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Quadrupole moments of the 11^- intruder isomers in ^{194,196}Pb in the particle-core coupling model

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Abstract

The recently measured spectroscopic quadrupole moments of the 11^- intruder isomers in ^{194,196}Pb are being discussed in the particle-core coupling model. The essential physics behind the nature of these isomers is revealed by a simple two-level-mixing calculation, showing that the coupling of the valence proton particles with vibrations of the underlying Hg core induces a factor of two increase of the quadrupole moments. © 2002 Elsevier Science B.V. All rights reserved.

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Over the last 20 years extensive studies of intruder states near to closed-shell regions, such as the Sn and the Pb nuclei, have been carried out [1,2]. The observation of low-lying excited 0^+ states is one of the essential experimental fingerprints [3,4]. These low-lying states have a typical excitation energy of $\simeq 1$ MeV, whereas, according to the shell model, exciting the nucleus into a spherical $2p$–$2h$ configuration costs almost twice the single-particle energy gap at $Z = 82$, i.e., $\simeq 7$ MeV. Meanwhile, it is understood that pairing and deformation play a crucial role in making these excitations to intrude in the neighborhood of the ground-state. Experimental support for the deformation of the intruder states comes from rotational-like bands built on top of the excited 0^+ states [5]. In addition, the broken pair $2p$ configurations, such as the $^A\mathrm{Pb}((h^{-2})_{0^+}1h_{9/2}1i_{13/2})_{11^-}$ states, carry important information on the orbitals involved and the way in which collectivity builds up. As these

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states have a nuclear spin larger than $1/2$ direct information on the deformation can be obtained by measuring the spectroscopic quadrupole moments.

In this Letter we report on a comparison between theoretical results obtained within the framework of the particle-core model and first experimental results on the quadrupole moments, Q_s, of such a $2p$–$2h$ configuration. We have measured the quadrupole moments of the 11^- isomers in ^{194,196}Pb [6,7], believed to be made out of the intruder $(3s_{1/2}^{-2}1h_{9/2}1i_{13/2})_{11^-}$ configuration. The quadrupole moments reported in this work not only confirm the intruder character of the isomers, but also allow to examine in some detail the shell-model mechanism which has been proposed to explain the appearance of intruder states [1,2]. To this aim we will compare the quadrupole moments of the 11^- intruder isomers to those of the corresponding regular $(1h_{9/2}1i_{13/2})_{11^-}$ states in the Po isotones, both experimentally and theoretically. Collectivity and deformation are shown to arise naturally from the increase in the number of valence particles ($2p$–$2h$ in the intruder states compared to only $2p$ for the regular ones) leading to an enhanced proton–neutron interaction energy and inducing 'core polarization' effects. Experimentally, we can disentangle the different contributions to the structure of intruder states using complementary information from magnetic and quadrupole moments. Magnetic moments can generally be used to fix the single-particle configuration of a state, but are rather insensitive to collective admixtures in the wave functions. This fact is rather well-known in connection with core polarization effects [8,9]. E.g., the experimental *magnetic moment* of the 11^- states is practically the same in all Po and Pb isotopes [10,17]. The quadrupole moments on the other hand allow to study the collective admixtures in the wave functions quite in detail.

The quadrupole moments of the isomers in ^{196}Pb ($I^\pi = 11^-$, $T_{1/2} = 72(4)$ ns, $\mu = 10.56(88)\mu_N$ [11, 12]) and ^{194}Pb ($I^\pi = 11^-$, $T_{1/2} = 122(10)$ ns [11,13]) have been measured at the Cyclone facility at Louvain-la-Neuve by applying the Level Mixing Spectroscopy (LEMS) technique [14]. The deduced quadrupole moments are $Q_s(^{196}\text{Pb}; 11^-) = (-)3.41(66)$ b and $Q_s(^{194}\text{Pb}; 11^-) = (-)4.48(86)$ b, respectively [6,7]. In order to derive a value for the quadrupole moment of the ^{194}Pb(11^-) isomer a new analysis technique, taking into account the influence of subsequent isomeric decays, has been applied. This is described extensively in Ref. [6]. Ref. [7] focuses entirely on how the value of the quadrupole moment of the 11^- isomer in Pb can been used to study the deformation of the 16^- magnetic rotational bandhead. In the present Letter a thorough discussion on the rather large deformation of the 11^- intruder states themselves is given.

