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A CP Violation Primer

This chapter is a primer on the subject of CP violation. It is intended as an introductory background for physicists joining the BABAR experiment. Much of the emphasis is on the physics relevant to that experiment. However other related topics are briefly reviewed and summarized.

The subject of CP symmetry and its violation is often referred to as one of the least understood in particle physics. Perhaps a better statement would be to say that it is experimentally one of the least constrained. CP symmetry violation is an expected consequence of the Standard Model with three quark generations, but is one of the least well-tested parts of that model. The only part of CP violation that currently is considered puzzling by theorists is the lack of CP violation in strong interactions. That subject is outside the realm of this document and of BABAR experiments. The CP violation that shows up in a small fraction of weak decays is accommodated simply in the three-generation Standard Model Lagrangian. All it requires is that CP is not imposed as a symmetry.

However, while it is known that CP violation occurs, because it has been observed in K decays [1], it is not yet known whether the pattern of CP violation predicted by the minimal Standard Model is the one found in nature. The K-decay observations, together with other measurements, place constraints on the parameters of the Standard Model mixing matrix (the CKM matrix [2, 3]) but do not yet provide any test. A multitude of CP-violating effects are expected in B decays, some of which are very cleanly predicted by the Standard Model. If enough independent observations of CP violation in B decays can be made then it will be possible to test the Standard Model predictions for CP violation. Either the relationships between various measurements will be consistent with the Standard Model predictions and fully determine the CKM parameters or there will be no single choice of CKM parameters that is consistent with all measurements.

This latter case, of course, would be much more interesting. It would indicate that there is a contribution of physics beyond the Standard Model. There may be enough information in the pattern of the inconsistencies to learn something about the nature of the new physics contributions. Thus the aim of the game is to measure enough quantities to impose redundant constraints on Standard Model parameters, including particularly the convention-independent combinations of CP-violating phases of CKM matrix elements.

One may well ask, after the many successes of the Standard Model, why one would expect violations to show up in such a low-energy regime. The best answer is simply that it has not yet been tested. Theorists will give a variety of further reasons. Many extensions of the Standard

Model have additional sources of CP-violating effects, or effects which change the relationship of the measurable quantities to the CP-violating parameters of the Standard Model.

In addition there is one great puzzle in cosmology that relates to CP violation, and that is the disappearance of antimatter from the Universe [4]. In grand unified theories, or even in the Standard Model at sufficiently high temperatures, there are baryon number-violating processes. If such processes are active then thermal equilibrium produces equal populations of particles and antiparticles. Thus in modern theories of cosmology the net baryon number of the universe is zero in the early high-temperature epochs. Today it is clearly not zero, at least in our local region. A full discussion of the cosmological arguments is not possible here. It suffices to remark that there is large class of theories in which the baryon number asymmetry is generated at the weak-phase transition [5]. Such theories, however, must include CP violation from sources beyond the minimal Standard Model. Calculations made in that model show that it does not generate a large enough matter-antimatter imbalance to produce the baryon number to entropy ratio observed in the universe today. This is a hint that CP violation from beyond Standard Model sources is worth looking for. It is by no means a rigorous argument. There are theories in which baryon number is generated at a much higher temperature and then protected from thermalization to zero by $B \Leftrightarrow L$ (baryon number minus lepton number) symmetry. Such theories do not in general require any new low-energy CP-violation mechanism. Neither do they forbid it.

More generally, since there is CP violation in part of the theory, any extension of the Standard Model cannot be required to be CP symmetric. Any additional fields in the theory bring possible additional CP-violating couplings. Even assumptions such as soft or spontaneous CP symmetry breaking leave a wide range of possibilities. Further experimental constraints, from experiments such as the B factory, are needed.

Section 1.1 begins by discussing the way CP violation appears in a field theory Lagrangian [6]. Sections 1.2–1.6, follow the discussion in [7].[1] Section 1.2 turns to the quantum mechanics and time dependence of neutral meson systems, and Section 1.3 gives a model-independent treatment of the possible types of CP violation. Following that, Section 1.4 presents the Standard Model version of CP violation, and Section 1.5 gives the predictions and relationships for various decays that arise from that theory. Finally, in Section 1.6, the situation for K-decays is reviewed.

1.1 CP Violation in Field Theories

1.1.1 Field Transformations

This section provides a basic introduction to the field theory basis of CP symmetry breaking. The fundamental point is that CP symmetry is broken in any theory that has complex coupling

[1]For a recent, excellent, and very detailed review see [8].

constants in the Lagrangian which cannot be removed by any choice of phase redefinition of the fields in the theory.

Three discrete operations are potential symmetries of a field theory Lagrangian [6]: Two of them, *parity* and *time reversal* are spacetime symmetries and constitute part of the Poincaré group. Parity, denoted by P, sends $(t, \mathbf{x}) \to (t, -\mathbf{x})$, reversing the handedness of space. Time reversal, denoted by T, sends $(t, \mathbf{x}) \to (-t, \mathbf{x})$, interchanging the forward and backward light-cones. A third (non-spacetime) discrete operation is *charge conjugation*, denoted by C. This operation interchanges particles and antiparticles. The combination CP replaces a particle by its antiparticle and reverses momentum and helicity. The combination *CPT* is an exact symmetry in any local Lagrangian field theory.

What is the status of these symmetry operations in the real world? From experiment, it is observed that electromagnetic and strong interactions are symmetric with respect to P, C and T. The weak interactions violate C and P separately, but preserve CP and T to a good approximation. Only certain rare processes, all involving neutral K mesons, have been observed to exhibit CP violation. All observations to date are consistent with exact *CPT* symmetry. (Gravitation couples to the energy-momentum tensor and is thus C, P, and T invariant. This is supported by the universality of the gravitational coupling for different types of matter, with different baryon number to mass ratios.)

To understand whether a given theory can accommodate CP violation, one needs to know the transformation properties of the fields under the various discrete symmetries. In particular for a Dirac spinor:

$$P\psi(t, \mathbf{x})P = \gamma^0 \psi(t, -\mathbf{x}), \tag{1.1}$$

$$T\psi(t, \mathbf{x})T = -\gamma^1\gamma^3 \psi(-t, \mathbf{x}), \tag{1.2}$$

$$C\psi(t, \mathbf{x})C = -i(\overline{\psi}(t, \mathbf{x})\gamma^0\gamma^2)^T. \tag{1.3}$$

The Lagrangian, being a Lorentz scalar, can only depend on terms bilinear in fermion fields (and not on single fermion fields). The transformation properties of various fermion bilinears under CP are summarized in the table below. Here the shorthand $(-1)^\mu \equiv 1$ for $\mu = 0$ and $(-1)^\mu \equiv -1$ for $\mu = 1, 2, 3$ (namely, $(-1)^\mu a^\mu = a_\mu$) is used.

$$\begin{array}{ccccc} \text{term} & \overline{\psi}_i\psi_j & i\overline{\psi}_i\gamma^5\psi_j & \overline{\psi}_i\gamma^\mu\psi_j & \overline{\psi}_i\gamma^\mu\gamma^5\psi_j \\ CP\text{-transformed term} & \overline{\psi}_j\psi_i & -i\overline{\psi}_j\gamma^5\psi_i & -(-1)^\mu\overline{\psi}_j\gamma^\mu\psi_i & -(-1)^\mu\overline{\psi}_j\gamma^\mu\gamma^5\psi_i \end{array} \tag{1.4}$$

Similarly, the CP transformation properties of scalar (H), pseudoscalar (A) and vector boson (W) fields, and also of the derivative operator are given by

$$\begin{array}{ccccc} \text{term} & H & A & W^{\pm\mu} & \partial_\mu \\ CP\text{-transformed term} & H & -A & -(-1)^\mu W^{\mp\mu} & (-1)^\mu\partial_\mu \end{array} . \tag{1.5}$$

Taking into account the Lorentz invariance and hermiticity of the Lagrangian, the above CP transformation rules imply that each of the combinations of fields and derivatives that appear in the Lagrangian transforms under CP to its hermitian conjugate. However, there are coefficients in front of these expressions which represent either coupling constants or particle masses and which do not transform under CP. If any of these quantities are complex, then the coefficients in front of CP-related terms are complex conjugates of each other. In such a case, CP is not necessarily a good symmetry of the Lagrangian. When the rates of physical processes that depend on these Lagrangian parameters are calculated, there can be CP-violating effects, namely rate differences between pairs of CP conjugate processes. Examples are given below.

