

SCHEMA RIASSUNTIVO

- quantità di moto
(momento lineare)

$$\vec{p} = m \vec{v}$$

$$[p] = \frac{[M][L]}{[T]} = \frac{\text{kg m}}{\text{s}}$$

- generalizzazione legge di Newton:

$$\sum_i \vec{F}_i = \frac{d\vec{p}}{dt}$$

infatti: $\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$

se $m \neq m(t) \rightarrow \frac{dm}{dt} = 0$

$$\sum_i \vec{F}_i = m \frac{d\vec{v}}{dt} = m\vec{a}$$

- Conservazione della quantità di moto

se $\sum_i \vec{F}_i = 0 \rightarrow$ sistema isolato

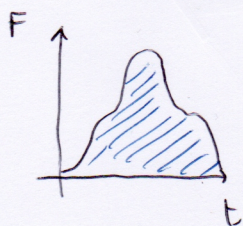
allora $\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{costante}$

- Impulso

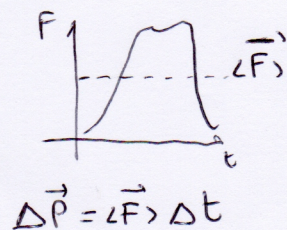
$$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow dp = F dt$$

$$\int_{p_1}^{p_2} dp = \int_0^t F(t) dt \rightarrow \Delta \vec{p} = \int_0^t \vec{F}(t) dt \stackrel{\text{def}}{=} \vec{I} \quad (\text{reazione diretta})$$

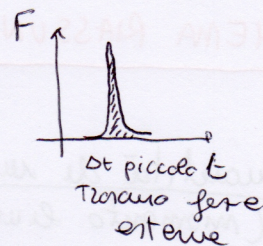
impulso



l'azione di una forza
in un intervallo di tempo
produce una variazione
della quantità di moto



Interazione tra due o più corpi tramite forze impulsive



$\Rightarrow$ Per ogni tipo di urto

$$\Delta \vec{P} = 0 \rightarrow \vec{P} \text{ si conserva}$$

• urto elastico

$$K_{\text{TOT}}^i = K_{\text{TOT}}^f \Rightarrow \Delta K = 0 \text{ l'energia cinetica si conserva}$$

• urto anelastico

$$K_{\text{TOT}}^i = K_{\text{TOT}}^f + E_1 + E_2 + \dots \text{ l'energia cinetica viene dissipata}$$

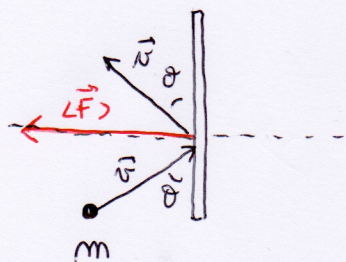
• urto completamente anelastico

$$K_{\text{TOT}}^i \neq K_{\text{TOT}}^f$$

$$M_f = \sum_i m_i$$



ESERCIZIO 1



$$m = 3 \text{ kg}$$

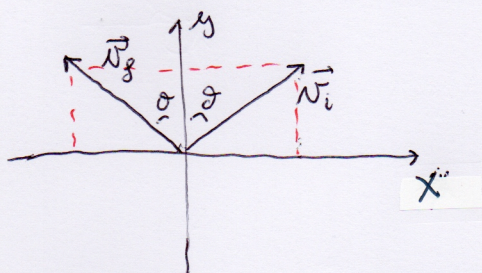
$$|\vec{v}| = 10 \text{ m/s}$$

$$\theta = 60^\circ$$

$$\Delta t = 0.2 \text{ s}$$

$$? = \langle \vec{F} \rangle$$

$$\Delta \vec{P} = \langle \vec{F} \rangle \Delta t \rightarrow \langle \vec{F} \rangle = \frac{\Delta \vec{P}}{\Delta t} \quad \Delta \vec{P} = ?$$



