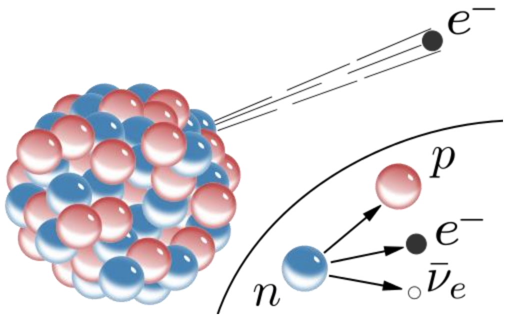
PART 1 – NUCLEAR PHYSICS

2 RADIOACTIVITY

2.1 Radioactive decay
2.2 Alpha decay
2.3 Beta decay
2.4 Gamma decay
2.5 Fission
2.6 Two-Proton Decay

Beta decay

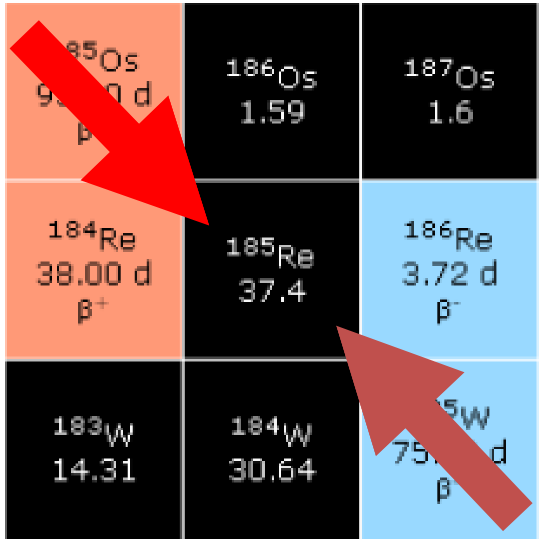


Beta Decay: universal term for all **weak-interaction transitions** between two neighboring isobars

three different forms
β⁻, β⁺ & EC (capture of an atomic electron)

β⁺: $p \rightarrow n + e^+ + \nu$

EC: $p + e^- \rightarrow n + \nu$



β⁻: $n \rightarrow p + e^- + \bar{\nu}$

a nucleon inside the nucleus is transformed into another

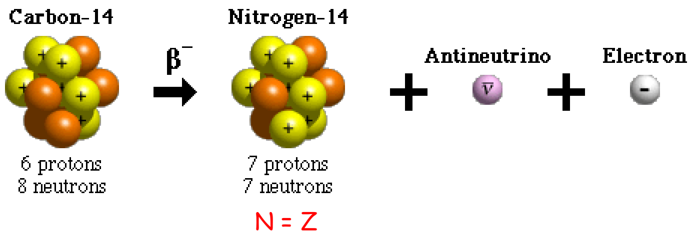
Beta decay

E' il processo che tende ad ottimizzare il rapporto N/Z.

Dovuto all' **interazione debole** che cambia un **neutrone in protone** (o viceversa) e crea una coppia **leptone-antileptone** (elettrone-antineutrino o positrone-neutrino) o trasforma un elettrone in un neutrino.

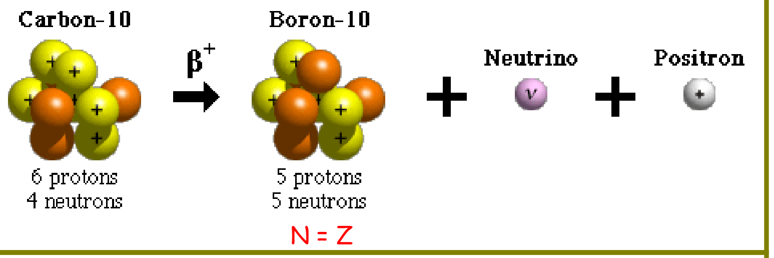
β^-

$$n \rightarrow p + e^- + \bar{\nu}$$



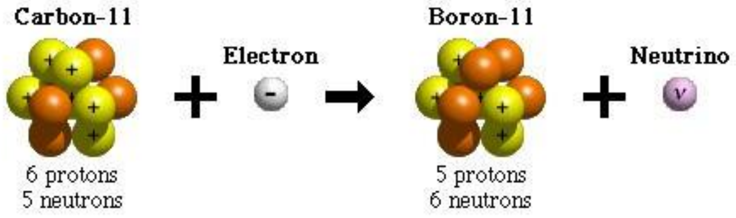
β^+

$$p \rightarrow n + e^+ + \nu$$

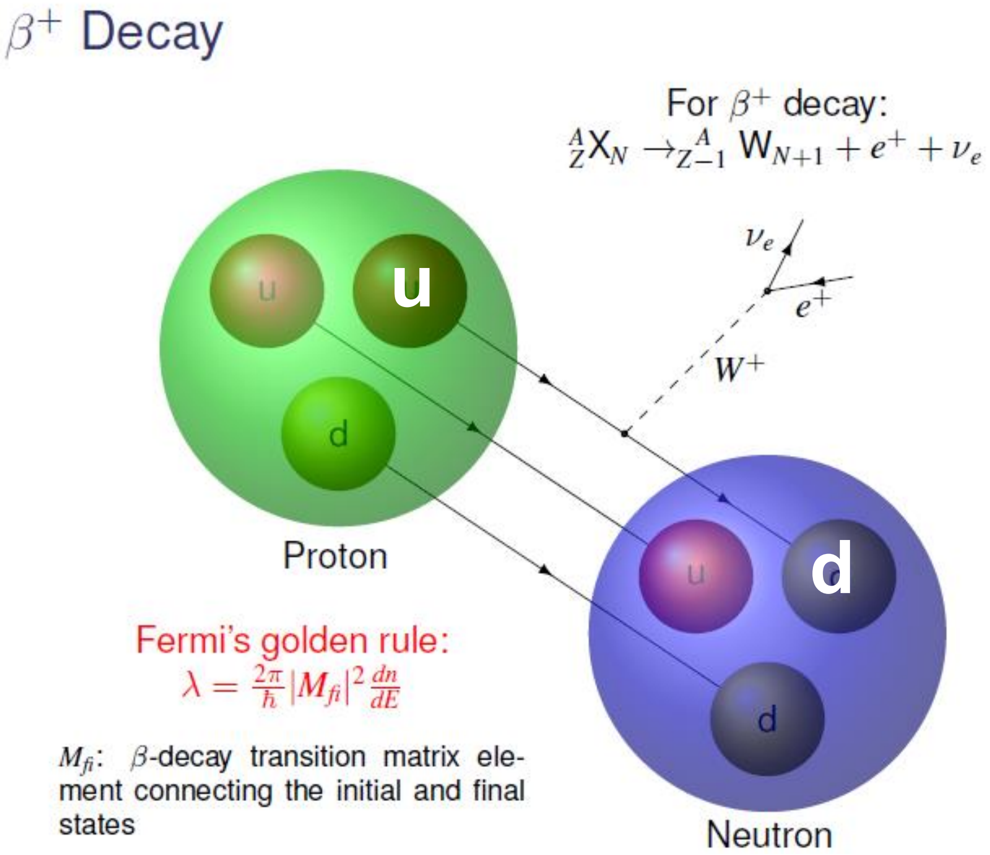


electron capture

$$p + e^- \rightarrow n + \nu$$



Beta decay of a proton



Lifetime of Free Proton

$\tau_p > 1.6 \cdot 10^{33}$ years (stable)

Age of Universe: $(13.72 \pm 0{,}12)\ 10^9$ years

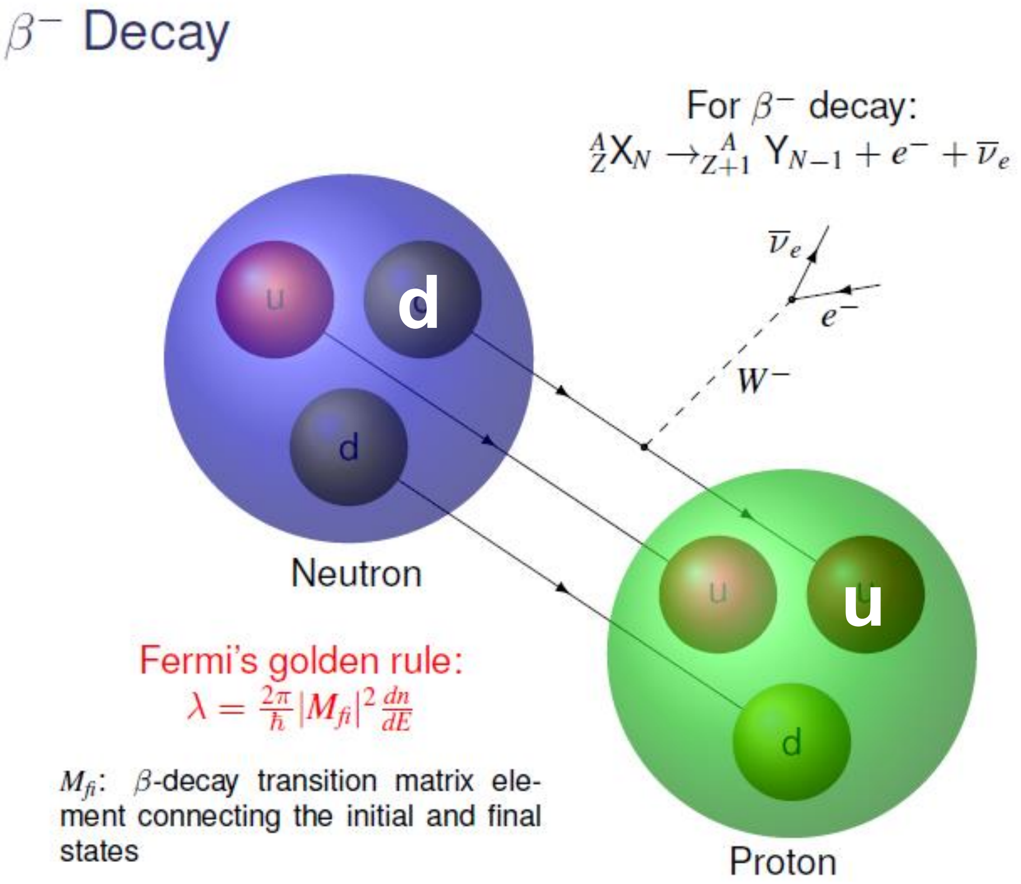
beta decay $p \rightarrow n + e^+ + \nu$

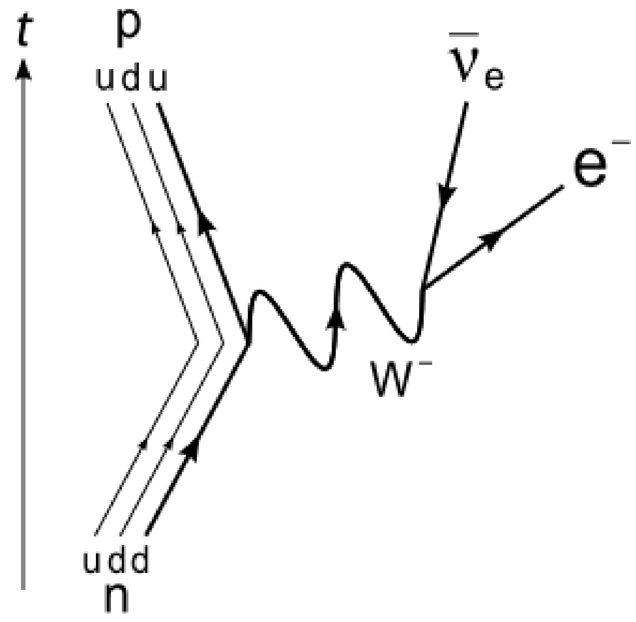
can only occur with ***bound protons***

m_p = 938.27208816(29) MeV/c2
m_n = 939.565378(21) MeV/c2

m_p < m_n

Beta decay of a neutron





$m_p = 1.673 \cdot 10^{-27}$ kg

938.272MeV/c^2

$m_n = 1.675 \cdot 10^{-27}$ kg

939.565MeV/c^2

The mean lifetime τ of **free** neutrons is 885.7(8) [s]

$$n \rightarrow p + e^- + \overline{\nu}_e + 0.782\ MeV$$

m_p = 938.27208816(29) MeV/c2
m_n = 939.565378(21) MeV/c2

m_p < m_n

While **bound neutrons in stable nuclei are stable**, **FREE neutrons are unstable**; they undergo beta decay with a lifetime of just under 15 minutes

$$n \rightarrow p + e^{-} + \bar{\nu}$$

$\tau_n = 885.7 \pm 0.8$ s $\approx$ *14.76 min*

Long lifetimes $\Rightarrow$ before decaying possibility to interact

$\Rightarrow$ n physics …

X Free neutrons are produced in nuclear **fission** and **fusion**

X Dedicated neutron sources like research reactors and spallation sources produce free neutrons for the use in irradiation in neutron scattering and neutron induced reaction experiments

N.B. *Vita media del protone: $t_p > 1.6 \cdot 10^{33}$ anni*
età dell'universo: $(13.72 \pm 0,12) \times 10^9$ anni.

beta decay $p \rightarrow n + e^{+} + \nu$ *can only occur with bound protons*

m_p = 938.27208816(29) MeV/c2
m_n = 939.565378(21) MeV/c2
m_p < m_n

The neutron lifetime puzzle

From 2016 Istitut Laue-Langevin (ILL, Grenoble) Annual Report

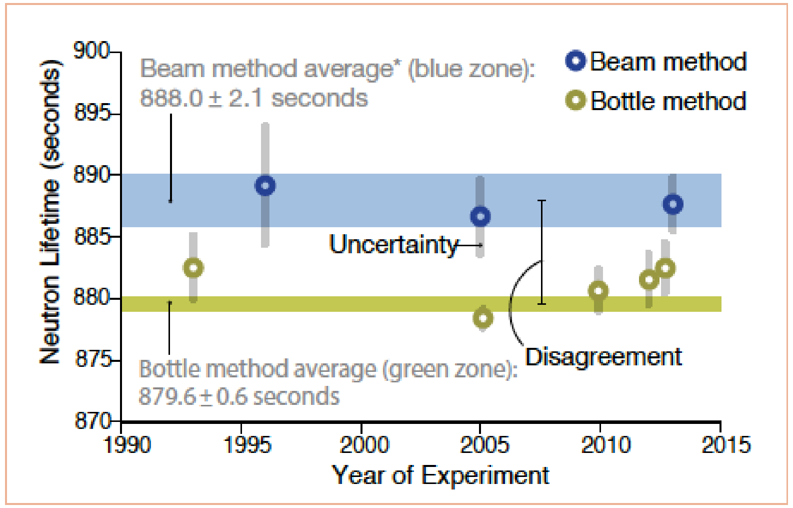


Figure 1

The history of recent neutron lifetime measurements. The green band represents the one standard deviation uncertainty for the average of all "Bottle" experiments. The blue band illustrates the same for all "Beam" experiments.

A discrepancy of more than 8 seconds (1%) !!!!

THE SCIENCES

Neutron Death Mystery Has Physicists Stymied

Conflicting results in measurements of how long neutrons live has physicists rethinking their experiments, because solving the riddle may point the way to exotic new physics

A possible source of dark matter ?

By Clara Moskowitz on May 13, 2014

A. Serebrov et al., Phys. Lett. B 605 (2005) 72
A.T. Yue et al., Phys. Rev. Lett. 111 (2013) 222501
Z. Berezhiani and L. Bento, Phys. Rev. Lett. 96 (2006) 081801
G.L. Greene and P. Geltenbort, Sci. Am. 314 (2016) 36

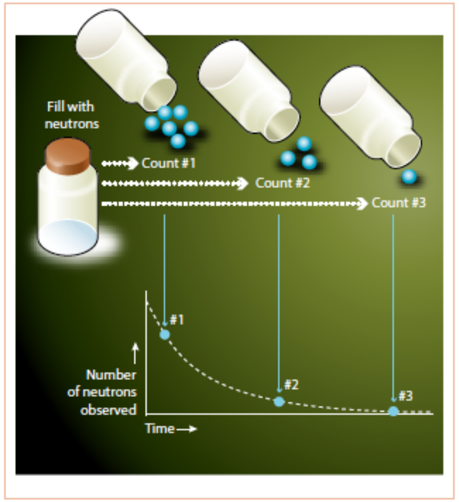


Figure 2

The Bottle Method: One way to measure how long neutrons live is to fill a container with neutrons and empty it after various time intervals under the same conditions to see how many remain. These tests fill in points along a curve that represents neutron decay over time. From this curve, scientists use a simple formula to calculate the average neutron lifetime. Because neutrons occasionally escape through the walls of the bottle, scientists vary the size of the bottle as well as the energy of the neutrons – both of which affect how many particles will escape from the bottle – to extrapolate to a hypothetical bottle that contains netrons perfectly with no losses.

