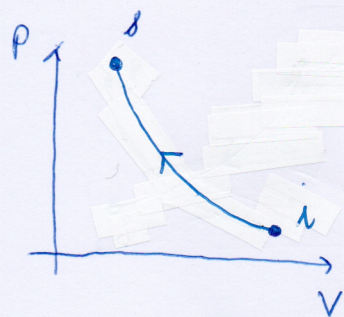


ESERCIZIO 4



$$n = 1$$

$$V_f = 0.025 \text{ m}^3$$

$$P_f = 1 \text{ atm}$$

$$W_{i \rightarrow f} = -3000 \text{ J}$$

$$? = i, f$$

$$? = P_i, V_i, T$$

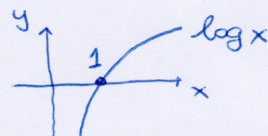
Lavoro negativo $\Rightarrow V_f < V_i \rightarrow$ compressione esterna

$$PV = nRT \Rightarrow P(V) = \frac{nRT}{V} \quad \text{con } nRT = \text{costante}$$

$$W_{i \rightarrow f} = \int_{V_i}^{V_f} P(V) dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \int_{V_i}^{V_f} \frac{1}{V} dV$$

$$W_{i \rightarrow f} = nRT (\log V_f - \log V_i) = nRT \log \frac{V_f}{V_i}$$

nota: $V_f < V_i \rightarrow \frac{V_f}{V_i} < 1 \rightarrow \log \frac{V_f}{V_i} < 0$



$$\log \frac{V_f}{V_i} = \frac{W_{i \rightarrow f}}{nRT} \quad \text{ma} \quad P_i V_i = P_f V_f = nRT$$

$$\log \frac{V_f}{V_i} = \frac{W_{i \rightarrow f}}{P_f V_f} \rightarrow \exp \left(\log \frac{V_f}{V_i} \right) = \exp \left(\frac{W_{i \rightarrow f}}{P_f V_f} \right)$$

$$\frac{V_f}{V_i} = \exp \left(\frac{W_{i \rightarrow f}}{P_f V_f} \right) \Rightarrow V_i = \frac{V_f}{\exp \left(\frac{W_{i \rightarrow f}}{P_f V_f} \right)} = V_f \exp \left(- \frac{W_{i \rightarrow f}}{P_f V_f} \right)$$

nota: $\frac{1}{e^x} = e^{-x}$

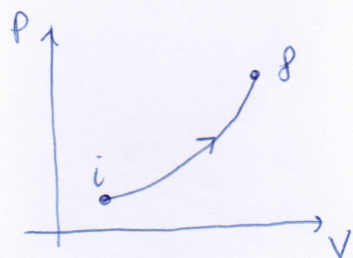
$$V_i = 0.025 \text{ m}^3 \exp \left(\frac{3000 \text{ J}}{1.013 \cdot 10^5 \text{ Pa} \cdot 0.025 \text{ m}^3} \right) = 0.08 \text{ m}^3$$

$$P_i V_i = P_f V_f \Rightarrow P_i = \frac{P_f V_f}{V_i} = \frac{1.013 \cdot 10^5 \text{ Pa} \cdot 0.025 \text{ m}^3}{0.08 \text{ m}^3} = 31650.25 \text{ Pa} = 0.3 \text{ atm}$$

$P_i < P_f$

$$T = \frac{P_f V_f}{nR} = \frac{1.013 \cdot 10^5 \text{ Pa} \cdot 0.025 \text{ m}^3}{1 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{mol K}}} = 305 \text{ K} = (305 - 273.15)^\circ \text{C} = 31.85^\circ \text{C}$$