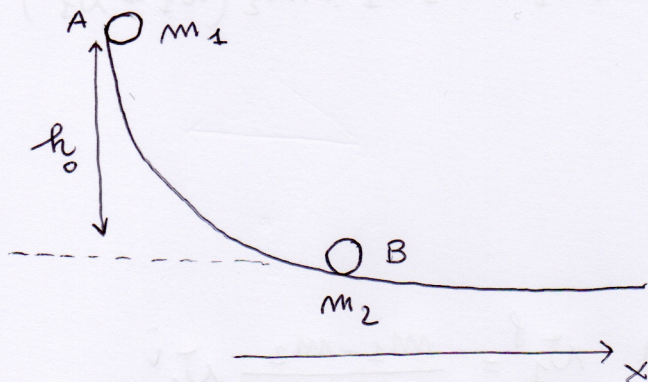
$$\Delta \vec{P}_y = \vec{P}_y^f - \vec{P}_y^i = m(\vec{v}_y^f - \vec{v}_y^i) = m(v \cos \theta \underline{\underline{1}} - v \cos \theta \underline{\underline{1}}) = 0 \underline{\underline{1}}$$

$$\begin{aligned} \Delta \vec{P}_x &= \vec{P}_x^f - \vec{P}_x^i = m(\vec{v}_x^f - \vec{v}_x^i) = m(-v \sin \theta \underline{\underline{1}} - v \sin \theta \underline{\underline{1}}) = \\ &= -2m v \sin \theta \underline{\underline{1}} \end{aligned}$$

$$\Delta \vec{P} = \Delta \vec{P}_x + \Delta \vec{P}_y = -2m v \sin \theta \underline{\underline{1}} + 0 \underline{\underline{1}} \equiv \Delta \vec{P}_x$$

$$\langle \vec{F} \rangle = \frac{\Delta \vec{P}_x}{\Delta t} = - \frac{2m v \sin \theta}{\Delta t} \underline{\underline{1}} = - \frac{2 \cdot 3 \text{ kg} \cdot 10 \text{ m/s} \cdot \sqrt{3}/2}{0.2 \text{ s}} \underline{\underline{1}} = -260 \text{ N} \underline{\underline{1}}$$

ESERCIZIO 2



$$m_1 = 5 \text{ kg}$$

$$m_2 = 10 \text{ kg}$$

=> URTO ELASTICO <=>

$h_{\max}$ di m_1 dopo urto?

v_1^i appena prima dell'urto

$$\frac{1}{2} m_1 v_1^{i2} = m_1 g h_0 \rightarrow v_1^i = \sqrt{2gh_0} = \sqrt{2 \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 5 \text{ m}} = 9.9 \frac{\text{m}}{\text{s}}$$

$v_1^f \rightarrow$ appena dopo urto

$$\begin{cases} \Delta \vec{P} = 0 \\ \Delta K = 0 \end{cases} \Rightarrow \begin{cases} \vec{P}_i = \vec{P}_f \\ K_i = K_f \end{cases}$$

$$\begin{cases} m_1 v_1^i + m_2 v_2^i = m_1 v_1^f + m_2 v_2^f \\ \frac{1}{2} m_1 v_1^{i2} + \frac{1}{2} m_2 v_2^{i2} = \frac{1}{2} m_1 v_1^{f2} + \frac{1}{2} m_2 v_2^{f2} \end{cases}$$

$$\begin{cases} m_1 v_1^i = m_1 v_1^f + m_2 v_2^f \\ m_1 v_1^{i2} = m_1 v_1^{f2} + m_2 v_2^{f2} \end{cases} \Rightarrow \begin{cases} m_1 (v_1^i - v_1^f) = m_2 v_2^f \\ m_1 (v_1^{i2} - v_1^{f2}) = m_2 v_2^{f2} \end{cases}$$

ma $v_1^{i2} - v_1^{f2} = (v_1^i - v_1^f)(v_1^i + v_1^f)$

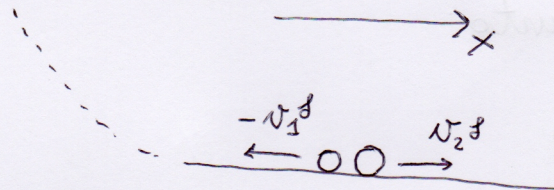
$$\begin{cases} m_1 (v_1^i - v_1^f) = m_2 v_2^f \\ m_1 (v_1^i - v_1^f)(v_1^i + v_1^f) = m_2 v_2^{f2} \end{cases} \xrightarrow{\text{RAPPORTO}} v_1^i + v_1^f = v_2^f$$