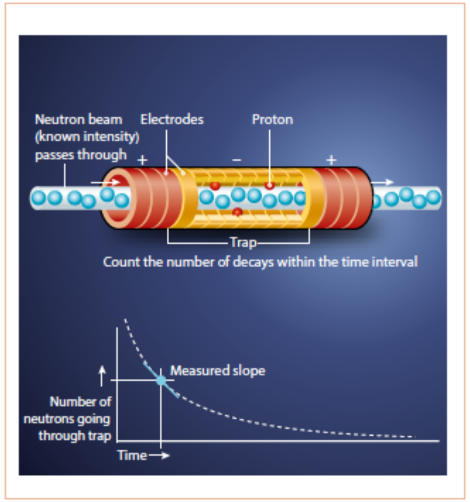


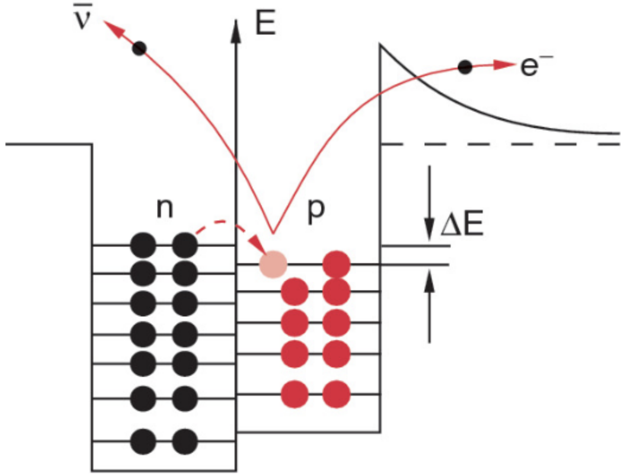
Figure 3

The Beam Method: In contrast to the bottle method, the beam technique looks not for neutrons but for one of their decay products, protons. Scientists direct a stream of neutrons through an electromagnetic 'trap' made of a magnetic field and ring-shaped high-voltage electrodes. The neutral neutrons pass right through, but if one decays inside the trap, the resulting positively charged protons will get stuck. The researchers know how many neutrons were in the beam, and they know how long they spent passing through the trap, so by counting the protons in the trap they can measure the number of neutrons that decayed in that span of time. This measurement is the decay rate, which is the slope of the decay curve at a given point in time and which allows the scientists to calculate the average neutron lifetime.

discrepancy explained by a neutron-decay I nto a dark matter channel?

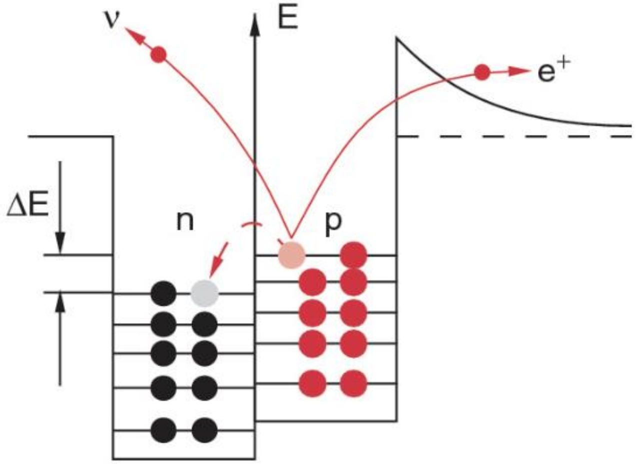
https://www.scientificamerican.com/article/neutron-lifetime-mystery-new-physics/

Beta decay of neutron and proton in nuclei



$$n \rightarrow p + e^- + \bar{\nu} + E_{kin}$$
$$^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^- + \bar{\nu}$$
$$\Delta E = \left[M(^{A}_{Z}X) - M(_{Z+1}^{A}Y)\right] \cdot c^2$$
$$\geq (m_e + m_{\bar{\nu}}) \cdot c^2 + E_{kin}$$

Fig. 3.22 Beta-decay of an unstable nucleus by conversion of a neutron into a proton



$$p \rightarrow n + e^+ + \nu$$
$$\Delta E = \left[M(^{A}_{Z}X) - M(_{Z-1}^{A}Y)\right] \cdot c^2$$
$$\geq (m_e + m_\nu) \cdot c^2$$

Fig. 3.23 Nuclear model of positron emission at the conversion of a proton in a neutron in ß$^+$-emitting unstable nuclei

Beta decay - overview

$^{A}Z \rightarrow A(Z\pm1)$ β-decay
$^{A}Z \rightarrow {}^{A}(Z\pm2)$ double β-decay

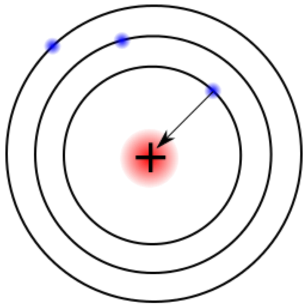
β⁻-decay $n \rightarrow p + e^- + \bar{\nu}_e$ if $Q_\beta > 0$

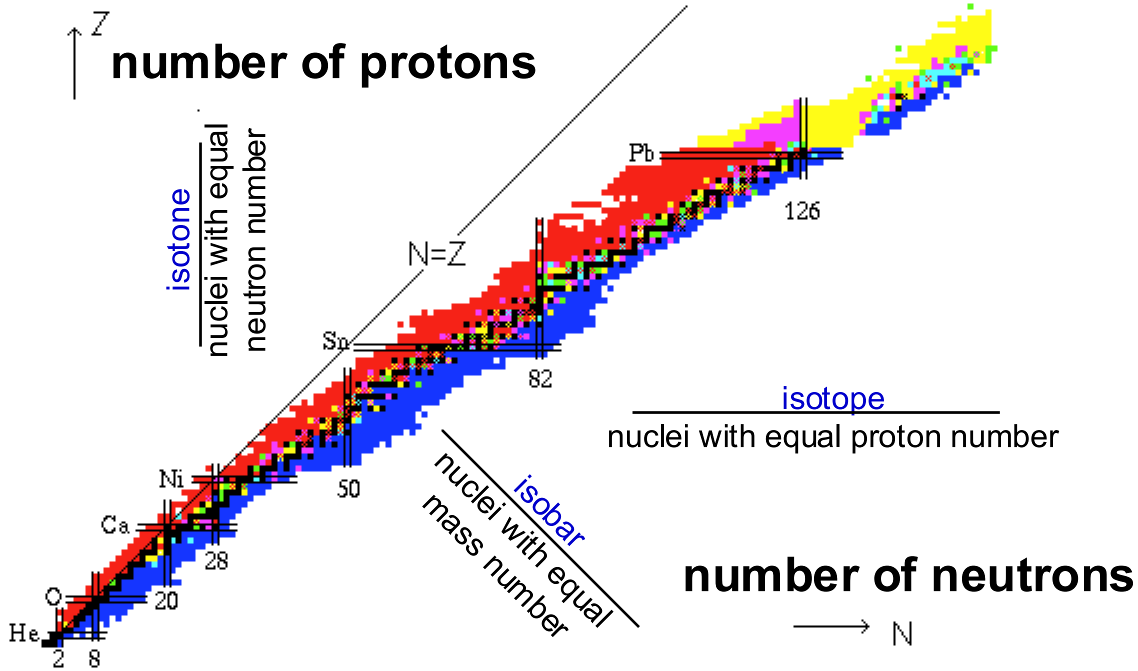
β⁺-decay $p \rightarrow n + e^+ + \nu_e$ if $Q_\beta > 1.02$ MeV

EC $p + e^- \rightarrow n + \nu_e$ if $Q_\beta > 0$

Electron capture: proton-rich nucleus absorbs an inner electron.

EC





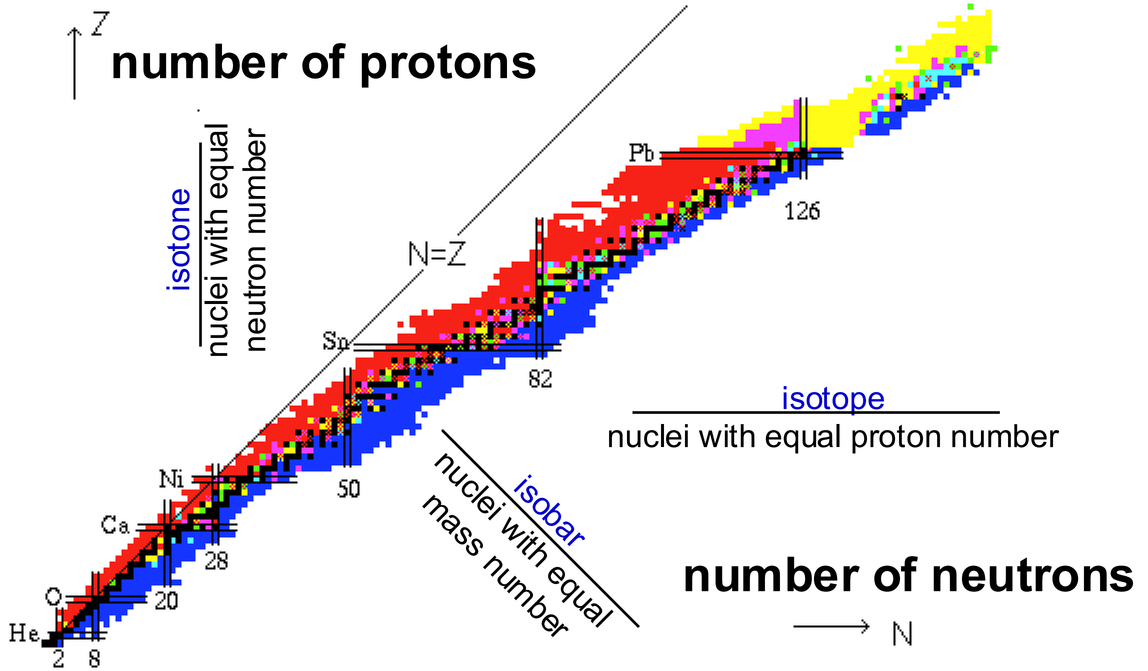
Motivation:

For the description of the nuclear synthesis in astrophysical environment one needs excellent knowledge about the **β-decay properties in unstable nuclei** far away from the valley of stability.

Beta decay - overview

- For isotopes with substantial **neutron excess** it is energetically favorable, if a neutron changes into a proton.
- For **neutron deficient nuclei** one has the opposite process: a proton changes into a neutron.

β⁻-decay	β⁺-decay
$n \rightarrow p + e^- + \bar{\nu}_e$ $d \rightarrow u + e^- + \bar{\nu}_e$	$p \rightarrow n + e^+ + \nu_e$ $u \rightarrow d + e^+ + \nu_e$



Necessary conditions for the different β decays:

$m(A,Z) > m(A,Z+1)$ β⁻-decay (the created electron is already considered)

$m(A,Z) > m(A,Z-1) + 2 \cdot m_e \cdot c^2$ β⁺-decay (the mother nucleus has one electron more and a positron is created)

$m(A,Z) > m(A,Z-1) + \epsilon$ electron capture EC (ε is the excitation energy of the hole in the daughter nucleus, EC can transform protons into neutrons at a lower energy)

in competition

β^+- decay versus electron capture (EC)

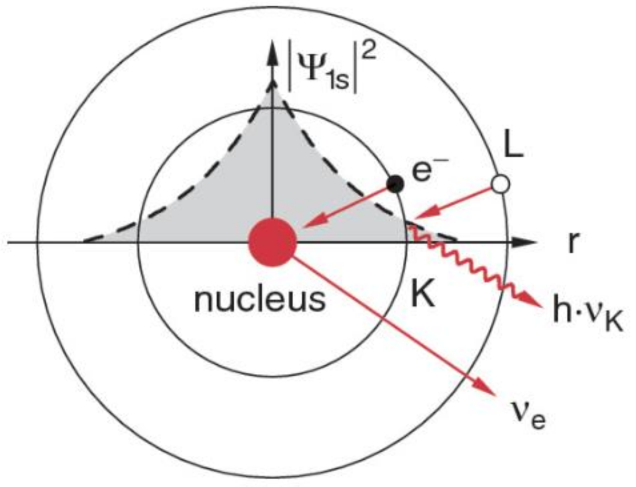
in EC: un nucleo **ricco di protoni** può **catturare un elettrone atomico** e **trasformare un protone in un neutrone**

⇨ Stesso effetto di un decadimento β^+ (energetically more favoured)

$$p + e^- \rightarrow n + \nu_e$$

⇨ L'elettrone viene tipicamente catturato dalla ***shell K*** che è caratterizzata da una funzione d'onda sensibilmente diversa da zero nel volume del nucleo:

K-electrons have a high probability inside of the nucleus and can be easily captured.



K-Capture

Osservazioni:

1) cattura elettronica ha un **Q-valore più alto** del decadimento β^+

→ più energia cinetica a disposizione delle particelle nello stato finale

2) Ci sono casi in cui la **differenza di massa tra (Z,A) e (Z-1,A) è troppo piccola (<1.022 MeV) per consentire il decadimento β^+**, ma la **cattura elettronica può invece avvenire**

Esempio: *^{83}Rb decade in ^{83}Kr (DM = 0.9 MeV) solo attraverso cattura elettronica*

Liquid drop mass formula

$$m(A,Z) = Z \cdot m_H + (A-Z) \cdot m_n - a_V \cdot A + a_S \cdot A^{2/3} + a_C \cdot \frac{Z \cdot (Z+1)}{A^{1/3}} + a_{asym} \cdot \frac{(Z-A/2)^2}{A} \pm \delta$$

1935 Carl von Weizsäcker

$$\delta = \begin{cases} +11.2\,MeV \text{ for } o-o \text{ nuclei} \\ 0\,MeV \text{ for odd}-A \text{ nuclei} \\ -11.2\,MeV \text{ for } e-e \text{ nuclei} \end{cases}$$

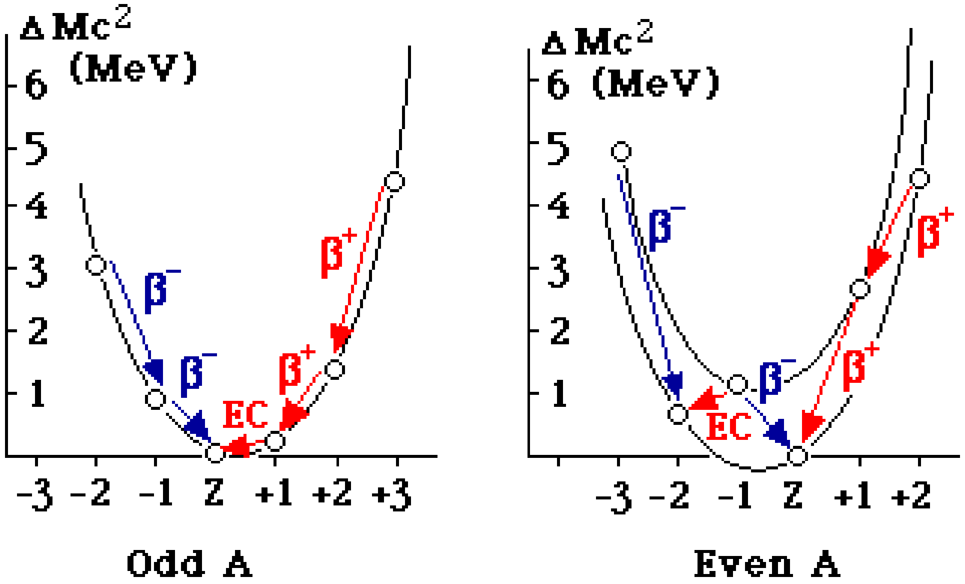
From the mass formula one obtains

$$m(A,Z) = \gamma \cdot Z^2 - \beta \cdot Z + \alpha \cdot A \pm \delta$$

$$\alpha = \left(m_n - a_V + \frac{a_S}{A^{1/3}} + \frac{a_{asym}}{4}\right) \qquad \beta = \left[a_{asym} + m_n - m(\,^1H)\right]$$

$$\gamma = \left(\frac{a_C}{A^{1/3}} + \frac{a_{asym}}{A}\right)$$

For isobar chains (nuclei with constant mass A) m(A,Z) is a quadratic function of Z.



