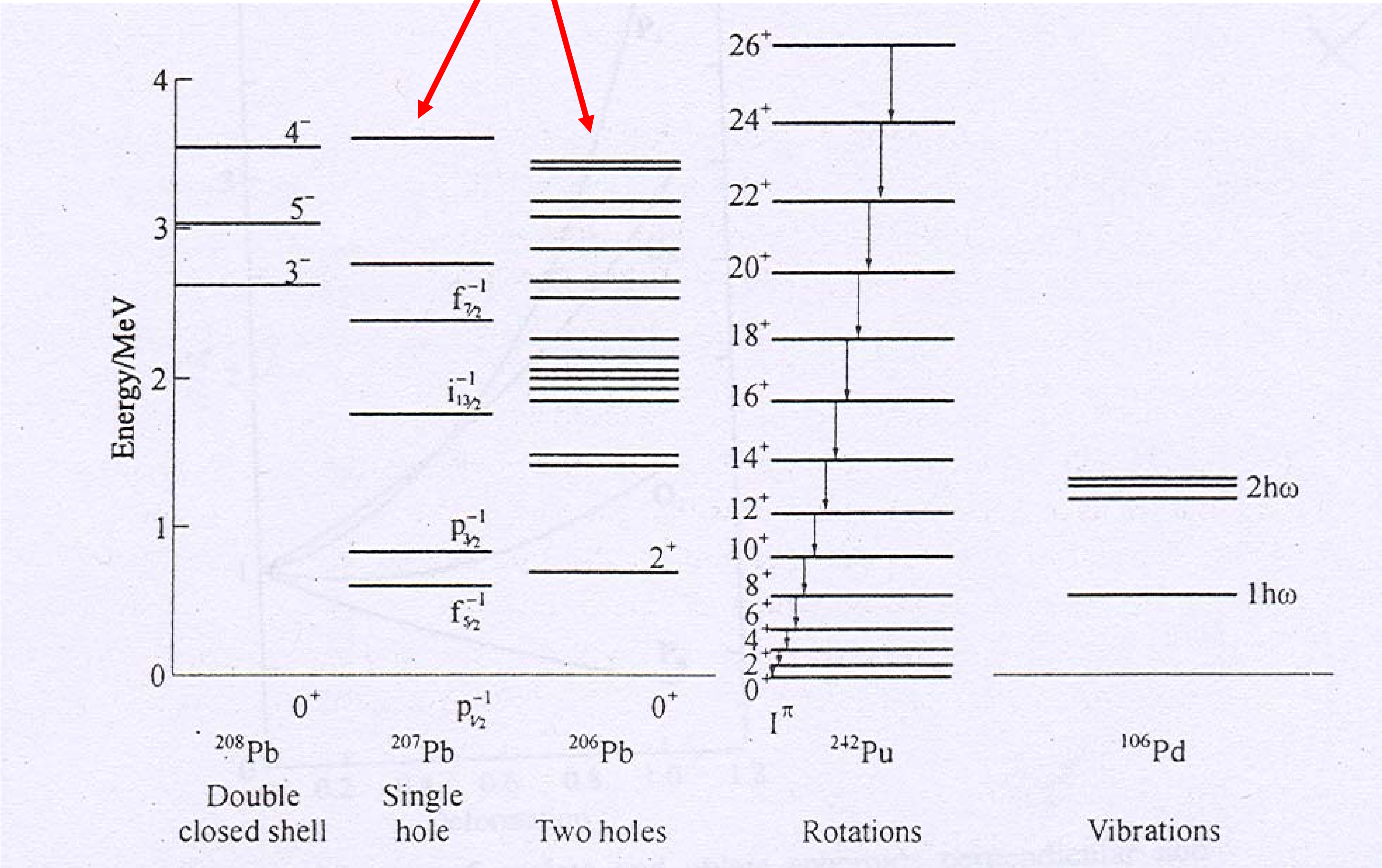
Nuclear Rotation

- Experimental evidences for collective modes
- Deformed Nuclei, shell model
- Indicators for rotational collectivity:
  moment of inertia, electric quadrupole moment,
  lifetime measurements

The energy spectra shope different structures
depending on the number **n** of nucleons outside the closed shell

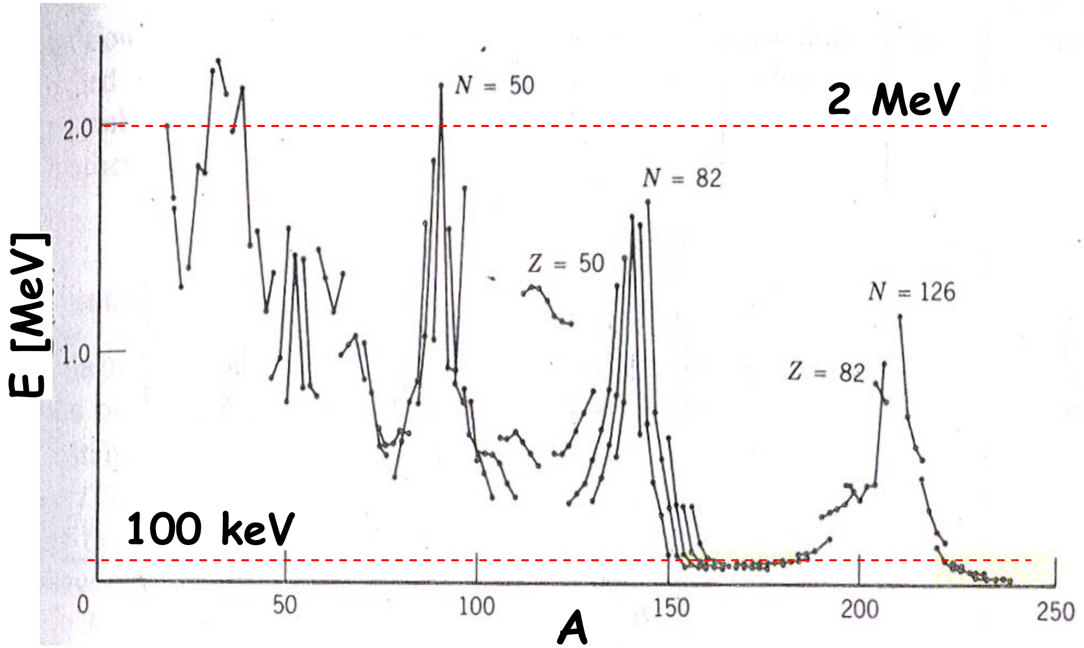
shell model states:
n = ± 1, ± 2 ...

middle shell:
simple spectra



Even-even Nuclei:

how can one interprete the energies of the 2^+ states?



energy for breaking a pair

Energy of 2^+ state decreases with A

0^+ (ground state): all nucleons are coupled to spin 0

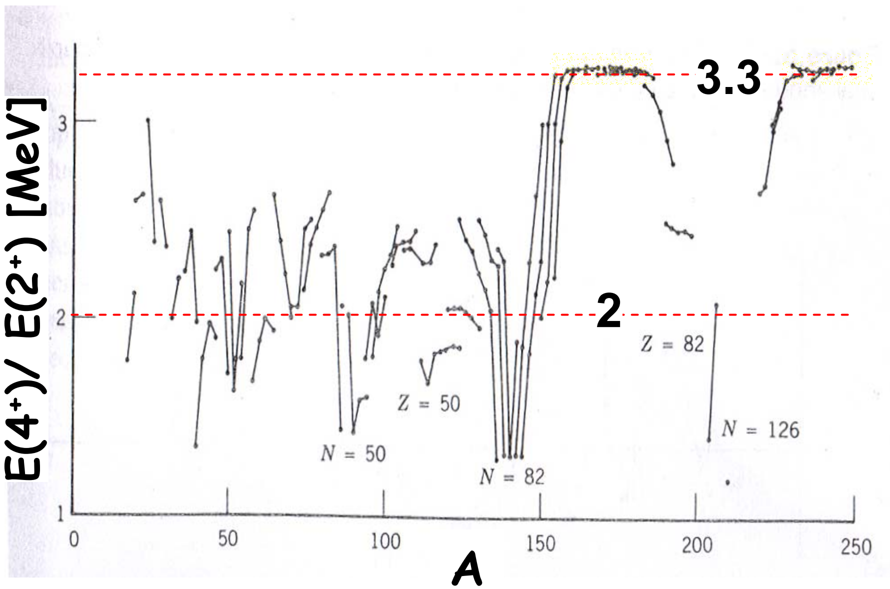
2^+ (first excited state): in middle shells the nucleus realizes an intrinsic configuration energetically more favoured than the breaking of a pair

even-even Nuclei: how to explain E(4+)/E(2+) and Q(2+) ?

E(4+)/E(2+) ratio

A<150: E(4+)/E(2+) ~ 2

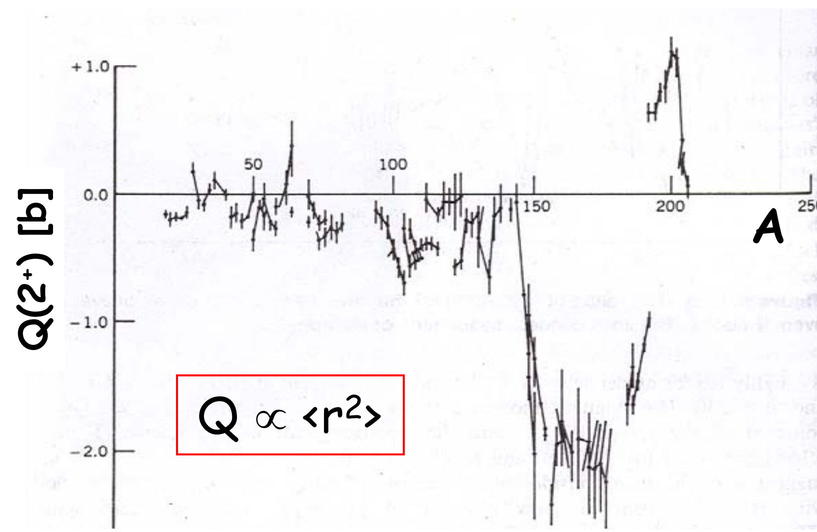
150<A<190 e A>230: E(4+)/E(2+) ~ 3.3



electric quadrupole $Q(2^+)$

A<150 $Q(2^+)$ ~ 0

150<A<190 e A>230: $Q(2^+)$ grande



Two different collective behaviour :

A<150	vibration of a stable spherical nucleus
150<A<190	rotation of a stable deformed nucleus

