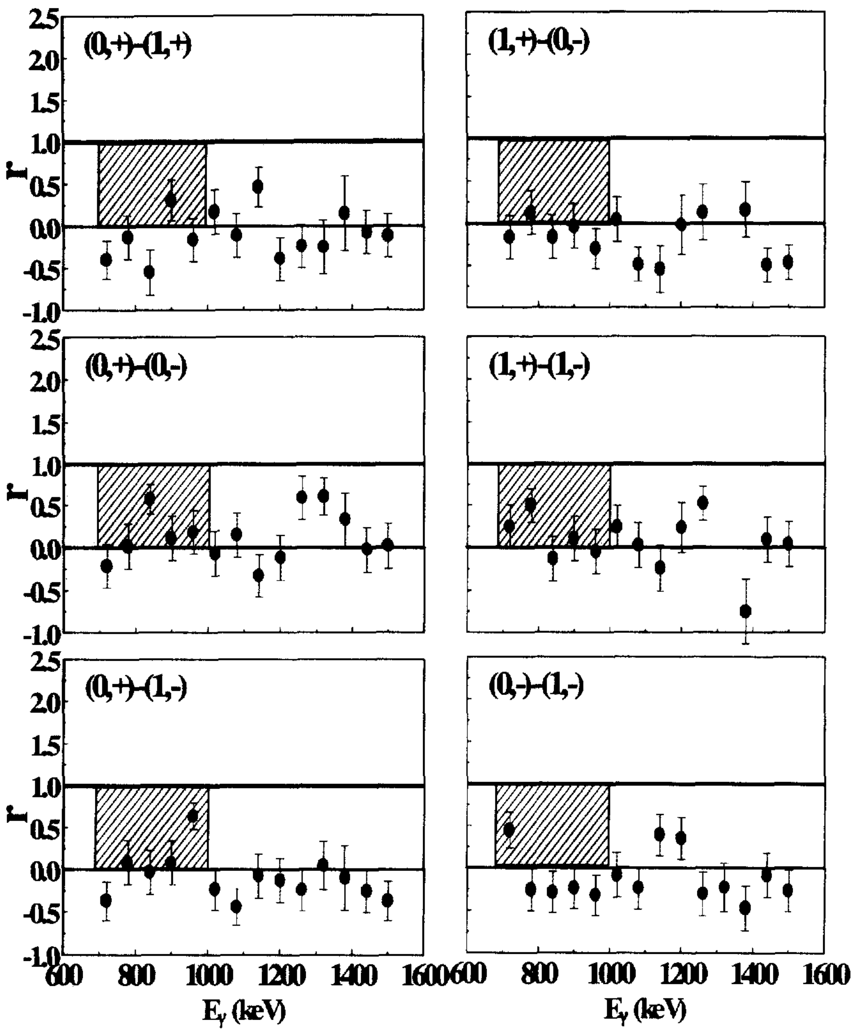

Figure 4. The correlation coefficient r extracted from the analysis of the valley region for all possible pairs of simulated spectra gated by the basic configuration (0,+), (0,-), (1,+) and (1,-) of ^{168}Yb. The shaded area indicate the transition energy regions covered by the gate-selected peaks, which compare with the experimental gating conditions.

$r \approx 1$. This may be interpreted in different ways: i) due to the gating conditions, we are probing a rather restricted region of excitation energy and spin, and the average decay flow extends beyond the region in which microscopically calculated levels are present (only these discrete levels can produce fluctuations); ii) although the previous simulations have been successful, it was not possible to test the quality of our strength function, nor the ratio between stretched and unstretched E1-transitions. In particular, there are only very few (2-3) E1-transitions per cascade, and they may not be enough to induce the correlations. Also, the accumulated statistics might not be sufficient. Finally, iii) other sources of correlations among the different configurations, besides the E1-decay, may play a role.

The general conclusion is that the comparison between the covariance analysis on data and simulation can offer the possibility to study in great detail the mechanism governing the quasi-continuum region in the nuclear many-body system.

4. Conclusions

Statistical analysis of the fluctuation of counts in γ-γ spectra can be used to obtain general information on the rotational decay of excited nuclei even in the region a few MeV above the yrast line. In the present paper the covariance technique between pair of 2D spectra, gated by different intrinsic configurations close to the yrast line, has been presented and discussed in connection with the experimental analysis of the quasi-continuum in ^{164}Yb. The method can give information on the transition between order to chaos in the nuclear system, in terms of loss of quantum numbers and validity of selection rules. The obtained results seem to indicate a possible weakening of the selection rules already at the moderate excitation energy ≥ 1 MeV above yrast, probed by the rotational decay-flow. Similar analysis on simulated spectra gives results different from experiments, suggesting that a more complex mechanism of mixing between single particle configurations is needed in order to interpret the experimental data.

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