The **minimum** can be obtained from

$$\partial M / \partial Z = 0$$

if Z(Z-1) instead of Z^2

$$Z_{min} = \frac{\left[m_n - m(^1H)\right] + a_c A^{-1/3} + 4a_{sym}}{2a_c A^{-1/3} + 8a_{sym} A^{-1}}$$

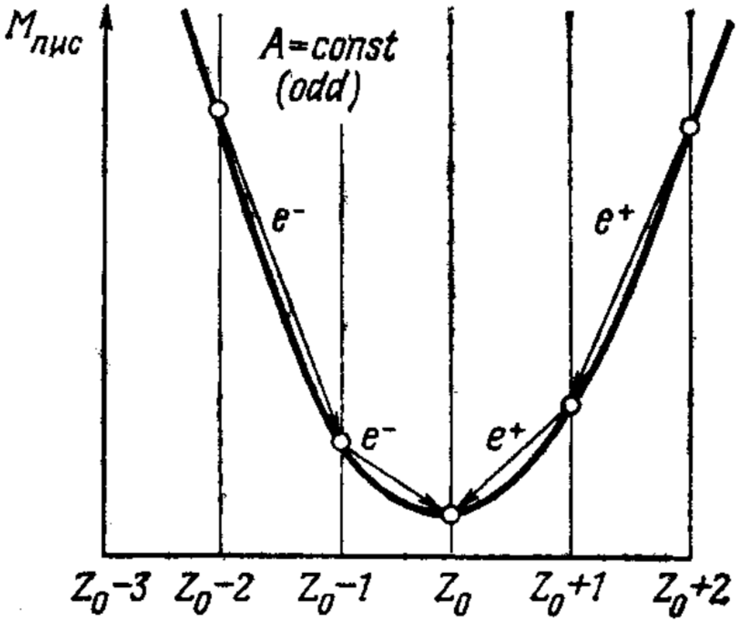
with a_c = 0.7 MeV, a_{sym} = 18.3 MeV
first two terms at numerator are negligible

$$Z_{min} \simeq \frac{A}{2} \frac{1}{1 + \frac{1}{4} A^{2/3} a_c / a_{sym}}$$

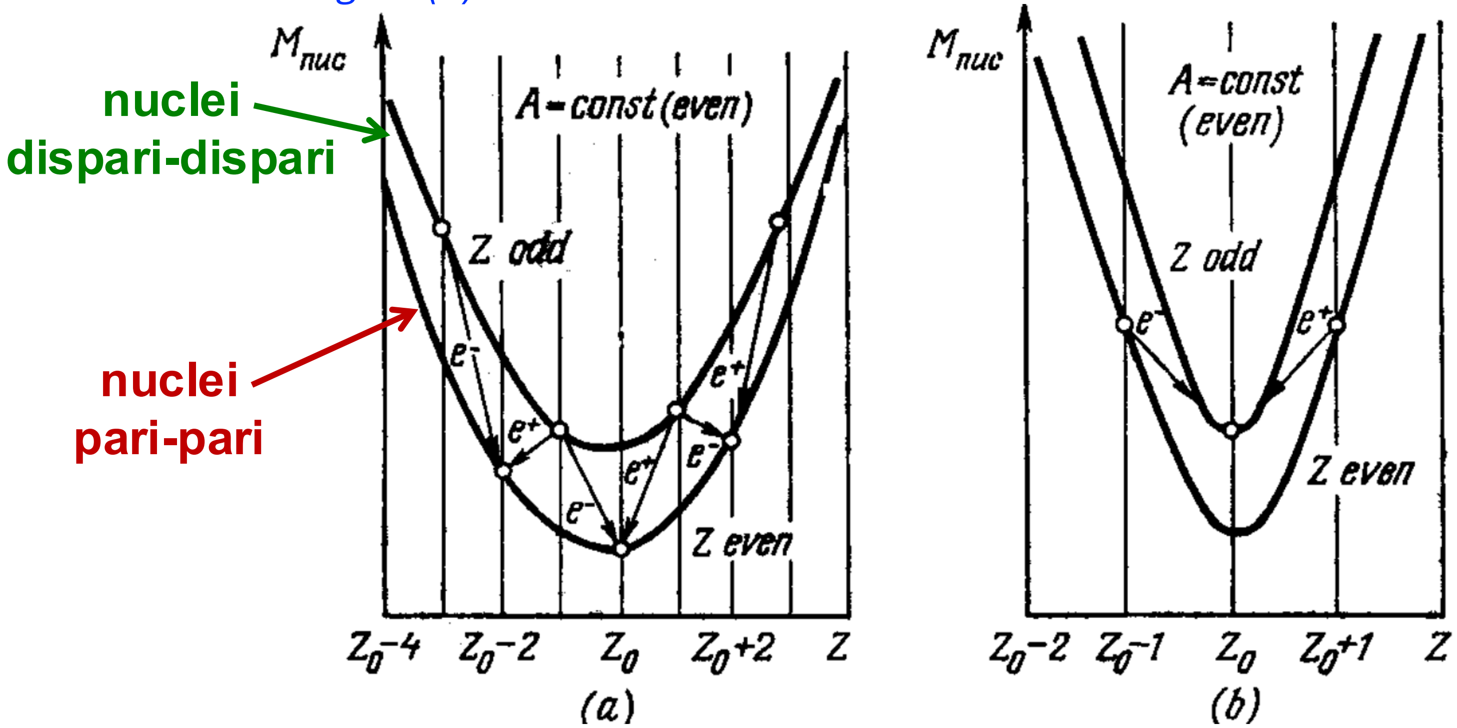
~ A/2 small A

< A/2 large A (~ 0.41)

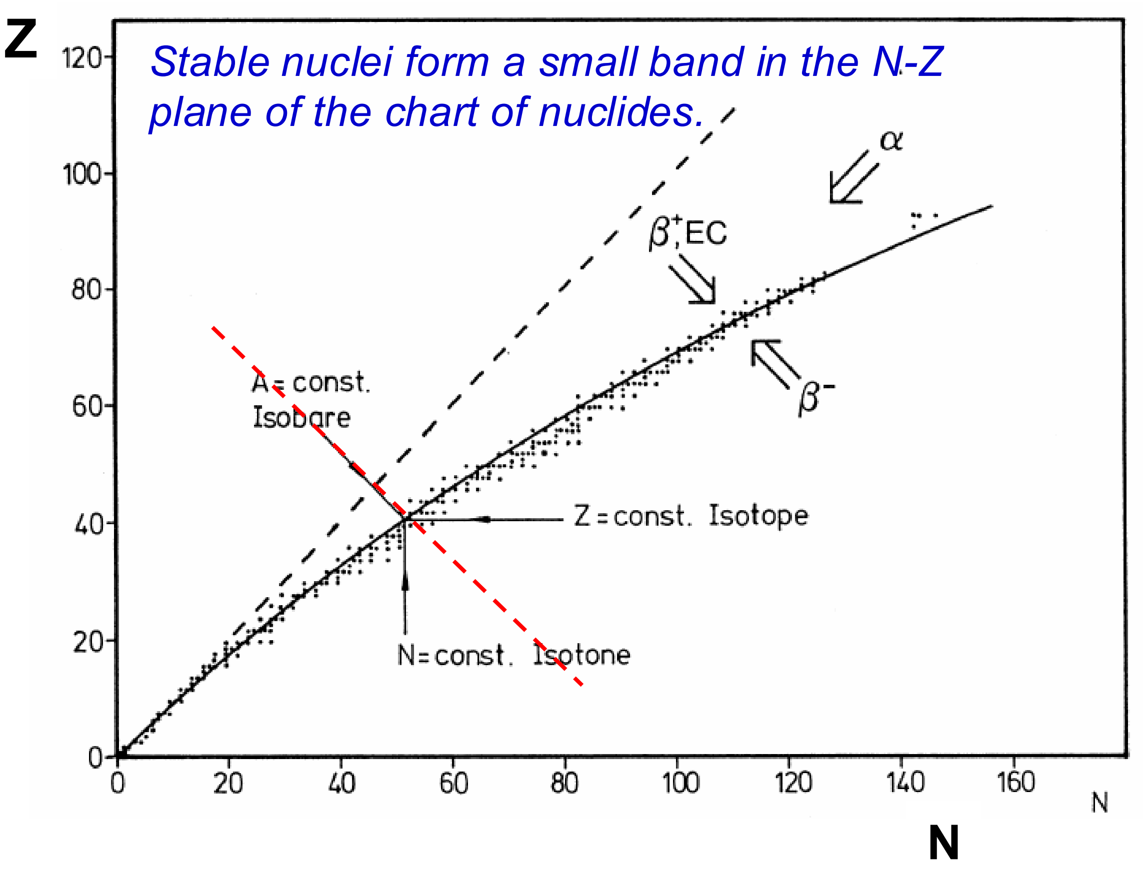
- **Nuclei con A dispari:**
  - Il parametro d vale 0 e quindi M(A,Z) ha un solo valore
  - Fissato A esiste un solo isobaro stabile con $Z=Z_0$

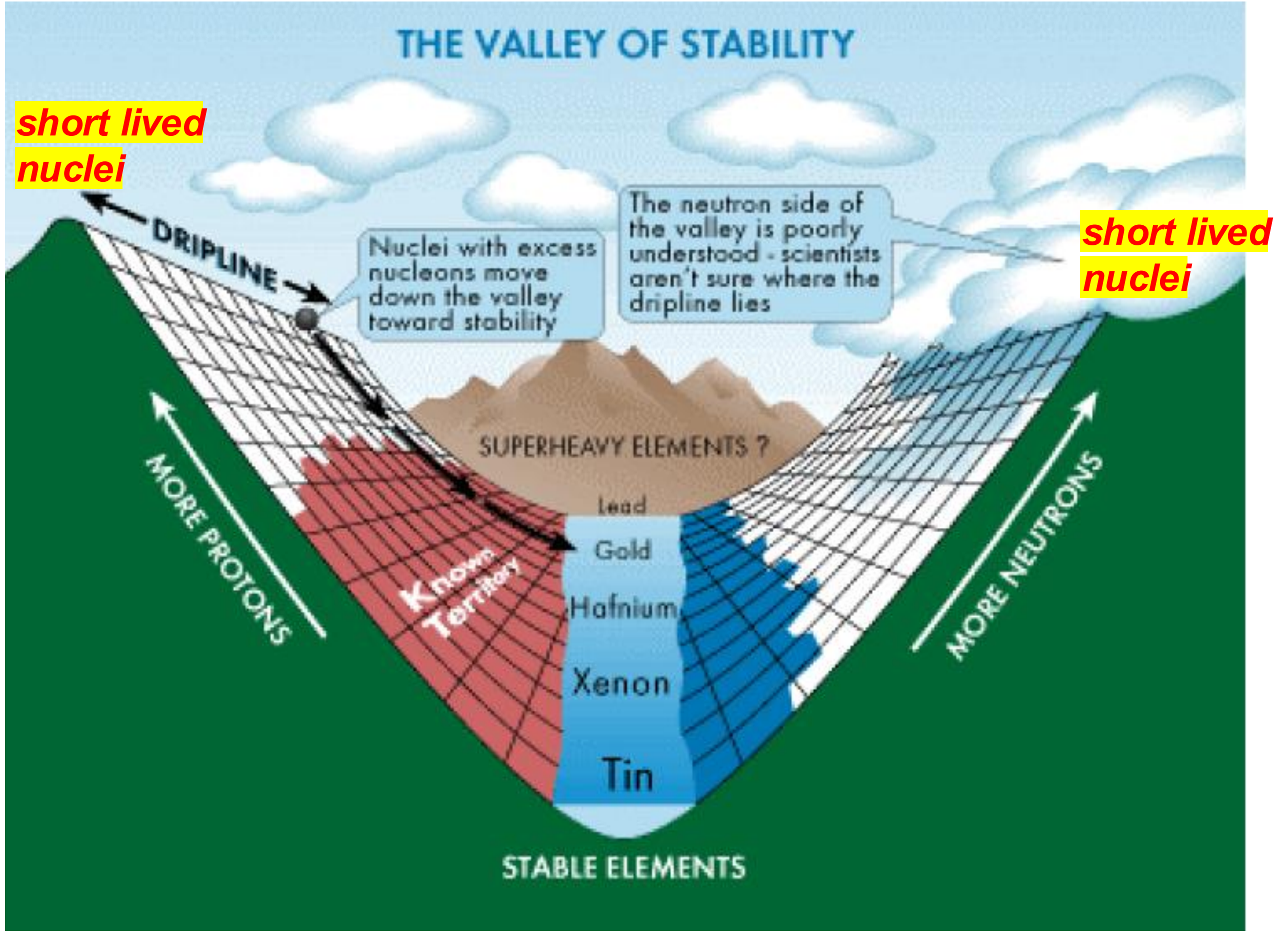


- **Nuclei con A pari**
  - ⇨M(A,Z) assume due valori diversi per nuclei pari-pari e dispari-dispari
  - ⇨Possono esserci fino a 3 isobari stabili per i nuclei pari-pari
  - ⇨Tutti i nuclei dispari-dispari devono essere instabili
    - *Uniche eccezioni sono: ^{2}H, ^{6}Li, ^{10}B e ^{14}N in cui le parabole sono disposte come in figura (b)*



Valley of Stability





Liquid drop mass parabola for odd-A isobars

In **β⁻ decay** a neutron changes into a proton: $n \rightarrow p + e^- + \bar{\nu}_e$

the mass balance for β⁻ decay is given by:

$$Q_\beta = \{[m(A,Z) - Z \cdot m_e] - [m(A,Z+1) - (Z+1) \cdot m_e + m_e]\} \cdot c^2$$
$$= \{m_N(A,Z) - m_N(A,Z+1)\} \cdot c^2$$

The rest mass of an anti-neutrino < 7 eV/c² can be neglected.

β⁻ decay is possible under the following condition:

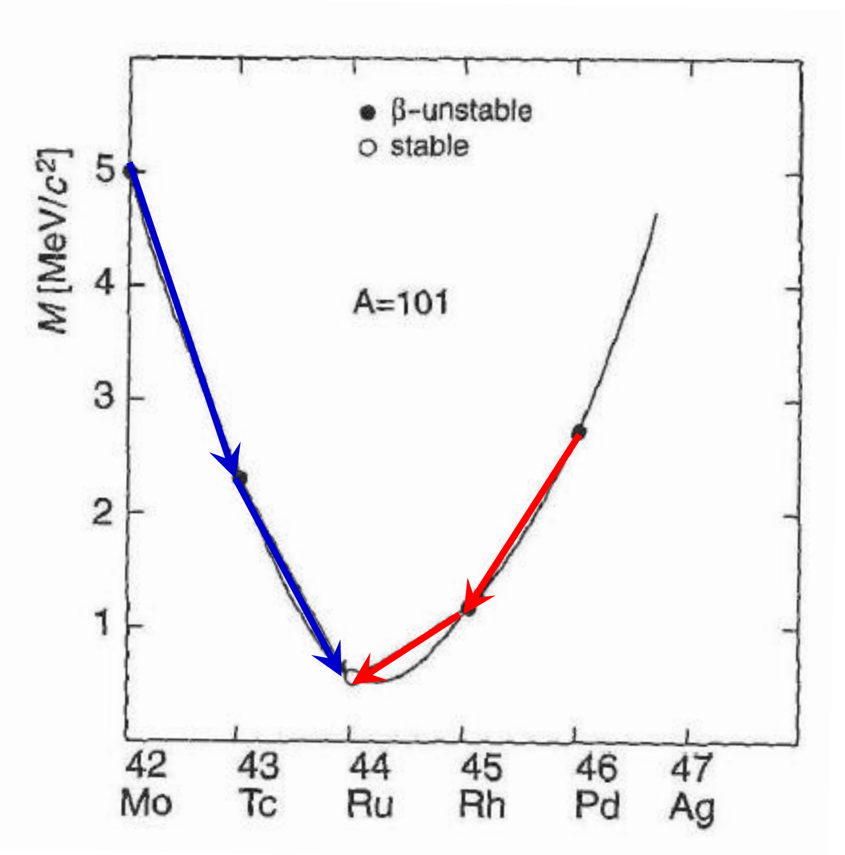
$$m(A,Z) > m(A,Z+1)$$

ESEMPIO: mass parabola for A=101 isobars

$${}^{101}_{42}Mo(14.61(3)\ min) \rightarrow {}^{101}_{43}Tc(14.123(3)\ min) + e^- + \bar{\nu}_e$$

$${}^{101}_{43}Tc(14.123(3)\ min) \rightarrow {}^{101}_{44}Ru(stable) + e^- + \bar{\nu}_e$$

$${}^{101}_{44}Ru\ \ is\ stable$$



→ What are the decays of ^{101}Pd und ^{101}Rh?

