Resume' on ^{196}Pb

➢ general considerations

Here we show the results we obtained for the 196Pb.

We describe first the single quantities involved in the analysis and then we show plots in the second part of this document.

In the first pages we explain again how we treated every single quantity. Most quantities are extracted by *K.Yoshida et al. NPA696 (2001) 85-122*, but in some cases we had to use other references, such as *Y.R.Shimizu et al. NPA557 (1993) 99c-108c*. We mention the source every time we introduce a new formula or quantity.

The pictures in this report are similar to the ones you find in these two articles.

In order to clarify the terminology used later on please use figure 1 where we report a schematic representation of the two wells and define the quantities used.

– E_γ-spin conversion

The spin → transition energy conversion is needed to compare the experimental data with theory. We have in fact to translate the theoretical results since these are given in terms of spin. The conversion is done by calculating the average spin-energy conversion over the four configurations using data in the TR files and then fitting this average with a linear function $E_\gamma = A + B \cdot I$. Values of A and B are given later.

– Moments of inertia

The moment of inertia are calculated using conventional formulas:

$\Im^{(1)} = 2\hbar^2 \frac{I}{E_\gamma}$ for the kinematic moment of inertia, and

$\Im^{(2)} = \frac{4\hbar^2}{\Delta E_\gamma}$ for the dynamic one.

– Level densities

In order to calculate the level densities for ND and SD we referred to the Fermi gas formula in *eq. No. 14 NPA 696.*

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{48} a^{-\frac{1}{4}} U^{-\frac{5}{4}} \exp(2\sqrt{aU})$$

a is: $a = \frac{A}{a_0}$. Clearly we used different values for a_{SD} and a_{ND}. These values are taken according to literature and will be specified later.

We calculated the level densities, both ND and SD, at increasing energy above the SD minimum, E_{exc}, going from 0 to 5 MeV with a step of 200 keV. This means that in the ND case $U = E_{ND} + E_{exc}$ (see fig.1), where E_{ND} is read in the .bap files.

– EM decay width

The Γ_{E2} [1] is calculated according to $\Gamma_{E2} = 3 \cdot 10^{-14} Q^2 E_\gamma^5$

The quadrupole moment is calculated as follows:

$Q = \frac{4}{5\sqrt{2}} ZR^2 \beta \cos(\gamma + 30)$, where $\beta = 1.057 \cdot \varepsilon$, $R = 1.26 \cdot A^{1/3}$ is the nuclear radius and Z is the atomic number. ε (ε_2) and γ are the deformation parameters read in the .bap files. We did not consider ε_4.

Γ_{E1} is calculated according to *eq. No. 17* in *NPA696*:

$\Gamma_{E1} = C_{E1} \cdot 2.3 \cdot 10^{-11} (A - Z) \cdot ZA^{1/3} \cdot \sqrt[5]{U/a}$,

where C_{E1} = 0.15 as suggested in *NPA696*. In this case the excitation energy U is calculated starting from the deepest ND minimum for each spin, and again at increasing energy E_{exc} above the SD minimum.

For ND minima then the EM decay width is calculating by summing

$\Gamma_{ND} = \Gamma_{E2} + \Gamma_{E1}$

, being Γ_{E2} and Γ_{E1} averaged over all the minima.

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[1] The factor in *eq. No.14* in *NPA696* is not correct, ten orders of magnitude off (misprint most probably), we therefore used the one quoted in *NPA557*

This is done for each spin and excitation energy above the SD minimum (E_{exc}), ranging from 0-5 MeV with steps of 0.2 MeV.

– TUNNELING

Starting from the action given in the .act files and the previously calculated spacing of the SD states, we calculate the tunneling probability following *eq. No.11 in NPA696*: $\Gamma_t = \frac{D_s}{2\pi}(1+\exp 2S)^{-1}$.

As is done in *NPA557* we decided to plot the ratio $\frac{\Gamma_{SD}}{\Gamma_{ND}}$ together with $\frac{\Gamma_t}{D_{ND}}$ being these the two factors that results into the P_{out} (P_{out} corresponds to Pout in *NPA696*), which is given by:

$P_{out} = \frac{1}{2\pi}\sqrt{\frac{\Gamma_t}{D_{ND}} \cdot \frac{\Gamma_{SD}}{\Gamma_{ND}}}$, in the weak coupling limit.

We thereafter calculated the P_{out} using the GOE.

– N_{paths}

We then finally calculated the N_{path} with and without considering the tunneling. Since the theoretical values do not match the experimental data we then introduced two rescaling factors (as done in *NPA696*), C_ρ and C_{mass}.

The two quantities enter in the evaluation of Pout as follows: the level density of ND states is changed as $\rho = C_\rho \cdot \rho$, while the square root of C_{mass} multiplies the action S, $S = \sqrt{C_{mass}} \cdot S$.

➢ Figures for ^{196}Pb

- Figure 2 shows the spin vs .transition energy conversion: top panel shows the conversion obtained by TR files for each single configuration (filled squares) compared to the yrast SD band of ^{196}Pb (stars). The average value obtained by TR is shown with the full line. The A and B coefficients from the fit (bottom panel) are:

  A= -16.03947

  B= 19.51974

  and will be used in the following analysis.
- Figure 3 Kinematic (top panel) and dynamic (bottom panel) moments of inertia. The values obtained for the yrast SD band (symbols) are compared to the ones obtained from the TR averaging over the four configurations (lines). We see a quite good agreement with theory.
- The level density results are shown in fig.4 for different spin values (20-40-60 ħ). The energy of the ND minimum is indicated close to the corresponding line. Level density parameters are: a_{ND} =8 and a_{SD} =13. In the bottom panel we show the level density of ND stases as function of spin for different excitation energies above the SD minimum.
- The calculated EM decay widths are shown in figure 5. Top panel refers to E_{exc} =0 MeV above SD minimum, while the picture in the bottom panel to E_{exc} =1 MeV. The EM decay widths are multiplied by a factor of 10^8 in the picture. One sees that the quantity Γ_{ND} is fluctuating as function of spin, probably owing to a discontinuity in the minima: this is in fact mainly present in the Γ_{E2} (black line), while fluctuations are smaller for Γ_{E1} (red line). The behaviour does not change with excitation energy.
- Figure 6 reports the action S as function of spin and excitation energy. The energy is increasing going from the top to the bottom of the picture, as indicated by the arrow, starting from 0 to 4 MeV with intervals of 0.5 MeV. The values are directly extracted from files .act. This picture is not very different from the one of ^{192}Hg reported in *NPA696*, even if in the ^{196}Pb case there seems to be a pronounced tendency of S to decrease again at higher spin

values. At contrast to Hg this starts at lower excitation energy, being visible in all curves.