In order to get a better feeling for the magnitude of the measured quadrupole moments of the 11^- intruder isomers we emphasize that the maximally aligned *neutron* states $(\nu 1i_{13/2})^2_{12^+}$ in the same nuclei, with a similar value of the angular momentum, have quadrupole moments nearly one order of magnitude smaller: $Q_s(12^+; ^{196}\text{Pb}) = 0.65(5)$ b [15] and $Q_s(12^+; ^{194}\text{Pb}) = 0.49(3)$ b [16]. The structure of the 11^- states must, therefore, be very different from that of neutron two quasiparticle states. The lowest possible proton excitation across the closed shell which gives a spin value 11^- is, as mentioned before, $\pi(3s_{1/2}^{-2}1h_{9/2}1i_{13/2})_{11^-}$, and the measured magnetic moment of the 11^- isomer in ^{196}Pb is very close to that expected for a rather pure excitation involving these proton orbitals [12]. The fact that the E1 transitions connecting the 11^- isomers in both ^{196}Pb and ^{194}Pb with the neutron $(1i_{13/2})^2$ two-quasiparticle states are highly retarded (of the order 10^{-8} W.u.), and two orders of magnitude smaller than the E1 transitions between known neutron two quasiparticle states in these nuclei [17], lends support to the proton two-particle two-hole assignment.

Further relevant to our discussion is to compare the measured quadrupole moments with the quadrupole moment of the pure $\pi(1h_{9/2}1i_{13/2})_{11^-}$ configuration (the "single-particle" estimate); a good approximation for this quantity is the quadrupole moment of the 11^-_1 state in the two-proton nucleus ^{210}Po: $Q_s(11^-_1) = (-)0.86(11)$ b [18]. The quadrupole moments of the 11^- isomers in Pb are thus four to five times larger than the single-particle value. This is in part because the neutron number is significantly smaller than that of the double magic ^{208}Pb, getting close to the neutron midshell ($N = 104$). As well known, this will cause modifications to the structure of the proton two-particle states via collective admixtures and renormalize their electromagnetic properties [8]. The effect of core polarization on the quadrupole moments of the $\pi(1h_{9/2})^2_{8^+}$ states in the Po isotopes has been success-

fully reproduced using the particle-vibration coupling hamiltonian and first-order perturbation theory [19].

However, the non-magic neutron number by itself is not sufficient to explain a factor of four difference between the single particle estimate and the experimental values, as will be shown below. A further important point is that the 11^- isomers in Pb involve an additional pair of proton holes besides the maximally aligned two-proton particle configuration in the corresponding Po nuclei. The shell-model view of intruder states [1,2] explains the large gain in energy of $2p$–$2h$ excitations across the closed shell by the additional proton–neutron interaction created with the additional valence particles. The attractive proton–neutron interaction leads to the onset of deformation as soon as the number of valence particles of both types is large enough.

This qualitative discussion can be quantified by comparing the measured quadrupole moments of the 11^- isomers in ^{196,194}Pb to the theoretical values of $Q_s(11^-)$ in both the Pb and the Po isotones. This allows us to deduce the influence of the additional two proton holes on the collectivity and deformation of these states. The comparison will be focused on the $N = 114$ isotones ^{198}Po and ^{196}Pb.

Two proton holes in the $Z = 50$–82 shell interacting with the valence neutrons give rise to the low-lying spectrum of the Hg isotopes. A description of the $2p$–$2h$ intruder states in ^{A}Pb as resulting from the coupling of two proton particles to $^{(A-2)}$Hg (two proton holes and the valence neutrons) should correctly account for the essential physics. The 'regular' states in the Po isotopes can be accurately described as two protons coupled to a Pb core. Particle-vibration coupling calculations have been performed for the Po isotopes, using a rather complete configuration space [20]. Furthermore, for the light isotopes ($A < 200$), the mixing between regular and intruder configurations has been studied and the unperturbed structures reconstructed [20].