Note, however, that not all Lagrangian phases are physically meaningful quantities. Consider the Lagrangian that contains the most general set of complex coupling constants consistent with all other symmetries in the theory. That is to say CP symmetry is not imposed, and hence any coupling is allowed to be complex (unless the Hermitian structure of the Lagrangian automatically requires it to be real). Now any complex field in the Lagrangian can be redefined by an arbitrary phase rotation; such rotations will not change the physics, but will change the phases of some set of terms in the Lagrangian. Consider for example a typical Yukawa-type term,

$$y_{ij} H \overline{\psi}_i \psi_j + \text{hermitian conjugate.} \tag{1.6}$$

The phase of y_{ij} can be changed by redefining the phase of any one of the three fields $H,\ \psi_i,\ \psi_j$ that enter this term. In general such redefinitions will also change the phase of any other terms in the Lagrangian that involve these same fields, unless the complex conjugate field appears with the same power in the same term. Some set of couplings can be made real by making such field redefinitions. However if any non-zero phases for couplings remain after all possible field redefinitions have been used to eliminate as many of them as possible, then there is CP violation. It is a matter of simple counting for any Lagrangian to see whether this occurs. If all phases can be removed in this way then that theory is automatically CP-conserving. In such a theory it is impossible to introduce any CP violations without adding fields or removing symmetries so that additional couplings appear. (This is the case for the Standard Model with only two generations and a single Higgs multiplet.)

If some phases survive the redefinitions, there is, in general, convention-dependence as to where the complex phases appear. One can choose to make certain terms real and leave others complex, but a different choice, related to the first by field redefinitions, has the same physical consequences. Only those differences between pairs of phases that are unchanged by such redefinitions are physically meaningful. How such phase differences manifest themselves as CP-violating effects will be shown below. First some conventions and notation for neutral B mesons need to be established.

1.2 Neutral B Mesons

1.2.1 Mixing of Neutral B Mesons

This section treats the quantum mechanics of the two state system of neutral B mesons. Unless otherwise specified, this discussion is completely model independent and does not depend on Standard Model specific results. It will however use features of the flavor and weak-interaction structure of the Standard Model, which will inevitably also be part of any extension of that theory.

The systems of interest are neutral self-conjugate pairs of mesons. There are two such systems involving b quarks: B_d mesons, made from one b-type quark or antiquark and one d type; and B_s mesons from one b and one s. Like the neutral K mesons, the neutral B mesons are complicated by the fact that different neutral states are relevant to the discussion of different physical processes: there are two flavor eigenstates, which have definite quark content and are most useful to understand particle production and particle decay processes; and there are eigenstates of the Hamiltonian, namely states of definite mass and lifetime, which propagate through space in a definite fashion. If CP were a good symmetry, the mass eigenstates would also be CP eigenstates, namely under a CP transformation they would transform into themselves with a definite eigenvalue ± 1. But since CP is not a good symmetry, the mass eigenstates can be different from CP eigenstates (see further discussion below). In any case the mass eigenstates are not flavor eigenstates, and so the flavor eigenstates are mixed with one another as they propagate through space. The flavor eigenstates for B_d are $B^0 = \overline{b}d$ and $\overline{B}^0 = \overline{d}b$. (The convention is that B^0 is the isospin partner of B^+; therefore it contains the $\overline{b}$ quark. This is similar to the K mesons, where K^0, the isospin partner of K^+, contains the $\overline{s}$ quark.) The conventional definitions for the B_s system are $B_s = \overline{b}s$ and $\overline{B}_s = \overline{s}b$. Unless explicitly stated the following general discussion applies to both B systems, and a similar notation can be used also for K^0 or D^0 mesons. However, the two neutral K mesons have very different lifetimes (while their masses are almost identical), so that it is more convenient to define the states by the half-life, K_L and K_S for the long-lived and short-lived state, respectively. For the neutral D mesons, the mixing rate is much slower than the decay rate so that flavor eigenstates are the most convenient basis.

An arbitrary linear combination of the neutral B-meson flavor eigenstates,

$$a|B^0\rangle + b|\overline{B}^0\rangle, \tag{1.7}$$

is governed by a time-dependent Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H \begin{pmatrix} a \\ b \end{pmatrix} \equiv (M \Leftrightarrow \frac{i}{2}\Gamma) \begin{pmatrix} a \\ b \end{pmatrix} \tag{1.8}$$

for which M and Γ are 2×2 Hermitian matrices. *CPT* invariance guarantees $H_{11} = H_{22}$.

The off-diagonal terms in these matrices, M_{12} and Γ_{12}, are particularly important in the discussion of CP violation. They are the dispersive and absorbtive parts respectively of the transition amplitude from B^0 to $\overline{B}^0$. (Note that both of these can be complex quantities because of complex coupling constants.) In the Standard model these contributions arise from the box diagrams with two W exchanges. The large mass of the B makes the QCD calculation of these quantities much more reliable than the corresponding calculation for K mixing. The dispersive part, M_{12}, is clearly short-distance dominated (*i.e.*, large-quark momenta in the box diagram) and long-distance effects are expected to be negligible. For the dispersive part, one calculates the cut of the quark box diagram and uses the argument of *quark-hadron duality* (see Chapter 2) to relate this quantity to the corresponding hadronic quantity. This is similar to the calculation of $R_{e^+e^-}$, the ratio of hadron to leptonic cross-sections for e^+e^- scattering. While there is no rigorous argument that quark-hadron duality holds at a single energy scale (known as local quark-hadron duality), it can be shown to be true when averaged over a sufficient range. However in a region where there are no thresholds the value of this cut does not vary rapidly with energy and hence one expects the quark calculation to be reliable. Combining heavy quark behavior with QCD calculation one obtains an estimate for Γ_{12} that is expected to be valid up to corrections of order $1/N_C$ and/or Λ/m_b where $N_C = 3$ is the number of colors and Λ is the scale that defines how the QCD coupling evolves with energy. New physics effects, that is physics from additional diagrams that arise in models beyond the Standard Model, are not expected to have significant effects on Γ_{12} because any additional particles in such theories are required to be massive and hence do not give new cut contributions at this scale, but such effects can significantly alter M_{12}, as is discussed in Chapter 13, Section 13.2.

The light B_L and heavy B_H mass eigenstates are given by

$$|B_L\rangle \;=\; p|B^0\rangle + q|\overline{B}^0\rangle, \tag{1.9}$$

$$|B_H\rangle \;=\; p|B^0\rangle \Leftrightarrow q|\overline{B}^0\rangle. \tag{1.10}$$

The complex coefficients p and q obey the normalization condition

$$|q|^2 + |p|^2 = 1. \tag{1.11}$$

Note that $\arg(q/p^*)$ is just an overall common phase for $|B_L\rangle$ and $|B_H\rangle$ and has no physical significance.

The mass difference Δm_B and width difference $\Delta\Gamma_B$ between the neutral B mesons are defined as follows:

$$\Delta m_B \equiv\; M_H \Leftrightarrow M_L, \quad \Delta\Gamma_B \equiv\; \Gamma_H \Leftrightarrow \Gamma_L, \tag{1.12}$$

so that Δm_B is positive by definition. Finding the eigenvalues of (1.8), one gets

$$(\Delta m_B)^2 \Leftrightarrow \frac{1}{4}(\Delta\Gamma_B)^2 = 4(|M_{12}|^2 \Leftrightarrow \frac{1}{4}|\Gamma_{12}|^2), \tag{1.13}$$

$$\Delta m_B \Delta\Gamma_B = 4\,\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{1.14}$$

The ratio q/p is given by

$$\frac{q}{p} = \Leftrightarrow \frac{\Delta m_B \Leftrightarrow \frac{i}{2}\Delta\Gamma_B}{2(M_{12} \Leftrightarrow \frac{i}{2}\Gamma_{12})} = \Leftrightarrow \frac{2(M_{12}^* \Leftrightarrow \frac{i}{2}\Gamma_{12}^*)}{\Delta m_B \Leftrightarrow \frac{i}{2}\Delta\Gamma_B}, \tag{1.15}$$

1.2.2 Phase Conventions

(This section follows the discussion in [9].) The states B^0 and $\overline{B}^0$ are related through CP transformation:

$$CP|B^0\rangle = e^{2i\xi_B}|\overline{B}^0\rangle, \quad CP|\overline{B}^0\rangle = e^{-2i\xi_B}|B^0\rangle. \tag{1.16}$$

The phase ξ_B is *arbitrary*. The freedom in defining it comes from the fact that flavor conservation (in particular b-flavor) is a symmetry of the strong interactions. A phase transformation,

$$|B^0_\zeta\rangle = e^{-i\zeta}|B^0\rangle, \quad |\overline{B}^0_\zeta\rangle = e^{+i\zeta}|\overline{B}^0\rangle, \tag{1.17}$$

has therefore no physical effects. In the new basis, CP transformations take the form

$$(CP)_\zeta|B^0_\zeta\rangle = e^{2i(\xi_B-\zeta)}|\overline{B}^0_\zeta\rangle, \quad (CP)_\zeta|\overline{B}^0_\zeta\rangle = e^{-2i(\xi_B-\zeta)}|B^0_\zeta\rangle. \tag{1.18}$$

The various quantities discussed in this chapter change with the phase transformation (1.17):

$$M_{12}^\zeta = e^{2i\zeta}M_{12}, \quad \Gamma_{12}^\zeta = e^{2i\zeta}\Gamma_{12}, \quad (q/p)_\zeta = e^{-2i\zeta}(q/p). \tag{1.19}$$

Decay amplitudes, defined by

$$A_f = \langle f|H|B^0\rangle, \tag{1.20}$$

$$\overline{A}_f = \langle f|H|\overline{B}^0\rangle, \tag{1.21}$$

are also affected by the phase transformation (1.17):

$$(A_f)_\zeta = e^{-i\zeta}A_f, \quad (\overline{A}_f)_\zeta = e^{+i\zeta}\overline{A}_f. \tag{1.22}$$

From the transformation of states (1.17), and the transformation of q/p in (1.19), one learns that

$$|B_{L\zeta}\rangle = e^{i\zeta'}|B_L\rangle, \quad |B_{H\zeta}\rangle = e^{i\zeta'}|B_H\rangle, \tag{1.23}$$

namely both mass eigenstates are rotated by a common phase factor, which has no physical significance.