- The tunneling probability Γ_t is shown in fig.7 for three different excitation energies above the SD minimum: E_{exc} =0, 1 and 2 MeV. Γ_t is multiplied by a factor 10^6 for sake of representation.
- The two factors $\frac{\Gamma_{SD}}{\Gamma_{ND}}$ and $\frac{\Gamma_t}{D_{ND}}$ are plotted in figure 8 for different excitation energies above the SD minimum, E_{exc} = 0, 1 and 2 MeV. The crossing point of the curves should correspond to the decay out spin. Again one can see that the EM widths display a zigzagging structure that, in the case of E_{exc}=1 MeV, can be misleading, giving values of Iout higher/lower than the ones one would obtain from a smoother curve.
- Figure 9 shows the P_{out} as function of spin and excitation energy calculated using the GOE matrices as function of spin and excitation energy. The curve seems to be quite steep E_{exc}=0 (black curve) and gets lower than 1 very rapidly, while the behaviour changes at higher E_{exc}, for which the curves show a deep and then regain higher values before getting definitely closer to 1 only at the very highest spins. The overall behaviour of the Pout at E_{exc}>0, which does not go to zero at increasing spin but,on the contrary, gains higher values for spin >40h (at E_{exc} =1MeV) and even lower spin values for E_{exc} =2 MeV can be explained by the nature of the action itself that does not monotonically increse at increasing spin but has a bell shape instead (cfr. Fig.6). The zigzagging pattern present are instead probably due to the not smooth behaviour of the EM widths for ND states. This is in fact confirmed by the next picture where we compare Pout values obtained averaging over all the minima or calculated only by using one of them.
- Figure 10 shows the Γ_{E2} EM width evaluated for each single minimum (symbols) compared to the average calculated weighting over the level density (black thick line). The minima are referred to with the number given in the pb196a.bap file. Only configuration a is here included for sake of simplicity. The trend of the average line tells us that some minima are locally

deeper and therefore count more even if they only exist in a limited spin range.

- Comparison of Pout evaluated over all the minima or including only one minimum is shown in figure 11. The data refer only to configuration “a” for sake of simplicity. Three different excitation energies are here represented, E_{exc} =0,1,2 MeV above the SD minimum (black, red and blue curves respectively). Open symbols refer to Pout evaluated using quantities averaged over all the minima, while filled ones are calculated only including minimum #29 (see file pb196a.bap). The filled symbols seem to follow a smoother trend than the open ones, particularly at increasing excitation energy above the minimum.
- In fig.12 we show the experimental N_{path} obtained in the total matrix (full squares) and in direct coincidence with the SD yrast band (open circles). They are shown together with predictions not including tunneling (red line).
- Figure 13 then shows comparison with predictions including tunneling. Experimental data obtained from the total matrix (symbols) compared to calculated Npath using different values for C_ρ and C_{mass}. When the two quantities are set to 1 (red curve) one sees that the N_{path} si underestimated and the decay-out spin is clearly too high, being 12hbar instead of the measured 6hbar (A. Wilson PRL95 (2005) 182501). By playing with C_ρ (magenta line C_ρ =2e-4) we can reproduce the data better, even if we slightly overestimate the values. We also show the calculations without tunneling (black line) for comparison. In addition we also show, with a green line, calculations obtained considering only one ND minimum for each configuration (the one extending over a wider spin range). No difference is visible comparing to the values obtained averaging over all minima (red line).

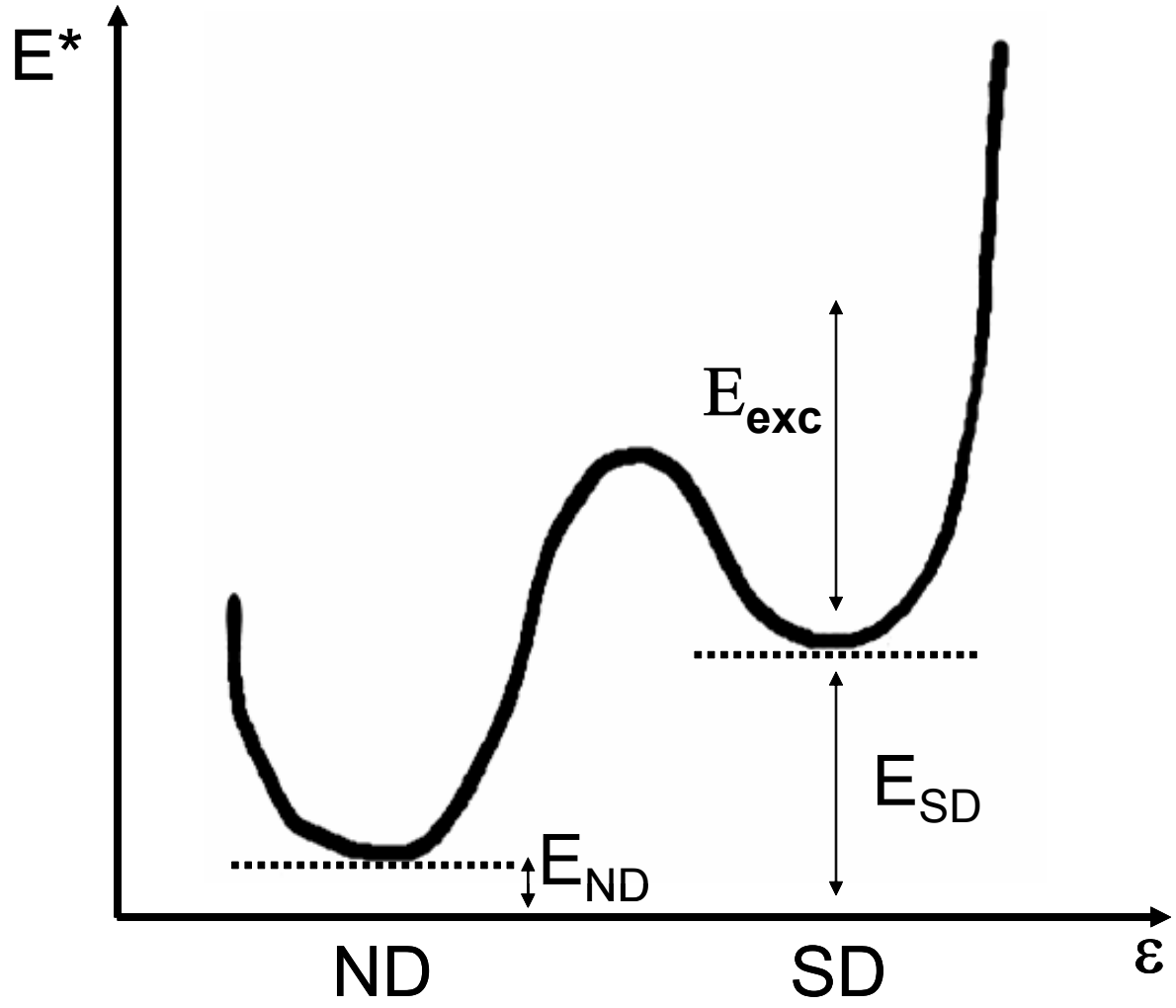

Figure 1 *Schematic representation of the two wells, ND and SD. In particular this picture defines the E_{exc} above the SD minimum that is used later on to evaluate all the quantities.*