β⁺ decay

In neutron deficient nuclei a proton changes into a neutron: $p \rightarrow n + e^{+} + \nu_e$

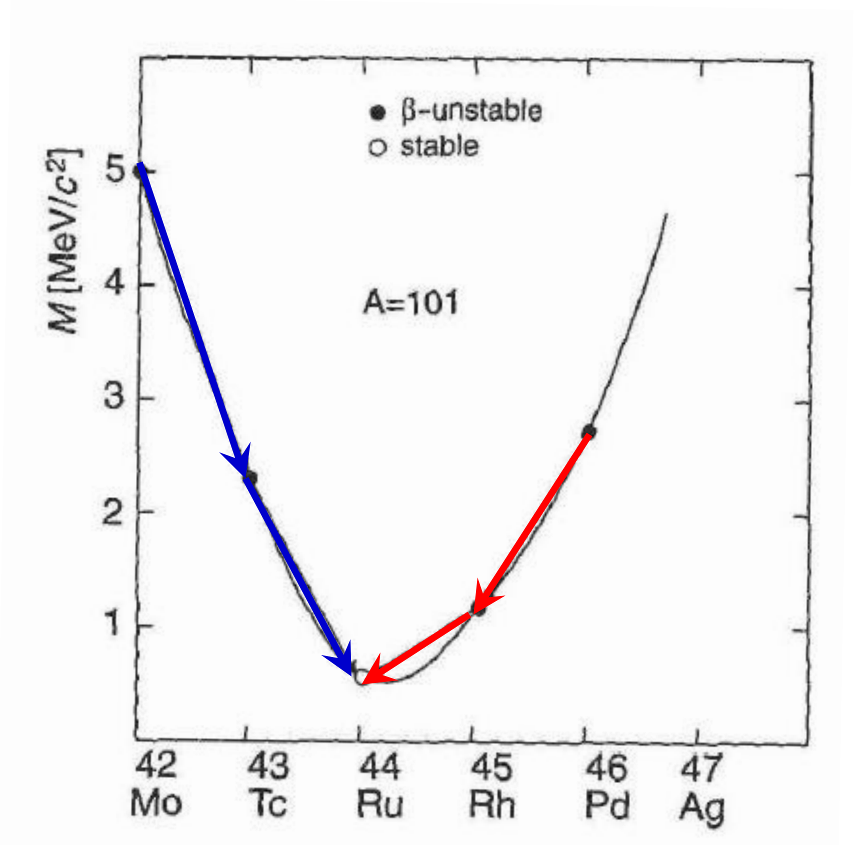
A positron e⁺, the anti-particle of an electron (positive charge, equal mass) and an electron-neutrino ν_e will be emitted.

The mass balance of the β⁺ decay is given by:

$$Q_{\beta} = \{[m(A,Z) - Z \cdot m_e] - [m(A,Z-1) - (Z-1) \cdot m_e + m_e]\} \cdot c^2$$
$$= \{m_N(A,Z) - m_N(A,Z-1) - 2 \cdot m_e\} \cdot c^2$$

Condition for β⁺ decay: m(Z,A) > m(Z-1,A) + $2 \cdot m_e$

The term $2 \cdot m_e$ considers that a positron is created and an
an electron is left over from the mother nucleus.



ESEMPIO: The following β⁺ decays are observed for A=101:

$${}^{101}_{46}Pd(8.47(6)\ h) \rightarrow {}^{101}_{45}Rh(4.34(1)\ d) + e^{+} + \nu_e$$

$${}^{101}_{45}Rh(4.34(1)\ d) \rightarrow {}^{101}_{44}Ru(stable) + e^{+} + \nu_e$$

$${}^{101}_{44}Ru\ is\ stable$$

Liquid drop mass parabola for even-A isobars

In **isobars with even mass** number two separate parabola exist because of the pairing energy: one for even-even nuclei and one higher lying for odd-odd nuclei.
The difference is 2δ, twice the pairing energy.

Consequence:

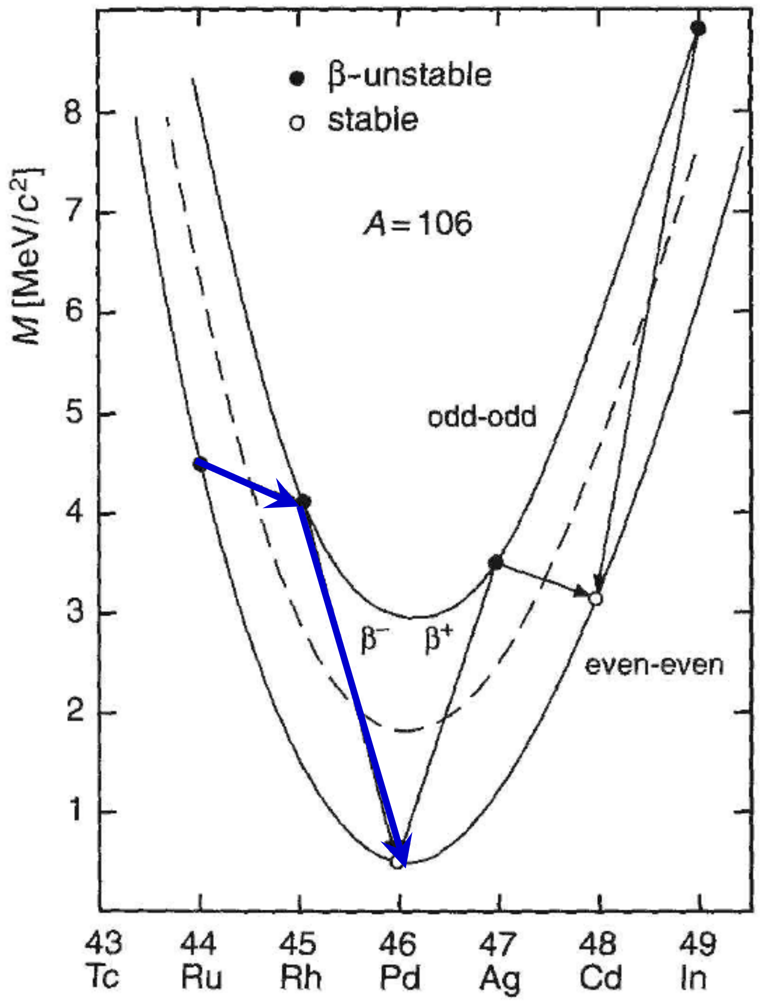
- All odd-odd nuclei have at least one stronger bound even-even nucleus (isobar) and are hence instable.
- Exceptions: ^{2}H, ^{6}Li, ^{10}B, ^{14}N because of the higher asymmetry energy.

EXAMPLE: mass parabola der A=106 isobars

$${}^{106}_{44}Ru(371.8(18)\ d) \rightarrow {}^{106}_{45}Rh(30.07(35)\ s) + e^- + \bar{\nu}_e$$

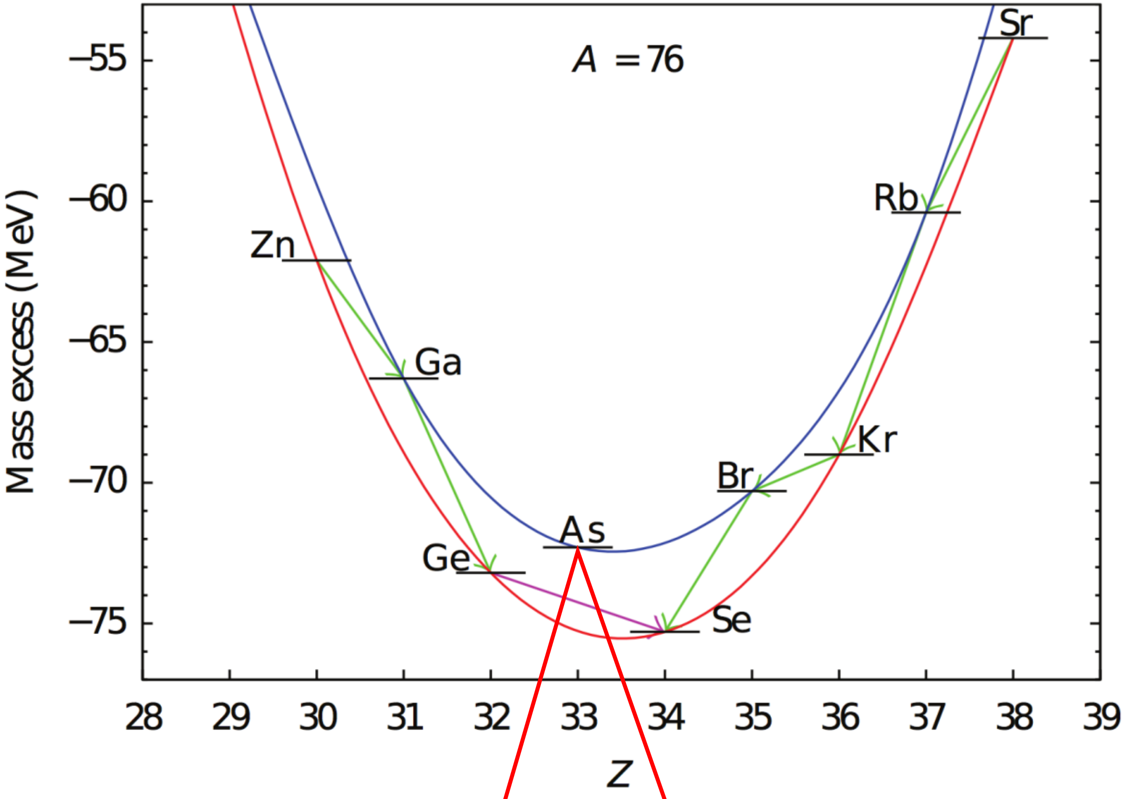
$${}^{106}_{45}Rh(30.07(35)\ s) \rightarrow {}^{106}_{46}Pd(stable) + e^- + \bar{\nu}_e$$

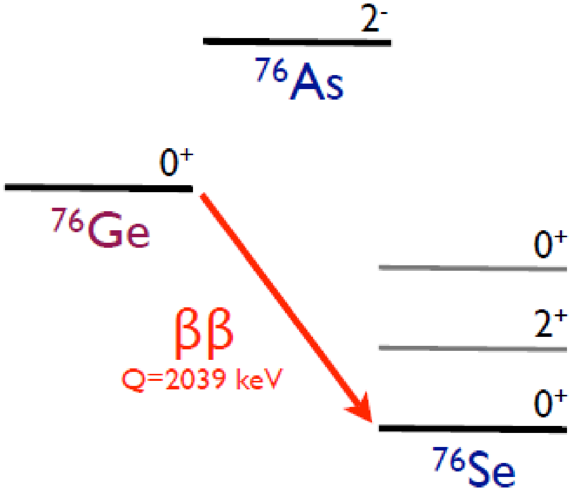
$${}^{106}_{46}Pd\ is\ stable$$



Two neutrino double-β decay

$$(Z, A) \rightarrow (Z+2, A) + 2e^- + 2\bar{\nu}_e$$





In some cases, the ***single beta decay is energetically forbidden***. Then the ***double beta decay* becomes relevant**

Two-neutrino double-β decay cases

Isotope	$T_{1/2}$ (2ν) (y)
^{48}Ca	$(4.4 \pm 0.6)\cdot 10^{19}$
^{76}Ge	$(1.5 \pm 0.1)\cdot 10^{21}$
^{82}Se	$(0.92 \pm 0.07)\cdot 10^{20}$
^{96}Zr	$(2.3 \pm 0.2)\cdot 10^{19}$
^{100}Mo	$(7.1 \pm 0.4)\cdot 10^{18}$
^{116}Cd	$(2.8 \pm 0.2)\cdot 10^{19}$
^{128}Te	$(1.9 \pm 0.4)\cdot 10^{24}$
^{130}Te	$(1.5 \pm 0.1)\cdot 10^{20}$
^{150}Nd	$(8.2 \pm 0.9)\cdot 10^{18}$
^{238}U	$(2.0 \pm 0.6)\cdot 10^{21}$
^{136}Xe	$(2.1 \pm 0.2)\cdot 10^{22}$

very long decay times

Two neutrino-less double-β decay

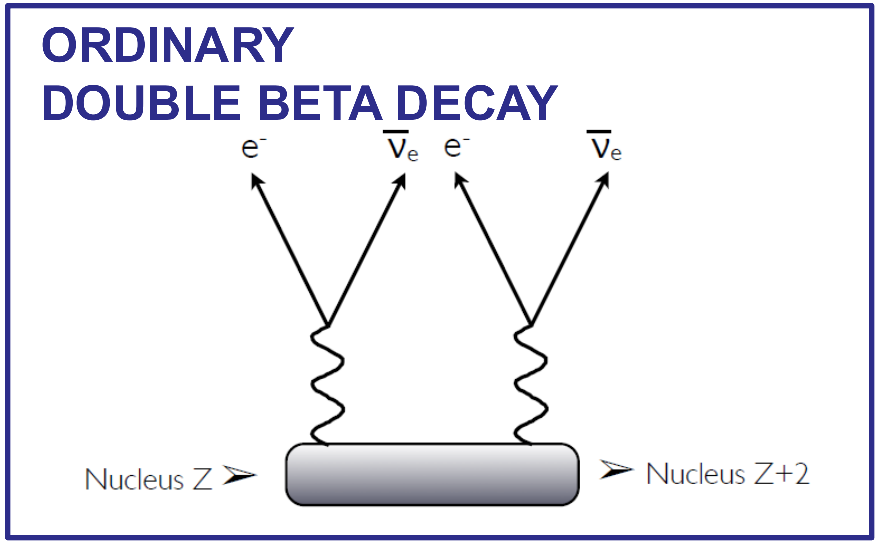
- **Under the conditions**:
  - ⇨neutrino has **zero mass**
  - ⇨neutrino is a **Majorana particle** (it coincides with its own anti-particle)
- it is possible to have a
  double-β decay without emission of neutrinos (0νββ)

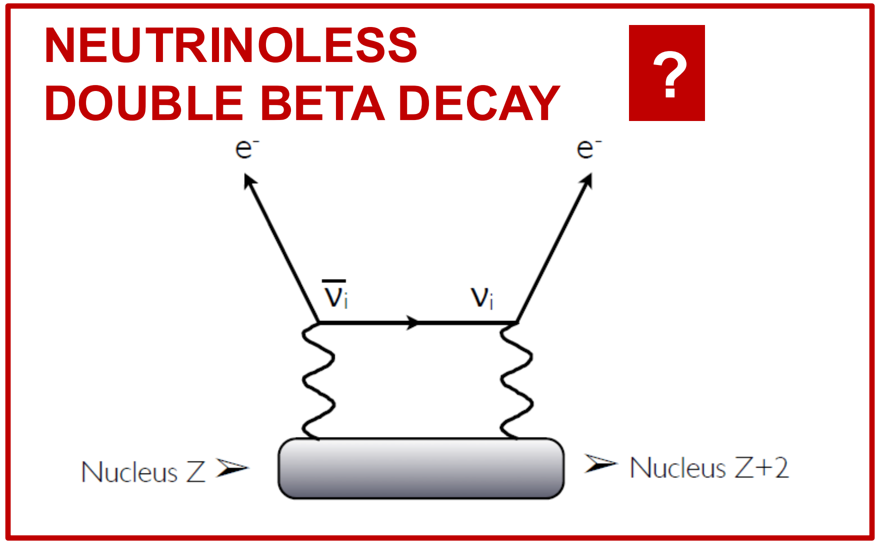
$$(Z, A) \rightarrow (Z+2, A) + 2e^-$$

Ettore Majorana
(1906-1938?)

The 0νββ decay violets the conservation of the lepton number

Not observed so far *(physics beyond the Standard Model, test of neutrino mass)*

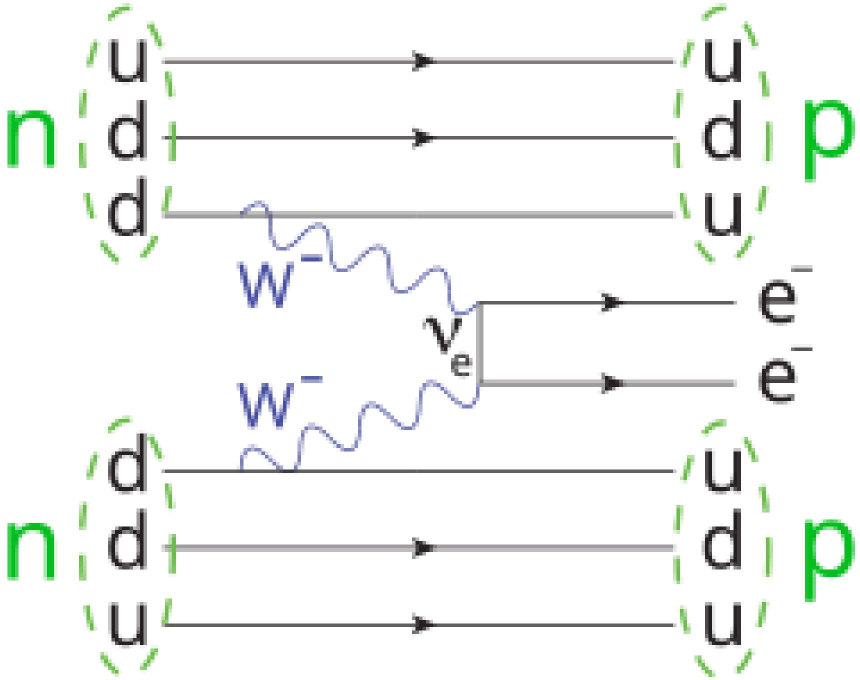




Major on-going Research Projects:

Europe: CUORE@LNGS
USA: LEGEND, MAJORANA
JAPAN: KamLAND-Zen
...