ESERCIZIO 5



$$P = \alpha V^2$$

$$\alpha = 0.1 \frac{\text{atm}}{\text{m}^6}$$

$$? = P_i, T_i$$

$$? = P_f, V_f, T_f$$

$$V_i = 1 \text{ m}^3$$

$$n = 1$$

$$V_f = 2 V_i$$

$$? = W_{i \rightarrow f}$$

$$P_i = \alpha V_i^2 = 0.1 \frac{\text{atm}}{\text{m}^6} \cdot 1 \text{ m}^6 = 0.1 \text{ atm} = 1.013 \cdot 10^4 \text{ Pa}$$

$$T_i = \frac{P_i V_i}{nR} = \frac{1.013 \cdot 10^4 \text{ Pa} \cdot 1 \text{ m}^3}{1 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}} = 1219 \text{ K} = 945.85 \text{ K}$$

$$V_f = 2 V_i = 2 \text{ m}^3$$

$$P_f = \alpha V_f^2 = \alpha (2 V_i)^2 = 4 \alpha V_i^2 = 4 \cdot 0.1 \frac{\text{atm}}{\text{m}^6} \cdot 1 \text{ m}^6 = 0.4 \text{ atm} = 4.05 \cdot 10^4 \text{ Pa}$$

$$T_f = \frac{P_f V_f}{nR} = \frac{4.05 \cdot 10^4 \text{ Pa} \cdot 2 \text{ m}^3}{1 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}} = 9747 \text{ K} = 9473^\circ \text{C}$$

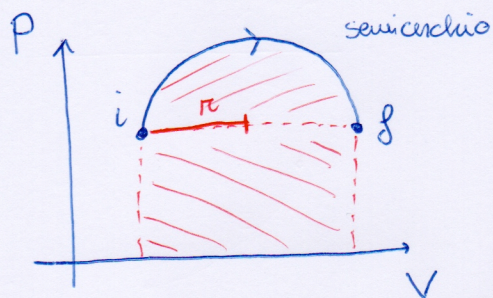
$$W_{i \rightarrow f} = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \alpha V^2 dV = \alpha \int_{V_i}^{V_f} V^2 dV$$

$$W_{i \rightarrow f} = \frac{1}{3} \alpha (V_f^3 - V_i^3) = \frac{1}{3} \alpha ((2 V_i)^3 - V_i^3) = \frac{1}{3} \alpha (8 V_i^3 - V_i^3) = \frac{7}{3} \alpha V_i^3$$

$$W_{i \rightarrow f} = \frac{7}{3} \cdot 0.1 \frac{\text{Pa}}{\text{m}^6} \cdot 1.013 \cdot 10^5 \cdot 1 \text{ m}^9 = 2.36 \cdot 10^4 \text{ J}$$

$$\text{nota: } \frac{\text{Pa} \cdot \text{m}^9}{\text{m}^6} = \text{Pa} \cdot \text{m}^3 = \frac{\text{N}}{\text{m}^2} \cdot \text{m}^3 = \text{N} \cdot \text{m} = \text{J}$$

ESERCIZIO 6



$n = 2$, monoatomico

$$V_i = 1.2 \text{ l}$$

$$P_i = 3 \cdot 10^5 \text{ Pa}$$

$$V_f = 6.0 \text{ l}$$

$$? = T_i, P_f, T_f$$

$$? = Q, W, \Delta E_{int}$$

$$T_i = \frac{P_i V_i}{nR} = \frac{1.2 \cdot 10^{-3} \text{ m}^3 \cdot 3 \cdot 10^5 \text{ Pa}}{2 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}} = 21.7 \text{ K} = -251.45^\circ \text{C}$$

$$P_f = P_i = 3 \cdot 10^5 \text{ Pa} = P$$

$$T_f = \frac{P_f V_f}{nR} = \frac{3 \cdot 10^5 \text{ Pa} \cdot 6 \cdot 10^{-3} \text{ m}^3}{2 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}} = 108.3 \text{ K} = -164.85^\circ \text{C}$$

$$\Delta E_{int} = n C_V \Delta T = n \cdot \frac{3}{2} R (T_f - T_i)$$

$$\Delta E_{int} = 2 \cdot \frac{3}{2} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot (108.3 - 21.7) \text{ K} = 2159 \text{ J}$$

$$W_{i \rightarrow f} = A_{\text{semic.}} + A_{\text{rett.}} = \frac{1}{2} \pi r^2 + (V_f - V_i) \cdot P$$

$$r = \frac{1}{2} (V_f - V_i)$$

$$W_{i \rightarrow f} = \frac{1}{2} \cdot \pi \cdot \frac{1}{4} (V_f - V_i)^2 + (V_f - V_i) \cdot P$$