^{196}Pb - Matsuo Calculations

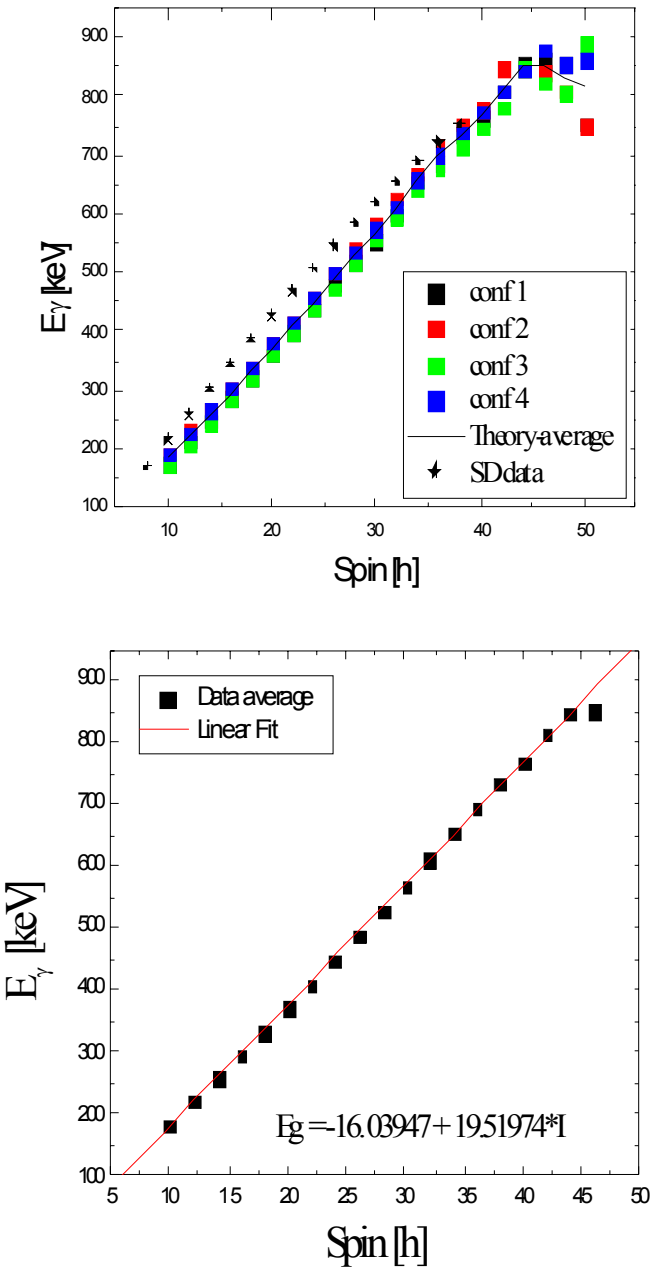

Figure 2 *Spin vs .transition energy conversion. Top panel shows the conversion obtained by TR files for each single configuration (filled squares) compared to the yrast SD band of ^{196}Pb (stars). The average value obtained by TR is shown with the full line. In the bottom panel we report the fit of the average distribution used later on in the analysis.*

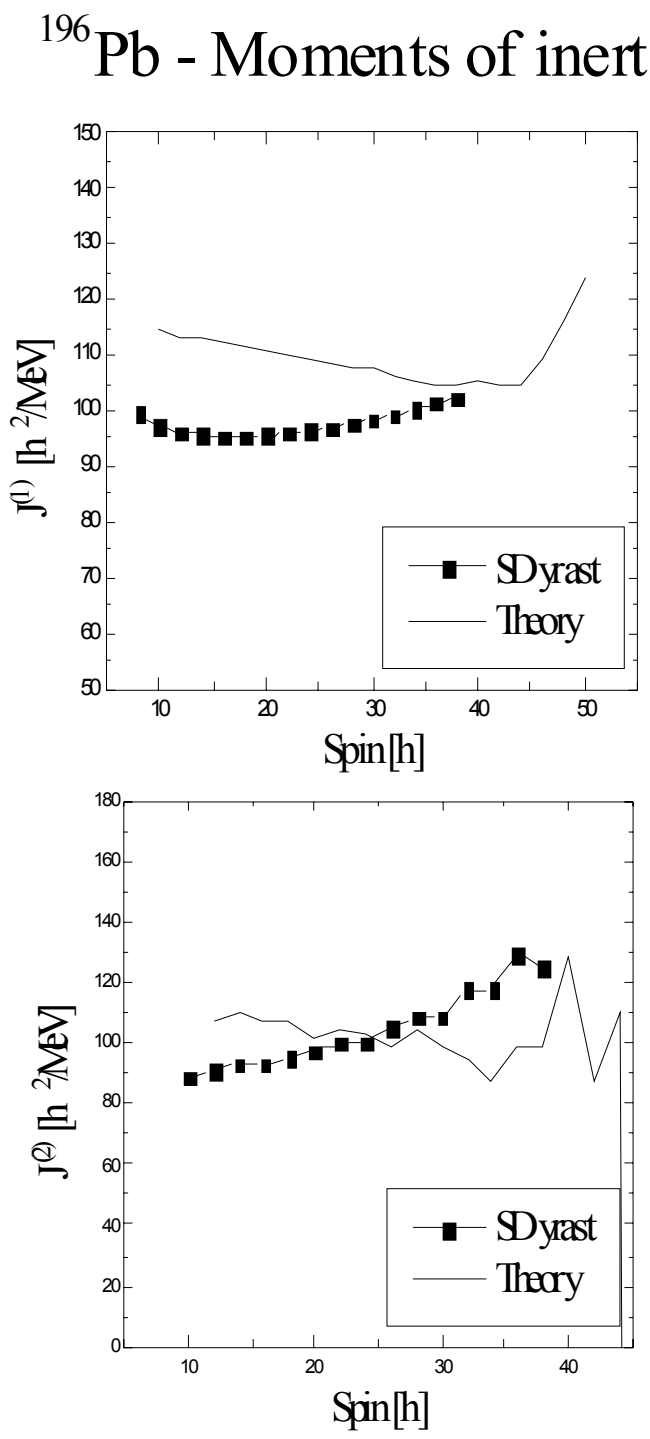

Figure 3 *Kinematic (top panel) and dynamic (bottom panel) moments of inertia for ^{196}Pb. The values obtained for the yrast SD band (symbols) are compared to the ones obtained from the TR averaging over the four configurations (lines). We see a quite good agreement with theory.*