Two neutrino-less double β decay



Candidates for two-neutrino-less double-β decay cases

Experimental limits (at least 90% C.I.)[7] on a collection of isotopes for 0νββ decay process mediated by the light neutrino mechanism, as shown in the Feynman diagram above.

Isotope	Experiment	lifetime $T^{0\nu}_{\beta\beta}$ [years]
$^{48}\mathrm{Ca}$	ELEGANT-VI	$> 1.4 \cdot 10^{22}$
$^{76}\mathrm{Ge}$	Heidelberg-Moscow[14]	$> 1.9 \cdot 10^{25}$ [14]
$^{76}\mathrm{Ge}$	GERDA	$> 1.8 \cdot 10^{26}$ [15]
$^{76}\mathrm{Ge}$	MAJORANA	$> 8.3 \cdot 10^{25}$ [16]
$^{82}\mathrm{Se}$	NEMO-3	$> 1.0 \cdot 10^{23}$
$^{82}\mathrm{Se}$	CUPID-0	$> 4.6 \cdot 10^{24}$ [17]
$^{96}\mathrm{Zr}$	NEMO-3	$> 9.2 \cdot 10^{21}$
$^{100}\mathrm{Mo}$	NEMO-3	$> 2.1 \cdot 10^{25}$
$^{116}\mathrm{Cd}$	Solotvina	$> 1.7 \cdot 10^{23}$
$^{128}\mathrm{Te}$	CUORE	$> 3.6 \cdot 10^{24}$ [18]
$^{130}\mathrm{Te}$	CUORE	$> 2.2 \cdot 10^{25}$
$^{136}\mathrm{Xe}$	EXO	$> 3.5 \cdot 10^{25}$ [19]
$^{136}\mathrm{Xe}$	KamLAND-Zen	$> 2.3 \cdot 10^{26}$ [20]
$^{150}\mathrm{Nd}$	NEMO-3	$> 2.1 \cdot 10^{25}$

experimental limits

$T_{1/2}$ > xx years

β-decay energy spectrum

In the β^- decay an electron and an electron anti-neutrino will be emitted simultaneously.

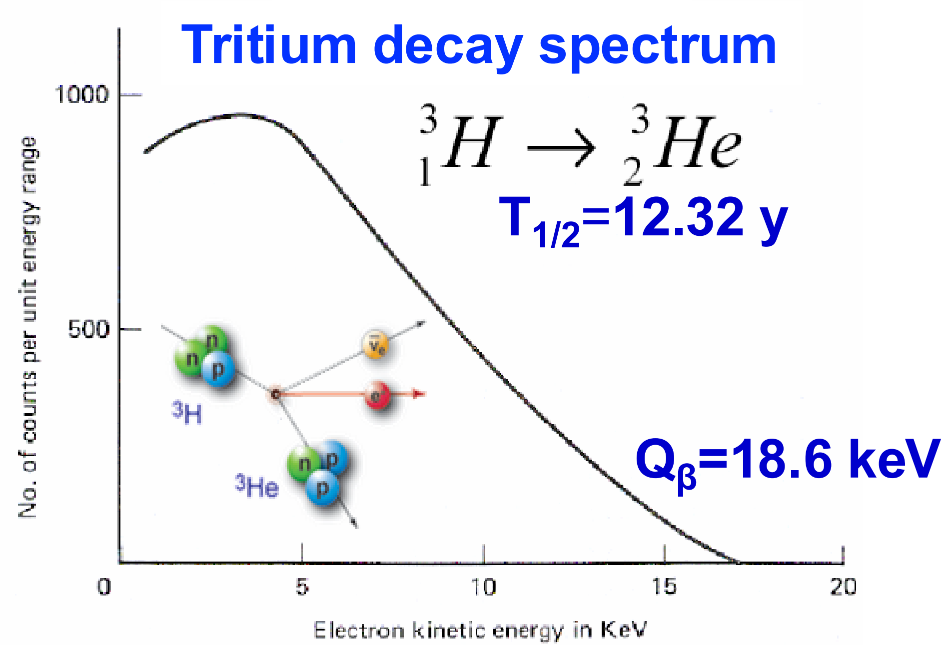
$$n \rightarrow p + e^- + \bar{\nu}_e$$

The β^- decay energy is given by the mass difference between mother and daughter nucleus.

The decay energy will be distributed as *kinetic energy on the emitting particles*, the electron and the anti-neutrino.

Hence, the electron spectrum is continuous.

It starts at zero energy and ends at the maximum possible energy $E_{max} = E_0 - m_\nu \cdot c^2$ (= Q_β).

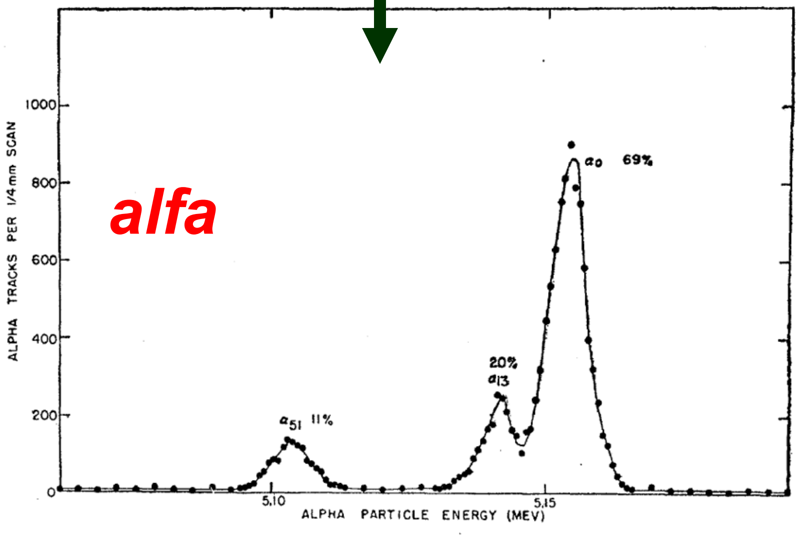


*β decays have a **long lifetime** and a **small decay probability**, the **related interaction is small** compared to other interactions in the nucleus, therefore **time dependent perturbation theory** is a good approximation.*

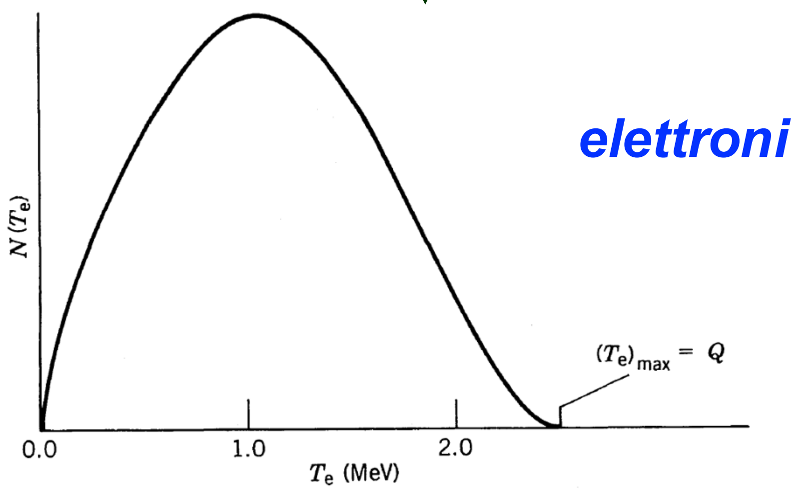
Nel nucleo non ci sono elettroni e quindi **l'elettrone** viene "creato" nel processo di decadimento β utilizzando l'energia a disposizione.
Viene immediatamente espulso fuori dal nucleo.

Nel decadimento α invece si assume che la particella esista gia' all'interno del nucleo

Particelle a nel decadimento alfa (energie discrete, ben definite)



Elettroni nel decadimento beta (spettro continuo)



Example:

$^{210}Bi \xrightarrow{\beta^-} {}^{210}Po + e^- + ...$

l'atomo di Bi ha 83 protoni (ed elettroni),
il Po ha 84 protoni (ed elettroni)

^{210}Bi M = 209.9841204 (16) u (unita' di massa atomica u = 931.494 MeV)

^{210}Po M = 209.9828737 (13) u

$\Delta M = Q = 0.0012467$ u = 1.16 MeV

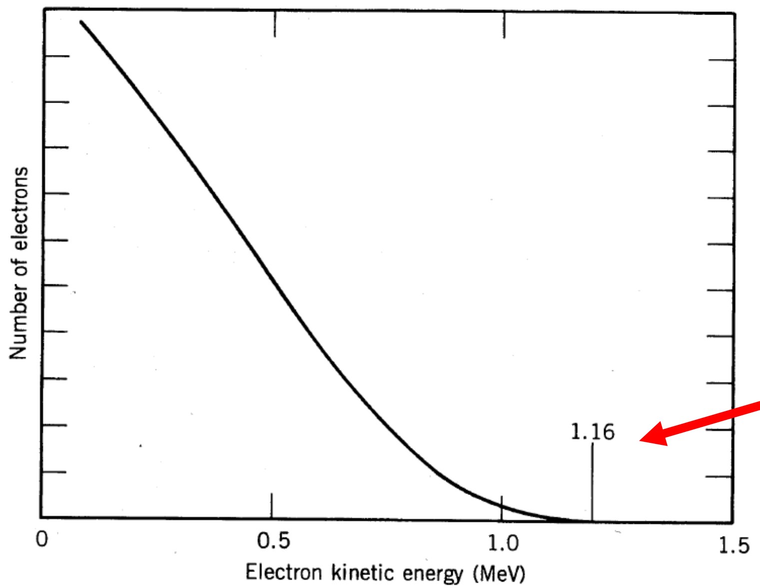


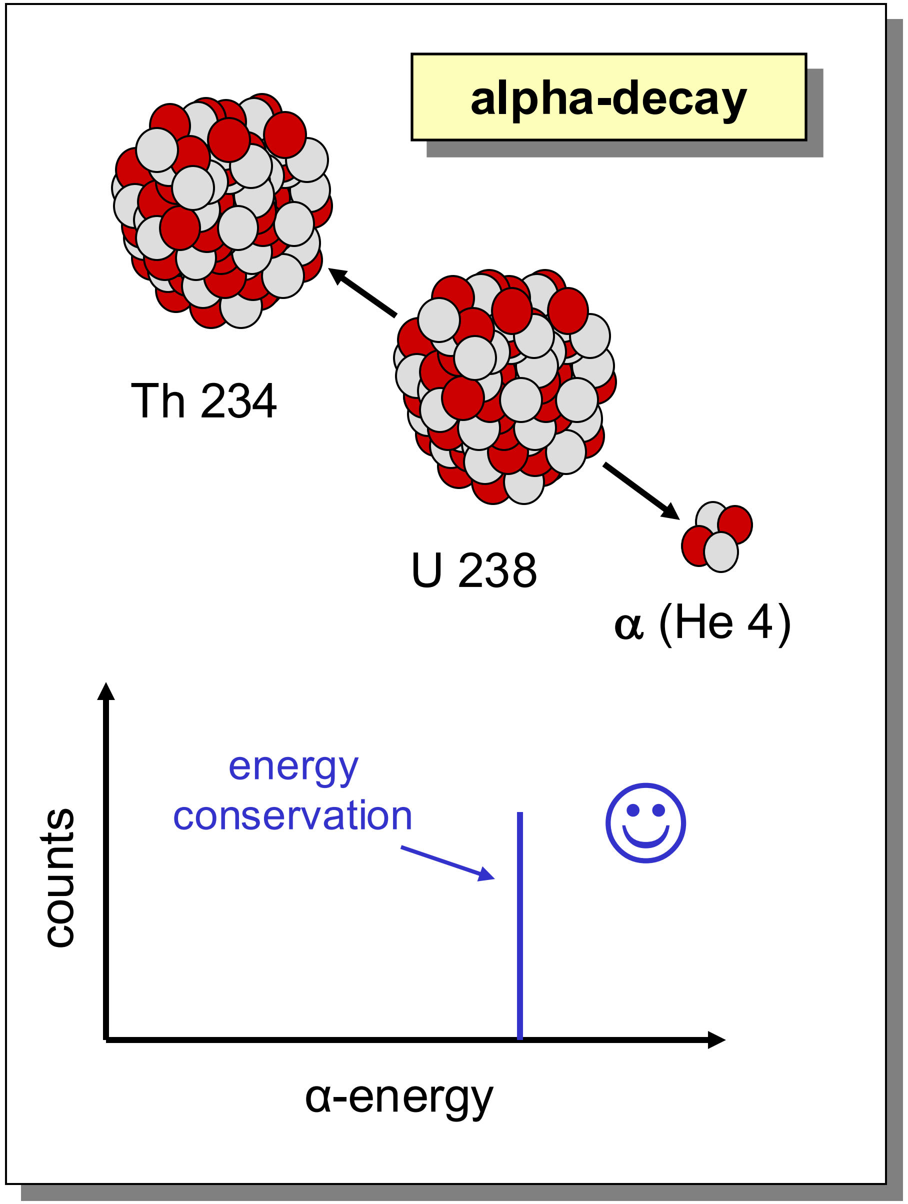
Figure 9.1 The continuous electron distribution from the β decay of ^{210}Bi (also called RaE in the literature).

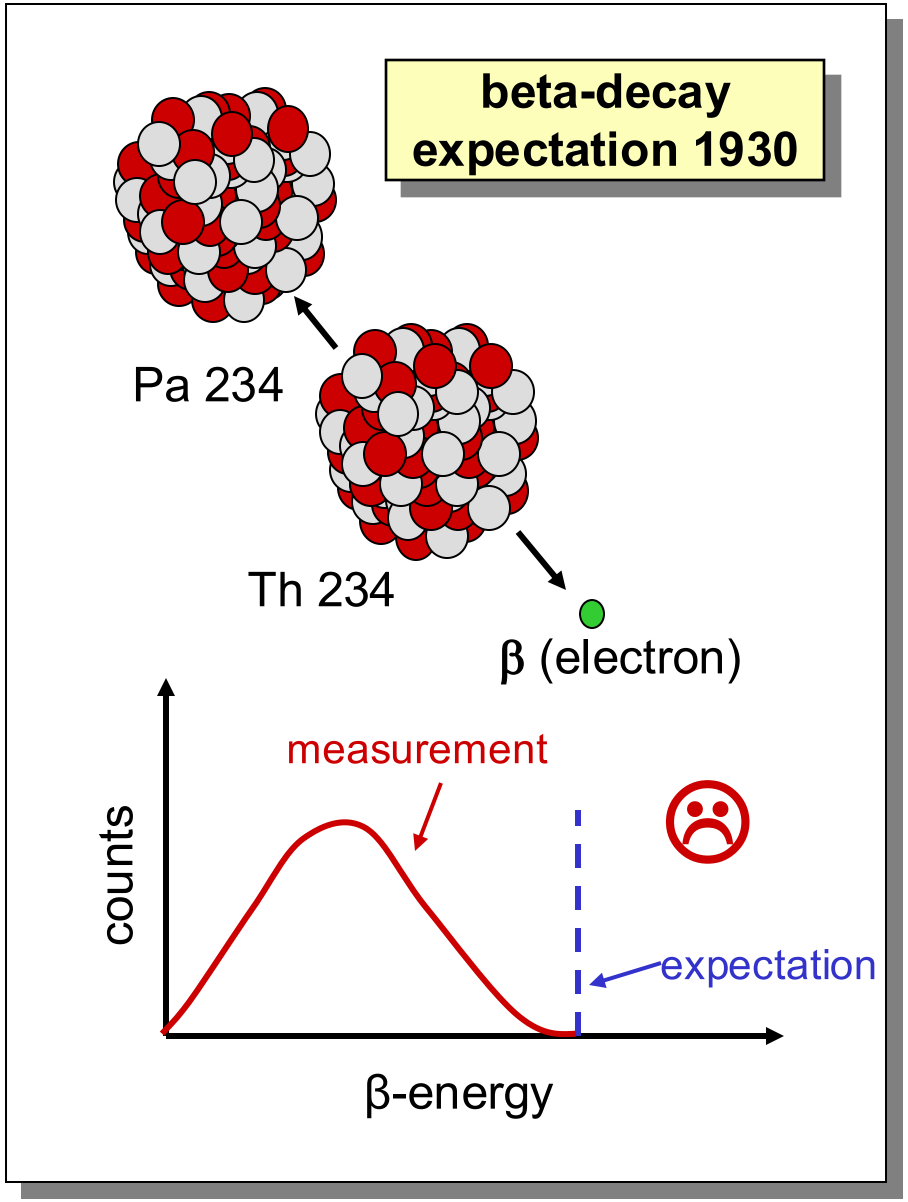
Se il processo fosse a due corpi,
come per la particella alfa,
ci si aspetterebbe un elettrone con energia ben definita pari a 1.16 MeV

