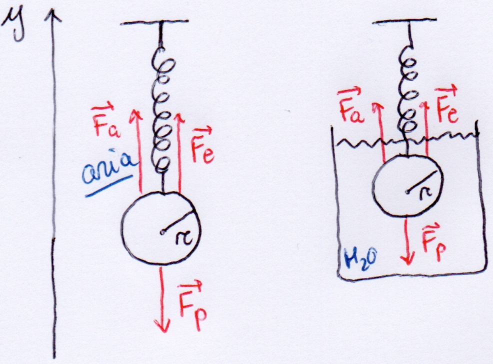
ESERCIZIO 3



$K = 250\ N/m$

$m = 1\ kg$

$r = 0.05\ m$

$\Delta y_{aria} = ?$ $\quad \rho_{aria} = 1.29\ kg/m^3$

$\Delta y_{H_2O} = ?$ $\quad \rho_{H_2O} = 10^3\ kg/m^3$

$\Rightarrow$ trascurare la massa della molla

in aria

$$-F_p + F_e + F_a = 0$$

$$-mg + k\Delta y + \rho_{aria} V g = 0 \quad \Rightarrow \quad \Delta y = \frac{mg - \rho_{aria} V g}{k}$$

$$\Delta y = \frac{1\,kg \cdot 9.8\,m/s^2 - 1.29\,kg/m^3 \cdot \frac{4}{3}\pi (0.05)^3 m^3 \cdot 9.8\,m/s^2}{250\,N/m} = 0.039\,m = 3.9\,cm$$

in acqua

$$-F_p + F_e + F_a = 0$$

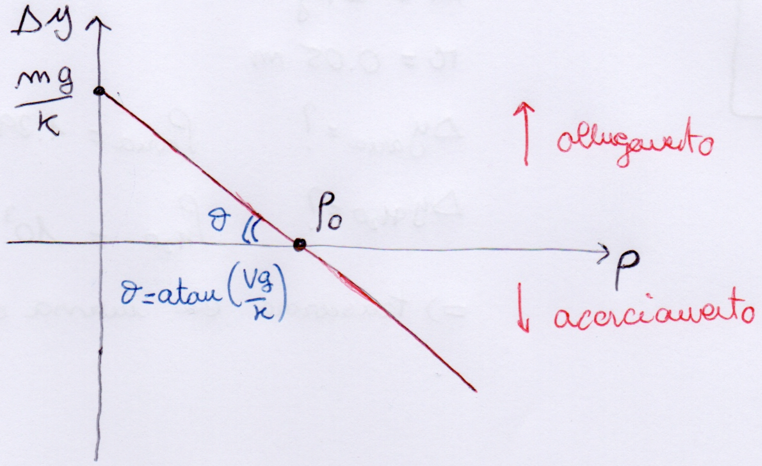
$$-mg + k\Delta y + \rho_{H_2O} V g = 0 \quad \Rightarrow \quad \Delta y = \frac{mg - \rho_{H_2O} V g}{k}$$

$$\Delta y = \frac{1\,kg \cdot 9.8\,m/s^2 - 10^3\,kg/m^3 \cdot \frac{4}{3}\pi (0.05)^3 m^3 \cdot 9.8\,m/s^2}{250\,N/m} = 0.019\,m = 1.9\,cm$$

$$\Delta y_{H_2O} < \Delta y_{aria}$$

nota

$$\Delta y = -\frac{Vg}{K}\rho + \frac{mg}{k}$$



$\rho < \rho_0 \Rightarrow$ allungamento

$\rho > \rho_0 \Rightarrow$ accorciamento

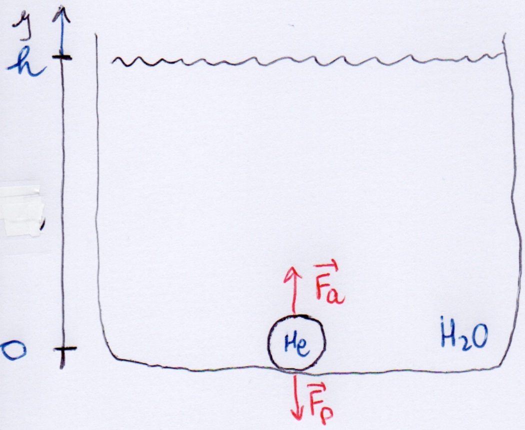
? ρ_0

$\Delta y = 0$

$$-\frac{V\cancel{g}}{\cancel{k}}\rho_0 + \frac{m\cancel{g}}{\cancel{k}} = 0 \Rightarrow \rho_0 = \frac{m}{V} = \text{densità oggetto}$$

ovviamente!!