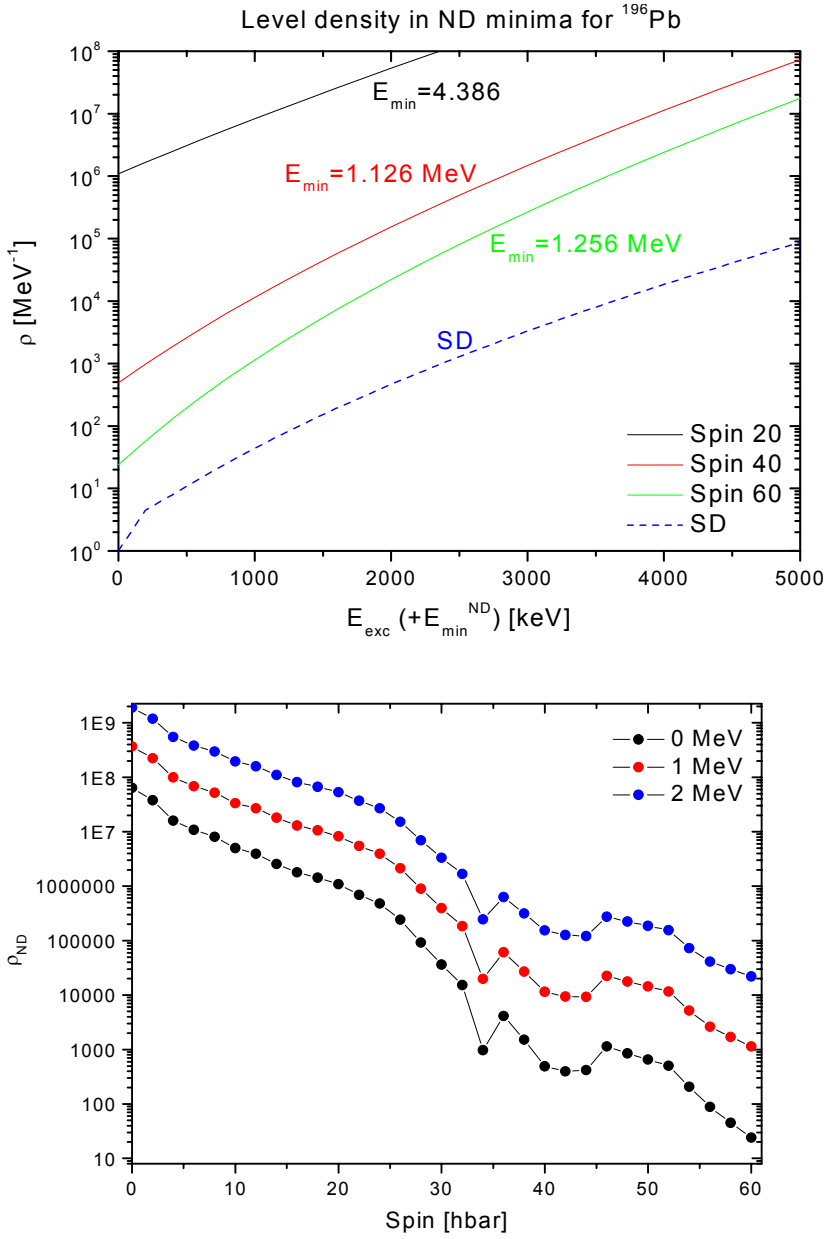

Figure 4 *Top panel: level density for both SD and ND states as function of the excitation energy above the SD minimum. The x axis corresponds to the E_{exc} in fig. 1. The level density for SD states clearly does not depend on the spin, since we only have one SD minimum. For the ND states, instead, we have different minima for different spin values (20-40-60 ħ). The energy U which enters in the Fermi gas formula for the ND case is the sum of the E_{exc} and the E_{ND}, energy of the minimum, tabulated in the .bap files. The energy of the corresponding ND minimum is indicated close to the corresponding line. Bottom panel shows the average ND level density as function of spin for three different excitation energies above the SD minimum.*