Similar phase freedom exists in defining the CP transformation law for a possible final state f and its CP conjugate $e^{2i\xi_f}\overline{f}$. The quantity ξ_f depends on the flavor content of f and is related to the quark flavor symmetries ($c,\ u,\ s,\ d$) of the strong interactions.

However, the freedom in defining the phase of the flavor eigenstates (which are defined through strong interactions only) does not mean that the full Lagrangian, which involves also weak interactions, is invariant under such phase redefinitions. Indeed, the differences of flavor redefinition phases appear as changes in the phases of the quark mixing matrix elements and of the Yukawa couplings of quarks to Higgs fields (or any other Lagrangian terms that cause couplings between different flavor eigenstates in more general models). While both (q/p) and A_f acquire overall phase redefinitions when these phase rotations are made, the quantity

$$\lambda = \frac{q}{p}\frac{\overline{A}_f}{A_f} \tag{1.24}$$

has a convention-independent phase that has physical significance, as will be seen when the possible types of CP violations are examined below.

Another subtle point that has to do with the arbitrariness of the phase ξ_B in the CP transformation law (1.16) is the following. If $|q/p| = 1$, it is always possible to choose a CP transformation (1.16) such that the mass eigenstates (1.9) and (1.10) are eigenstates of this transformation. Such a definition is, however, not meaningful, because there is no relationship between the so-called CP quantum numbers for state with different flavor content. For example, the state B_H can be chosen to be odd under such an appropriately defined transformation, but can decay into a final two-pion state (which is even under the conventionally defined CP transformation) even without CP violating phases in the decay amplitude.

1.2.3 Time Evolution of Neutral B_d Mesons

The two neutral B_d mesons are expected to have a negligible difference in lifetime,

$$\Delta\Gamma_{B_d}/\Gamma_{B_d} = \mathcal{O}(10^{-2}). \tag{1.25}$$

Note that $\Delta\Gamma_{B_d}$ has not been measured. The difference in width is produced by decay channels common to B^0 and $\overline{B}^0$. The branching ratios for such channels are at or below the level of 10^{-3}. As various channels contribute with differing signs, one expects that their sum does not exceed the individual level, hence $\Delta\Gamma_{B_d} \ll \Gamma_{B_d}$ is a rather safe and model-independent assumption [10]. (For B_s^0 mesons the lifetime difference may be significant [11].)

On the other hand, Δm_{B_d} has been measured [12],

$$x_d \equiv \Delta m_{B_d}/\Gamma_{B_d} = 0.73 \pm 0.05. \tag{1.26}$$

From (1.25) and (1.26) one learns that, model-independently,

$$\Delta\Gamma_B \ll \Delta m_B. \tag{1.27}$$

Equations (1.25) and (1.27) imply that, to $\mathcal{O}(10^{-2})$ accuracy, Eqs. (1.13), (1.14) and (1.15) simplify into

$$\Delta m_B = 2|M_{12}|, \quad \Delta\Gamma_B = 2\,\mathcal{R}e(M_{12}\Gamma_{12}^*)/|M_{12}|, \tag{1.28}$$

$$q/p = \Leftrightarrow |M_{12}|/M_{12}. \tag{1.29}$$

Any B state can then be written as an admixture of the states B_H and B_L, and the amplitudes of this admixture evolve in time

$$a_H(t) = a_H(0)e^{-iM_Ht}e^{-\frac{1}{2}\Gamma_Ht}, \quad a_L(t) = a_L(0)e^{-iM_Lt}e^{-\frac{1}{2}\Gamma_Lt}. \tag{1.30}$$

A state which is created at time $t = 0$ as initially pure B^0, is denoted $|B^0_{\rm phys}\rangle$, it has $a_L(0) = a_H(0) = 1/(2p)$. Similarly an initially pure $\overline{B}^0$, $|\overline{B}^0_{\rm phys}\rangle$, has $a_L(0) = \Leftrightarrow a_H(0) = 1/(2q)$. The time evolution of these states is thus given by

$$|B^0_{\rm phys}(t)\rangle = g_+(t)|B^0\rangle + (q/p)g_-(t)|\overline{B}^0\rangle, \tag{1.31}$$

$$|\overline{B}^0_{\rm phys}(t)\rangle = (p/q)g_-(t)|B^0\rangle + g_+(t)|\overline{B}^0\rangle, \tag{1.32}$$

where

$$g_+(t) = e^{-iMt}e^{-\Gamma t/2}\cos(\Delta m_B\, t/2), \tag{1.33}$$

$$g_-(t) = e^{-iMt}e^{-\Gamma t/2}i\sin(\Delta m_B\, t/2), \tag{1.34}$$

and $M = \frac{1}{2}(M_H + M_L)$.

For some purposes, it is useful to go beyond the leading approximation for $\frac{q}{p}$, the relevant expression is:

$$\frac{q}{p} = \Leftrightarrow \frac{M_{12}^*}{|M_{12}|}\left[1 \Leftrightarrow \frac{1}{2}\,\mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]. \tag{1.35}$$

1.2.4 Two-Time Formalism for Coherent $B\overline{B}$ States

At a B factory, that is an e^+e^- collider operating at the $\Upsilon(4S)$ resonance, the B^0 and $\overline{B}^0$ mesons produced from the decay of the Υ are in a coherent $L = 1$ state. One way to view this state is that each of the two particles evolve in time as described above for a single B. However they evolve in phase, so that at any time, until one particle decays, there is always exactly one B^0 and one $\overline{B}^0$ present. (This is yet one more particle physics case of the classic Einstein-Podolsky-Rosen situation.) However once one of the particles decays the other continues to evolve, and thus there are possible events with two B or two $\overline{B}$ decays, whose probability is governed by the time between the two decays.

The two particles from the Upsilon decay are identified by the angle θ that they make with the e^- beam direction in the Upsilon rest frame. Then the two-B state

$$\begin{aligned} S(t_f, t_b) &= \tfrac{1}{\sqrt{2}}\{B^0_{\rm phys}(t_f, \theta, \phi)\overline{B}^0_{\rm phys}(t_b, \pi \Leftrightarrow \theta, \phi + \pi) \\ &\Leftrightarrow \overline{B}^0_{\rm phys}(t_f, \theta, \phi)B^0_{\rm phys}(t_b, \pi \Leftrightarrow \theta, \phi + \pi)\} \sin(\theta) \end{aligned} \tag{1.36}$$

can be written as

$$\begin{aligned} S(t_f, t_b) = \tfrac{1}{\sqrt{2}} e^{-(\Gamma/2 + iM)(t_f + t_b)} \{ &\cos[\Delta m_B(t_f \Leftrightarrow t_b)/2](B^0_f \overline{B}^0_b \Leftrightarrow \overline{B}^0_f B^0_b) \\ &\Leftrightarrow i \sin[\Delta m_B(t_f \Leftrightarrow t_b)/2](\tfrac{p}{q} B^0_f B^0_b \Leftrightarrow \tfrac{q}{p} \overline{B}^0_f \overline{B}^0_b)\} \sin(\theta_f), \end{aligned} \tag{1.37}$$

where t_f is the proper time of the B_f, the B particle in the forward half-space at angle $(\theta_f < \pi/2, \phi_f)$ and t_b is the proper time for the backward-moving B_b, at $(\pi \Leftrightarrow \theta_f, \phi_f + \pi)$. Since the B's have equal (though back-to-back) momenta in this frame, until such time as one or the other of these particles decays $t_f = t_b$ and Eq. (1.37) contains one B^0 and one $\overline{B}^0$. However decay stops the clock for the decayed particle. Then the terms that depend on $\sin[\Delta m_B(t_f \Leftrightarrow t_b)/2]$ begin to play a role.