We calculate the spectroscopic quadrupole moments of the 8^+ and 11^- isomers in $^{198-210}$Po using the particle-vibration coupling model. For the heavier isotopes, $^{200-210}$Po, we use the published parameters [20]. The two-proton $\pi(1h_{9/2}1i_{13/2})_{11^-}$ states in ^{196,198}Po are known, but no quadrupole moments have been measured. However, the quadrupole moment of the isomeric $\pi(1h_{9/2}^2)_{8^+}$ states have been measured in

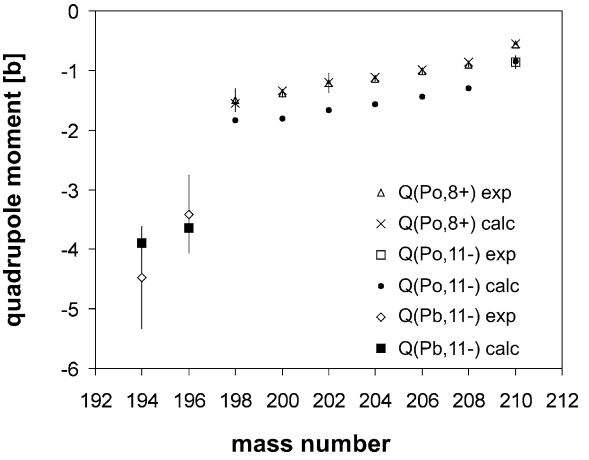

Fig. 1. Systematics of the calculated and experimental quadrupole moments of the 8_1^+ states in Po and the 11_1^- states in the Po and Pb isotopes. The experimental values $Q_s(11^-)$ for Pb are from the present work. The other data are taken from Refs. [17,19].

a number of Po isotopes, reaching down to ^{200}Po. As seen from Fig. 1, the smooth behavior of the quadrupole moments $Q_s(8_1^+)$ with mass allows to extrapolate to ^{198}Po: $Q_s(8_1^+; {}^{198}\text{Po})_{\text{extr.}} \simeq 1.5(2)$ b. This information, together with the reconstructed energy spectrum of the 'regular' states [20], allows us to pin down the model parameters involved in the particle-vibration coupling calculation for ^{198}Po. We show in Fig. 1 the results of the calculations regarding the quadrupole moments of the first 8^+ and 11^- states for the Po isotopes, as well as the experimental values where data are available (for 11^-, only in ^{210}Po) and the extrapolated value for $Q_s(8^+; {}^{198}\text{Po})$. The core contribution to the quadrupole moments in Po is calculated from the reduced transition probabilities $B(\text{E2}; 0^+ \rightarrow 2_1^+)$(Pb). Experimental values, listed in Table 1, are only available for $^{204-208}$Pb. For the lighter isotopes, the measured value of $Q_s(8^+; \text{Po})$ was used to estimate $B(\text{E2}) \uparrow$ (Pb) and calculate $Q_s(11^-; \text{Po})$. The same effective charge, $e_{\text{eff}} = 1.4e$, has been used for single-particle E2 contributions in all isotopes with an open neutron shell. For ^{210}Po, with a double magic core, the effective charge was $e_{\text{eff}} = 1.0e$.

Using the parameters taken from the Po calculation for the proton single-particle states, we can describe ^{194,196}Pb as ^{192,194}Hg to which two protons in the shell $Z > 82$ are coupled. The strength of the particle-vibration coupling can be estimated in the

Table 1
Results for the calculated quadrupole moments of the 8^+ and 11^- isomers in Po and the 11^- isomers in Pb. $Q_s^{\rm calc}$ are the quadrupole moments as calculated with the full diagonalization method, $Q_s^{\rm 2dim}$ the quadrupole moments as calculated with the 2-dimensional model