Si osserva una distribuzione continua dell'energia da zero al valore massimo $T_{max} = Q = 1.16$ MeV

dove Q e' la differenza delle masse atomiche del nucleo padre e di quello figlio

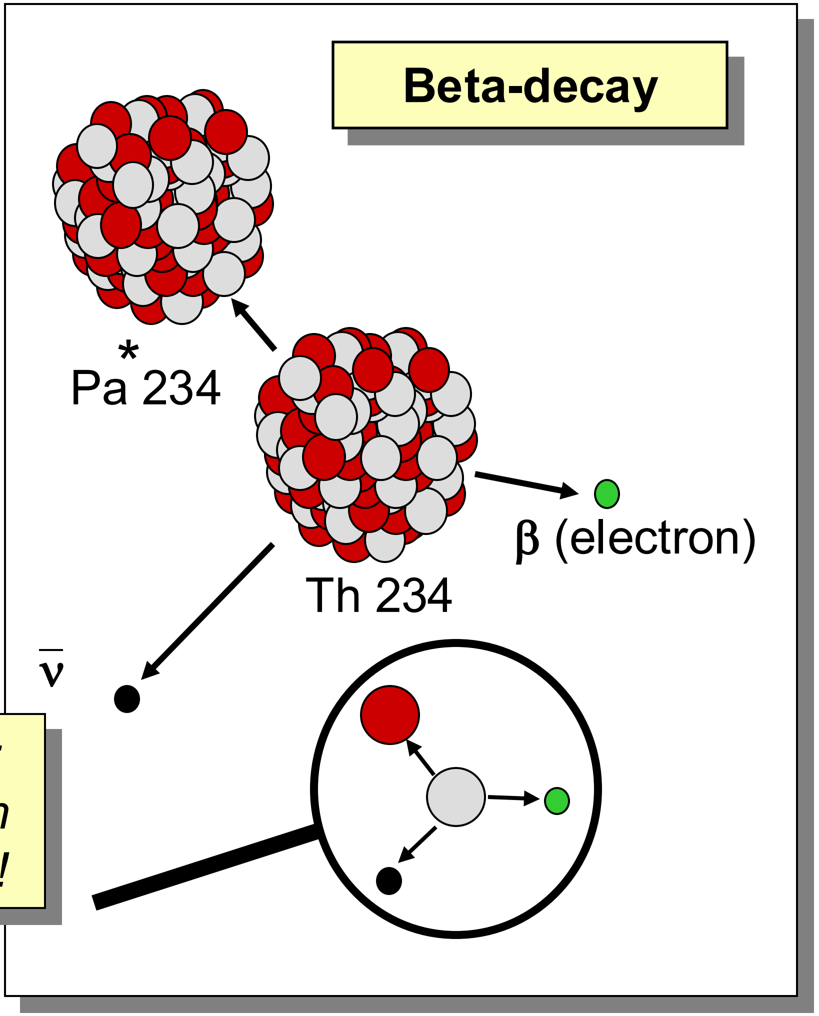
1930: Energy conservation violated in β-decay ?





The new particle proposed by Pauli

Wolfgang Pauli



*In addition to an electron a **light neutral particle** is created which carries away the missing energy!*

Leptons	
Charge –1	Charge 0
Electron e	e-Neutrino ν_e
Muon μ	μ-Neutrino ν_μ
Tauon τ	τ-Neutrino ν_τ

*for the **lepton number conservation** the **electron** must be accompanied by an **antineutrino** and not neutrino!*

„Today I have done something, **what one should not do in theoretical physics**. I have explained something, what is not understood, by something, which can not be observed!"

W. Pauli predicted in a letter to E. Fermi the existence of neutrinos (1930)

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

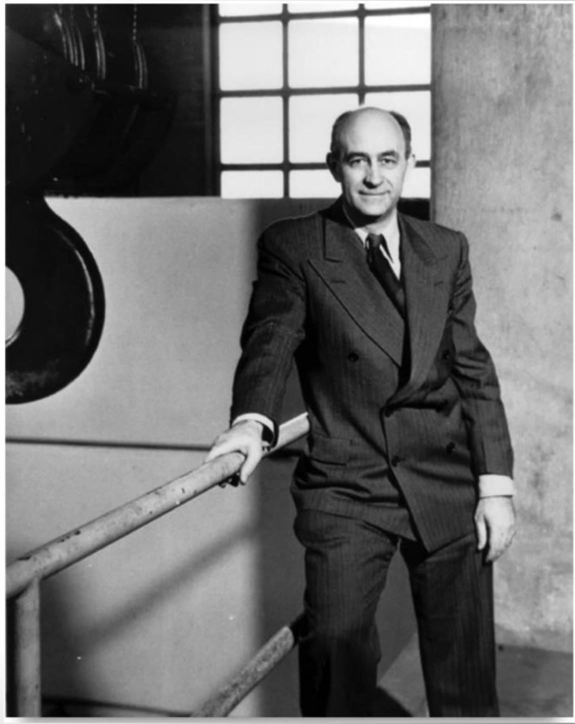
Pauli Letter Collection, CERN

Pauli did not believe energy-momentum conservation was violated
He proposed a desperate way out: a new 'invisible' particle
He called it the neutron.
He also stayed away from the conference because of a ball in Zurich..

1934
Enrico Fermi, father of the world's first nuclear reactor, coined the term "neutrino" which is Italian for "little neutral"
The name neutron had meanwhile been claimed for the discovered nucleon partner of the proton

He proposed a theory for Beta-decay including the neutrino, a first formulation of the weak force…

Funny enough his paper got refused by Nature magazine
(criticism: nothing practical in this paper)



Pauli (1931): ipotesi del neutrino

(particella che possiede l'energia mancante)

Conservazione della carica $\rightarrow q_{neutrino} = 0$

Conservazione del momento angolare $\rightarrow spin_{neutrino} = 1/2$

Esempio: $^{14}_{6}C \rightarrow {}^{14}_{7}N + e^{-}$

Senza la presenza della nuova particella **NON** si conserva il momento angolare

I = 0 1 1/2

(in ogni caso, per A pari I deve essere intero)

Stato iniziale con I intero,
Stato finale con I semi-intero (somma di un I intero, di uno semi-intero, e di un eventuale momento angolare orbitale del sistema ^{14}N - elettrone, comunque intero).

il neutrino ha spin semi-intero I = 1/2

Dopo la scoperta del neutrone, il decadimento beta e tutti i fatti ad esso connessi vennero spiegati con l'ipotesi del neutrino di Pauli (1931) che portò alla teoria di Fermi del 1934.

L'ipotesi di Pauli è che la trasformazione sia data da:

$$ {}_{Z}^{A}X \rightarrow {}_{Z\pm 1}^{A}Y + e^{\mp} + \nu $$

dove la particella ν (**neutrino**) è neutra per la conservazione della carica, ha spin semi-intero per la conservazione del momento angolare, ed ha massa a riposo molto piccola per la conservazione dell'energia (altrimenti la massima energia dell'elettrone emesso dovrebbe dipendere dalla massa del neutrino).

La **forma dello spettro energetico della emissione beta** si ricava studiando in che modo l'energia a disposizione nella transizione può essere **ripartita tra la particella beta ed il neutrino**.

Assunzioni:

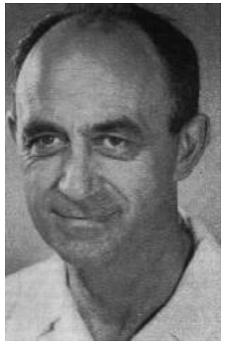
a) si trascura l'interazione coulombiana tra l'elettrone ed il nucleo (vale almeno per i nuclei più leggeri, con $Z < 10$);

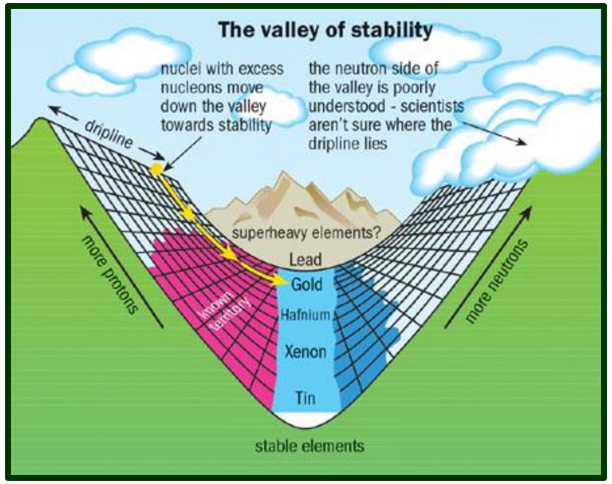
b) si trascura il rinculo del nucleo residuo (sempre valida, essendo $m_e << M_{nucleo}$);

c) si considera nulla la massa del neutrino ($m_n = 0$);

d) si considerano egualmente probabili tutte le possibili suddivisioni dell'energia tra elettrone e neutrino.

The Fermi Theory





La teoria di Fermi da una spiegazione ai seguenti fatti:

i) la **forma degli spettri beta**;

ii) la relazione tra **energia massima** del decadimento e la **vita media**;

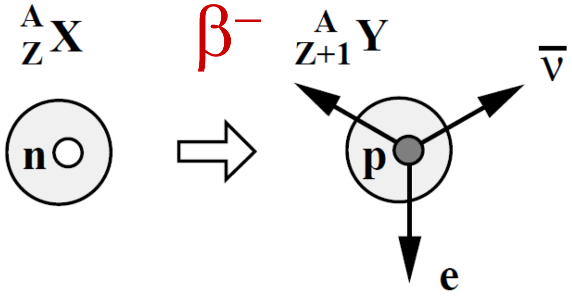
iii) la **classificazione delle transizioni beta** e la definizione di **regole di selezione**.

Le vite medie dei decadimenti beta variano da alcuni ms a 10^{17} anni!
Le regole di selezione della teoria lo spiegano. Ci sono i decadimenti
super-permessi e quelli proibiti
(in funzione del momento angolare dell'elettrone/positrone e del neutrino)

	$T_{1/2}$
n	10.23 min
^{3}H	12.32 y
^{11}Li	8.5 ms
^{50}V	$1.4 \cdot 10^{17}$ Y

Il decadimento β dipende da:

- energia ottenuta nel processo (Q)
- momento angolare delle particelle emesse
- carica nucleare
- natura degli stati iniziali (i) e finali (f)



Spettro energetico di elettrone

E' uno spettro continuo:

emissione di un'altra particella oltre l'elettrone (o positrone).
Antineutrino o neutrino di massa trascurabile

$$m_\nu = m_{\bar{\nu}} \approx 0$$

$M_{nucleo} >> m_e \rightarrow E_{rinculo} \approx 0$

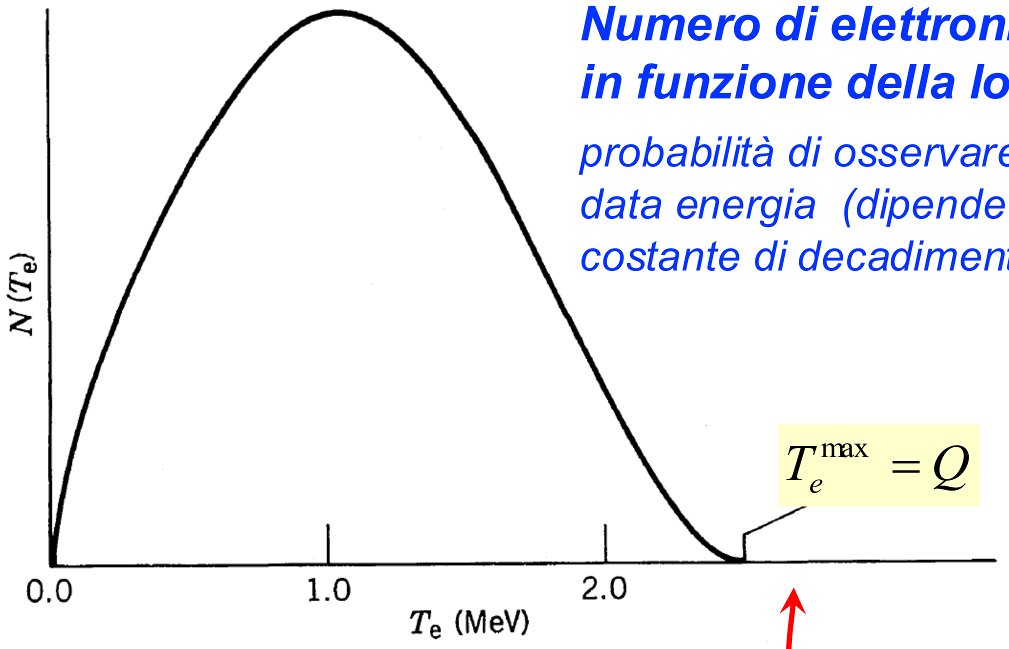
Il Q-valore del processo rappresenta l'energia a disposizione da suddividere tra elettrone e neutrino ($Q = T_e + E_\nu$).

L'energia totale del sistema elettrone-neutrino sara' quindi

$$E_0 = E_e + E_\nu$$

$$E_e = T_e + m_e c^2 \qquad E_\nu = c p_\nu$$

Ponendo zero l'energia del neutrino,
si ottiene il valore massimo per l'energia cinetica dell'elettrone



Numero di elettroni misurati in funzione della loro energia:

probabilità di osservare elettroni di data energia (dipende dalla costante di decadimento)

e : e^- o e^+
ν : neutrino o antineutrino

per massa nulla del neutrino

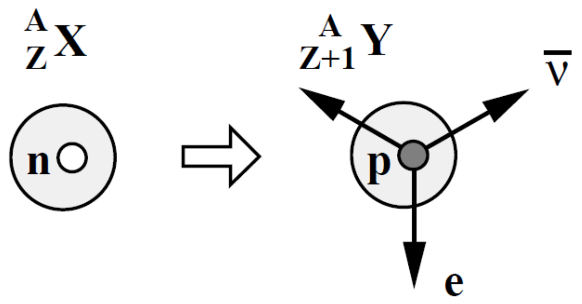
$E^2 = (pc)^2 + (mc^2)^2$

$$T_e^{max} = Q = E_0 - m_e c^2$$

end point energy

Regola fondamentale: Golden Rule di Fermi

β^-



la probabilità di transizione λ da uno stato iniziale i ad uno finale f

$\lambda = 1/\tau$

$$\lambda = \frac{2\pi}{\hbar}|M|^2\frac{dn}{dE}$$

$$M = \langle \psi_f | H | \psi_i \rangle = \int \psi_f^* H \psi_i d\vec{r}$$

(volume del nucleo)

interazione che causa la transizione

↓

informazioni sulla struttura nucleare

M e' chiamato **elemento di matrice** dell'operatore di transizione H

Fattore dovuto alla spazio delle fasi: densità di stati finali disponibili

dn : stati finali tra E e E + dE

fattore cinematico: determina la forma della distribuzione energetica

- ***la funzione d'onda ψ_i (iniziale)*** *descrive il nucleo prima del decadimento.*
- ***la funzione d'onda ψ_f (finale)*** *è il prodotto della funzione d'onda del nucleo nello stato finale e le funzioni d'onda dell'elettrone e del neutrino.*

Fermi introduce un modello, detto **a gas di Fermi**, che esprimere la densità di stati in termini del momento (quantita' di moto) di una particella.
Si ha che il numero di **stati dell'elettrone** tra p_e e $p_e + dp_e$ in un volume V e'

$$dN_e = V \frac{4\pi \cdot p_e^2 dp_e}{h^3}$$

Elettrone come onda piana

$$\Psi_e(\vec{r}) = \frac{1}{V^{1/2}} \cdot e^{i\vec{k}_e \cdot \vec{r}_e}$$

$$\vec{k} = \frac{\vec{p}}{\hbar}$$

Analogamente, trattando il **neutrino come indipendente** (non c'e' correlazione tra e e n, salvo la conservazione dell'energia e della quantità di moto)

$$dN_\nu = V \frac{4\pi \cdot p_\nu^2 dp_\nu}{h^3}$$

E quindi, il **numero totale di stati dn**

$$dn = dN_e \cdot dN_\nu = \left(\frac{4\pi V}{h^3}\right)^2 p_e^2 dp_e p_\nu^2 dp_\nu$$

Vincolo sull'energia totale (divisa tra elettrone e neutrino)

$$E_\nu = p_\nu c = E_0 - E_e$$ e fissata E_e $$dp_\nu = \frac{dE_0}{c}$$

si ottiene ***dn/dE_0*** *= numero di stati con energia totale tra* ***E_0*** *e* ***$E_0 + dE_0$***

The state of a particle is determined by its position and its momentum $\Delta x \cdot \Delta y \cdot \Delta z \cdot \Delta p_x \cdot \Delta p_y \cdot \Delta p_z \approx h^3$

Each particle has a volume of h^3 in phase space.