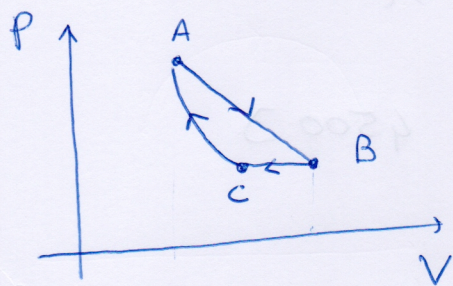
$$W_{i \rightarrow f} = \frac{1}{2} \cdot \pi \cdot \frac{1}{4} \cdot (6 \cdot 10^{-3} - 1.2 \cdot 10^{-3})^2 \text{ m}^6 + (6 \cdot 10^{-3} - 1.2 \cdot 10^{-3}) \text{ m}^3 \cdot 3 \cdot 10^5 \text{ Pa}$$

$$W_{i \rightarrow f} = 1440 \text{ J}$$

$$\Delta E_{int} = Q - W \Rightarrow Q_{i \rightarrow f} = \Delta E_{int} + W_{i \rightarrow f}$$

$$Q_{i \rightarrow f} = 2159 \text{ J} + 1440 \text{ J} = 3599 \text{ J}$$

ESERCIZIO 7



$n = 2$, monoatomico

$$P_A = 2 \cdot 10^5 \text{ Pa}$$

$$V_A = 10 \text{ dm}^3$$

$$P_B = \frac{1}{2} P_A$$

$$V_B = 4 V_A$$

$A \rightarrow B$: lineare
 $B \rightarrow C$: isobara
 $C \rightarrow A$: isoterma

$$? = \Delta E_{A \rightarrow B}^{\text{int}}$$

$$? = Q_{\text{TOT}}$$

? = coordinate
termodinamiche

Coordinate termodinamiche:

$$T_A = \frac{P_A V_A}{nR} = \frac{2 \cdot 10^5 \text{ Pa} \cdot 10^{-2} \text{ m}^3}{2^{\text{mol}} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}} = 120.3 \text{ K}$$

$$P_B = \frac{1}{2} P_A = \frac{1}{2} \cdot 2 \cdot 10^5 \text{ Pa} = 10^5 \text{ Pa}$$

$$V_B = 4 V_A = 40 \text{ dm}^3 = 4 \cdot 10^{-2} \text{ m}^3$$

$$T_B = \frac{P_B V_B}{nR} = \frac{\frac{1}{2} P_A \cdot 4 V_A}{nR} = 2 \frac{P_A V_A}{nR} = 2 T_A = 240.6 \text{ K}$$

$$P_C = P_B = 10^5 \text{ Pa}$$

$$T_C = T_A = 120.3 \text{ K}$$

$$P_C V_C = P_A V_A \quad (C \rightarrow A \text{ isoterma, } PV = \text{cost})$$

$$V_C = \frac{P_A V_A}{P_C} = \frac{P_A V_A}{P_B} = \frac{P_A V_A}{\frac{1}{2} P_A} = 2 V_A = 20 \text{ dm}^3 = 2 \cdot 10^{-2} \text{ m}^3$$

$$\Delta E_{\text{int}}^{A \rightarrow B} = n C_V \Delta T_{AB} = 2 \cdot \frac{3}{2} R (T_B - T_A) = 3 R (T_B - T_A) = 3 R (2 T_A - T_A) = 3 R T_A$$

$$\Delta E_{\text{int}}^{A \rightarrow B} = 1^{\text{mol}} \cdot 3 \cdot 8.31 \frac{\text{J}}{\text{mol K}} \cdot 120.3 \text{ K} = 3000 \text{ J}$$

$$\Delta E_{\text{int}}^{\text{ciclo}} = 0 \Rightarrow 0 = Q - W \Rightarrow Q = W$$

($\Delta T = 0$)

$$Q = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow A}$$

$$W_{A \rightarrow B} = (P_A + P_B)(V_B - V_A) \cdot \frac{1}{2} = \frac{1}{2} (P_A + \frac{1}{2} P_A)(4V_A - V_A)$$

$$W_{A \rightarrow B} = \frac{9}{4} P_A V_A = \frac{9}{4} \cdot 2 \cdot 10^5 \text{ Pa} \cdot 10^{-2} \text{ m}^3 = 4500 \text{ J}$$

$$W_{B \rightarrow C} = P_B(V_C - V_B) = \frac{1}{2} P_A (2V_A - 4V_A) = -P_A V_A$$