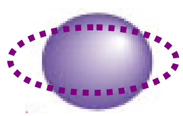
Collective Vibrations and Rotations

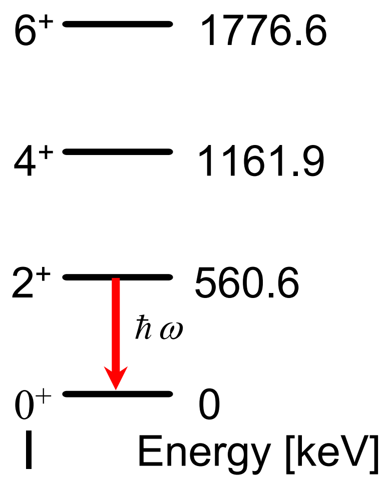
Vibrational Nucleus

$$\boxed{E_n = n \cdot \hbar\omega}$$

E(4+)/E(2+) ~ 2

^{120}Te



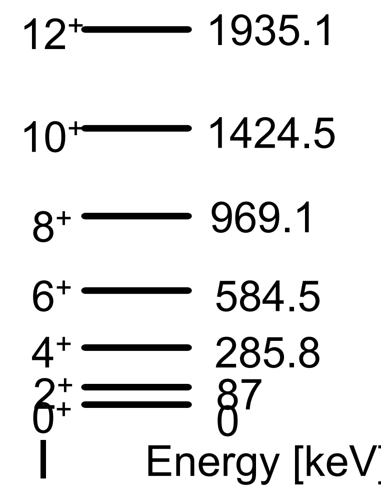


Rotational Nucleus

$$\boxed{E = \frac{\hbar^2}{2\Im} I(I+1)}$$

E(4+)/E(2+) ~ 3.3

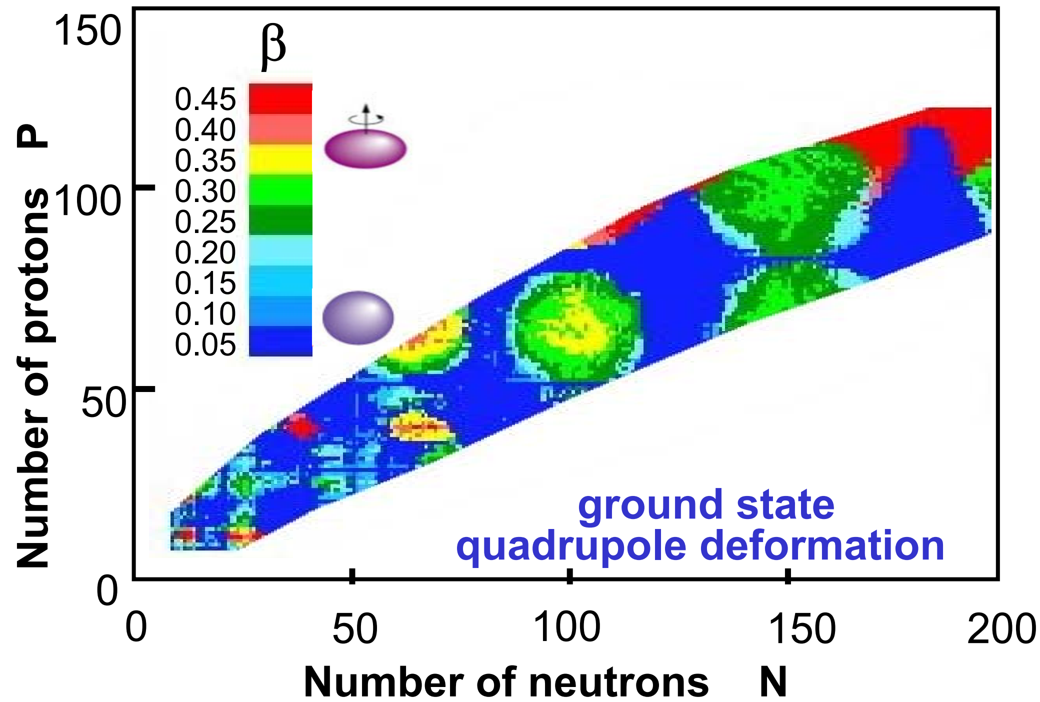
^{168}Yb



Rotational Motion:

It can be observed only in nuclei
with stable equilibrium deformation

deformed nuclei



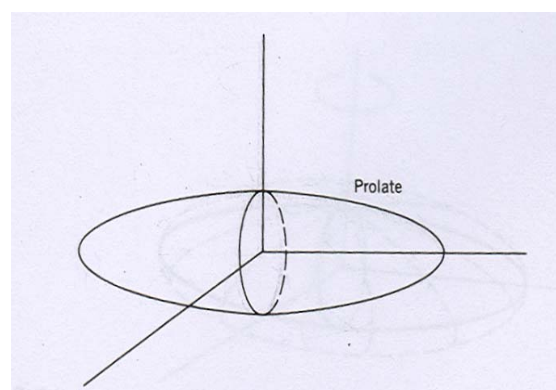
The shape of the nucleus is represented by
an ellipsoid of revolution:

$$R(\theta,\phi) = R_{av}\left[1 + \beta Y_{20}(\theta,\phi)\right]$$

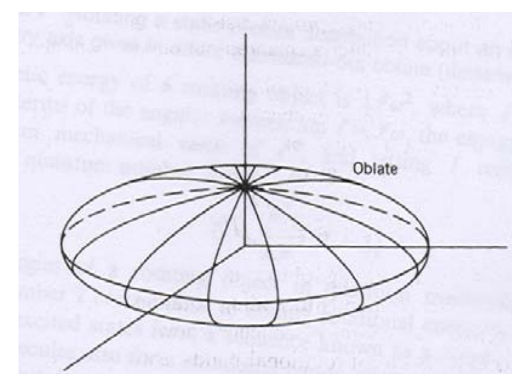
deformation parameter β
(ellipsoid eccentricity):

$$\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}}$$

$$R_{av} = R_0 A^{1/3}$$

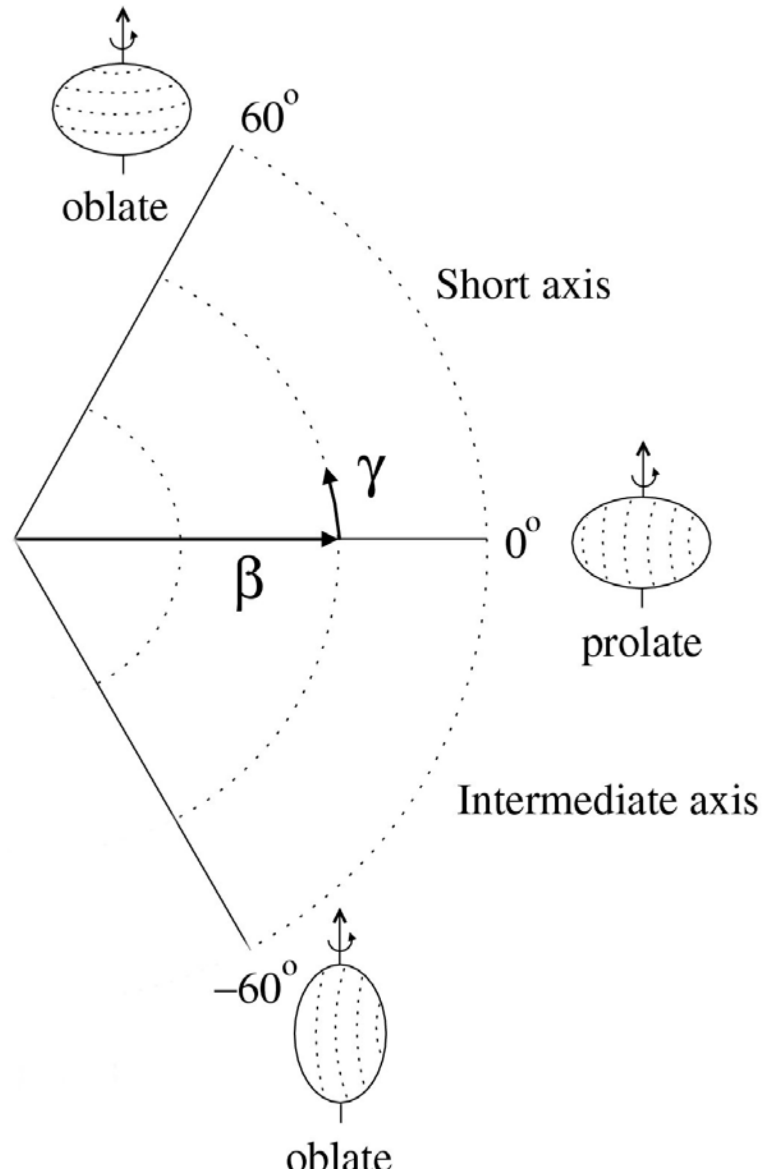


$\beta>0$ prolate



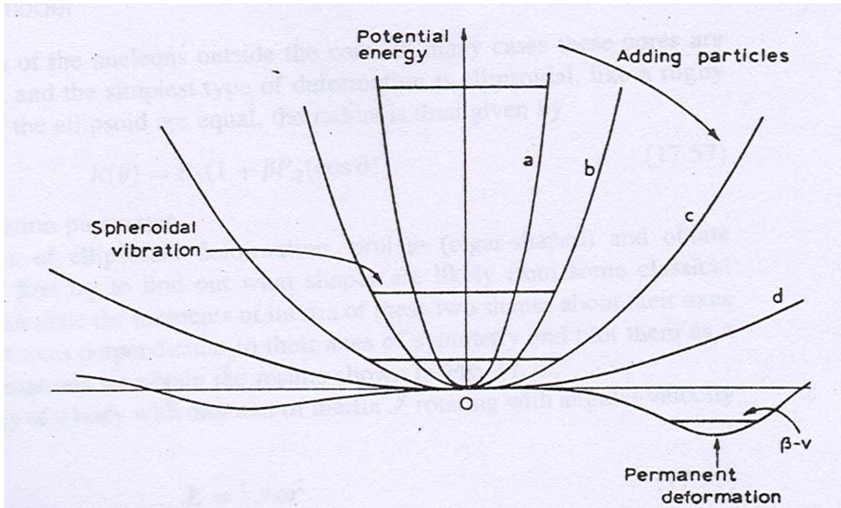
$\beta<0$ oblate

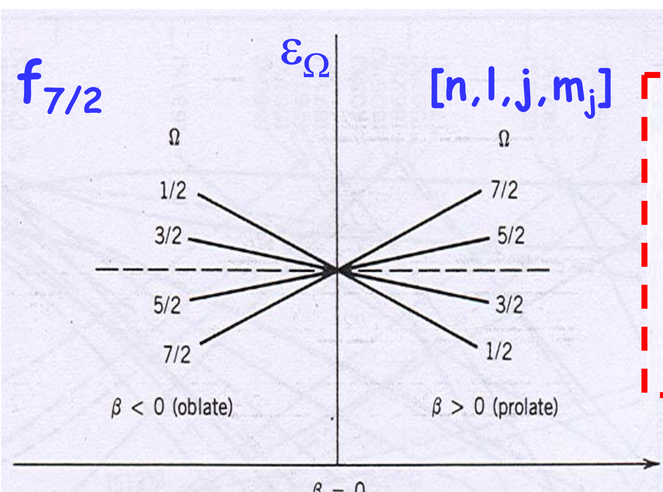
Nuclear Shapes

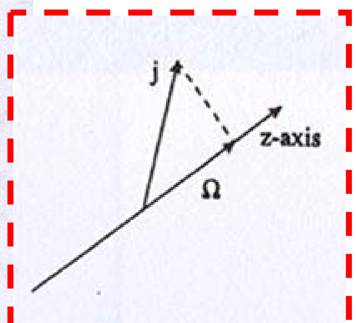


the deformed potential gives
the nuclear shape

(Bohr & Mottelson, 1950)