From Eq. (1.37) one can derive the amplitude for decays where one of the two B's decays to any state f_1 at time t_1 and the other decays to f_2 at time t_2. One obtains

$$\begin{aligned} A(t_1, t_2) = \tfrac{1}{\sqrt{2}} e^{-(\Gamma/2 + iM)(t_1 + t_2)} \zeta(t_1, t_2) \{ &\cos[\Delta m_B(t_1 \Leftrightarrow t_2)/2](A_1 \overline{A}_2 \Leftrightarrow \overline{A}_1 A_2) \\ &\Leftrightarrow i \sin[\Delta m_B(t_1 \Leftrightarrow t_2)/2](\tfrac{p}{q} A_1 A_2 \Leftrightarrow \tfrac{q}{p} \overline{A}_1 \overline{A}_2)\} \sin(\theta_1), \end{aligned} \tag{1.38}$$

where A_i is the amplitude for a B^0 to decay to the state f_i, $\overline{A}_i$ is the amplitude for a $\overline{B}^0$ to decay to the *same* state f_i (see Eqs. (1.20) and (1.21)). Any state that identifies the flavor of the parent B ('tagging decays') has either A_f or $\overline{A}_f = 0$. (The fact that sin(2$\pi \Leftrightarrow \theta$) = -sin($\theta$) is used to write Eq. (1.38) with θ_1 running over angles $(0, \pi)$.) In Eq. (1.38) to keep signs consistent with Eq. (1.37) the shorthand

$$\zeta(t_1, t_2) = \begin{cases} +1 & t_1 = t_f,\ t_2 = t_b, \\ \Leftrightarrow 1 & t_1 = t_b,\ t_2 = t_f \end{cases} \tag{1.39}$$

is introduced, but this overall sign factor will disappear in the rate.

It is now straightforward to calculate the time-dependent rate for producing the combined final states f_1, f_2. One finds

$$\begin{aligned} R(t_1, t_2) = C e^{-\Gamma(t_1 + t_2)} \{ &(|A_1|^2 + |\overline{A}_1|^2)(|A_2|^2 + |\overline{A}_2|^2) \Leftrightarrow 4\, \mathcal{R}e(\frac{q}{p} A_1^* \overline{A}_1)\, \mathcal{R}e(\frac{q}{p} A_2^* \overline{A}_2) \\ &\Leftrightarrow \cos(\Delta m_B(t_1 \Leftrightarrow t_2))[(|A_1|^2 \Leftrightarrow |\overline{A}_1|^2)(|A_2|^2 \Leftrightarrow |\overline{A}_2|^2) + 4\, \mathcal{I}m(\frac{q}{p} A_1^* \overline{A}_1)\, \mathcal{I}m(\frac{q}{p} A_2^* \overline{A}_2)] \\ &+ 2 \sin(\Delta m_B(t_1 \Leftrightarrow t_2))[\mathcal{I}m(\frac{q}{p} A_1^* \overline{A}_1)(|A_2|^2 \Leftrightarrow |\overline{A}_2|^2) \Leftrightarrow (|A_1|^2 \Leftrightarrow |\overline{A}_1|^2)\, \mathcal{I}m(\frac{q}{p} A_2^* \overline{A}_2)]\}. \end{aligned} \tag{1.40}$$

Here an integral over all directions for either B has been performed, so the angular dependence has dropped out of the expressions, and an overall normalization factor C has appeared. The approximation $|q/p| = 1$ has also been used.

To measure CP asymmetries one looks for events where one B decays to a final CP eigenstate f_{CP} at time t_f, while the second decays to a tagging mode, that is a mode which identifies its b-flavor, at time $t_{\rm tag}$. For example, take a tagging mode with $A_2 = 0$, $\overline{A}_2 = \overline{A}_{\rm tag}$. This then identifies the *other* B-particle as a B^0 at time $t_2 = t_{\rm tag}$ at which the tag decay occurs. Note that this is true even when the tag decay occurs after the CP eigenstate decay. In this case the state of the other B at any time $t_f < t_{\rm tag}$ must be just that mixture that, if it had not decayed, would have evolved to become a B^0 at time $t_f = t_{\rm tag}$. The double time expression reduces to the form

$$R(t_{\rm tag}, t_{f_{CP}}) = Ce^{-\Gamma(t_{\rm tag}+t_{f_{CP}})}|\overline{A}_{\rm tag}|^2|A_{f_{CP}}|^2\{1 + |\lambda_{f_{CP}}|^2 + \cos[\Delta m_B(t_{f_{CP}} \Leftrightarrow t_{\rm tag})](1 \Leftrightarrow |\lambda_{f_{CP}}|^2) \Leftrightarrow 2\sin[\Delta m_B(t_{f_{CP}} \Leftrightarrow t_{\rm tag})]\,\mathcal{I}m(\lambda_{f_{CP}})\} \quad (1.41)$$

where

$$\lambda_{f_{CP}} \equiv \frac{q}{p}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}}\frac{q}{p}\frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}. \quad (1.42)$$

The second form for $\lambda_{f_{CP}}$ here uses the property

$$\overline{A}_{f_{CP}} = \eta_{f_{CP}}\overline{A}_{\overline{f}_{CP}}, \quad (1.43)$$

where $\eta_{f_{CP}}$ is the CP eigenvalue of the state f_{CP}. The amplitudes $A_{f_{CP}}$ and $\overline{A}_{\overline{f}_{CP}}$ are related by CP and differ only in the signs of the weak phase for each term, while $\eta_{f_{CP}} = \pm 1$, so the second form is useful in calculating the expected asymmetries, and explains the extra minus sign that appears for a CP odd final state.

For the case where the tag final state has $\overline{A}_2 = 0$, $A_2 = A_{\rm tag}$, which identifies the second particle as a $\overline{B}$ at time $t_{\rm tag}$, an expression similar to Eq. (1.41) applies, except that the signs of both the cosine and the sine terms are reversed. The fact that $|q/p| = 1$ means that the amplitudes for the two opposite tags are the same. Thus the difference of these rates divided by their sum, which measures the time-dependent CP asymmetry[13], is given by

$$a_{f_{CP}} = \frac{(1 \Leftrightarrow |\lambda_{f_{CP}}|^2)\cos(\Delta m_B t) \Leftrightarrow 2\,\mathcal{I}m\,\lambda_{f_{CP}}\sin(\Delta m_B t)}{1 + |\lambda_{f_{CP}}|^2}, \quad (1.44)$$

where $t = t_{f_{CP}} \Leftrightarrow t_{\rm tag}$.

It is useful to note that the above expressions can be integrated over the variable $(t_1 + t_2)$, which for $t_1 \geq 0$ and $t_2 \geq 0$ can take values between $|t_1 \Leftrightarrow t_2|$ and infinity. Thus one can fit the dependence on the variable $t_1 \Leftrightarrow t_2$ without having to measure the Υ decay time. The fact that the variable $t_1 \Leftrightarrow t_2$ can be related to the distance between the locations of the two decays is of course the prime reason for building an energy-asymmetric collider for the B factory. If one had to integrate over this variable as well all information on the coefficient of $\sin(\Delta m_B(t_1 \Leftrightarrow t_2))$ would be lost in the above expressions, and the experiment would be sensitive only to those CP-violating effects that give $|\lambda| \neq 1$. (Note that this is a consequence of the coherent production of the two B states, in a hadronic environment, where the B's are produced incoherently, time-integrated rates are always integrals from $t = 0$ to infinity and hence retain information about the $\sin(\Delta m_B t)$ behavior.)

1.3 The Three Types of CP Violation in B Decays

The possible manifestations of CP violation can be classified in a model-independent way:

1. CP violation in decay, which occurs in both charged and neutral decays, when the amplitude for a decay and its CP conjugate process have different magnitudes;
2. CP violation in mixing, which occurs when the two neutral mass eigenstates cannot be chosen to be CP eigenstates;
3. CP violation in the interference between decays with and without mixing, which occurs in decays into final states that are common to B^0 and $\overline{B}^0$. (It often occurs in combination with the other two types but, important for BABAR, there are cases when, to an excellent approximation, it is the only effect.)

In each case it is useful to identify a particular CP-violating quantity that is independent of phase conventions and discuss the types of processes that depend on this quantity.

1.3.1 CP Violation in Decay

For any final state f, the quantity $|\frac{\overline{A}_{\overline{f}}}{A_f}|$ is independent of phase conventions and physically meaningful. There are two types of phases that may appear in A_f and $\overline{A}_{\overline{f}}$.

Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP conjugate amplitude. Thus their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model these phases occur only in the CKM matrix which is part of the electroweak sector of the theory, hence these are often called "weak phases." The weak phase of any single term is convention dependent. However the difference between the weak phases in two different terms in A_f is convention independent; the initial and final states are the same for every term and thus any phase rotation of the fields that appear in these states will affect all terms in the same way.