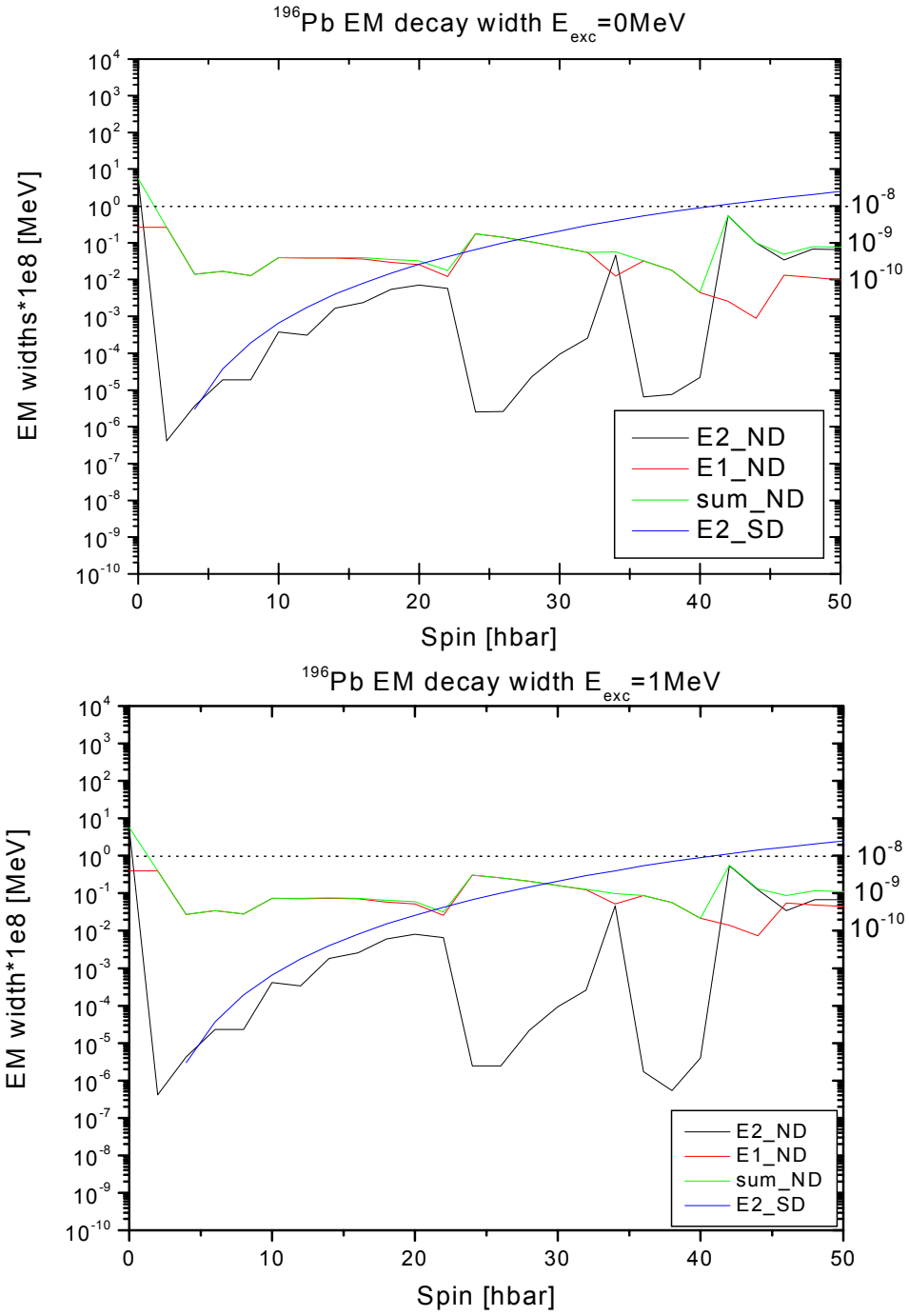

Figure 5 *Calculated quantities Γ_{E1} (red line), Γ_{E2} (black line) and the sum of the two (green line) for the ND minima and Γ_{E2} (blue line) for SD minimum as function of spin. Top panel refers to E_{exc} =0 MeV while bottom panel to E_{exc} =1 MeV. In the case of ND minima we calculated the average Γ_{E2} and Γ_{E1} over all the minima weighting over the level density.*

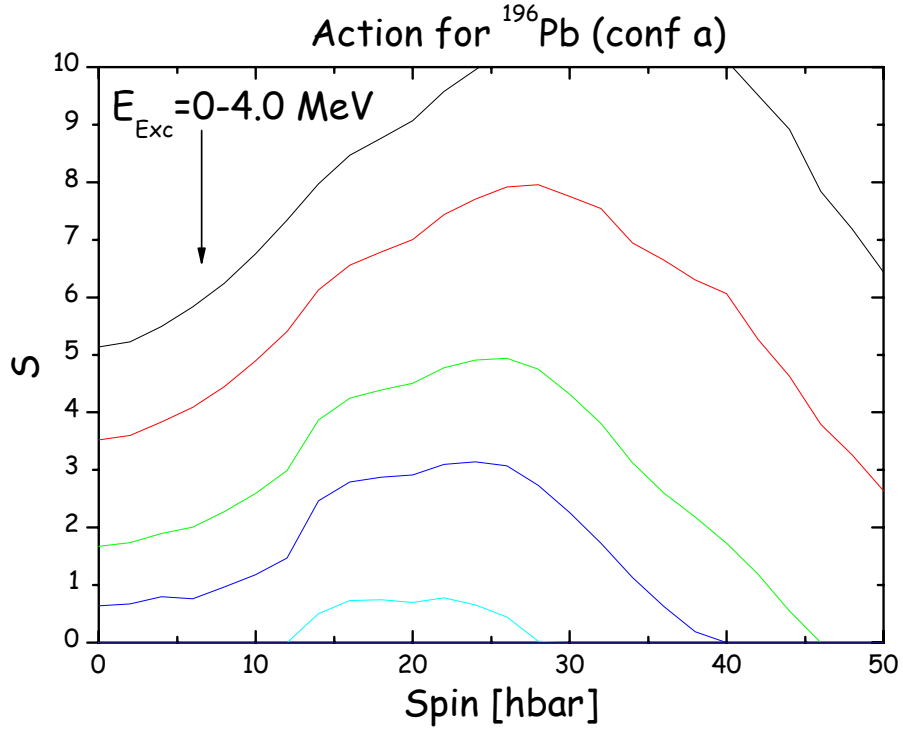

Figure 6 *Action S as function of spin and excitation energy. The energy is increasing going from the top to the bottom of the picture as indicated by the arrow with intervals of 0.5 MeV. The values are directly extracted from files .act.*

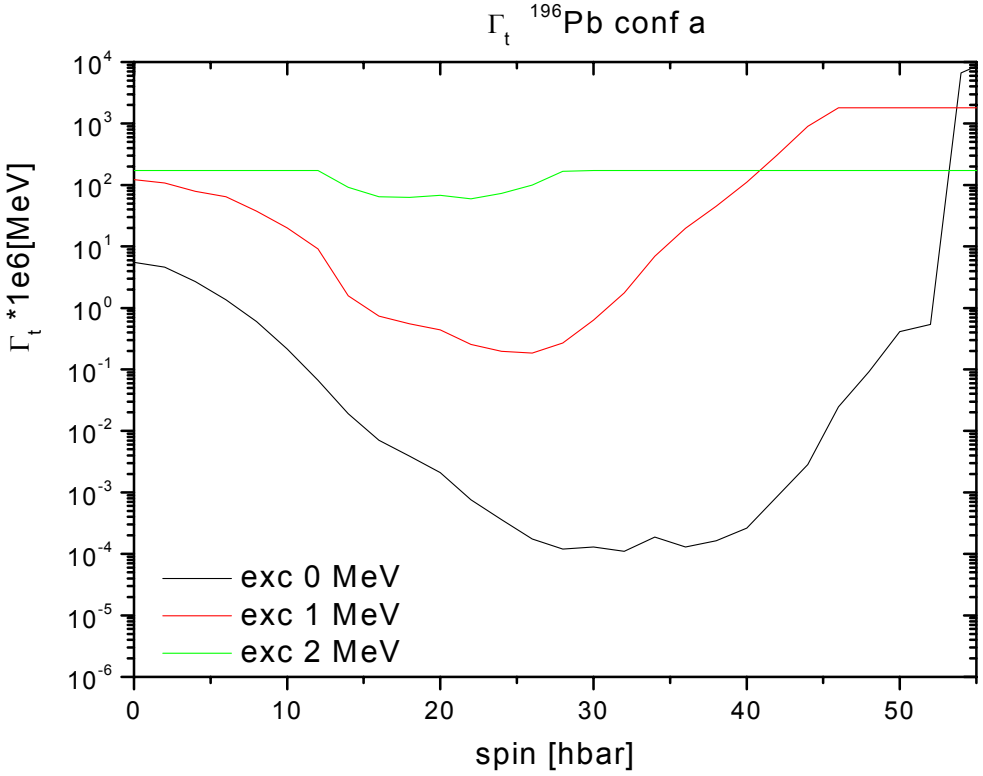

Figure 7 *Γ_t as function of spin for three different excitation energies E_{exc} above the SD minimum: 0, 1 and 2 MeV. Γ_t is multiplied by a factor 10^6 for sake of representation.*