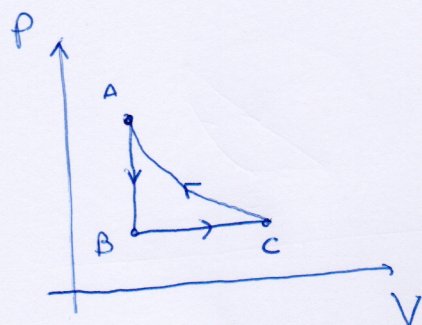
$$W_{B \rightarrow C} = -2 \cdot 10^5 \text{ Pa} \cdot 10^{-2} \text{ m}^3 = -2000 \text{ J}$$

$$W_{C \rightarrow A} = n R T \log \frac{V_A}{V_C} = 2 \cdot 8.31 \frac{\text{J}}{\text{mol K}} \cdot 120.3 \text{ K} \cdot \log \frac{1}{2} = -1386 \text{ J}$$

$$\log \frac{V_A}{2V_A} = \log \frac{1}{2}$$

$$Q_{\text{TOT}} = 4500 \text{ J} - 2000 \text{ J} - 1386 \text{ J} = 1114 \text{ J}$$

ESERCIZIO 8



A → B: isocora
B → C: isobara
C → A: adiabatrica
M = 2, monatomico

$$P_A = 4 \text{ atm}$$

$$V_A = 3 \text{ l}$$

$$P_B = \frac{1}{4} P_A$$

? = coordinate
termodinamiche

? = Q, W

$$P_A = 4 \text{ atm} = 4.013 \cdot 10^5 \text{ Pa}$$

$$V_A = 3 \text{ l} = 3 \cdot 10^{-3} \text{ m}^3$$

$$T_A = \frac{P_A V_A}{nR} = \frac{4.013 \cdot 10^5 \text{ Pa} \cdot 3 \cdot 10^{-3} \text{ m}^3}{2 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}} = 72.2 \text{ K}$$

$$P_B = \frac{1}{4} P_A = 1.013 \cdot 10^5 \text{ Pa} = 1 \text{ atm}$$

$$V_B = V_A = 3 \cdot 10^{-3} \text{ m}^3 = 3 \text{ l}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{\frac{1}{4} P_A V_A}{nR} = \frac{1}{4} \frac{P_A V_A}{nR} = \frac{1}{4} T_A = 18.1 \text{ K}$$

$$P_C = P_B = 1.013 \cdot 10^5 \text{ Pa}$$

$$\text{da } P_A V_A^\gamma = P_C V_C^\gamma \rightarrow V_C^\gamma = \frac{P_A}{P_C} V_A^\gamma$$

$$V_C = V_A \left(\frac{P_A}{P_C} \right)^{\frac{1}{\gamma}}$$

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3} \quad \begin{cases} C_P = \frac{5}{2} R \\ C_V = \frac{3}{2} R \end{cases}$$

$$V_C = V_A \sqrt[5]{\left(\frac{P_A}{P_C} \right)^3} = V_A \sqrt[5]{\left(\frac{P_A}{\frac{1}{4} P_A} \right)^3} = V_A \sqrt[5]{4^3} = 3 \cdot 10^{-3} \text{ m}^3 \sqrt[5]{4^3} = 6.9 \cdot 10^{-3} \text{ m}^3$$

$$T_C = \frac{P_C V_C}{nR} = \frac{\frac{1}{4} P_A \cdot \sqrt[5]{4^3} V_A}{nR} = \frac{\sqrt[5]{4^3}}{4} \frac{P_A V_A}{nR} = \frac{\sqrt[5]{4^3}}{4} T_A = 41.6 \text{ K}$$

$$\Delta E_{int}^{ciclo} = 0 \Rightarrow Q = W$$

$$W_{TOT} = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow D}$$

$$W_{A \rightarrow B} = 0$$

$$W_{TOT} = W_{B \rightarrow C} + W_{C \rightarrow D}$$

$$W_{B \rightarrow C} = (V_C - V_B) P_B = \left(\sqrt[5]{4^3} V_A - V_A \right) \cdot \frac{1}{4} P_A = \frac{\sqrt[5]{4^3} - 1}{4} V_A P_A$$