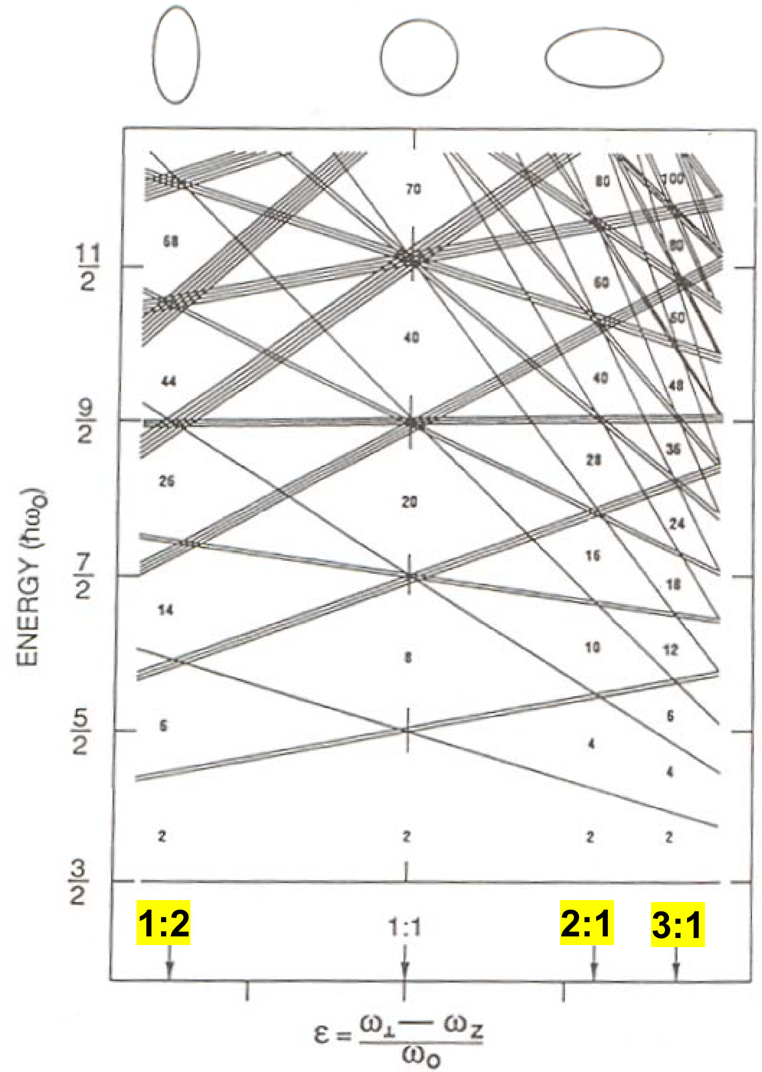






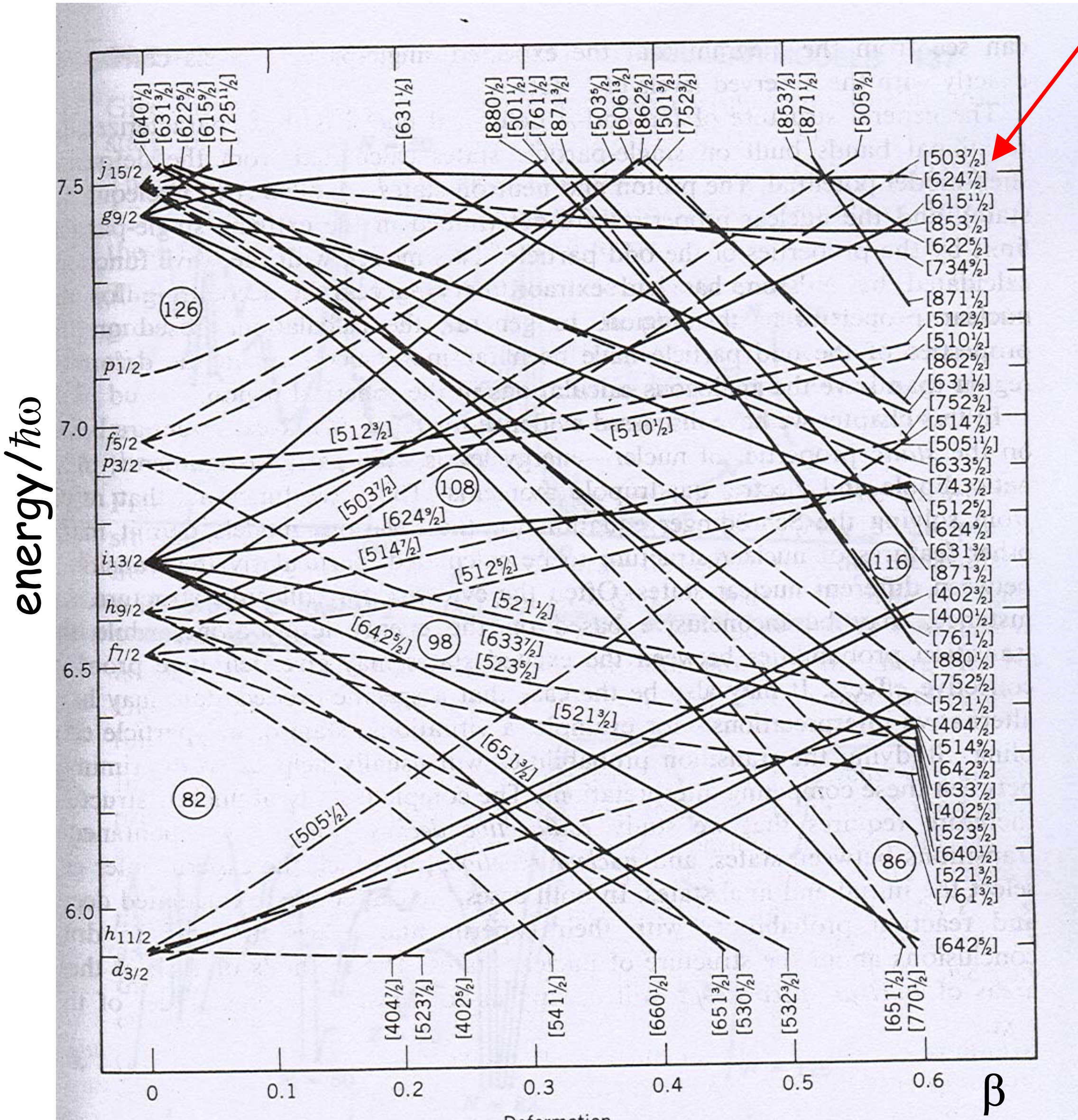
the energy levels loose
the (2j+1) degeneracy

deformed harmonic oscillator



appearance of new magic numbers:
- superdeformed nuclei (2:1)
- hyperdeformed nuclei (3:1)