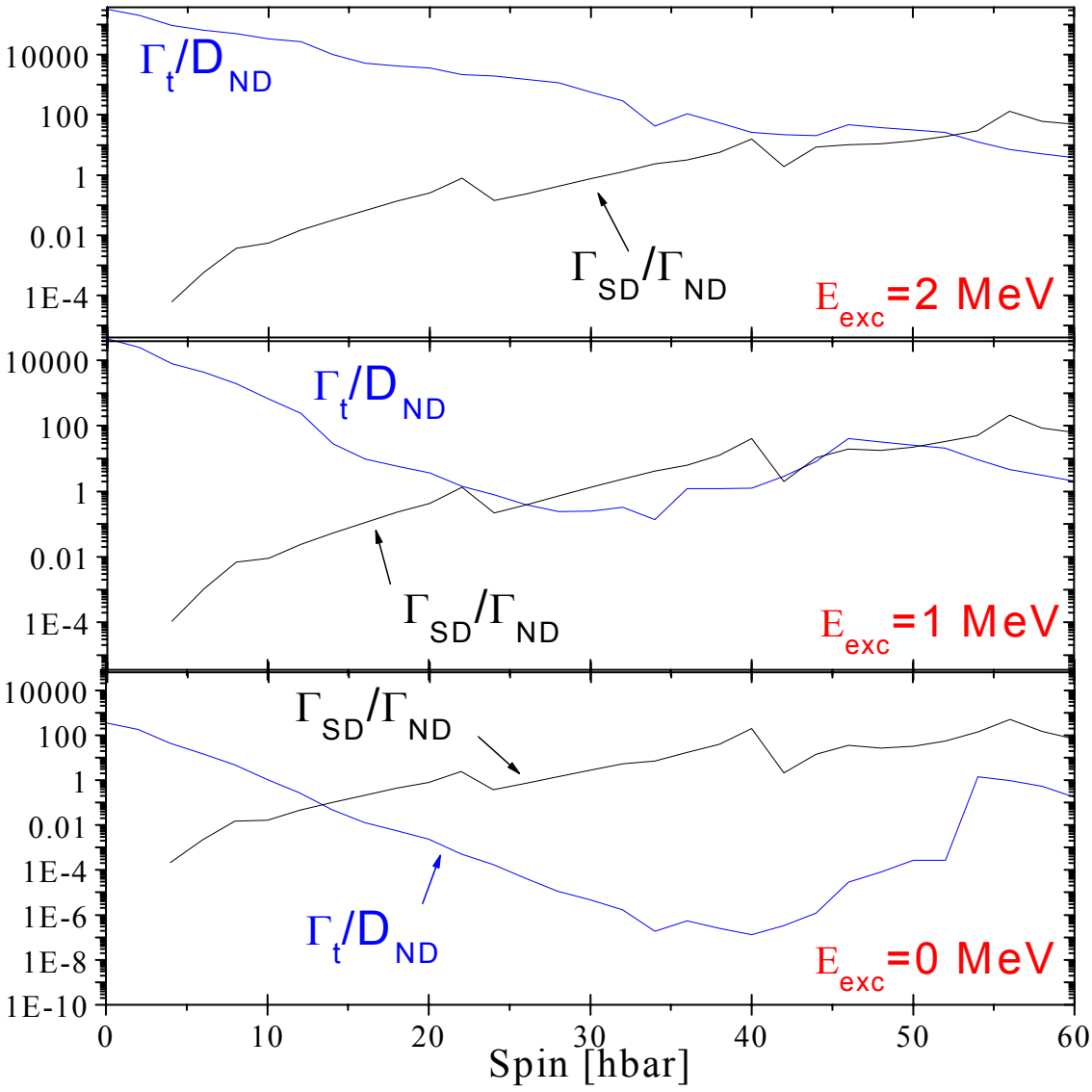

Figure 8 *Comparison of the two factors contributing to P_{out}. They are plotted as function of spin for different excitation energies above the SD minimum, E_{exc} = 0, 1 and 2 MeV.*

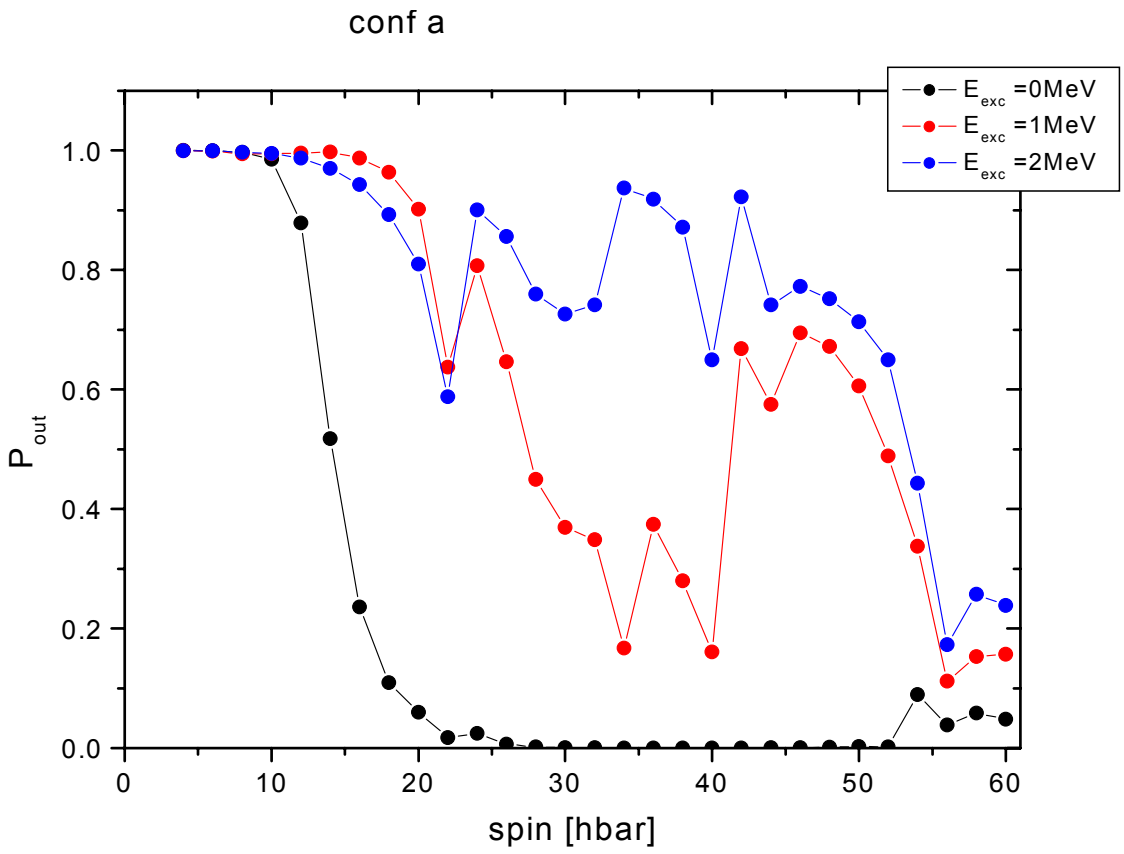

Figure 9 *P_{out} as function of spin and excitation energy. The curve seems to be quite steep E_{exc}=0 (black curve) and gets lower than 1 very rapidly, while the behaviour changes at higher E_{exc}, for which the curves show a deep and then regain higher values before getting closer to zero only at the very highest spins.*