$$\begin{cases} \boxed{v_1^i + v_1^f} = v_2^f \\ m_1 v_1^i = m_1 v_1^f + m_2 v_2^f \end{cases} \Rightarrow m_1 v_1^i = m_1 v_1^f + m_2 (v_1^i + v_1^f)$$

$$m_1 v_1^i = m_1 v_1^f + m_2 v_1^i + m_2 v_1^f$$

$$(m_1 + m_2) v_1^f = (m_1 - m_2) v_1^i \Rightarrow v_1^f = \frac{m_1 - m_2}{m_1 + m_2} v_1^i$$

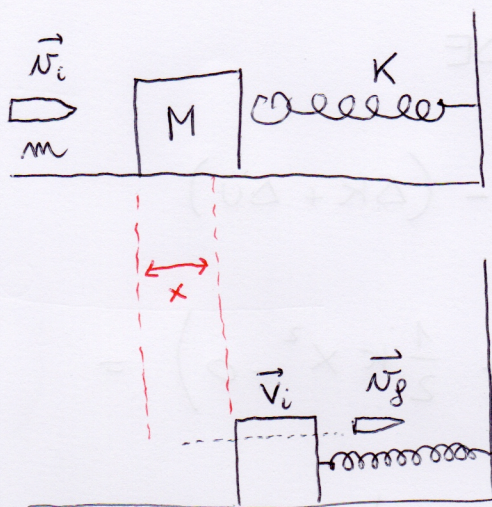
$$v_1^f = \frac{(5 - 10) \text{ kg}}{(5 + 10) \text{ kg}} \cdot 9.9 \text{ m/s} = -3.3 \text{ m/s}$$



$$m_1 g h_{\max} = \frac{1}{2} m_1 v_1^f{}^2 \rightarrow h_{\max} = \frac{v_1^f{}^2}{2g}$$

$$h_{\max} = \frac{(3.3)^2 \text{ m}^2/\text{s}^2}{2 \cdot 9.8 \text{ m/s}^2} = 0.56 \text{ m}$$

ESERCIZIO 3



$$\begin{aligned} m &= 5 \text{ g} \\ |\vec{v}_i| &= 400 \text{ m/s} \\ M &= 1 \text{ kg} \\ K &= 900 \text{ N/m} \\ x &= 5 \text{ cm} \\ ? &= |\vec{v}_g| \\ ? &= \Delta E \end{aligned}$$

$$\text{into} \rightarrow \Delta \vec{P} = 0 \rightarrow \vec{P}_i = \vec{P}_f$$

$$m \vec{v}_i = M \vec{v}_i + m \vec{v}_g$$

Quando il blocco ha raggiunto V_i , il blocco comprime la molla e si arresta

=> approssimazione: Δt molto piccolo

$$\frac{1}{2} M V_i^2 = \frac{1}{2} K x^2 \rightarrow V_i = \sqrt{\frac{K}{M} x^2} = 1 \times 1 \sqrt{\frac{K}{M}}$$

$$V_i = 0.05 \text{ m} \sqrt{\frac{900 \text{ N/m}}{1 \text{ kg}}} = 1.5 \text{ m/s} \quad \text{nota: } \frac{\text{N}}{\text{m kg}} = \frac{\text{kg m}}{\text{m kg s}^2} = \frac{1}{\text{s}^2}$$

$$m \vec{v}_i = M \vec{v}_i + m \vec{v}_g \Rightarrow \vec{v}_g = \frac{m \vec{v}_i - M \vec{v}_i}{m}$$

$$\vec{v}_g = \frac{0.005 \text{ kg} \cdot 400 \text{ m/s} - 1 \text{ kg} \cdot 1.5 \text{ m/s}}{0.005 \text{ kg}} = 100 \text{ m/s}$$