Isomer	A	Core	$A_{\rm core}$	B(E2; core) ($e^2\,b^2$)[a]	$Q_s(2_1^+;$ core) (b)[b]	$Q_s^{\rm exp}$ (b)[b]	$Q_s^{\rm calc}$ (b)	$Q_s^{\rm 2dim}$ (b)
Po(8^+)	210	Pb	208	0.290	−0.7	−0.568(3)	−0.55	−0.73
	208		206	0.100	0.05	−0.90(4)	−0.86	−0.79
	206		204	0.162	0.23	−1.02(4)	−0.99	−0.88
	204		202			−1.14(5)	−1.11	
	202		200			−1.21(16)	−1.20	
	200		198			−1.38(7)	−1.34	
	198		196			−1.5(2)[c]	−1.55	
Po(11^-)	210	Pb	208	0.290	−0.7	−0.86(11)	−0.85	−1.27
	208		206	0.100	0.05		−1.30	−1.29
	206		204	0.162	0.23		−1.44	−1.39
	204		202				−1.57	
	202		200				−1.66	
	200		198				−1.80	
	198		196				−1.83	
Pb(11^-)	206	Hg	204	0.427	0.40			−2.09
	204		202	0.612	0.784			−2.57
	202		200	0.853	0.652			−2.64
	200		198	0.990	0.76			−2.82
	196		194	1.39[c]		(−)3.41(66)	−3.64	
	194		192	1.59[c]		(−)4.48(86)	−3.90	

[a] Taken from Ref. [21].
[b] Taken from Ref. [10].
[c] Extrapolated value.

usual way [9], from the energy and B(E2) value of the quadrupole phonon 2_1^+ state. Since the B(E2) values are not known for ^{192,194}Hg, we use a linear extrapolation of the variation of the B(E2) ↑ (Hg) with mass in the heavier Hg isotopes. This results in $B(\mathrm{E2};{}^{194}\mathrm{Hg})\uparrow \simeq 1.4\ e^2\,b^2$ and $B(\mathrm{E2};{}^{192}\mathrm{Hg})\uparrow \simeq 1.6\ e^2\,b^2$. The coupling strengths were slightly adjusted to reproduce the energies in the intruder band of ^{194,196}Pb. The coupling is *strong* (coupling constant $\xi_2 = 4.0$ for ^{196}Pb), as opposed to the *intermediate* coupling ($\xi_2 = 1.5$) for the isotone ^{198}Po. For comparison, the particle-vibration coupling in ^{210}Po is *weak*, with $\xi_2 = 0.3$. Using the extrapolated B(E2) ↑ for the Hg core and an effective charge of 1.4 for the single-particle contribution, we calculate a value $Q_s(11^-;{}^{196}\mathrm{Pb})_{\rm calc.} = -3.64$ b, to be compared to the experimental one $Q_s(11^-;{}^{196}\mathrm{Pb})_{\rm exp.} = (-)3.41(66)$ b and with the value calculated for ^{198}Po, $Q_s(11^-;{}^{198}\mathrm{Po})_{\rm calc.} = -1.83$ b. For ^{194}Pb the calculated value is $Q_s(11^-;{}^{194}\mathrm{Pb})_{\rm calc.} = -3.90$ b, to be compared to the experimental one $Q_s(11^-;{}^{194}\mathrm{Pb})_{\rm exp.} = (-)4.48(86)$ b. Both calculated and experimental values for the two Pb isotopes are shown in Fig. 1.

The large increase in the quadrupole moment for the intruder 11^- state in Pb, compared to the regular 11^- state in Po (see Fig. 1) is extremely robust to variations of the parameters in a physically meaningful range. The increase is due to the strong coupling of the two protons to the Hg core (resulting in the intruder states in Pb), as opposed to the moderate coupling of the two protons to Pb (resulting in the regular Po states). This in turn is due to the significant increase in quadrupole collectivity when going from Pb to Hg. The contribution of the core (core polarization) to the calculated quadrupole moment of the 11^- isomer in ^{196}Pb is roughly 4 times larger than the single-particle contribution and will further increase with increasing quadrupole collectivity of the Hg core towards mid-shell. In ^{198}Po the two contributions are roughly equal. In the semi-magic ^{210}Po ($N = 126$) the single-particle contribution is about three times larger than the core contribution.

The intruder mechanism thus receives a clear confirmation: starting with unperturbed proton two-particle states, as in ^{210}Po, the interaction with the valence neutrons (open shell Pb core) leads to core polarization and a subsequent increase in the quadrupole moment from $Q_s(N = 126) \cong -0.9\ e\,\mathrm{b}$ to $Q_s(N = 114) \cong -1.8\ e\,\mathrm{b}$. Further increasing the number of valence protons by addition of a proton hole pair, as is the case in the Pb intruder states, leads to an even stronger interaction of two proton particles but now with the Hg core. The increase in the quadrupole moments due to core polarization can be interpreted as an increase in deformation due to enhanced proton–neutron interaction.