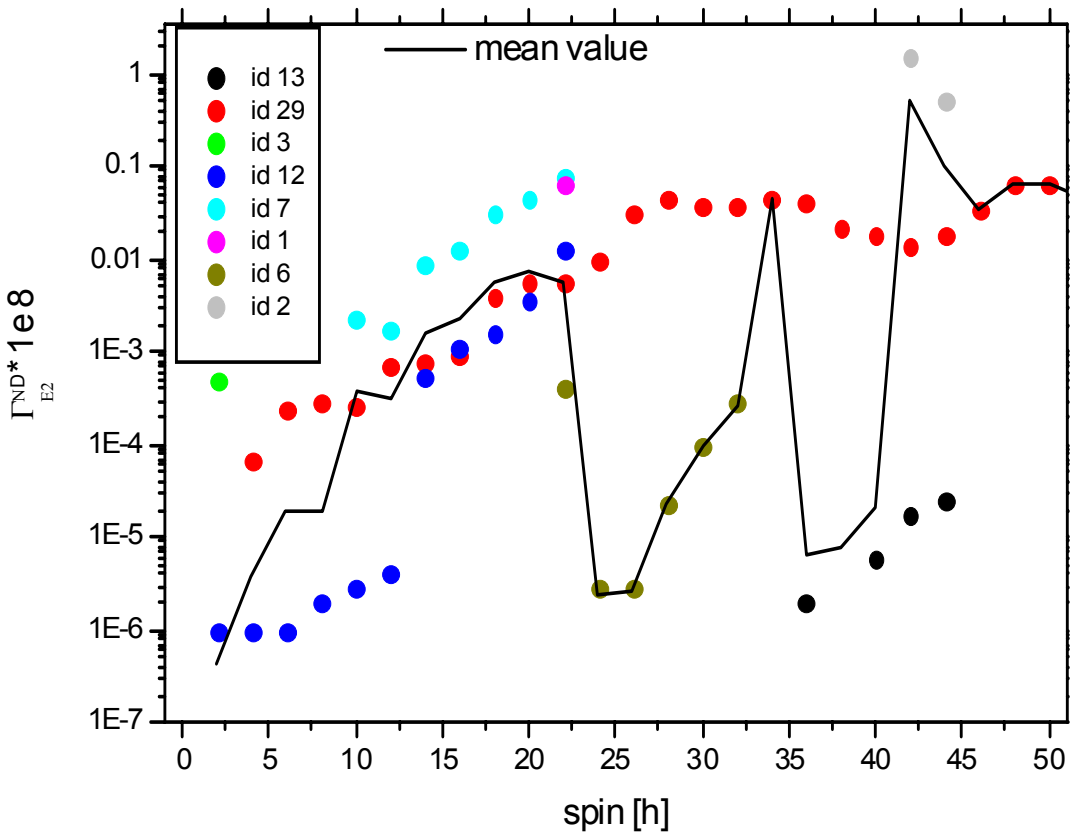

Figure 10 *Comparison between the Γ_{E2} evaluated for each single ND minimum present in conf.a file pb196a.bap. The numbering of the minima reflect that of the file. The black full line shows the average calculated by weighting over the level density of each minimum. As it is clear from the picture some minima drive the average away from a smooth line since they are locally deeper.*

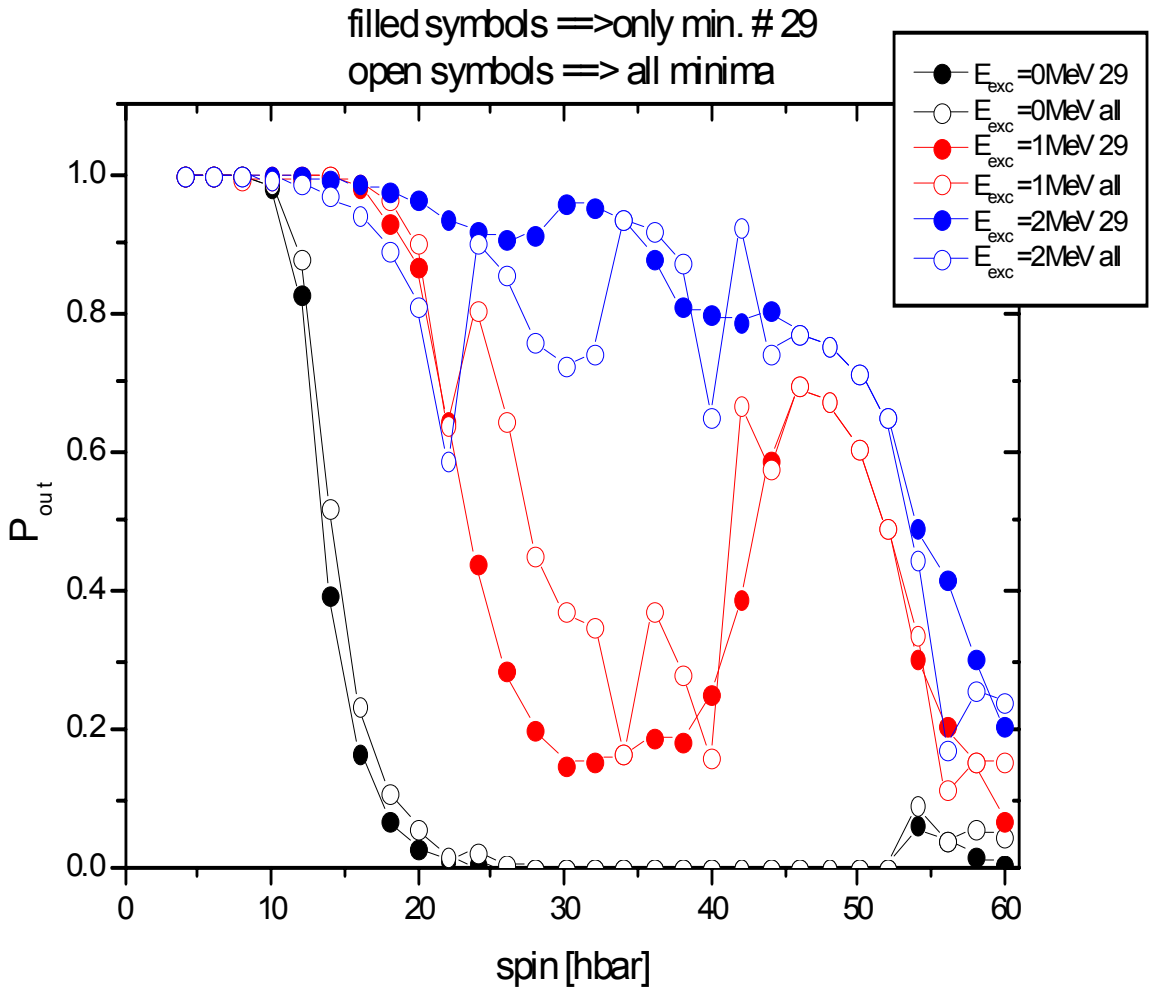

Figure 11 *Comparison of P_{out} evaluated over all the minima or including just one minimum, number 29 as listed in file pb196a.bap. The data refer only to configuration "a" for sake of simplicity. Three different excitation energies are here represented, E_{exc} =0,1,2 MeV above the SD minimum (black, red and blue curves respectively). Open symbols refer to P_{out} evaluated using quantities averaged over all the minima, while filled ones are calculated only including minimum #29. Filled symbols seem to follow a smoother trend than the open ones, particularly at increasing excitation energy above the minimum. The P_{out} does not go smoothly to zero, but instead increases back at spins > 40 (for E_{exc}=1MeV). This is due to the particular behaviour of the action itself.*