The number of states in momentum shell $\Delta p = 4\pi \cdot p^2 dp$ *is given by* $dn = \frac{V \cdot 4\pi \cdot p^2}{h^3} dp$ *level density of free particles*

$$\Delta x \cdot \Delta y \cdot \Delta z = V$$

Modello a gas di Fermi

$$\frac{dn}{dE_0} = \left(\frac{4\pi V}{h^3}\right)^2 \frac{1}{c^3}(E_0 - E_e)^2 p_e^2 dp_e$$ (FORMULA 1)

$$\lambda = \frac{2\pi}{\hbar}|M|^2 \frac{dn}{dE}$$

Energia totale
tra E_0 e $E_0 + dE_0$

momento elettrone
tra p_e e $p_e + dp_e$
indipendente dal neutrino

> Dalla regola di Fermi

$$d\lambda(p_e) = C|M|^2 (E_0 - E_e)^2 p_e^2 dp_e$$

costante

probabilità di disintegrazione con p_e tra p_e e $p_e + dp_e$

> da cui integrando

$$\lambda = C\int_0^{p_e^{max}} |M|^2 (E_0 - E_e)^2 p_e^2 dp_e$$

$$p_e^{max} = \frac{1}{c}(E_0^2 - m_e^2 c^4)^{1/2}$$

quando l'energia cinetica
del neutrino e' nulla

L'elemento di matrice nucleare viene assunto indipendente da p_e

$$\lambda \cong C|M|^2 f(E_0)$$

Il **campo coulombiano** ha effetti di **distorsione sullo spettro energetico di elettroni o positroni** (rispetto alla previsione teorica).

Esempio ^{64}Cu

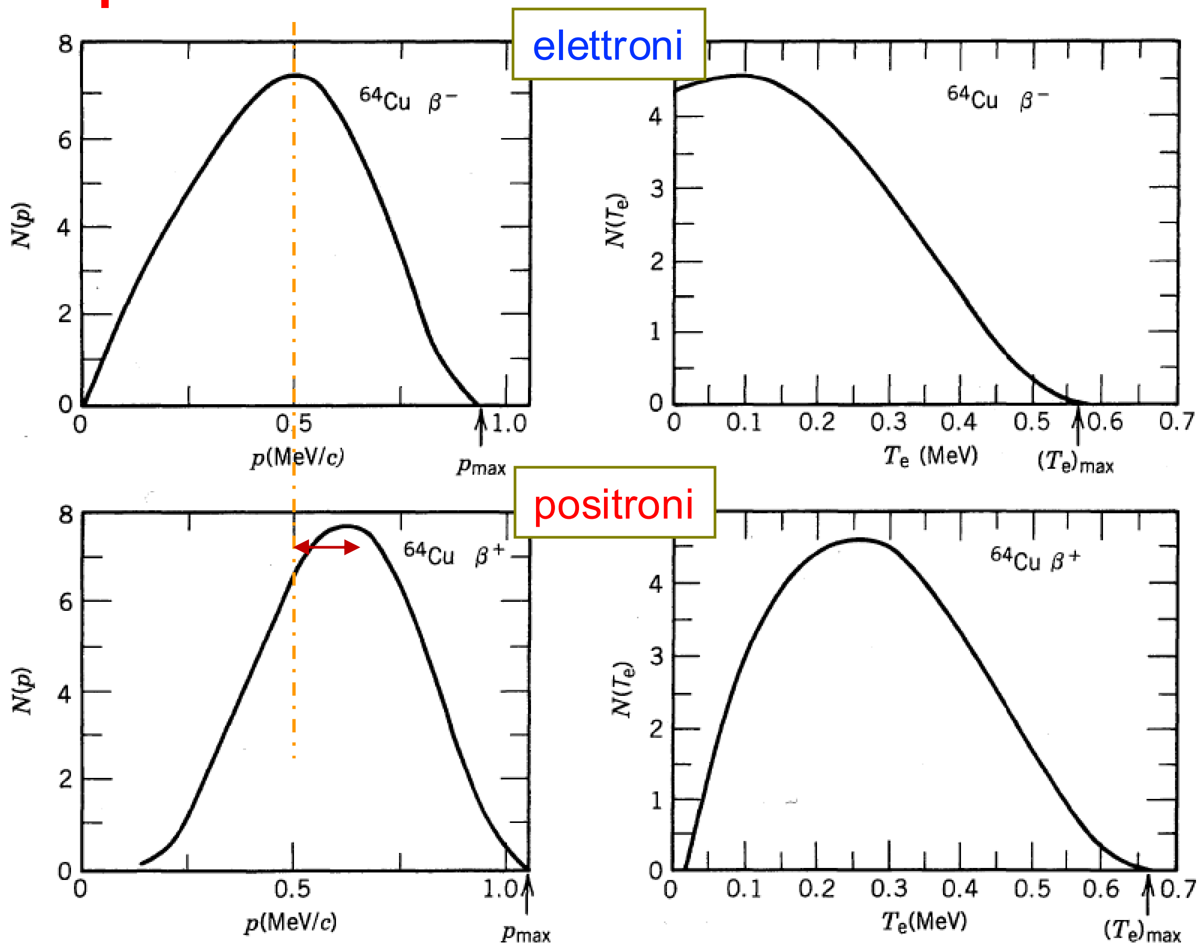


Figure 9.3 Momentum and kinetic energy spectra of electrons and positrons emitted in the decay of ^{64}Cu. Compare with Figure 9.2; the differences arise from the Coulomb interactions with the daughter nucleus. From R. D. Evans, *The Atomic Nucleus* (New York: McGraw-Hill, 1955).

ci sono differenze, rispetto all'andamento teorico aspettato, tra la distribuzione degli elettroni e dei positroni

repulsione coulombiana dei positroni (dal nucleo)

→ *meno positroni di bassa energia*

attrazione coulombiana degli elettroni ((dal nucleo)

→ *piu' elettroni di bassa energia*

C'e' un fattore correttivo (**funzione di Fermi**) che tiene conto di questo effetto

$$F(Z,E_e) \approx \frac{2\pi\eta}{1-e^{-2\pi\eta}}$$

$$\eta = \mp \frac{Ze^2}{\hbar v_e}$$

numero atomico del nucleo figlio

parametro di Sommerfeld

velocità asintotica elettrone (o positrone)

$$\eta << 1 \rightarrow F(Z,E_e) \approx 1$$

Più in dettaglio

funzione di Fermi tiene conto degli effetti di distorsione causati del campo Coulombiano

$$F(Z,E_e) \approx \frac{2\pi\eta}{1-e^{-2\pi\eta}}$$

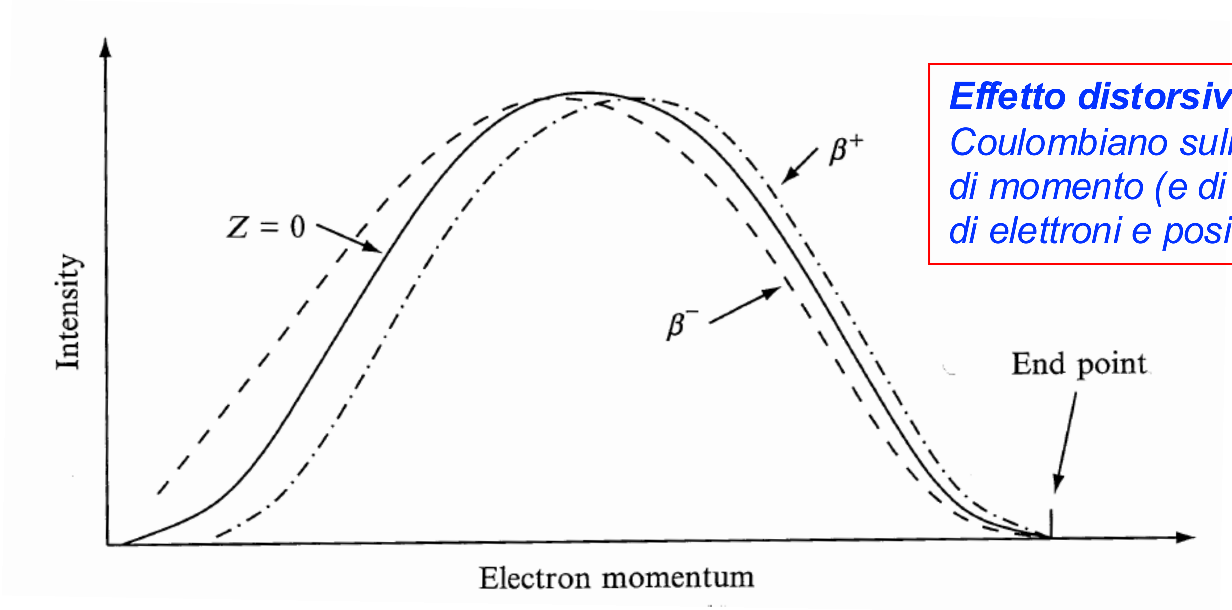
numero atomico del nucleo figlio

$$\eta << 1 \rightarrow F(Z,E_e) \approx 1$$

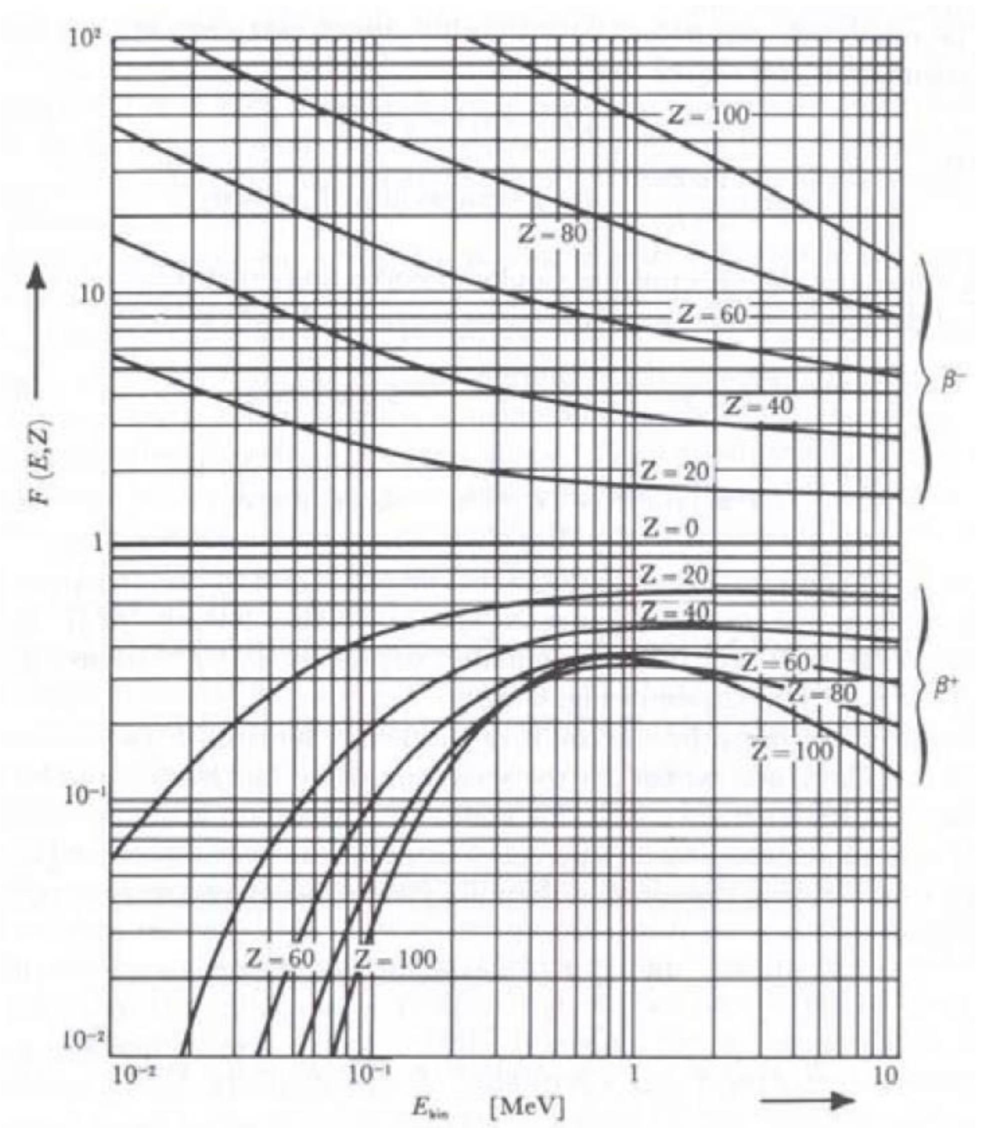
$$\eta = \mp \frac{Ze^2}{\hbar v_e}$$

parametro
di Sommerfeld

velocità asintotica
elettrone (o positrone)



Effetto distorsivo del campo
Coulombiano sulla distribuzione
di momento (e di energia)
di elettroni e positroni



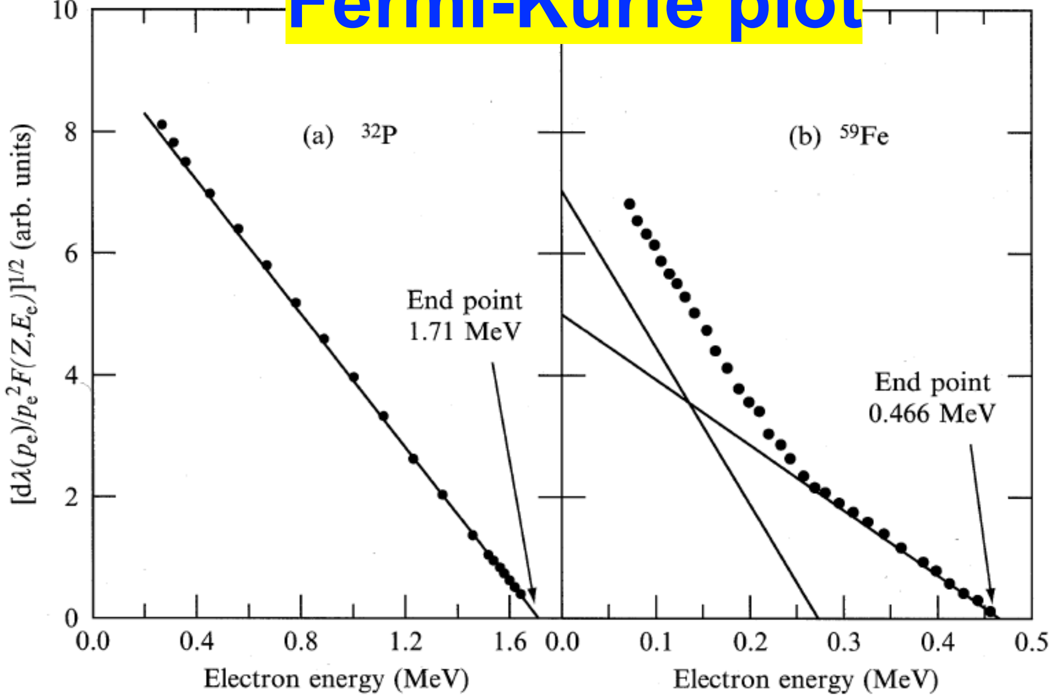
Tornando alla probabilità di disintegrazione differenziale (**regola d'oro di Fermi**) con inclusione dell **nuovo termine correttivo:**

$$d\lambda(p_e) = C|M|^2 F(Z,E_e)(E_0 - E_e)^2 p_e^2 dp_e$$

definisco il rapporto $K(E_e) = \left[\frac{\frac{d\lambda(p_e)}{dp_e}}{p_e^2 F(Z,E_e)} \right]^{1/2} \propto E_0 - E_e$

e' lineare in E_e

Fermi-Kurie plot



Permette di determinare con precisione *end point dello spettro energetico*

Dall'analisi del Fermi-Kurie plot si ottengono informazioni su

a - **massa neutrino**

b - transizioni multiple

c - transizioni proibite

Se il neutrino ha massa ≠ 0

Dall'analisi del Fermi-Kurie plot si ottengono informazioni su

a - **massa neutrino**

b - transizioni multiple

c - transizioni proibite

$$E_0 = E_e + E_\nu$$

$$E_\nu = \sqrt{p_\nu^2 c^2 + m_\nu^2 c^4}$$

Per un valore fissato di E_e

$$p_\nu^2 c^2 = (E_0 - E_e)^2 - m_\nu^2 c^4$$

$$p_\nu = \frac{\sqrt{(E_0 - E_e)^2 - m_\nu^2 c^4}}{c}$$

differenziando

$$p_\nu dp_\nu = \frac{(E_0 - E_e)dE_0}{c^2}$$

$$p_\nu^2 dp_\nu = \frac{1}{c^3}\sqrt{(E_0 - E_e)^2 - m_\nu^2 c^4} \cdot (E_0 - E_e) dE_0$$