Nilsson diagram for neutrons in a prolate deformed potential

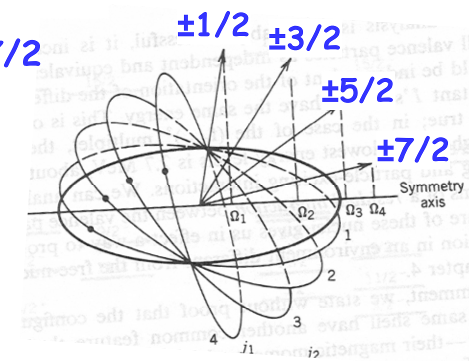


$[N,l,j,m_j]$

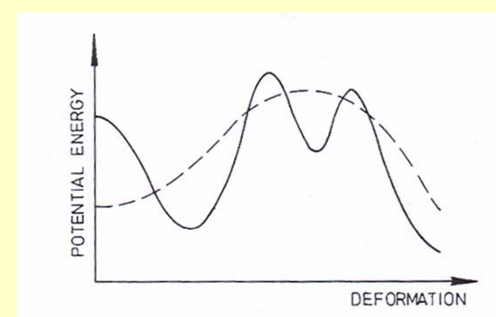
$\pi = (-1)^N$

$\Omega = m_j$

$f_{7/2}$

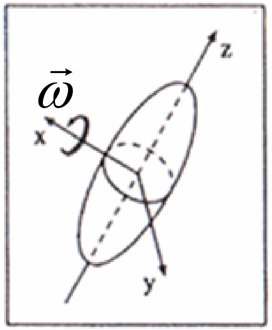


at ω=0 the energy levels show a 2 fold degeneracy: $\pm m_j$

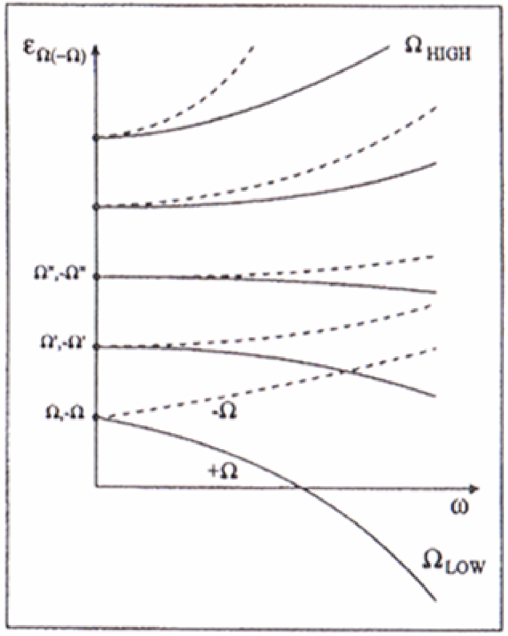


- New magic numbers
- New minima at larger deformations

Rotation removes the time-reversal invariance



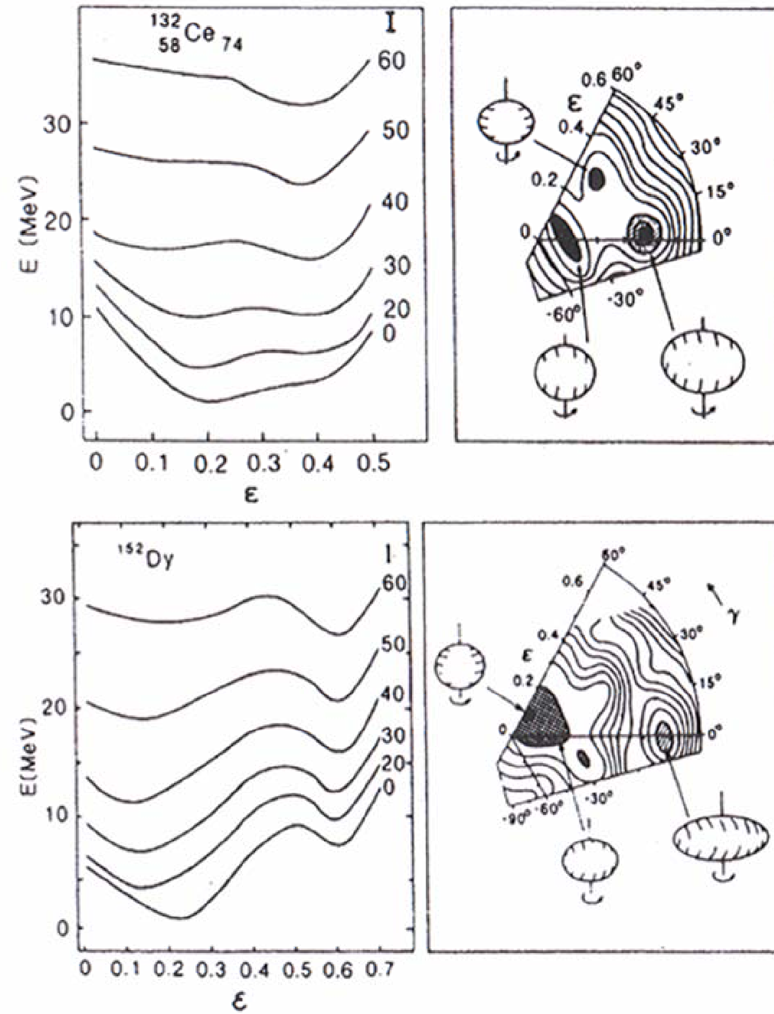
$H_\omega = H_o - \hbar\omega j_x$



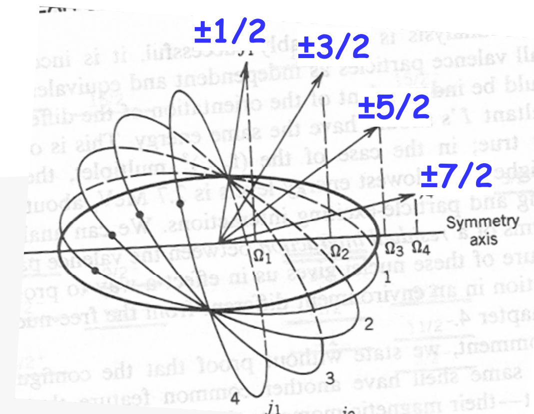
The **Coriolis interaction** gives rise to forces of opposite sign, depending whether a nucleon moves clockwise or anti-clockwise

$\Rightarrow$ splitting of $\pm m_j$ energy levels

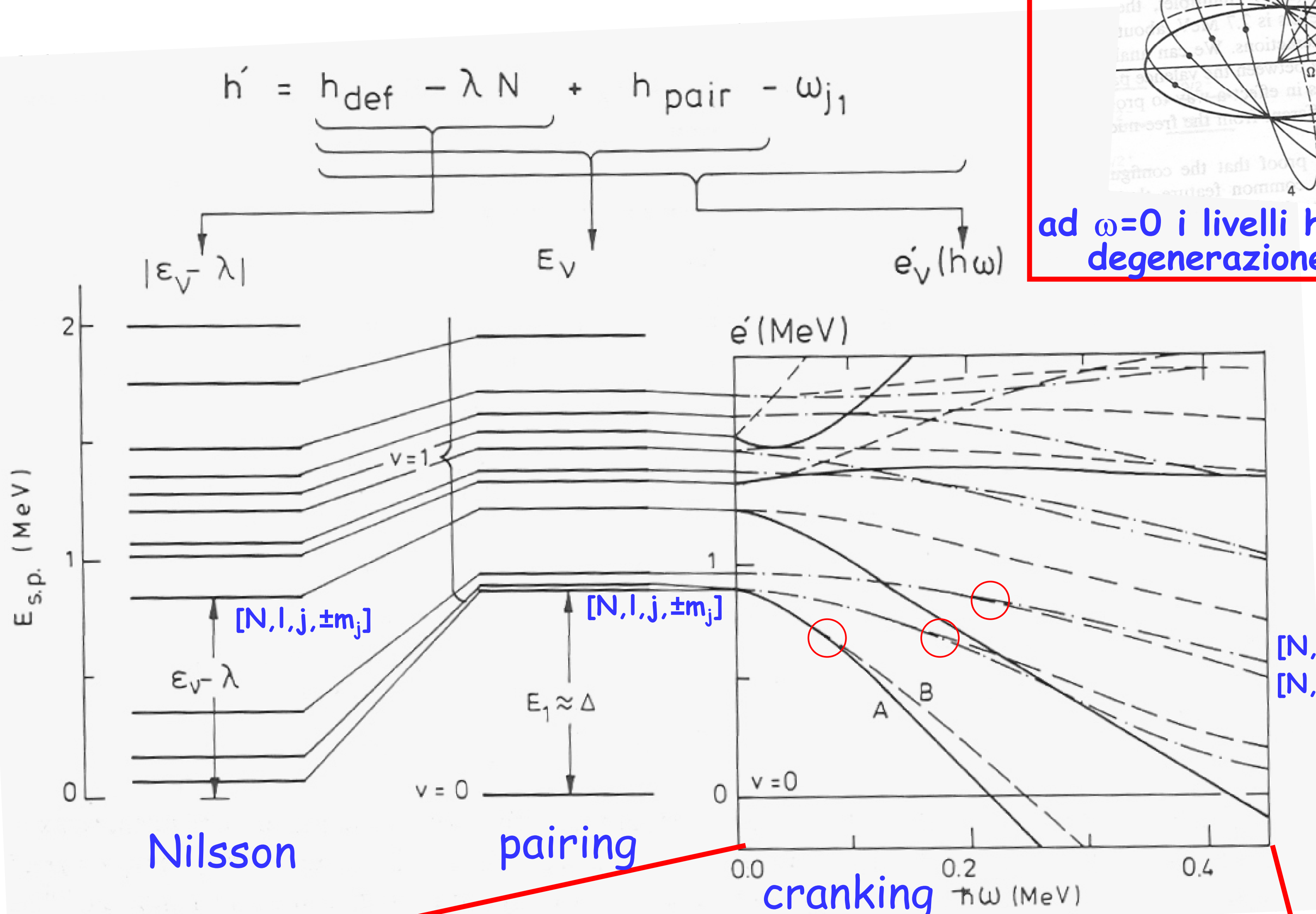
$\Rightarrow$ **changes of shell structures with rotation**



Appearance of **favorite deformed** minima at high spins

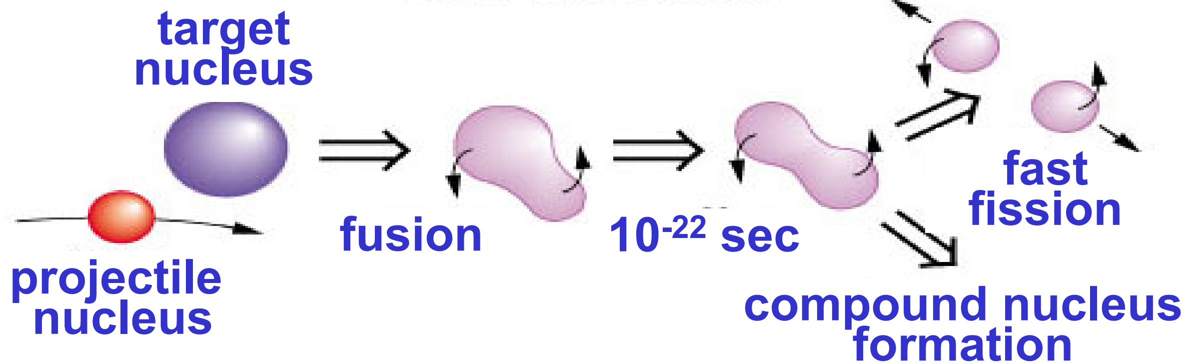


ad ω=0 i livelli hanno degenerazione 2

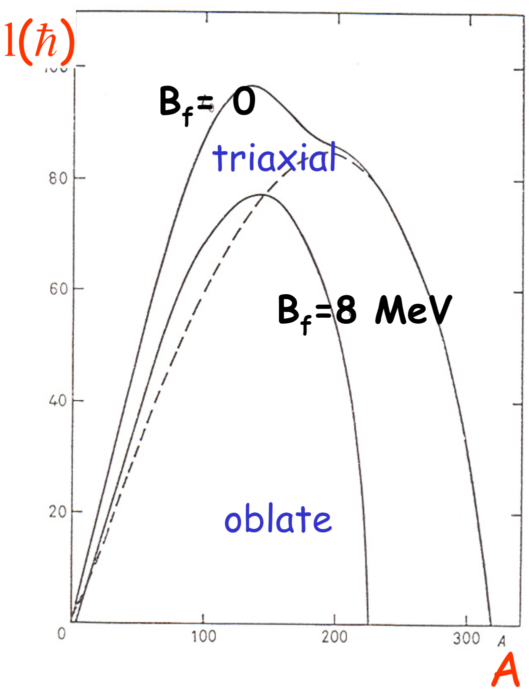


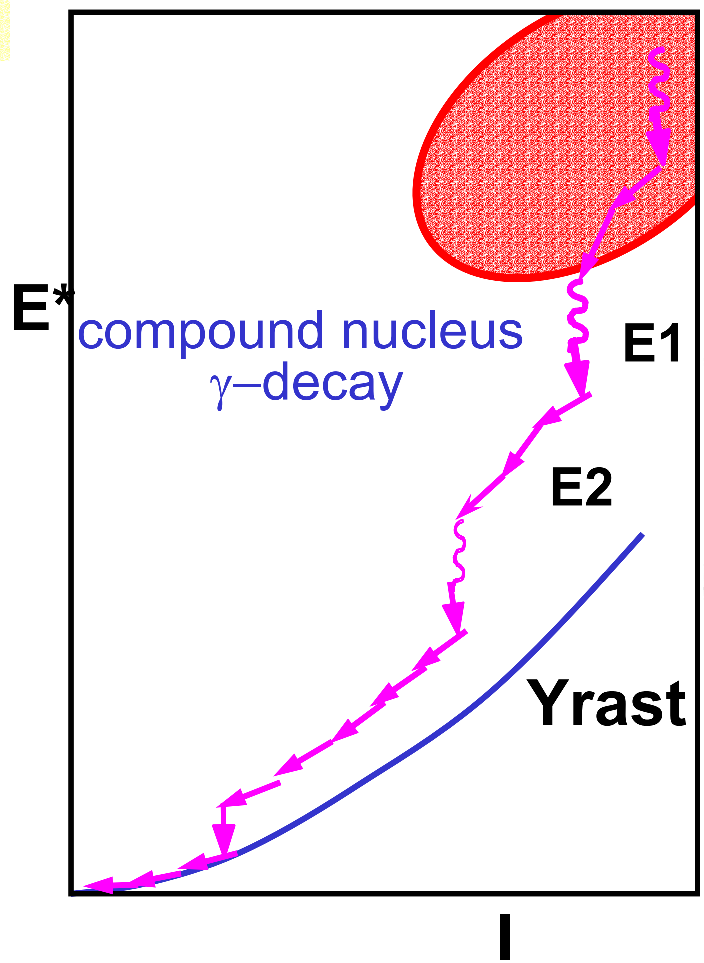
la rotazione provoca una rottura nella degenerazione in m_j