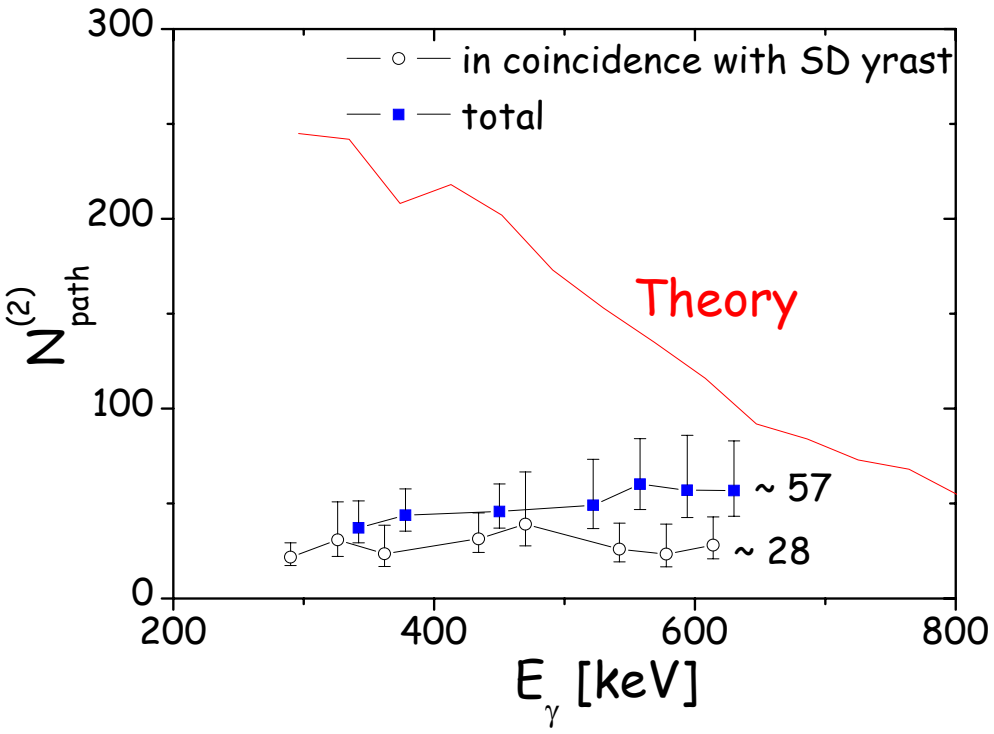

Figure 12 *Experimental N_{path} obtained in the total matrix (full squares) and in direct coincidence with the SD yrast band (open circles) together with predictions not including tunneling (red line).*

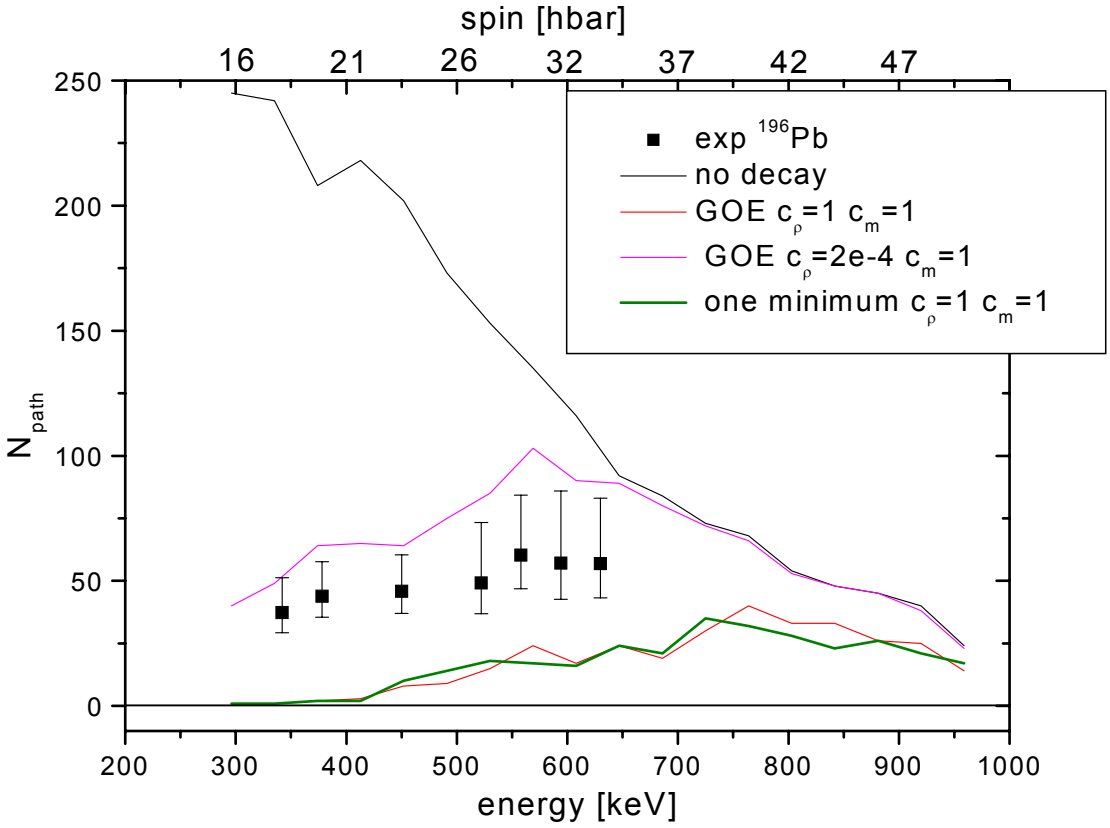

Figure 13 *Experimental data obtained from the total matrix (symbols) compared to calculated Npath using different values for C_ρ and C_{mass}. When the two quantities are set to 1 (red curve) one sees that the Npath si underestimated and the decay-out spin is clearly too high. By playing with C_ρ (magenta line C_ρ=2e-4) we can reproduce the data better, even if we slightly overestimate the values.*

We also show the calculations without tunneling (black line) for comparison. In addition we also show, with a green line, calculations obtained considering only one ND minimum for each configuration (the one extending over a wider spin range). No difference is visible comparing to the values obtained averaging over all minima (red line).