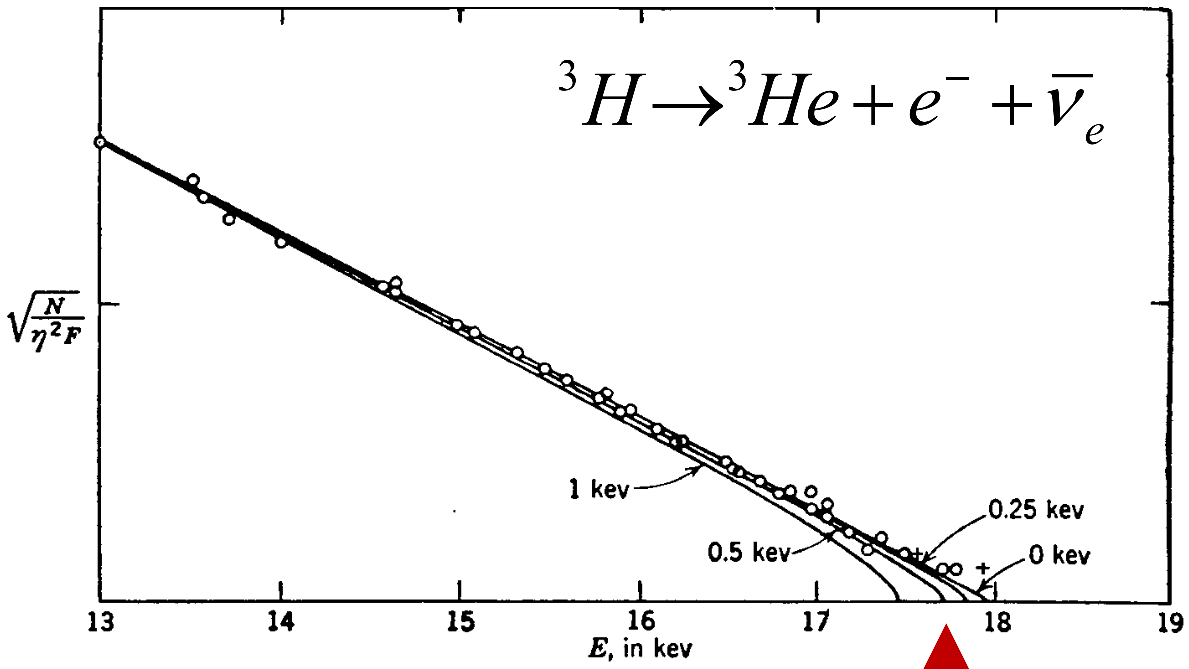
nell'espressione *(FORMULA 1)* questo termine sostituisce $(E_0 - E_e)^2$

$$\frac{dn}{dE_0} = \left(\frac{4\pi V}{h^3}\right)^2 \frac{1}{c^3} (E_0 - E_e)^2 p_e^2 dp_e$$

Se il neutrino ha massa ≠ 0

nel Fermi-Kurie plot K(E_e) **non e' piu' lineare** in funzione di E_e.

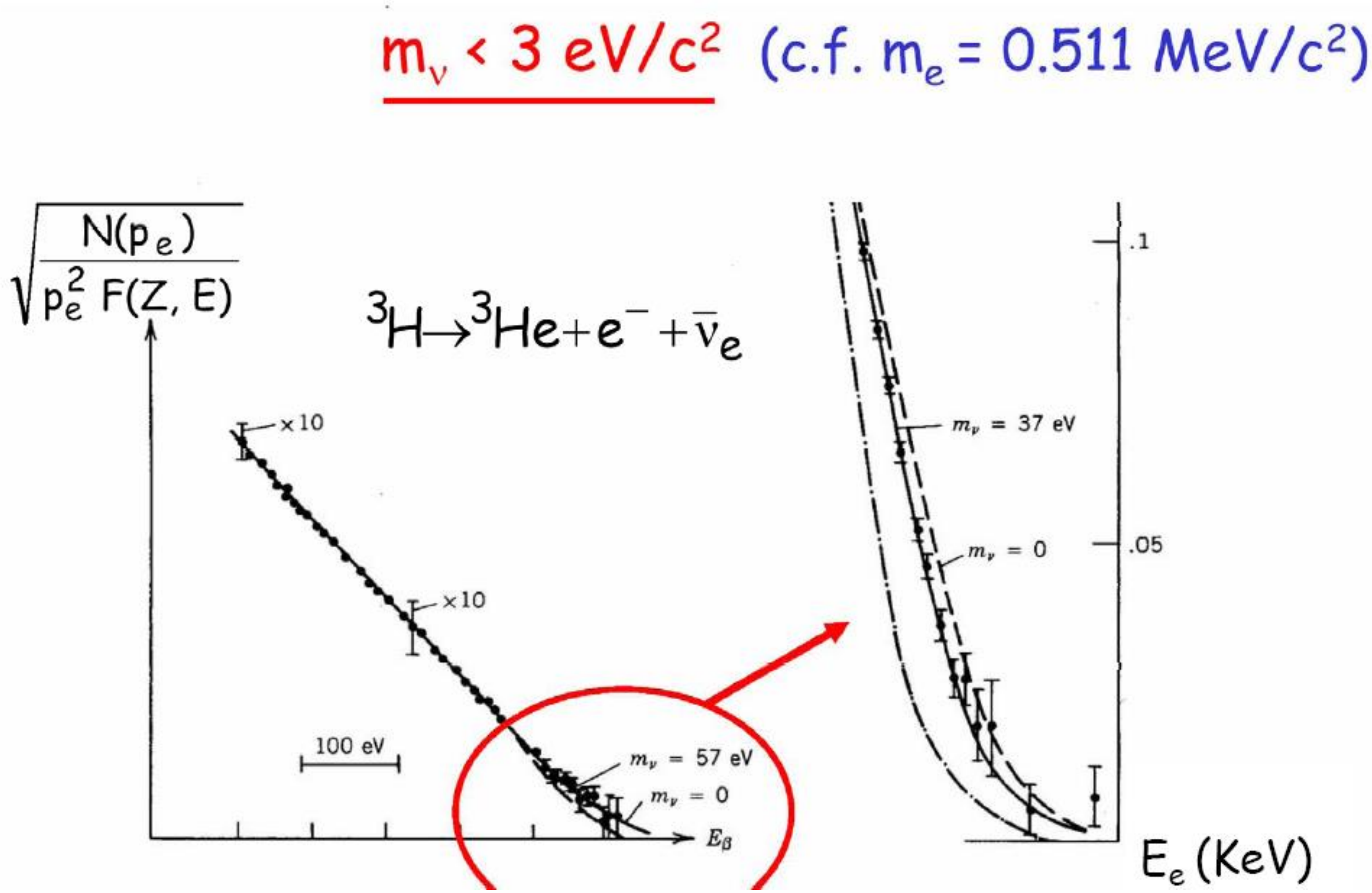
$$\sqrt{\frac{N(p_e)}{F(Z,\varepsilon)\cdot p_e^2}} \neq const\cdot(\varepsilon_0-\varepsilon)$$



valori diversi della ***massa del neutrino***
*portano a valori diversi dell'****end point energy***

metodo per misurare la massa del neutrino

Decadimento del Trizio ^3H, end-point energy = 18.6 keV



La misura della distribuzione vicino all'end-point (E_{max}) della distribuzione, fornisce un metodo per stimare la massa del neutrino.

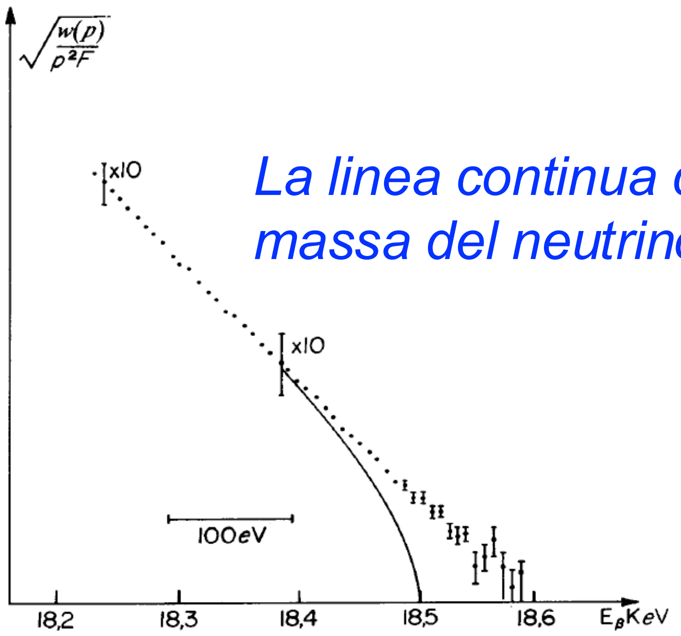
La **misure più precise** sono state fatte studiando il **decadimento del Trizio**:

- *Nuclei semplici*, correzioni facili da valutare
- *Energia disponibile* nello stato finale è piccola

($E_0 = 530\ keV \to E_\nu = 18.6\ keV$)

→ *aumenta la sensibilità della misura*

Decadimento del Trizio ^{3}H, end-point energy = 18.6 keV



Plot di Kurie del ^{3}H

La linea continua corrisponde ad una massa del neutrino pari a ***100 eV***

Vari esperimenti su decadimento trizio hanno portato negli anni a ***risultati diversi***:

Langer, Moffatt	1952	m_ν < 10 keV
Bergkvist	1972	m_ν < 65 eV
Tretyakov	1976	m_ν < 35 eV
Lyubimov	1980	m_ν < 30 eV
Fritschi	1986	m_ν < 18 eV
Robertson	1991	m_ν < 9.3 eV
Stoeffl	1995	m_ν < 7 eV
Weinheimer	1999	m_ν < 2.8 eV
Lobashev	1999	m_ν < 2.5 eV

Latest Press Release by INFN on Katrin (2025 data): Link

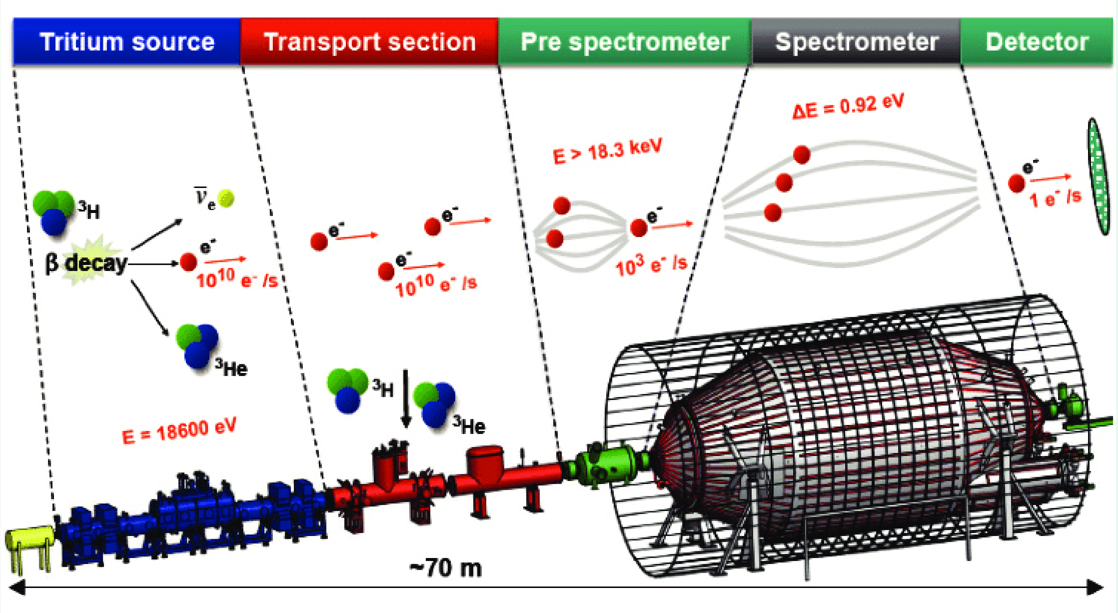
KATRIN Collaboration et al., Science 388, 180–185 (2025)

Esperimento Katrin m_ν < 0.45 eV = 8 10^{-37} kg (2025 data)

Karlsruhe Institute of Technology in Germania

spettro di energia dell'elettrone misurato ad altissima precisione

spettrometro di 70 m



Nature Physics (2019)

https://www.nature.com/articles/s41567-021-01463-1

Neutrino Detection – «Project Poltergeist»

(experiment performed at **nuclear reactor**, 1953-1956)

Instant signal

$$\bar{\nu}_e + p \rightarrow n + e^+$$

p from water tanks

$$e^+ + e^- \rightarrow \gamma + \gamma$$

e+ annihilation

$$\gamma + e^- \rightarrow \gamma' + e^-$$

e- Compton scattering

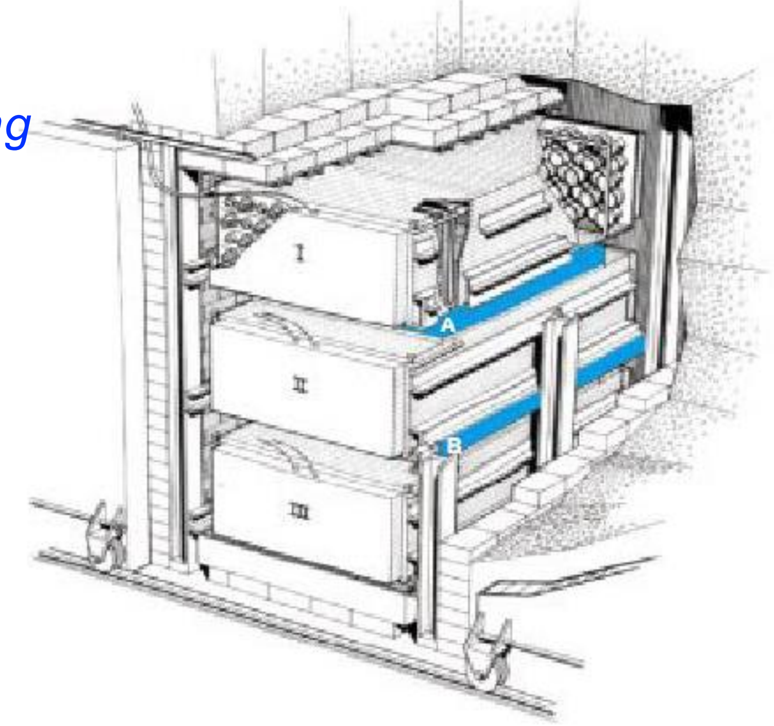
Retarded signal (1 ms)

$$n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X + \gamma$$

neutron capture on p ((n,p)→ d+ γ 2.2 MeV) and (n,Cd)→ Cd+γ*

β⁻: $n \rightarrow p + e^- + \bar{\nu}$

5×10^{13} cm²/s *(much higher flux than from radioactive sources)*



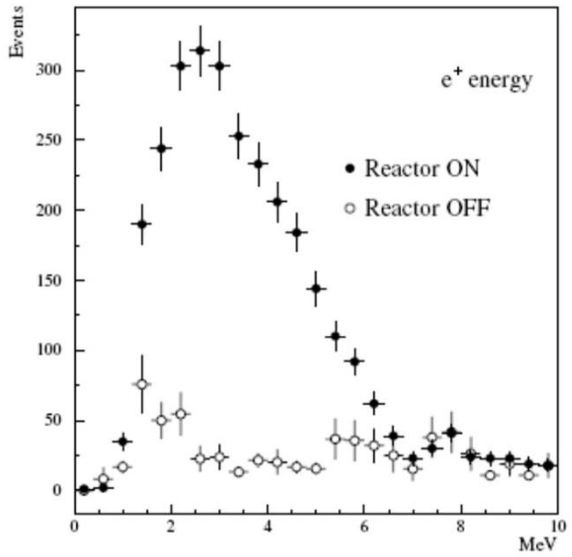


Fig. 46. Experimental positron spectra for reactor-on and reactor- periods after application of all selection criteria. The errors shown are statistical

Frederik Reines

Clyde L Cowan

F. Reines Nobel Prize in Physics 1995 (Cowan dies in 1974)

*La **sezione d'urto predetta** per la reazione era circa **6×10^{-44} cm²** e la **sezione d'urto misurata** fu **6.3×10^{-44} cm²***

next lecture on beta decay selection rules:

After discussion on SHELL MODEL