ESERCIZIO 4



$m = 4$ kg

$r = 0.1$ m

$\rho_{He} = 0.179$ kg/m^3

$h = 4$ m

$|\vec{a}| = ?$

$t_{sup} = ?$

=) trascurando attrito H_2O

$$\vec{F_p} = \vec{F_p}^m + \vec{F_p}^{He}$$

proiettando $\sum \vec{F} = m\vec{a}$ lungo y

$$-F_p^m - F_p^{He} + F_a = m'a \qquad m' = m + m_{He}$$

$$-mg - m_{He}g + \rho_{H_2O}Vg = (m + m_{He})a$$

dove $m_{He} = \rho_{He}V$

e $V = \frac{4}{3}\pi r^3$

quindi:

$$-mg - \rho_{He}Vg + \rho_{H_2O}Vg = (m + m_{He})a$$

$$g\left[-m + V(\rho_{H_2O} - \rho_{He})\right] = (m_1 + m_{He})a$$

$$a = \frac{\frac{4}{3}\pi r^3(\rho_{H_2O} - \rho_{He}) - m}{m + \frac{4}{3}\pi r^3 \rho_{He}}\, g = \frac{\frac{4}{3}\pi(0.1)^3 m^3 (10^3 - 0.179)\frac{kg}{m^3} - 4\,kg}{4\,kg + \frac{4}{3}\pi(0.1)^3 m^3 \cdot 0.179\,\frac{kg}{m^3}}\, 9.8\,\frac{m}{s^2}$$

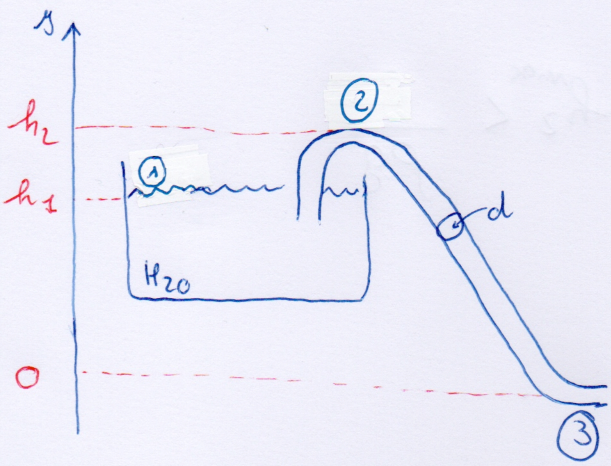
$$a = 0.46\ \frac{m}{s^2}$$

se non c'è attrito l'accelerazione è costante e il palloncino sale!

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$h = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \cdot 4\,m}{0.46\,m/s^2}} = 4.2\ s$$

ESERCIZIO 5



$h_1 = 1\ m$

d = costante

$v_3 = ?$

$h_2^{max} = ?$

ipotesi: flusso costante in 3

Applichiamo Bernoulli in 1 e 3

$$\cancel{P_1} + \cancel{\frac{1}{2}\rho v_1^2} + \cancel{\rho} g h_1 = \cancel{P_3} + \frac{1}{2}\cancel{\rho} v_3^2 + \cancel{\rho g h_3}$$

dove

$P_1 = P_3 = P_0$ pressione atmosferica

$v_1 << v_3 \Rightarrow v_1 \simeq 0$

$h_3 = 0$

$$g h_1 = \frac{1}{2} v_3^2 \Rightarrow v_3 = \sqrt{2 g h_1} = \sqrt{2 \cdot 9.8\ m/s^2 \cdot 1\ m} = 4.4\ m/s$$

Applichiamo Bernoulli in 2 e 3

$$P_2 + \cancel{\frac{1}{2}\rho v_2^2} + \rho g h_2 = P_3 + \cancel{\frac{1}{2}\rho v_3^2} + \cancel{\rho g h_3}$$

dove

$v_2 = v_3$ poiché $A_2 v_2 = A_3 v_3$ e $A_2 = A_3 = d$ | eq. di continuità

$P_3 = P_0$ pressione atmosferica

$h_3 = 0$

$$P_2 = P_0 - \rho g h_2$$

Per avere flusso in 3 devo avere $P_2 > 0$

$$P_2 > 0 \Rightarrow P_0 - \rho g h_2^{max} > 0 \Rightarrow h_2^{max} < \frac{P_0}{\rho g}$$

$$h_2 < \frac{1.013 \cdot 10^5 \, Pa}{10^3 \, kg/m^3 \cdot 9.8 \, m/s^2} = 10.3 \, m$$