$$W_{B \rightarrow C} = \frac{\sqrt[5]{4^3} - 1}{4} 3 \cdot 10^{-3} \text{ m}^3 \cdot 4.013 \cdot 10^5 \text{ Pa} = 390.5 \text{ J}$$

$$W_{C \rightarrow A} = -\Delta E_{int}^{C \rightarrow A} \quad \text{perché } Q_{C \rightarrow A} = 0$$

$$\Delta E_{int}^{C \rightarrow A} = m C_V \Delta T_{CA} = 2 \cdot \frac{3}{2} R \cdot (T_A - T_C)$$

$$\Delta E_{int}^{C \rightarrow A} = 2 \cdot \frac{3}{2} R (72.4 - 41.6) \text{ K} = 767.8 \text{ J}$$

$$W_{C \rightarrow A} = -767.8 \text{ J}$$

$$W_{TOT} = 390.5 \text{ J} - 767.8 \text{ J} = -377.3 \text{ J}$$

altro metodo,

$$Q_{TOT} = Q_{A \rightarrow B} + Q_{B \rightarrow C} + Q_{C \rightarrow A}$$

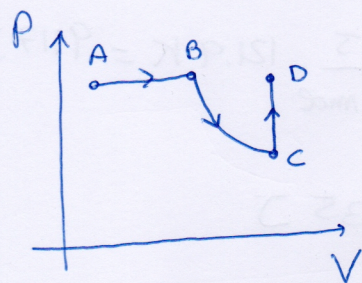
$$Q_{C \rightarrow A} = 0$$

$$Q_{TOT} = Q_{A \rightarrow B} + Q_{B \rightarrow C} = m C_V (T_B - T_A) + m C_P (T_C - T_B) =$$

$$= m \frac{3}{2} R (T_B - T_A) + m \frac{5}{2} R (T_C - T_B) = \frac{mR}{2} (3T_B - 3T_A + 5T_C - 5T_B) =$$

$$= \frac{mR}{2} (5T_C - 3T_A - 2T_B) = \frac{2 \cdot 8.31 \frac{\text{J}}{\text{mol K}}}{2} (208 - 217.2 - 36.2) \text{ K} = -377.3 \text{ J} \quad (2)$$

ESERCIZIO 9



A → B: isobara

B → C: isoterma

C → D: isobara

? = Q, W, ΔE_{int}

n = 2, mono

P_A = 2 atm

V_A = 10 l

V_B = 2 V_A

V_C = 2 V_B

P_D = P_A

$$T_A = \frac{P_A V_A}{nR} = \frac{2 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 10^{-2} \text{ m}^3}{2 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}} = 121.9 \text{ K}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{P_A 2V_A}{nR} = 2 \frac{P_A V_A}{nR} = 2T_A$$

$$T_C = T_B = 2T_A$$

$$T_D = \frac{P_D V_D}{nR} = \frac{P_A 4V_A}{nR} = 4 \frac{P_A V_A}{nR} = 4T_A$$

$$Q_{\text{TOT}} = Q_{AB} + Q_{BC} + Q_{CD}$$

$$Q_{AB} = n C_p (T_B - T_A) = n C_p (2T_A - T_A) = n C_p T_A$$

$$Q_{AB} = 2 \cdot \frac{5}{2} R \cdot T_A = 5 R T_A = \overset{1 \text{ mol}}{5} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot 121.9 \text{ K} = 5065 \text{ J}$$

$$Q_{BC} = W_{BC} = n R T_B \log \frac{V_C}{V_B} = n R \cdot 2T_A \cdot \log \frac{4V_A}{2V_A} = 2 n R T_A \log 2$$

$$Q_{BC} = 2 \cdot 2 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot 121.9 \text{ K} \cdot \log 2 = 2809 \text{ J}$$

$$Q_{CD} = n C_v (T_D - T_C) = n C_v (4T_A - 2T_A) = 2 n C_v T_A$$

$$Q_{CD} = 2 \cdot n \cdot \frac{3}{2} R T_A = 3 n R T_A = 3 \cdot 2 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot 121.9 \text{ K} = 6078 \text{ J}$$