A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Such phases do not violate *CP*, since they appear in A_f and $\overline{A}_{\overline{f}}$ with the same sign. Their origin is the possible contribution from intermediate on-shell states in the decay process, that is an absorptive part of an amplitude that has contributions from coupled channels. Usually the dominant rescattering is due to strong interactions, hence the designation "strong phases" for the phase shifts so induced. Again only the relative strong phases of different terms in a scattering amplitude have physical content, an overall phase rotation of the entire amplitude has no physical consequences.

Thus it is useful to write each contribution to A in three parts: its magnitude A_i, its weak-phase term $e^{i\phi_i}$, and its strong phase term $e^{i\delta_i}$. Then, if several amplitudes contribute to $B^0 \to f$, the amplitude A_f (see (1.20)) and the *CP* conjugate amplitude $\overline{A}_{\overline{f}}$ (see (1.21)) are given by:

$$A_f = \sum_i A_i e^{i(\delta_i+\phi_i)}, \quad \overline{A}_{\overline{f}} = e^{2i(\xi_f-\xi_B)} \sum_i A_i e^{i(\delta_i-\phi_i)}, \tag{1.45}$$

where ξ_f and ξ_B are defined in 1.2.2. (If f is a *CP* eigenstate then $e^{2i\xi_f} = \pm 1$ is its *CP* eigenvalue.) The convention-independent quantity is then

$$\left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| = \left|\frac{\sum_i A_i e^{i(\delta_i-\phi_i)}}{\sum_i A_i e^{i(\delta_i+\phi_i)}}\right|. \tag{1.46}$$

When *CP* is conserved, the weak phases ϕ_i are all equal. Therefore, from Eq. (1.46) one sees that

$$|\overline{A}_{\overline{f}}/A_f| \neq 1 \quad \Longrightarrow \quad CP \text{ violation.} \tag{1.47}$$

This type of *CP* violation is here called *CP violation in decay*. It is often also called *direct CP violation*. It results from the *CP*-violating interference among various terms in the decay amplitude. From Eq. (1.46) it can be seen that a *CP* violation of this type will not occur unless at least two terms that have different weak phases acquire different strong phases, since:

$$|A|^2 - |\overline{A}|^2 = -2 \sum_{i,j} A_i A_j \sin(\phi_i - \phi_j) \sin(\delta_i - \delta_j). \tag{1.48}$$

Any *CP* asymmetries in charged B decays,

$$a_f = \frac{\Gamma(B^+ \to f) - \Gamma(B^- \to \overline{f})}{\Gamma(B^+ \to f) + \Gamma(B^- \to \overline{f})}, \tag{1.49}$$

are from *CP* violation in decay. In terms of the decay amplitudes

$$a_f = \frac{1 - |\overline{A}/A|^2}{1 + |\overline{A}/A|^2}. \tag{1.50}$$

CP violation in decays can also occur for neutral meson decays, where it competes with the other two types of *CP* violation effects described below. There is as yet no unambiguous experimental evidence for CP violation in decays. (As explained in 1.6.3, a measurement of $\mathcal{R}e\ \varepsilon'_K \neq 0$ would constitute such evidence.)

The magnitude and strong phase of any amplitude involve long distance strong interaction physics, and cannot be calculated from first principles. Thus quantities that depend only on the weak phases are much cleaner than those that require knowledge of the relative magnitudes or strong phases of various amplitude contributions. There is however a large literature and considerable theoretical effort that goes into the calculation of amplitudes and strong phases. In many cases one can only relate experiment to Standard Model parameters through such calculations. The techniques that are used are expected to be more accurate for B decays than for K decays because of the larger B mass, but theoretical uncertainty remains significant. The calculations generally contain two parts. First, the operator product expansion and QCD perturbation theory are used to write any underlying quark process as a sum of local quark operators with well-determined coefficients. Second, the matrix elements of the operators between the initial and final hadron states must be calculated. This is where the theory is weakest and the results most model dependent. Ideally lattice calculations should be able to provide accurate determinations for the matrix elements, and in certain cases this is already true, but much remains to be done. In the following chapter an overview of the principal methods used in such calculations is given. Further details on the status of various theoretical approaches are presented in relevant chapters and in the appendices.

1.3.2 *CP* Violation in Mixing

A second quantity that is independent of phase conventions and physically meaningful is

$$\left|\frac{q}{p}\right|^2 = \left|\frac{M_{12}^* \Leftrightarrow \frac{i}{2}\Gamma_{12}^*}{M_{12} \Leftrightarrow \frac{i}{2}\Gamma_{12}}\right|. \tag{1.51}$$

When CP is conserved, the mass eigenstates must be CP eigenstates. In that case the relative phase between M_{12} and Γ_{12} vanishes. Therefore, Eq. (1.51) implies

$$|q/p| \neq 1 \quad \Longrightarrow \quad CP \text{ violation.} \tag{1.52}$$

This type of CP violation is here called *CP violation in mixing*; it is often referred to as *indirect CP violation*. It results from the mass eigenstates being different from the CP eigenstates. CP violation in mixing has been observed unambiguously in the neutral kaon system.

For the neutral B system, this effect could be observed through the asymmetries in semileptonic decays:

$$a_{\rm sl} = \frac{\Gamma(\overline{B}^0_{\rm phys}(t) \to \ell^+\nu X) \Leftrightarrow \Gamma(B^0_{\rm phys}(t) \to \ell^-\nu X)}{\Gamma(\overline{B}^0_{\rm phys}(t) \to \ell^+\nu X) + \Gamma(B^0_{\rm phys}(t) \to \ell^-\nu X)}. \tag{1.53}$$

In terms of $|q/p|$,

$$a_{\rm sl} = \frac{1 \Leftrightarrow |q/p|^4}{1 + |q/p|^4}, \tag{1.54}$$

which follows from

$$\langle \ell^- \nu X | H | B^0_{\rm phys}(t) \rangle = (q/p) g_-(t) A^*, \quad \langle \ell^+ \nu X | H | \overline{B}^0_{\rm phys}(t) \rangle = (p/q) g_-(t) A. \tag{1.55}$$

As can be seen from the discussion in Section 1.2.3, effects of CP violation in mixing in neutral B_d decays, such as the asymmetries in semileptonic decays, are expected to be small, $\mathcal{O}(10^{-2})$. Moreover, to calculate the deviation of q/p from a pure phase, one needs to calculate Γ_{12} and M_{12}. This involves large hadronic uncertainties, in particular in the hadronization models for Γ_{12}. The overall uncertainty is easily a factor of 2–3 in $|q/p| \Leftrightarrow 1$ [10]. Thus even if such asymmetries are observed, it will be difficult to relate their rates to fundamental CKM parameters.

1.3.3 CP Violation in the Interference Between Decays With and Without Mixing

Finally, consider neutral B decays into final CP eigenstates, f_{CP} [14, 15, 16]. Such states are accessible in both B^0 and $\overline{B}^0$ decays. The quantity of interest here that is independent of phase conventions and physically meaningful is λ of Eq. (1.42), $\lambda = \eta_{f_{CP}} \frac{q}{p} \frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}$. When CP is conserved, $|q/p| = 1$, $|\overline{A}_{\overline{f}_{CP}}/A_{f_{CP}}| = 1$, and furthermore, the relative phase between (q/p) and $(\overline{A}_{\overline{f}_{CP}}/A_{f_{CP}})$ vanishes. Therefore, Eq. (1.42) implies

$$\lambda \neq \pm 1 \quad \Longrightarrow \quad CP \text{ violation.} \tag{1.56}$$

Note that both CP violation in decay (1.47) and CP violation in mixing (1.52) lead to (1.56) through $|\lambda| \neq 1$. However, it is possible that, to a good approximation, $|q/p| = 1$ and $|\overline{A}/A| = 1$, yet there is CP violation:

$$|\lambda| = 1, \quad \mathcal{I}m\,\lambda \neq 0. \tag{1.57}$$

This type of CP violation is called *CP violation in the interference between decays with and without mixing* here; sometimes this is abbreviated as "interference between mixing and decay." As explained in Section 1.6, this type of CP violation has also been observed in the neutral kaon system.