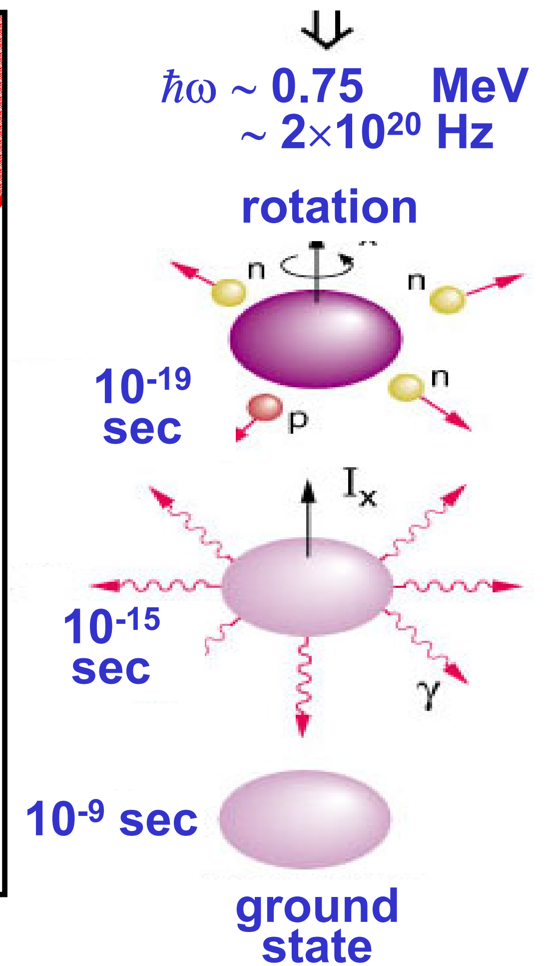
Heavy ion reactions
allow to populate
nuclear states
at high angular momenta
(≥40 $\hbar$)

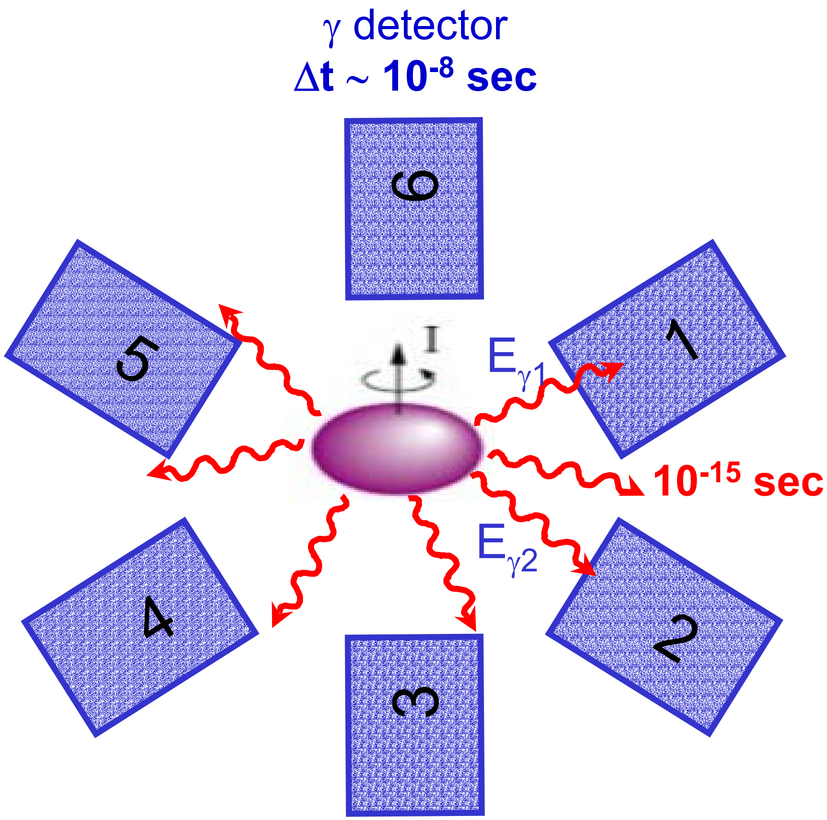


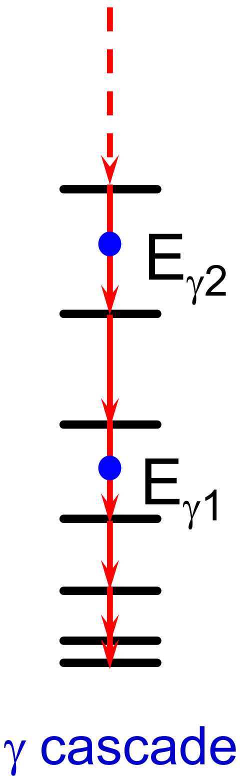
Angular momentum limits
(liquid drop calculations)

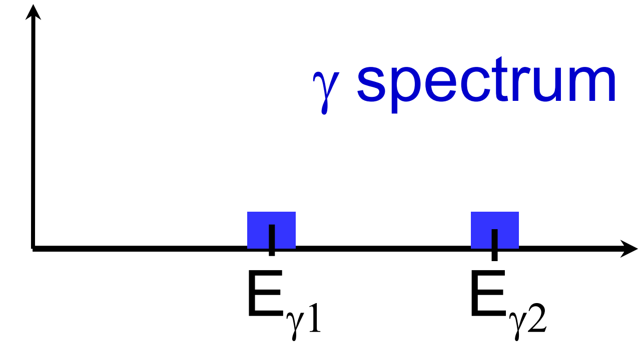


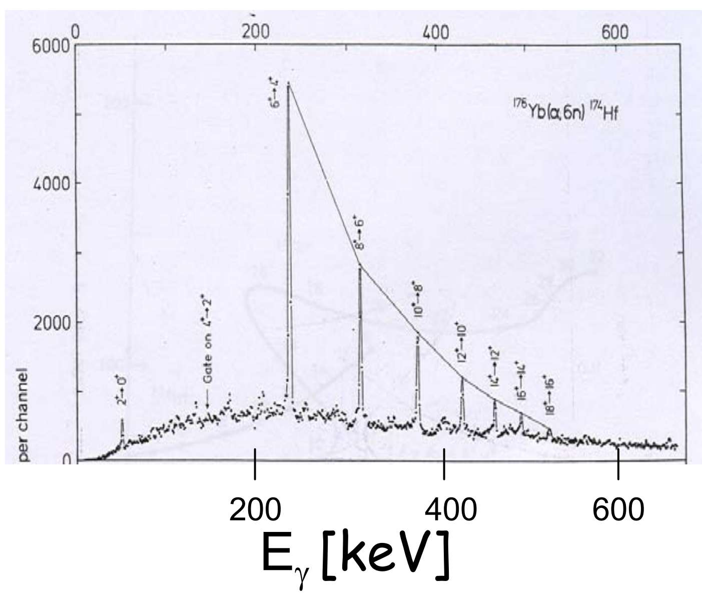





γ detector
Δt ~ 10-8 sec
6
5
1
Eγ1
10-15 sec
Eγ2
4
3
2


Eγ2
Eγ1
γ cascade


γ spectrum
Eγ1
Eγ2


0
200
400
600
6000
4000
2000
0
per channel
176Yb(α,6n) 174Hf
200
400
600
Eγ [keV]

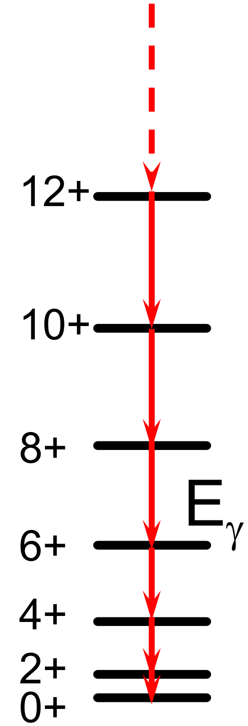
rotational energy of a body with

moment of inerzia $\Im$
angular velocity $\omega = I/\Im$

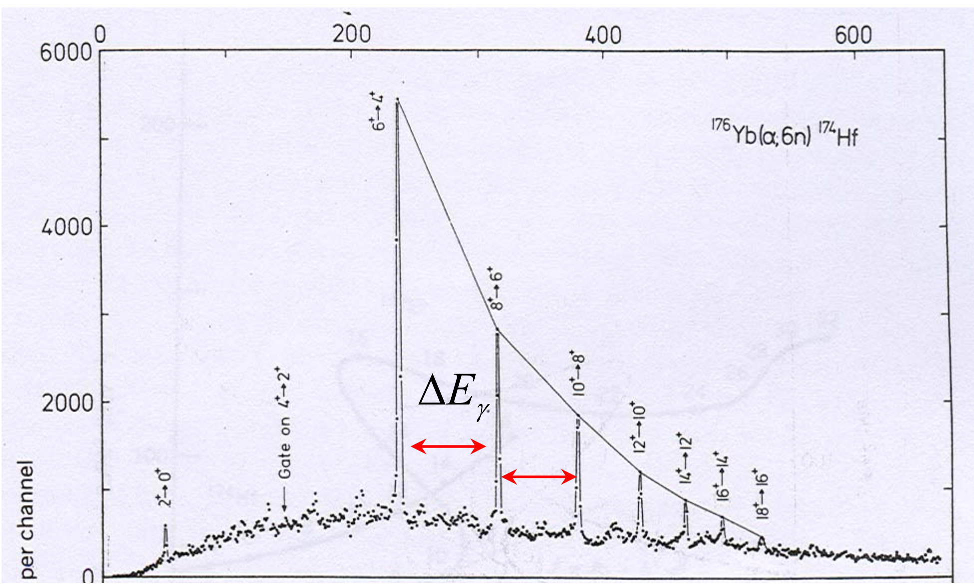
$$E = \frac{1}{2}\Im\omega^2 = \frac{I^2}{2\Im} \rightarrow \frac{\hbar^2}{2\Im} I(I+1)$$