Conservazione dell'energia

$$K_i + U_i = K_f + U_f + \Delta E$$

$$\Delta E = K_i - K_f + U_i - U_f = -(\Delta K + \Delta U)$$

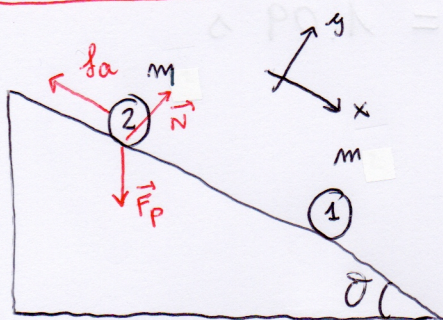
$$\Delta E = - \left(\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + \frac{1}{2} k x^2 - 0 \right) =$$

$$= - \frac{1}{2} \left(m v_f^2 - m v_i^2 + k x^2 \right) =$$

$$= - \frac{1}{2} \left(0.005 \text{ kg} \cdot (100)^2 \frac{\text{m}^2}{\text{s}^2} - 0.005 \text{ kg} (400)^2 \frac{\text{m}^2}{\text{s}^2} + 900 \frac{\text{N}}{\text{m}} \cdot (0.05)^2 \text{m}^2 \right)$$

$$\Rightarrow \Delta E = - 374 \text{ J}$$

ESERCIZIO 4



$$\theta = 30^\circ$$

$$m = 1 \text{ kg}$$

$$\mu_2 = 0.1 ; \mu_1 = 0.3$$

$$d = 1 \text{ m}$$

$$t_{\text{urto}} = ?$$

$$V_{\text{urto}} = ?$$

URTO COMPLETAMENTE ANELASTICO

$$\textcircled{2} \quad \begin{cases} x: mg \sin \theta - f_{a2} = ma_2 \\ y: -mg \cos \theta + N = 0 \end{cases}$$

$$\textcircled{1} \quad \begin{cases} x: mg \sin \theta - f_{a1} = ma_1 \\ y: -mg \cos \theta + N = 0 \end{cases}$$

$$1, 2: \quad N = mg \cos \theta \quad \begin{cases} f_{a1} = \mu_1 mg \cos \theta \\ f_{a2} = \mu_2 mg \cos \theta \end{cases}$$

$$\textcircled{2}: \quad mg \sin \theta - \mu_2 mg \cos \theta = ma_2 \quad \Rightarrow \quad a_2 = g (\sin \theta - \mu_2 \cos \theta)$$

$$\textcircled{1}: \quad mg \sin \theta - \mu_1 mg \cos \theta = ma_1 \quad \Rightarrow \quad a_1 = g (\sin \theta - \mu_1 \cos \theta)$$

$$\textcircled{2} \quad \begin{cases} x_2(t) = \frac{1}{2} a_2 t^2 \\ x_1(t) = d + \frac{1}{2} a_1 t^2 \end{cases} \quad \xrightarrow{\text{urto}} \quad x_2(t') = x_1(t')$$

$$\frac{1}{2} a_2 t'^2 = d + \frac{1}{2} a_1 t'^2 \quad \Rightarrow \quad t'^2 (a_2 - a_1) = 2d$$

$$t' = \sqrt{\frac{2d}{a_2 - a_1}} = \sqrt{\frac{2d}{g (\sin \theta - \mu_2 \cos \theta - \sin \theta + \mu_1 \cos \theta)}}$$

$$t' = \sqrt{\frac{2d}{g \cos \theta (\mu_1 + \mu_2)}} = \sqrt{\frac{2 \cdot 1 \text{ m}}{9.8 \frac{\text{m}}{\text{s}^2} \cdot \frac{\sqrt{3}}{2} \cdot 0.2}} = 1.09 \text{ s}$$

dopo l'urto

$$m v_1 + m v_2 = 2m V \Rightarrow V = \frac{v_1 + v_2}{2}$$

$$v_1 = a_1 t'$$

$$v_2 = a_2 t'$$

$$V = \frac{t'}{2} (a_1 + a_2) = \frac{t'}{2} g (m \sin \theta - \mu_1 \cos \theta + m \sin \theta - \mu_2 \cos \theta) =$$

$$= \frac{t'}{2} g [2m \sin \theta - \cos \theta (\mu_1 + \mu_2)]$$

$$V = \frac{1.09 \text{ s}}{2} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \left[2 \cdot \frac{1}{2} - 0.4 \frac{\sqrt{3}}{2} \right] = 3.49 \frac{\text{m}}{\text{s}}$$