For the neutral B system, CP violation in the interference between decays with and without mixing can be observed by comparing decays into final CP eigenstates of a time-evolving neutral B state that begins at time zero as B^0 to those of the state that begins as a $\overline{B}^0$:

$$a_{f_{CP}} = \frac{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) \Leftrightarrow \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})}{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) + \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})}. \tag{1.58}$$

It was shown above (1.44) that this time-dependent asymmetry is given by:

$$a_{f_{CP}} = \frac{(1 \Leftrightarrow |\lambda_{f_{CP}}|^2)\cos(\Delta m_B t) \Leftrightarrow 2\,\mathcal{I}m\,\lambda_{f_{CP}}\sin(\Delta m_B t)}{1 + |\lambda_{f_{CP}}|^2}. \tag{1.59}$$

This asymmetry will be non-vanishing if any of the three types of *CP* violation are present. However, for decays such that $|\lambda| = 1$ (the 'clean' modes — see below), (1.44) simplifies considerably:

$$a_{f_{CP}} = \Leftrightarrow \mathcal{I}m\,\lambda_{f_{CP}}\sin(\Delta m_B\, t)\;. \tag{1.60}$$

One point concerning this type of asymmetries is worth clarifying. Consider the decay amplitudes of B^0 into two different final *CP* eigenstates, A_a and A_b. A non-vanishing difference between $\eta_a\lambda_a$ and $\eta_b\lambda_b$,

$$\eta_a\lambda_a \Leftrightarrow \eta_b\lambda_b = \frac{q}{p}\left(\frac{\overline{A}_{\overline{a}}}{A_a} \Leftrightarrow \frac{\overline{A}_{\overline{b}}}{A_b}\right) \neq 0, \tag{1.61}$$

would establish the existence of *CP* violation in $\Delta b = 1$ processes. For this reason, this type of *CP* violation is also called sometimes "direct *CP* violation." Yet, unlike the case of *CP* violation in decay, no nontrivial strong phases are necessary. The richness of possible final *CP* eigenstates in B decays makes it very likely that various asymmetries will exhibit (1.61). (A measurement of $\mathcal{B}(K_L \to \pi\nu\overline{\nu}) \gtrsim 10^{-11}$ can establish the existence [17, 18, 19] of a similar effect, a $\Delta s = 1$ *CP* violation that does not depend on strong phase shifts.) Either this type of observation or the observation of *CP* violation in decay would rule out superweak models for *CP* violation.

CP violation in the interference between decays with and without mixing can be cleanly related to Lagrangian parameters when it occurs with no *CP* violation in decay. In particular, for B_d decays that are dominated by a single *CP*-violating phase, so that the effect of *CP* violation in decay is negligible, $a_{f_{CP}}$ is cleanly translated into a value for $\mathcal{I}m\,\lambda$ (see (1.60)) which, in turn, is cleanly interpreted in terms of purely electroweak Lagrangian parameters. (As discussed below, $\mathcal{I}m\,\varepsilon_K$ which describes *CP* violation in the interference between decays with and without mixing in the K system, is cleanly translated into a value of ϕ_{12}, the phase between $M_{12}(K)$ and $\Gamma_{12}(K)$. It is difficult, however, to interpret ϕ_{12} cleanly in terms of electroweak Lagrangian parameters.)

When there is *CP* violation in decay at the same time as in the interference between decays with and without mixing, the asymmetry (1.58) depends also on the ratio of the different amplitudes and their relative strong phases, and thus the prediction has hadronic uncertainties. In some cases, however, it is possible to remove any large hadronic uncertainties by measuring several isospin-related rates (see *e.g.*, [20, 21, 22]) and thereby extract a clean measurement of CKM phases. This is discussed in further detail in Chapters 5 and particularly 6.

There are also many final states for B decay that have *CP* self-conjugate particle content but are not *CP* eigenstates because they contain admixtures of different angular momenta and hence different parities. In certain cases angular analyses of the final state can be used to determine the amplitudes for each different *CP* contribution separately. Such final states can then also be used for clean comparison with theoretical models [23]. This is discussed in more detail in Chapter 5.

1.4 CP Violation in the Standard Model

1.4.1 The CKM Picture of CP Violation

In the Standard Model (SM) [24] of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry with three fermion generations, CP violation arises from a single phase in the mixing matrix for quarks [3]. Each quark generation consists of three multiplets:

$$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix} = (3,2)_{+1/6}, \quad u_R^I = (3,1)_{+2/3}, \quad d_R^I = (3,1)_{-1/3}, \tag{1.62}$$

where $(3,2)_{+1/6}$ denotes a triplet of $SU(3)_C$, doublet of $SU(2)_L$ with hypercharge $Y = Q \Leftrightarrow T_3 = +1/6$, and similarly for the other representations. The interactions of quarks with the $SU(2)_L$ gauge bosons are given by

$$\mathcal{L}_W = \Leftrightarrow \frac{1}{2} g \overline{Q_{Li}^I} \gamma^\mu \tau^a \mathbf{1}_{ij} Q_{Lj}^I W_\mu^a, \tag{1.63}$$

where γ^μ operates in Lorentz space, τ^a operates in $SU(2)_L$ space and $\mathbf{1}$ is the unit matrix operating in generation (flavor) space. This unit matrix is written explicitly to make the transformation to mass eigenbasis clearer. The interactions of quarks with the single Higgs scalar doublet $\phi(1,2)_{+1/2}$ of the Standard Model are given by

$$\mathcal{L}_Y = \Leftrightarrow G_{ij} \overline{Q_{Li}^I} \phi d_{Rj}^I \Leftrightarrow F_{ij} \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I + \text{Hermitian conjugate}, \tag{1.64}$$

where G and F are general *complex* 3×3 matrices. Their complex nature is the source of CP violation in the Standard Model. With the spontaneous symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$ due to $\langle \phi \rangle \neq 0$, the two components of the quark doublet become distinguishable, as are the three members of the W^μ triplet. The charged current interaction in (1.63) is given by

$$\mathcal{L}_W = \Leftrightarrow \sqrt{\frac{1}{2}} g \overline{u_{Li}^I} \gamma^\mu \mathbf{1}_{ij} d_{Lj}^I W_\mu^+ + \text{h.c.}. \tag{1.65}$$

The mass terms that arise from the replacement $\mathcal{R}e(\phi^0) \to \sqrt{\frac{1}{2}}(v + H^0)$ in (1.64) are given by

$$\mathcal{L}_M = \Leftrightarrow \sqrt{\frac{1}{2}} v G_{ij} \overline{d_{Li}^I} d_{Rj}^I \Leftrightarrow \sqrt{\frac{1}{2}} v F_{ij} \overline{u_{Li}^I} u_{Rj}^I + \text{Hermitian conjugate}, \tag{1.66}$$

namely

$$M_d = Gv/\sqrt{2}, \quad M_u = Fv/\sqrt{2}. \tag{1.67}$$

The phase information is now contained in these mass matrices. To transform to the mass eigenbasis, one defines four unitary matrices such that

$$V_{dL} M_d V_{dR}^\dagger = M_d^{\rm diag}, \quad V_{uL} M_u V_{uR}^\dagger = M_u^{\rm diag}, \tag{1.68}$$

where $M_q^{\rm diag}$ are diagonal and real, while V_{qL} and V_{qR} are complex. The charged current interactions (1.65) are given in the mass eigenbasis by

$$\mathcal{L}_W = \Leftrightarrow\sqrt{\frac{1}{2}} g \overline{u_{Li}} \gamma^\mu \overline{V}_{ij} d_{Lj} W_\mu^+ + \text{h.c.}. \qquad (1.69)$$

(Quark fields with no superscript denote mass eigenbasis.) The matrix $\overline{V} = V_{uL} V_{dL}^\dagger$ is the (unitary) mixing matrix for three quark generations. As such, it generally depends on nine parameters: three can be chosen as real angles (like the Cabibbo angle) and six are phases. However, one may reduce the number of phases in $\overline{V}$ by a transformation

$$\overline{V} \implies V = P_u \overline{V} P_d^*, \qquad (1.70)$$

where P_u and P_d are diagonal phase matrices. This is a legitimate transformation because it amounts to redefining the phases of the quark-mass-eigenstate fields, as was discussed earlier:

$$q_{Li} \to (P_q)_{ii} q_{Li}, \quad q_{Ri} \to (P_q)_{ii} q_{Ri}, \qquad (1.71)$$

which does not change the real diagonal mass matrix $M_q^{\rm diag}$. The five phase differences among the elements of P_u and P_d can be chosen so that the transformation (1.70) eliminates five of the six independent phases from $\overline{V}$; thus V has one irremovable phase. This phase is called the Kobayashi-Maskawa phase [3], $\delta_{\rm KM}$, and the mixing matrix is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. It is interesting to note that the same procedure applied to a two-generation Standard Model Lagrangian with a single Higgs field would remove all CP-violating phases — that theory could not accommodate CP violation without the addition of extra fields. It was this observation that led Kobayashi and Maskawa to suggest a third quark generation long before there was any experimental evidence for it.

The irremovable phase in the CKM matrix allows possible CP violation. To see this, recall the CP transformation laws (1.4) and (1.5),

$$\overline{\psi}_i \psi_j \to \overline{\psi}_j \psi_i, \quad \overline{\psi}_i \gamma^\mu W_\mu (1 \Leftrightarrow \gamma_5) \psi_j \to \overline{\psi}_j \gamma^\mu W_\mu (1 \Leftrightarrow \gamma_5) \psi_i. \qquad (1.72)$$

Thus the mass terms and gauge interactions are obviously CPinvariant if all the masses and couplings are all real. In particular, consider the coupling of $W^\pm$ to quarks. It has the form

$$g V_{ij} \overline{u}_i \gamma_\mu W^{+\mu} (1 \Leftrightarrow \gamma_5) d_j + g V_{ij}^* \overline{d}_j \gamma_\mu W^{-\mu} (1 \Leftrightarrow \gamma_5) u_i. \qquad (1.73)$$

The CP operation interchanges the two terms except that V_{ij} and V_{ij}^* are not interchanged. Thus, CP is a good symmetry only if there is a mass basis and choice of phase convention where all couplings and masses are real.