even-even nuclei: $0^+, 2^+, 4^+, 6^+, \ldots$

E(4+)/E(2+) ~ 3.3



rotational band



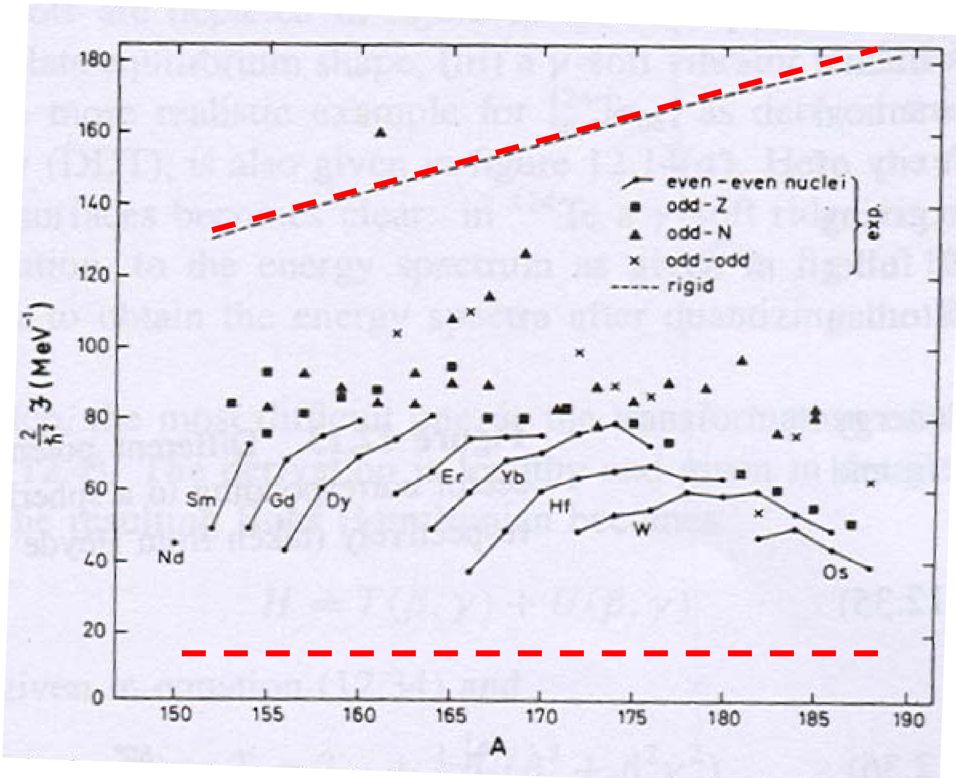
Channel number

$$E(I) = \frac{\hbar^2}{2\Im} I(I+1)$$

$$E_\gamma(I) = E(I+2) - E(I) = \frac{\hbar^2}{2\Im} 4I$$

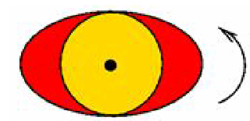
$$\Delta E_\gamma = E_\gamma(I+2) - E_\gamma(I) = \frac{4\hbar^2}{\Im}$$

The nucleus is NON a rigid body, is NON an irrotational fluid



$$\Im_{rigid} = \frac{2}{5} M R_{av}^{2} (1 + 0.31\beta)$$

$$\Im_{fluid} < \Im_{exp} < \Im_{rigid}$$

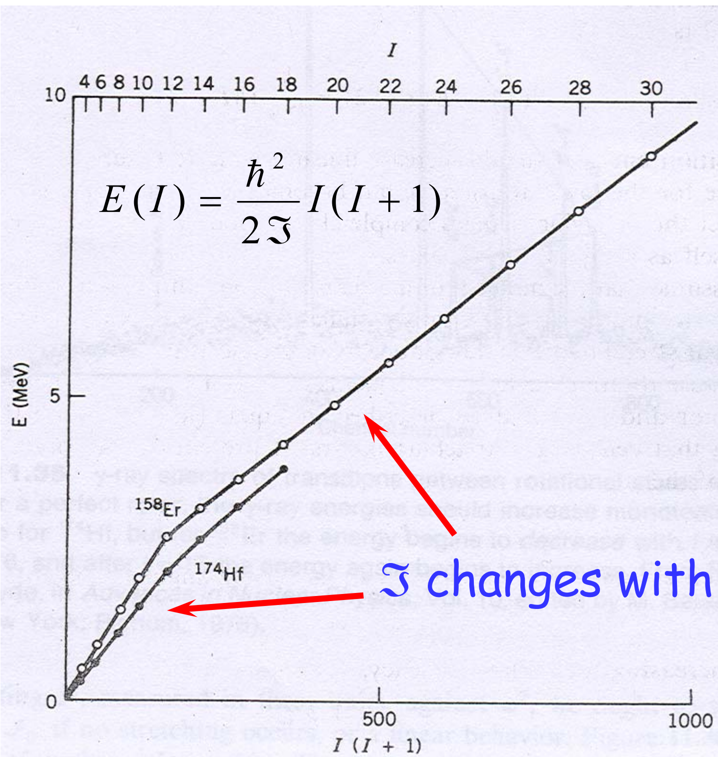


$$\Im_{fluid} = \frac{9}{8\pi} M R_{av}^{2} \beta$$

this is a consequence of the short range nature of the nuclear force:
strong forces exist only among close nucleons
$\rightarrow$ The nucleus does not show the long range structure typical of rigid bodies

Additional evidence for **lack of rigididy**:

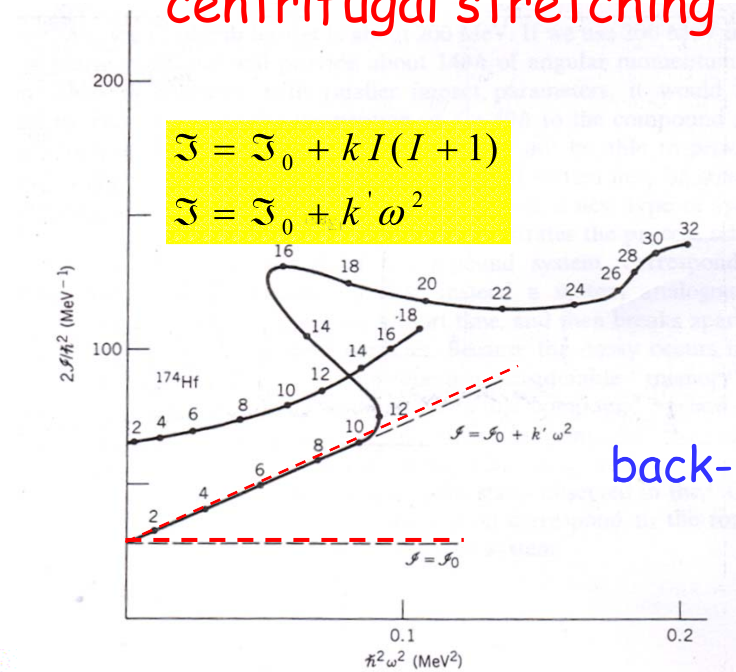
$\Im$ is NON constant with I



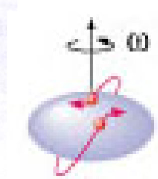
$$E(I) = \frac{\hbar^2}{2\Im} I(I+1)$$

$\Im$ changes with I

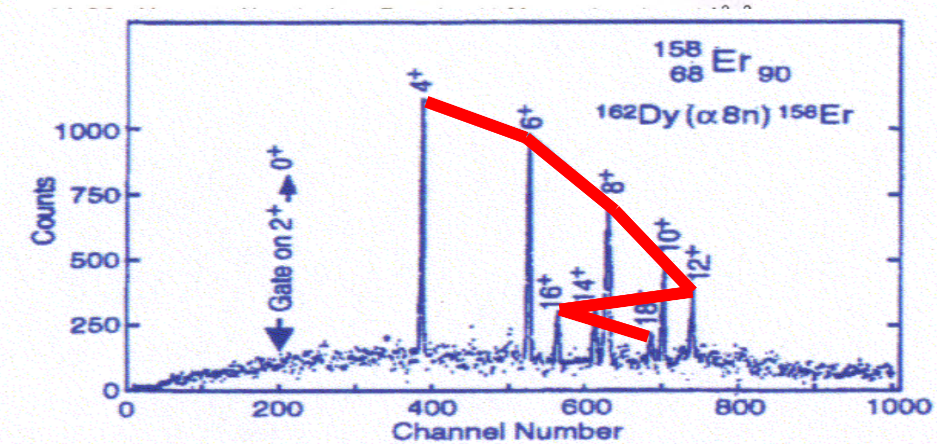
$\Im$ increases with the rotation
(as it happens in fluids, but not in rigid bodies)
"centrifugal stretching"



$$\Im = \Im_0 + k\,I(I+1)$$
$$\Im = \Im_0 + k'\omega^2$$



back-bending



back-bending takes places when
the rotational energy exceeds the energy
required to break a pair of nucleons

Unpaired nucleons go to different orbits
and change the momet of inertia

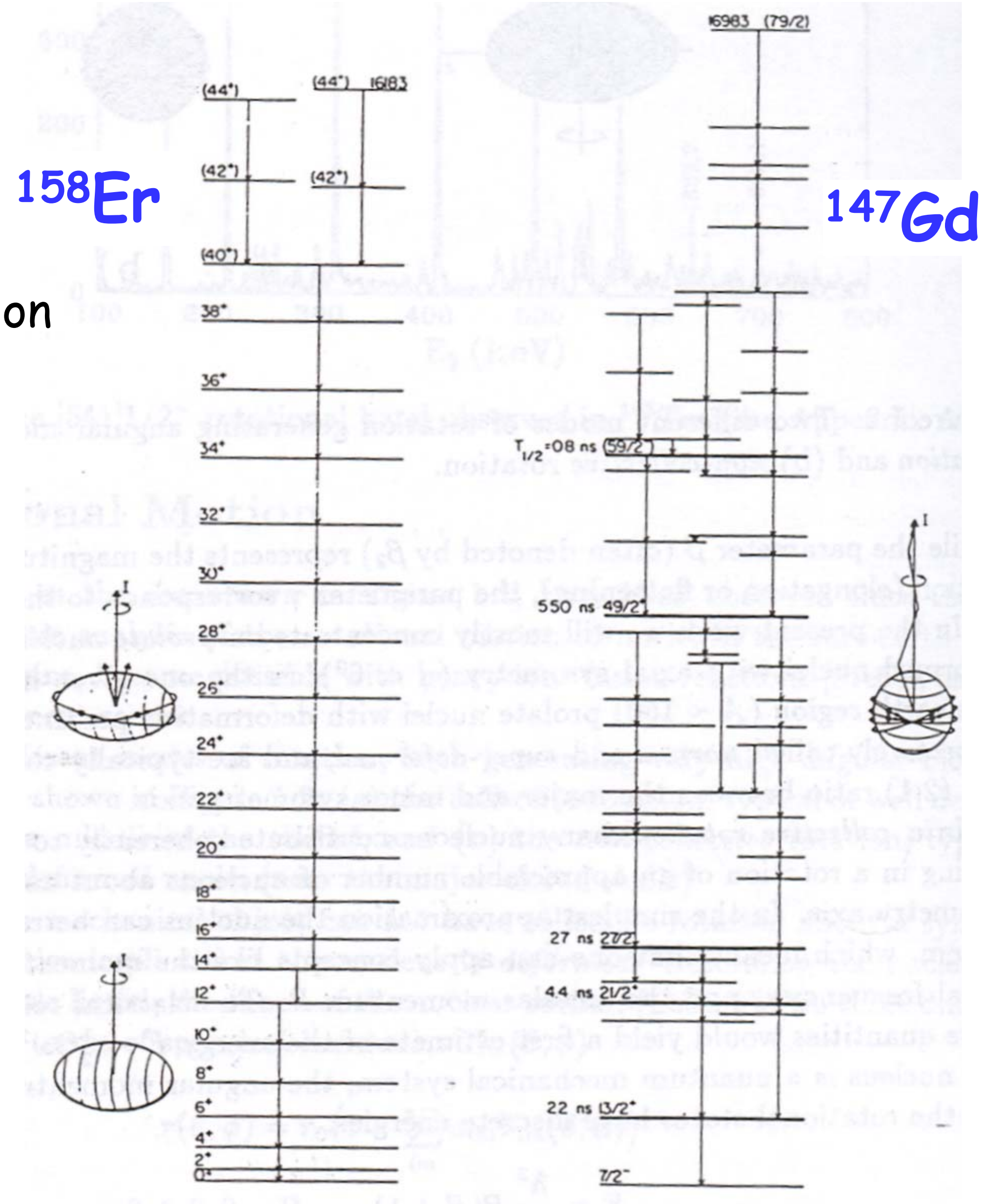
The nucleus can constract the rotation in two different ways:

$$I = R + i = \Im\,\omega + i$$

collective motion

single particle motion

$$E(I) = \frac{\hbar^2}{2\Im} R(R+1) + E(i)$$



Deformed nucleus

quasi-spherical nucleus

Indicator of collectivity of the nuclear system: the quadrupole moment Q_0

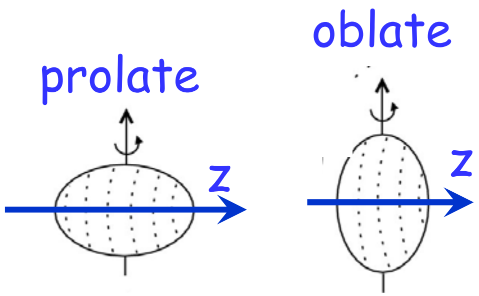
Q_0 measures the deviation from a symmetric distribution of the nuclear charge distribution

$$Q = \int \rho \left[3z^2 - (x^2 + y^2 + z^2)\right] d\tau$$
$$= Z\left[3 < z^2 > -(< x^2 > + < y^2 > + < z^2 >)\right]$$

$= 0$ spherical shape $\langle z^2 \rangle = \langle x^2 \rangle = \langle y^2 \rangle$

> 0 prolate: elongation along z $\langle z^2 \rangle > \langle x^2 \rangle = \langle y^2 \rangle$

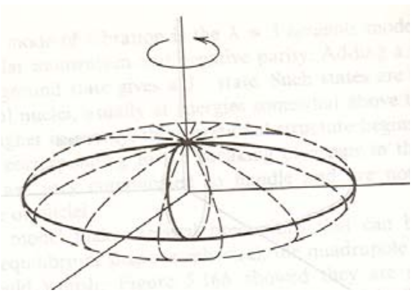
< 0 oblate: flattening along z $\langle z^2 \rangle < \langle x^2 \rangle = \langle y^2 \rangle$



Large electric quadrupole moments indicate a stable deformation :

$$Q_0 = \frac{3}{\sqrt{5\pi}} R_{av}^2 Z\beta(1 + 0.16\beta)$$

intrinsec quadrupole momenti
(osservable only in the intrinsic reference frame)



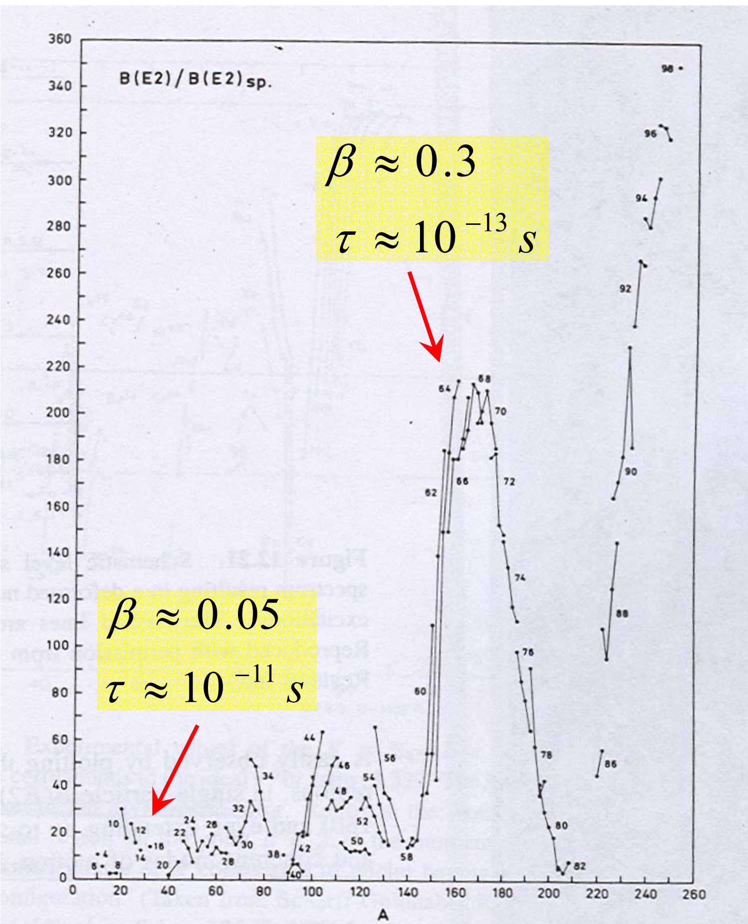
measured quadrupole moment

$$Q = -\frac{2}{7} Q_0 \quad for \quad 2^+$$

Q_0 can be obtained from the B(E2) reduced transition probability:

$$B(E2:J_i \rightarrow J_f) = \frac{5}{16\pi} e^2 Q_0^2 \left\langle J_i 020 \middle| J_f 0 \right\rangle^2$$

$$B(E2:2^+ \rightarrow 0^+) = \frac{5}{16\pi} e^2 Q_0^2 \propto \beta^2$$



$$B(E2) = \frac{0.08156\ B_\gamma}{E_\gamma^5 \tau (1+\alpha_{tot})} e^2 b^2$$

B_γ = γ decay probability

α_{tot} = probability for internal conversion

High Spin limit: $B_\gamma = 1, \quad \alpha_{tot} = 0$

$$\tau = \frac{0.08156}{B(E2) E_\gamma^5} \times 10^{-12}\ s$$

$B(E2)$ in W.U.