Finally, we compare the values calculated above in the complete configuration space with the results provided by a simple picture, which incorporates the main physics and can be quickly adapted for other mass regions too. The leading components of the wave functions of the 11^- isomers in Po or Pb can be calculated by diagonalising the Hamiltonian $H = H_{\mathrm{s.p.}} + H_{\mathrm{coll}} + H_{\mathrm{int}}$ in the 2-dimensional Hilbert space spanned by the wave functions $|11^-_{\mathrm{s.p.}}\rangle$ and $|11^-_{\mathrm{s.p.}} \otimes 2^+_1;\ 11^-\rangle$. Here, $|11^-_{\mathrm{s.p.}}\rangle$ is a shorthand notation for $|1h_{9/2}1i_{13/2};\ 11^-\rangle$ for Po or $|(h^{-2})_{0^+}1h_{9/2}1i_{13/2};\ 11^-\rangle$ for Pb, which is an eigenfunction of $H_{\mathrm{s.p.}}$, and $|2^+_1\rangle$ denotes the one-phonon quadrupole vibrational state of the underlying core, which is an eigenfunction of H_{coll}. The interaction Hamiltonian, H_{int}, as defined in Ref. [9], accounts for the coupling of the valence nucleons with the vibrational states of the core. The solution of the eigenvalue problem, $|\widetilde{11}\rangle = a|11^-_{\mathrm{s.p.}}\rangle + b|11^-_{\mathrm{s.p.}} \otimes 2^+_1;\ 11^-\rangle$, is used to calculate the expectation value of the quadrupole operator. This results in the following expression for the spectroscopic quadrupole moments of the 11^- isomers in Po and Pb:

$$\begin{aligned} Q_s = {} & a^2 Q_s\left(11^-_{\mathrm{s.p.}}\right) + b^2 \cdot 0.932 \cdot Q_s\left(11^-_{\mathrm{s.p.}}\right) \\ & - b^2 \cdot 0.856 \cdot Q_s\left(2^+_1;\ \mathrm{core}\right) \\ & + 2ab \cdot 1.24 \cdot \sqrt{B\left(\mathrm{E2};\ 0^+ \to 2^+_1\right)}, \end{aligned} \tag{1}$$

in which $Q_s(2_1;\ \mathrm{core})$ is the spectroscopic quadrupole moment of the first excited 2^+_1 state in the underlying core (the precise origin of the various numerical coefficients in Eq. (1) is discussed in Ref. [22]). The quadrupole moments calculated with the two-state mixing method are listed in Table 1. Unfortunately, neither the $B(\mathrm{E2};\ 0^+ \to 2^+_1)$ nor the $Q_s(2^+_1)$ values are known for the lighter Pb and Hg nuclei, so that the quadrupole moments can be calculated only for a few Pb and Po isotopes with this simple model. The full diagonalization method has the advantage of allowing to determine the coupling strengths (which is related to the values of $B(\mathrm{E2};\ \mathrm{core})$) from more readily available data, like level energies. The values obtained within the truncated configuration space come rather close to those obtained from the full diagonalization (see Table 1). This proves that the 2-dimensional approach works well and, hence, that the increase in the quadrupole moment of the two-particle two-hole states in Pb, compared to two-particle states in Po, is mainly due to the coupling of the two valence protons with the quadrupole vibrational state of the more collective underlying core.

In conclusion, the first measurements of quadrupole moments of $2p$–$2h$ excitations across the $Z = 82$ shell gap give unambiguous experimental information on the precise nature of the 11^- intruder states in the neutron-deficient Pb nuclei. A comparison of the experimental numbers with calculations in a rather complete model space and simple 2-dimensional calculations shows that the coupling of the valence protons with a more collective underlying core is causing a substantial increase of the quadrupole moments of the 11^- isomers in Pb compared to the corresponding quadrupole moments in Po. It demonstrates the validity of the shell-model mechanism proposed [1,2,23] to be responsible for the appearance of intruder states.

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