*nota

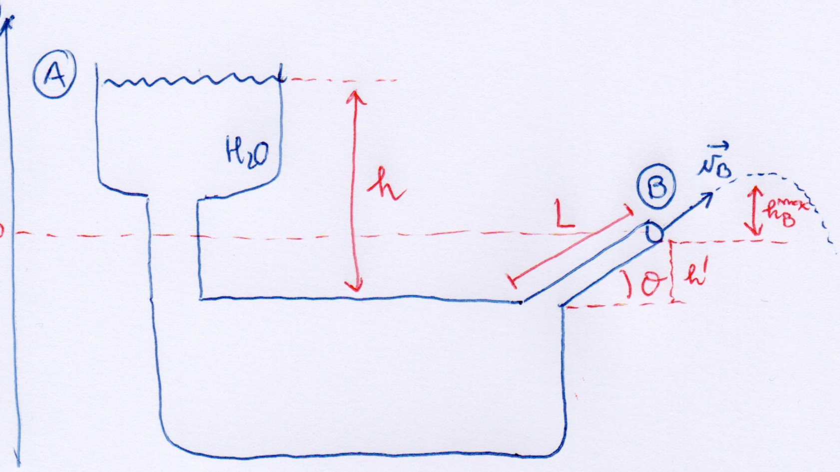
Per $h_2 = h_2^{max}$

$$P_2 = P_0 - \rho g h_2^{max} = 0 \Rightarrow P_0 = \rho g h_2^{max}$$

Quindi chiedere che $P_2 > 0$ equivale a chiedere che P_2 non sia inferiore di P_0, cioè della pressione atmosferica –

$$P_2 > P_0$$

ESERCIZIO 6



$h = 10\ m$

$L = 2\ m$

$\theta = 30°$

$A_A \gg A_B$

$h_B^{max} = ?$

Applichiamo Bernoulli in (A) e (B)

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

dove

$v_A \ll v_B \Rightarrow v_A \simeq 0$

$h_A = h - h' = h - L\sin\theta$

$h_B = 0$

$$P_A + \rho g(h - L\sin\theta) = P_B + \frac{1}{2}\rho v_B^2$$

ma $P_A = P_B = P_0$ pressione atmosferica

$$\cancel{\rho} g(h - L\sin\theta) = \frac{1}{2}\cancel{\rho} v_B^2 \Rightarrow v_B = \sqrt{2g(h - L\sin\theta)}$$

$$v_B = \sqrt{2 \cdot 9.8\ m/s^2 \cdot \left(10\ m - 2\ m \cdot \frac{1}{2}\right)} = 13.3\ m/s$$

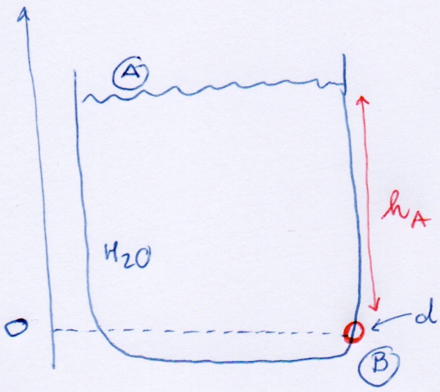
$h_B^{max} \rightarrow v_y = 0$ moto parabolico

$$v_y = v_{0y} - gt \rightarrow 0 = v_B\sin\theta - gt \rightarrow t = \frac{v_B\sin\theta}{g}$$

$$h_B^{max} = v_{0y}t - \frac{1}{2}gt^2 = v_B\sin\theta \cdot \frac{v_B\sin\theta}{g} - \frac{1}{2}g \cdot \frac{v_B^2\sin^2\theta}{g^2}$$

$$h_B^{max} = \frac{v_B^2\sin^2\theta}{g} - \frac{1}{2}\frac{v_B^2\sin^2\theta}{g} = \frac{v_B^2\sin^2\theta}{2g} = \frac{(13.3)^2\ m^2/s^2 \cdot \frac{1}{4}}{2 \cdot 9.8\ m/s^2} = 2.3\ m$$

ESERCIZIO 7



$h_A = 16\ m$

$\varphi_B = 2.5 \cdot 10^{-3} \frac{m^3}{min}$

$v_B = ?$

$d = ?$

Per Bernoulli

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

$P_A = P_B = P_0$ atmosfera

$A_A \gg A_B \qquad v_A A_A = v_B A_B \rightarrow \frac{A_A}{A_B} = \frac{v_B}{v_A} \rightarrow v_A \simeq 0$

$h_B = 0$

$$\cancel{\rho} g h_A = \frac{1}{2}\cancel{\rho} v_B^2 \qquad v_B = \sqrt{2 g h_A} = \sqrt{2 \cdot 9.8 \frac{m}{s^2} \cdot 16\ m} = 17.7\ m/s$$

cos'è il flusso φ?

$$\varphi = A v \qquad [\varphi] = [L^2] \cdot \frac{[L]}{[T]} \rightarrow \frac{m^3}{s}$$

$$\varphi_B = 2.5 \cdot 10^{-3} \frac{m^3}{min} = 4.17 \cdot 10^{-5} \frac{m^3}{s}$$

$$\varphi_B = A_B v_B = \pi \left(\frac{d}{2}\right)^2 v_B \rightarrow d = 2\sqrt{\frac{\varphi_B}{\pi v_B}}$$

$$d = 2\sqrt{\frac{4.17 \cdot 10^{-5} \frac{m^3}{s}}{\pi \cdot 17.7 \frac{m}{s}}} = 1.73 \cdot 10^{-3}\ m = 1.73\ mm$$