E_γ in MeV

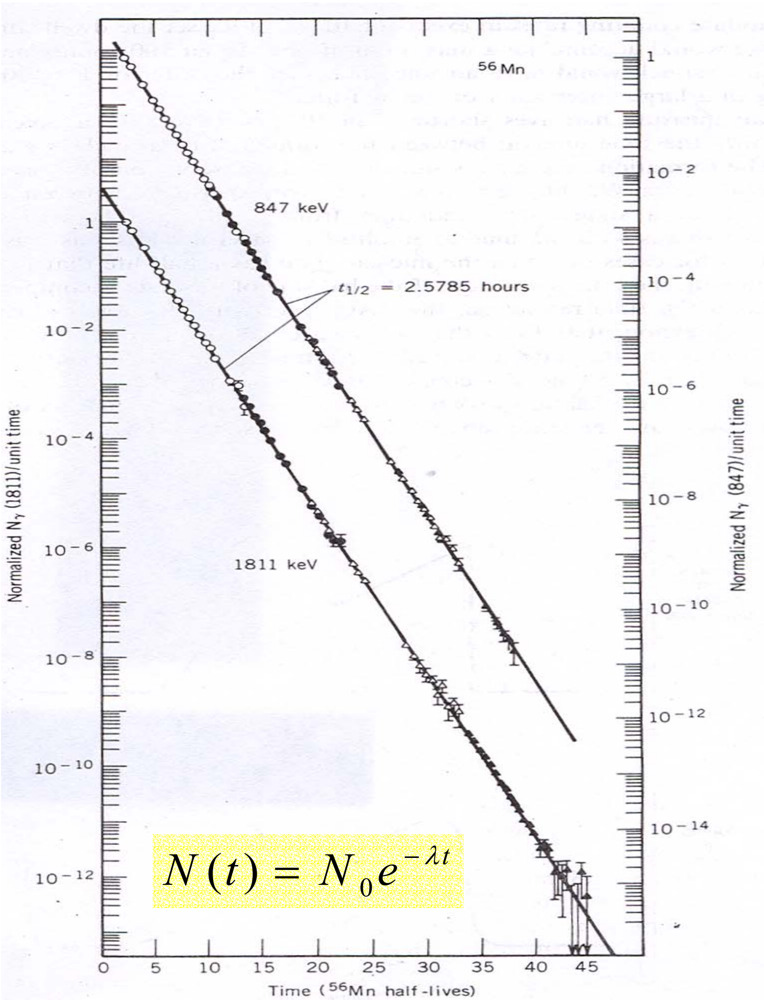
$1\,WU = 0.0594\ A^{4/3} e^2\ fm^4$

expected intensity for 1 transition involving only 1 nucleon

Measurement of nuclear lifetimes τ

$\tau > 10^{-3}$ s

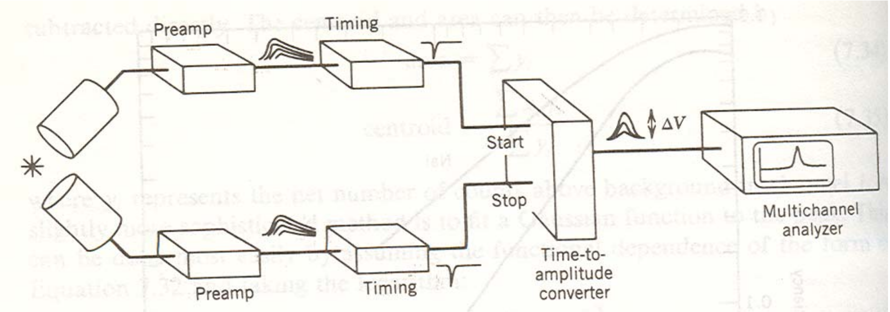
direct measure

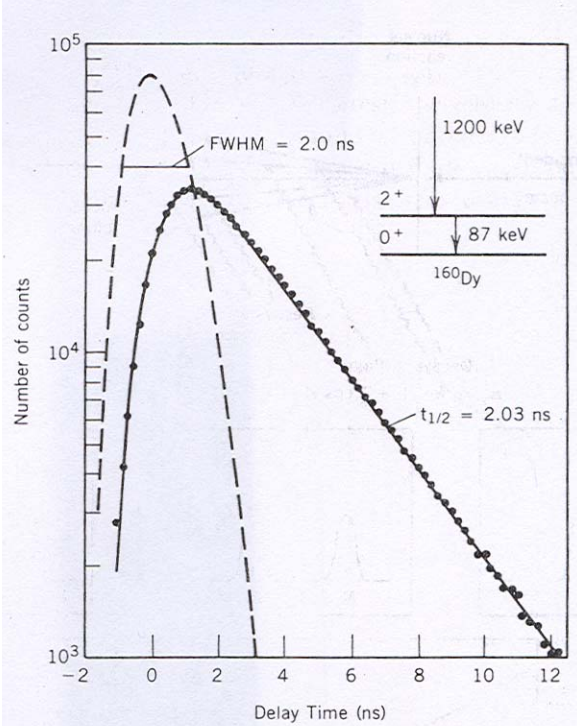


$$N(t) = N_0 e^{-\lambda t}$$

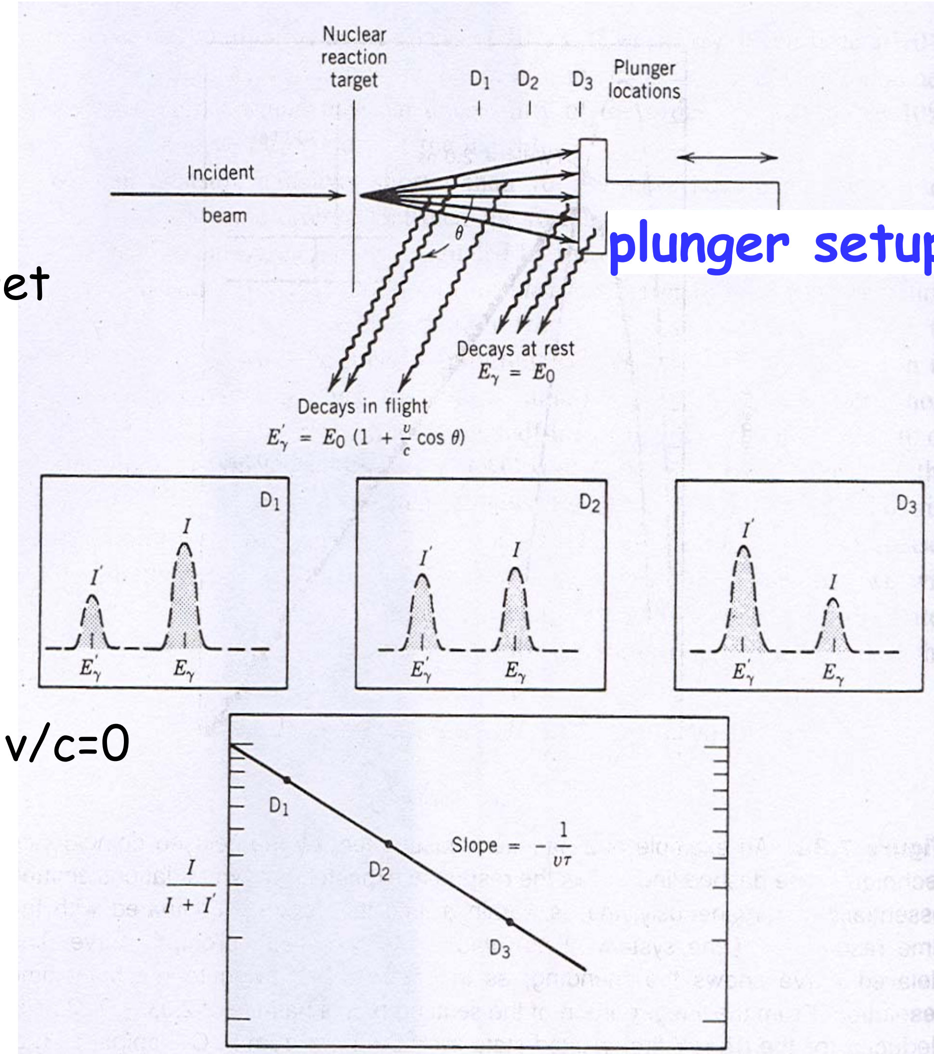
$10^{-3} < \tau < 10^{-11}$ s

Delayed coincidence technique





$10^{-10} < \tau < 10^{-12}$ s
Doppler-recoil method



- the produced nuclei escape from the taget with a velocity v/c
- they decay emitting γ's which are Doppler shifted

$$E'_\gamma = E_\gamma\,(1 + \frac{v}{c}\cos\,\theta)$$

- the nuclei qre finally completely stopped into a stopper, therefore decaying with v/c=0

$$E'_\gamma = E_\gamma$$

- two peaks are observed:
  shifted $\rightarrow$ in flight decay
  stopped $\rightarrow$ decay at rest

Tipical values:
v/c ~ 0.1, τ ~ 10^{-12} s $\rightarrow$ d ~ 0.03 mm

The ratio of the peak intensities depends on the lifetime τ of the state

10^{-15} s $< \tau < 10^{-12}$ s
Doppler-shift attenuation method (DSAM)

- the produced nuclei immediately penetrate into a solid backing (Pb o Au)

- they immediately start to slow down and at the end they stop

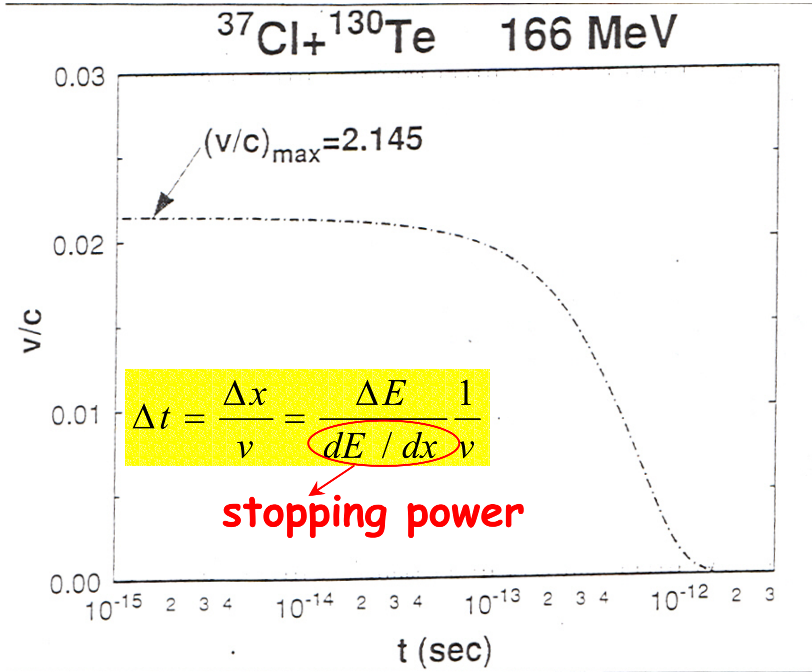
$0 < v/c < (v/c)_{max}$

- the γ emission varies continously in energy

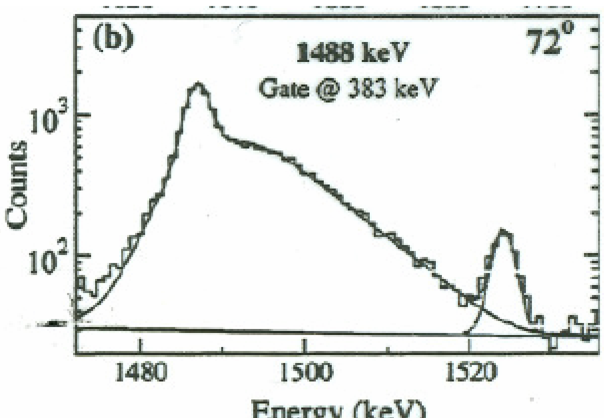
$E_\gamma < E_\gamma' < E_\gamma(1+ v/c \cos\theta)$

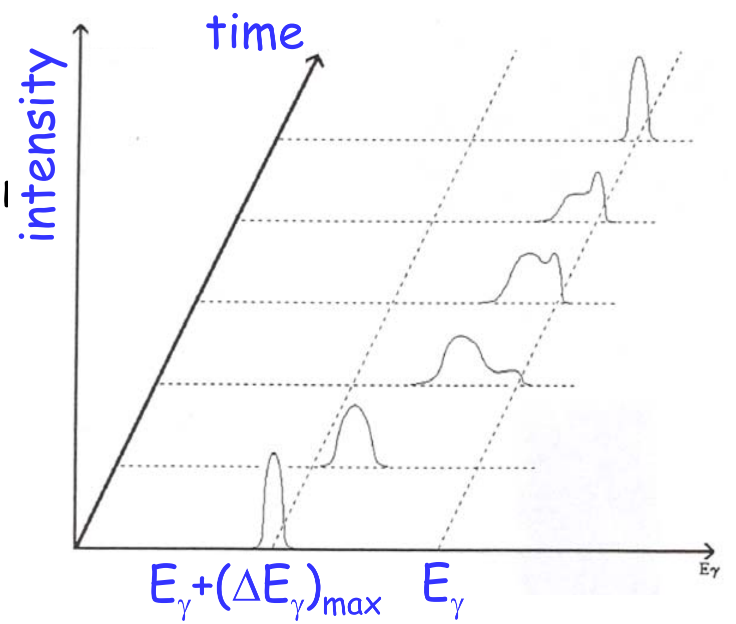
- from the shape of the peak one extract v/c and therefore τ

(once the dE/dx energy loss nechanism is knowi



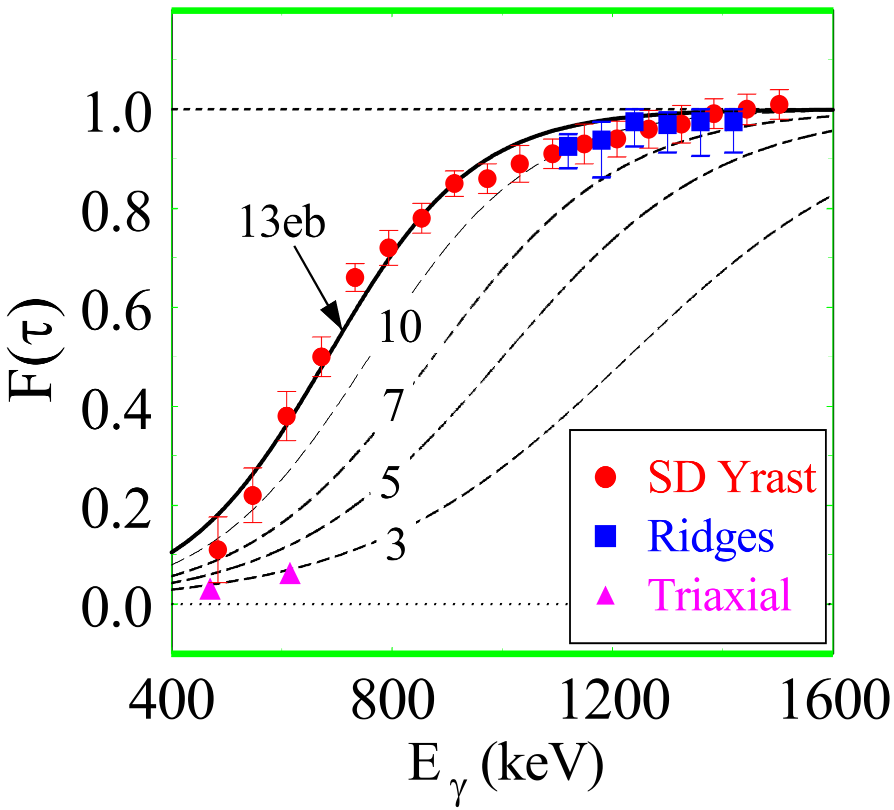
$$\Delta t = \frac{\Delta x}{v} = \frac{\Delta E}{dE\ /\ dx}\frac{1}{v}$$





DSAM technique

^{143}Eu: lifetime analysis



$$F(\tau) = \frac{(v/c)}{(v/c)_{MAX}}$$