CP is not necessarily violated in the three generation Standard Model. If two quarks of the same charge had equal masses, one mixing angle and the phase could be removed from V. This can be written as a condition on quark mass differences: CP violation requires

$$(m_t^2 \Leftrightarrow m_c^2)(m_c^2 \Leftrightarrow m_u^2)(m_t^2 \Leftrightarrow m_u^2)(m_b^2 \Leftrightarrow m_s^2)(m_s^2 \Leftrightarrow m_d^2)(m_b^2 \Leftrightarrow m_d^2) \neq 0. \qquad (1.74)$$

(The squared masses appear here because the sign of a fermion mass term is not physical.) Likewise, if the value of any of the three mixing angles were 0 or $\pi/2$, then the phase could be removed. Finally, CP would not be violated if the value of the single phase were 0 or π. These last eight conditions are elegantly incorporated into one, parameterization-independent, condition [25]. To find this condition, note that unitarity of the CKM matrix, $VV^\dagger = \mathbf{1}$, requires that for any choice of $i, j, k, l = 1, 2, 3$

$$\mathcal{I}m[V_{ij} V_{kl} V_{il}^* V_{kj}^*] = J \sum_{m,n=1}^{3} \epsilon_{ikm} \epsilon_{jln}. \tag{1.75}$$

Then, the conditions on the mixing parameters are summarized by

$$J \neq 0. \tag{1.76}$$

The fourteen conditions incorporated in (1.74) and (1.76) can all be written as a single requirement of the mass matrices in the interaction basis [25]:

$$\mathcal{I}m\{\det[M_d M_d^\dagger, M_u M_u^\dagger]\} \neq 0 \;\Leftrightarrow\; CP \text{ violation}. \tag{1.77}$$

This is a convention-independent condition. The quantity J is of much interest in the study of CP violation from the CKM matrix. The maximum value that J could in principle assume is $1/(6\sqrt{3}) \approx 0.1$, but it is found to be $\lesssim 4 \times 10^{-5}$, providing a concrete meaning to the notion that CP violation in the Standard Model is small.

The fact that the three generation Standard Model with a single Higgs multiplet contains only a single independent CP-violating phase makes the possible CP-violating effects in this theory all very closely related. It is this that makes the pattern of CP violations in B decays strongly constrained in this model. The goal of the B factory is to test whether this pattern occurs.

1.4.2 Unitarity of the CKM Matrix

The unitarity of the CKM matrix is manifest using an explicit parameterization. There are various useful ways to parameterize it, but the standard choice is the following [26]:

$$V = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}, \tag{1.78}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. In this parameterization

$$J = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta. \tag{1.79}$$

This shows explicitly the requirement that all mixing angles are different from $0, \pi/2$ and $\delta \neq 0, \pi$.

The unitarity of the CKM matrix implies various relations among its elements. A full list of these relations can be found in Ref. [8]. Three of them are very useful for understanding the Standard Model predictions for CP violation:

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{1.80}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{1.81}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{1.82}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles." Note that the term "Unitarity Triangle" is reserved for the relation (1.82) only (for reasons soon to be understood).

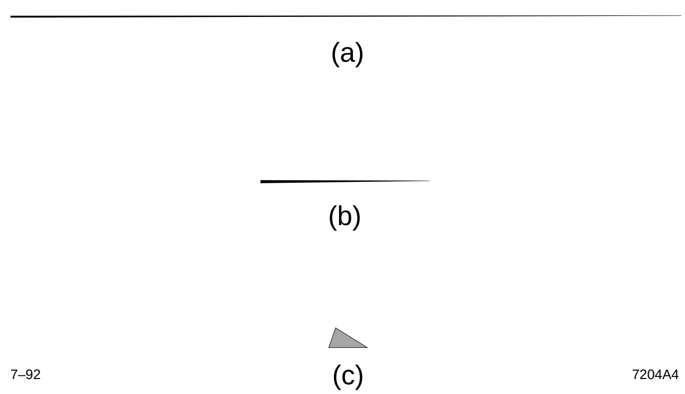

Figure 1-1. *The three unitarity triangles a) $V_{id}V_{is}^* = 0$, b)$V_{is}V_{ib}^* = 0$, and c) $V_{id}V_{ib}^* = 0$, drawn to a common scale.*

It is instructive to draw the three triangles, knowing the experimental values (within errors) for the various $|V_{ij}|$. This is done in Fig. 1-1. In the first two triangles, one side is much shorter than the other two, and so they almost collapse to a line. This would give an intuitive understanding of why CP violation is small in the leading K decays (the first triangle) and in the leading B_s decays (the second triangle). Decays related to the short sides of these triangles (for example, $K_L \to \pi\nu\overline{\nu}$) are rare but could exhibit significant CP violation. The most exciting physics of CP violation lies in the B system, related to the third triangle. The openness of this triangle predicts large CP asymmetries in B decays.

Equation (1.75) has striking implications for the unitarity triangles:

1. All unitarity triangles are equal in area.

2. The area of each unitarity triangle equals $|J|/2$.

3. The sign of J gives the direction of the complex vectors.

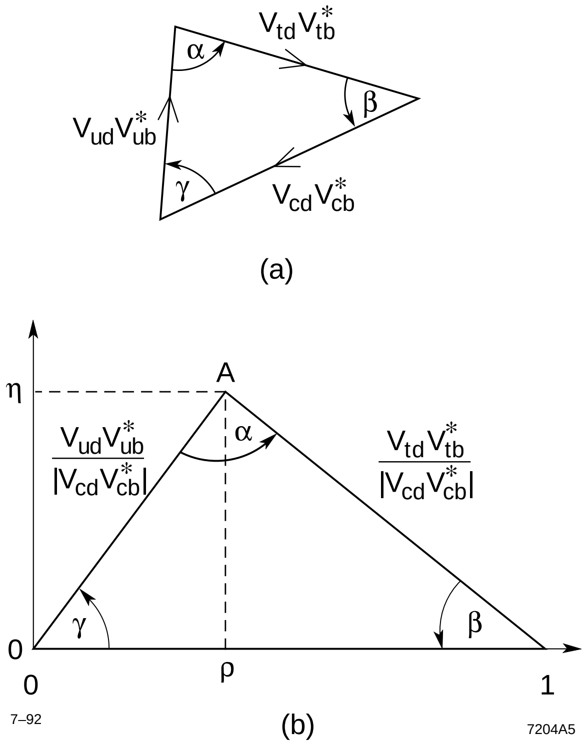

Figure 1-2. *The rescaled Unitarity Triangle, all sides divided by* $V_{cb}^* V_{cd}$.

The rescaled Unitarity Triangle (Fig. 1-2) is derived from (1.82) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$; (a) aligns one side of the triangle with the real axis, and (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled Unitarity Triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex are denoted by (ρ, η). It is customary these days to express the CKM-matrix in terms of four Wolfenstein parameters (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and η representing the CP-violating phase [27]:

$$V = \begin{pmatrix} 1 \Leftrightarrow \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho \Leftrightarrow i\eta) \\ \Leftrightarrow\lambda & 1 \Leftrightarrow \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 \Leftrightarrow \rho \Leftrightarrow i\eta) & \Leftrightarrow A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{1.83}$$

λ is small, and for each element in V, the expansion parameter is actually λ^2. Hence it is sufficient to keep only the first few terms in this expansion. The relation between the parameters of (1.78) and (1.83) is given by

$$s_{12} \equiv \lambda, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho \Leftrightarrow i\eta). \tag{1.84}$$

This specifies the higher order terms in (1.83).

The definition of (λ, A, ρ, η) given in (1.84) is useful because it allows an elegant improvement of the accuracy of the original Wolfenstein parameterization. In particular, up to O(λ^6) corrections,

$$V_{us} = \lambda, \quad V_{cb} = A\lambda^2, \quad V_{ub} = A\lambda^3(\rho - i\eta), \tag{1.85}$$

$$V_{td} = A\lambda^3(1 - \overline{\rho} - i\overline{\eta}), \tag{1.86}$$

$$\mathcal{I}m\, V_{cd} = -A^2\lambda^5\eta, \quad \mathcal{I}m\, V_{ts} = -A\lambda^4\eta, \tag{1.87}$$

where

$$\overline{\rho} = \rho(1 - \lambda^2/2), \quad \overline{\eta} = \eta(1 - \lambda^2/2). \tag{1.88}$$

These are excellent approximations to the exact expressions [28]. Depicting the rescaled Unitarity Triangle in the $(\overline{\rho}, \overline{\eta})$ plane, the lengths of the two complex sides are

$$R_b \equiv \sqrt{\overline{\rho}^2 + \overline{\eta}^2} = \frac{1 - \lambda^2/2}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|, \quad R_t \equiv \sqrt{(1 - \overline{\rho})^2 + \overline{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{1.89}$$

The three angles of the Unitarity Triangle are denoted by α, β and γ [29]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \tag{1.90}$$

The third angle is then

$$\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] \equiv \pi - \alpha - \beta. \tag{1.91}$$

These are physical quantities and, as discussed below, can be measured by CP asymmetries in various B decays. The consistency of the various measurements provide tests of the Standard Model.