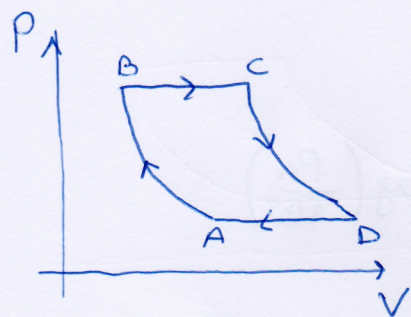
$$Q_{\text{TOT}} = 5065 \text{ J} + 2809 \text{ J} + 6078 \text{ J} = 13952 \text{ J}$$

$$\Delta E_{int}^{A \rightarrow D} = m c_v (T_D - T_A) = m c_v (4T_A - T_A) = 3 m c_v T_A$$

$$\Delta E_{int}^{A \rightarrow D} = 3 m \frac{3}{2} R T_A = \frac{9}{2} m R T_A = \frac{9}{2} \cdot 2 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot 12.9 \text{ K} = 9117 \text{ J}$$

$$W_{tot} = Q_{tot} - \Delta E_{int}^{A \rightarrow D} = 13952 \text{ J} - 9117 \text{ J} = 4835 \text{ J}$$

ESERCIZIO 10



A → B : isoterma

B → C : isobara

C → D : isoterma

D → A : isobara

? = Quante e quali
P, V, T
mi servono per
calcolare W?

$$W = \int_{V_A}^{V_B} P dV + \int_{V_B}^{V_C} P dV + \int_{V_C}^{V_D} P dV + \int_{V_D}^{V_A} P dV$$

$$W = nRT_A \int_{V_A}^{V_B} \frac{1}{V} dV + P_B \int_{V_B}^{V_C} dV + nRT_C \int_{V_C}^{V_D} P dV + P_D \int_{V_D}^{V_A} dV$$

$$W = nRT_A \log \frac{V_B}{V_A} + P_B (V_C - V_B) + nRT_C \log \frac{V_D}{V_C} + P_D (V_A - V_D)$$

$$P_A V_A = P_B V_B \quad e \quad P_C V_C = P_D V_D \quad e \quad P_B = P_C \quad e \quad P_D = P_A$$

$$\begin{aligned} P_B (V_C - V_B) + P_D (V_A - V_D) &= P_B V_C - P_B V_B + P_D V_A - P_D V_D = \\ &= \cancel{P_C V_C} - P_B V_B + P_D V_A - \cancel{P_D V_D} = \\ &= P_D V_A - P_B V_B = P_A V_A - P_A V_A = 0 \end{aligned}$$

$$W = nRT_A \log \frac{V_B}{V_A} + nRT_C \log \frac{V_D}{V_C}$$

$$\frac{V_B}{V_A} = \frac{P_A}{P_B} \quad e \quad \frac{V_D}{V_C} = \frac{P_C}{P_D} = \frac{P_B}{P_A}$$

$$\begin{aligned} W &= nRT_A \log \left(\frac{P_A}{P_B} \right) + nRT_C \log \left(\frac{P_B}{P_A} \right) = \\ &= nRT_A \log \left(\frac{P_A}{P_B} \right) - nRT_C \log \left(\frac{P_A}{P_B} \right) = \\ &= nR \log \left(\frac{P_A}{P_B} \right) (T_A - T_C) \end{aligned}$$

$$mRT_A = P_A V_A$$

$$mRT_C = P_C V_C = P_D V_D = P_A V_D$$

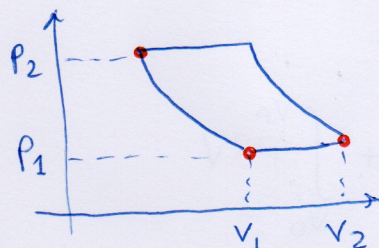
$$W = (P_A V_A - P_A V_D) \log \left(\frac{P_A}{P_B} \right) = P_A (V_A - V_D) \log \left(\frac{P_A}{P_B} \right)$$

$$P_A = P_1$$

$$P_B = P_2$$

$$V_A = V_1$$

$$V_D = V_2$$



$$W = P_1 (V_1 - V_2) \log \left(\frac{P_1}{P_2} \right)$$