The angle β gives, to a good approximation, the Standard Model phase between the neutral B mixing amplitude and its leading decay amplitudes. It is interesting to define the analog phases for the B_s meson, β_s, and the K meson, β_K:

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{1.92}$$

The angles β_s and β_K can be seen to be the small angles of the second and first unitarity triangles, (1.81) and (1.80), respectively.

It is straightforward to express the angles of the triangle in terms of $\overline{\rho}$ and $\overline{\eta}$. For example, the following two relations are useful:

$$\sin 2\alpha = \frac{2\overline{\eta}[\overline{\eta}^2 + \overline{\rho}(\overline{\rho} - 1)]}{[\overline{\eta}^2 + (1 - \overline{\rho})^2][\overline{\eta}^2 + \overline{\rho}^2]}, \quad \sin 2\beta = \frac{2\overline{\eta}(1 - \overline{\rho})}{\overline{\eta}^2 + (1 - \overline{\rho})^2}. \tag{1.93}$$

Note that unitarity is a fundamental property of any field theory. When one speaks of testing the unitarity of the CKM matrix one is not looking for violations of unitarity, but for violations of the consequences of unitarity in the three generation theory. Such violations would simply imply the presence of other channels, particles not included in the Standard Model theory, contributing in some way to the decays under study. To call these effects "unitarity violations" is perhaps misleading, but it is the common terminology of the field.

1.4.3 Measuring CKM Parameters with *CP*-Conserving Processes

Six of the nine absolute values of the CKM entries are measured directly, namely by tree-level processes. (All numbers below are taken from [12].) Nuclear beta decays give

$$|V_{ud}| = 0.9736 \pm 0.0010. \tag{1.94}$$

Semileptonic kaon and hyperon decays give

$$|V_{us}| = 0.2205 \pm 0.0018. \tag{1.95}$$

Neutrino and antineutrino production of charm off valence d quarks give

$$|V_{cd}| = 0.224 \pm 0.016. \tag{1.96}$$

Semileptonic D decays give

$$|V_{cs}| = 1.01 \pm 0.18 \tag{1.97}$$

Semileptonic exclusive and inclusive B decays give

$$|V_{cb}| = 0.041 \pm 0.003. \tag{1.98}$$

The endpoint spectrum in semileptonic B decays gives

$$|V_{ub}/V_{cb}| = 0.08 \pm 0.02. \tag{1.99}$$

Using unitarity constraints, one can narrow some of the above ranges (most noticeably, that of $|V_{cs}|$) and put constraints on the top mixings $|V_{ti}|$. The full information on the absolute values of the CKM elements (as given by [12]) from both direct measurements and three generation unitarity is summarized by

$$|V| = \begin{pmatrix} 0.9745 \Leftrightarrow 0.9757 & 0.219 \Leftrightarrow 0.224 & 0.002 \Leftrightarrow 0.005 \\ 0.218 \Leftrightarrow 0.224 & 0.9736 \Leftrightarrow 0.9750 & 0.036 \Leftrightarrow 0.046 \\ 0.004 \Leftrightarrow 0.014 & 0.034 \Leftrightarrow 0.046 & 0.9989 \Leftrightarrow 0.9993 \end{pmatrix}. \tag{1.100}$$

Note that the only large uncertainties are in $|V_{ub}|$ and $|V_{td}|$. However, the two are related through (1.82). Thus, the unitarity triangle is a very convenient tool for presenting constraints from indirect measurements on the most poorly determined parameters.

The most useful CP conserving indirect measurement, namely a Standard Model loop level process, is mixing in the $B^0 \Leftrightarrow \overline{B}^0$ system. The experimental result is

$$x_d \equiv \frac{\Delta m_B}{\Gamma_{B^0}} = 0.73 \pm 0.05. \tag{1.101}$$

Note that this value averages over measurements at the $\Upsilon(4S)$ of

$$\chi_d \equiv \frac{\Gamma(B^0 \to \ell^- X)}{\Gamma(B^0 \to \ell^\pm X)} = \frac{x_d^2}{2(1+x_d^2)}, \tag{1.102}$$

and measurements at the Z^0 of

$$\Delta m_B = x_d/\tau_{B^0}. \tag{1.103}$$

The Standard Model accounts for this quantity by the box diagrams with intermediate top quarks [30]:

$$M_{12} = \Leftrightarrow \frac{G_F^2}{12\pi^2} \eta m_B (B_B f_B^2) m_t^2 f_2(m_t^2/m_W^2)(V_{tb}V_{td}^*)^2 e^{-2i\xi_B}, \tag{1.104}$$

$$x_d = 2\tau_b |M_{12}|. \tag{1.105}$$

In Eq. (1.104) the quantity η is a QCD correction factor and $f_2(y)$ is a kinematic function calculated from the box diagrams. Both are positive quantities. Using [12] $B_B f_B^2 = (1.2 \pm 0.2)(173 \pm 40\ MeV)^2$ and $m_t = 174 \pm 16\ GeV$ as input, (1.105) gives

$$|V_{tb}^* V_{td}| = 0.009 \pm 0.003, \tag{1.106}$$

which significantly improves over the unitarity constraint (1.100).

The above ranges for the V_{ij}'s give the following 90% CL range for the CP-violating measure $|J|$:

$$|J| = (3.0 \pm 1.3) \times 10^{-5}\ \sin\delta. \tag{1.107}$$

1.5 Expected *CP* Asymmetries — Standard Model Predictions

1.5.1 *CP* Violation in Mixing

As mentioned above, in the B_d system the result $\Gamma_{12} \ll M_{12}$ is model independent. Moreover, within the Standard Model and assuming that the box diagram (with a cut) is appropriate to estimate Γ_{12}, one can actually calculate the two quantities from the quark diagrams of Fig. 1-3. The calculation gives [10]

$$\frac{\Gamma_{12}}{M_{12}} = \Leftrightarrow \frac{3\pi}{2} \frac{1}{f_2(m_t^2/m_W^2)} \frac{m_b^2}{m_t^2} \left(1 + \frac{8}{3} \frac{m_c^2}{m_b^2} \frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}\right). \tag{1.108}$$

This confirms the order of magnitude estimate, $|\Gamma_{12}/M_{12}| \lesssim 10^{-2}$. The deviation of $|q/p|$ from unity is proportional to $\mathcal{I}m(\Gamma_{12}/M_{12})$ which is even further suppressed by another order of magnitude. Thus, to a very good approximation,

$$\frac{q}{p} = \Leftrightarrow \frac{M_{12}^*}{|M_{12}|} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} e^{2i\xi_B} . \qquad (1.109)$$

Note that (1.108) allows an estimate of CP violation in mixing, namely

$$1 \Leftrightarrow \left|\frac{q}{p}\right| = \frac{1}{2}\mathcal{I}m\,\frac{\Gamma_{12}}{M_{12}} = \frac{4\pi}{f_2(m_t^2/m_W^2)}\frac{m_c^2}{m_t^2}\frac{J}{|V_{tb}V_{td}^*|^2} \sim 10^{-3}. \qquad (1.110)$$

The last term is the ratio of the area of the Unitarity Triangle to the length of one of its sides squared, so it is $\mathcal{O}(1)$. The only suppression factor is then (m_c^2/m_t^2). The uncertainty in the calculation comes from the use of a quark diagram to describe Γ_{12} and could easily be of order 30%, but not three orders of magnitude. (A similar expression to (1.109) holds for B_s, except that the last terms is $J/|V_{tb}V_{ts}^*|^2 \sim 10^{-2}$, as can be seen from the relevant unitarity triangle in Fig. 1-1.)

1.5.2 Decay-Amplitude Weak-Phase Structure

Most channels have contributions from both tree and three types of penguin diagrams [31]. The latter are classified according to the identity of the quark in the loop, as diagrams with different intermediate quarks may have both different strong phases and different weak phases. On the other hand, the subdivision of tree processes into spectator, exchange, and annihilation diagrams is unimportant in this respect since they all carry the same weak phase. In addition to gluonic penguins there are also electroweak penguin contributions, with a photon or Z boson. In certain cases the latter contribution can be significant because it is enhanced by a factor M_t^2/M_Z^2 which partially compensates the relative suppression of electroweak versus QCD couplings.

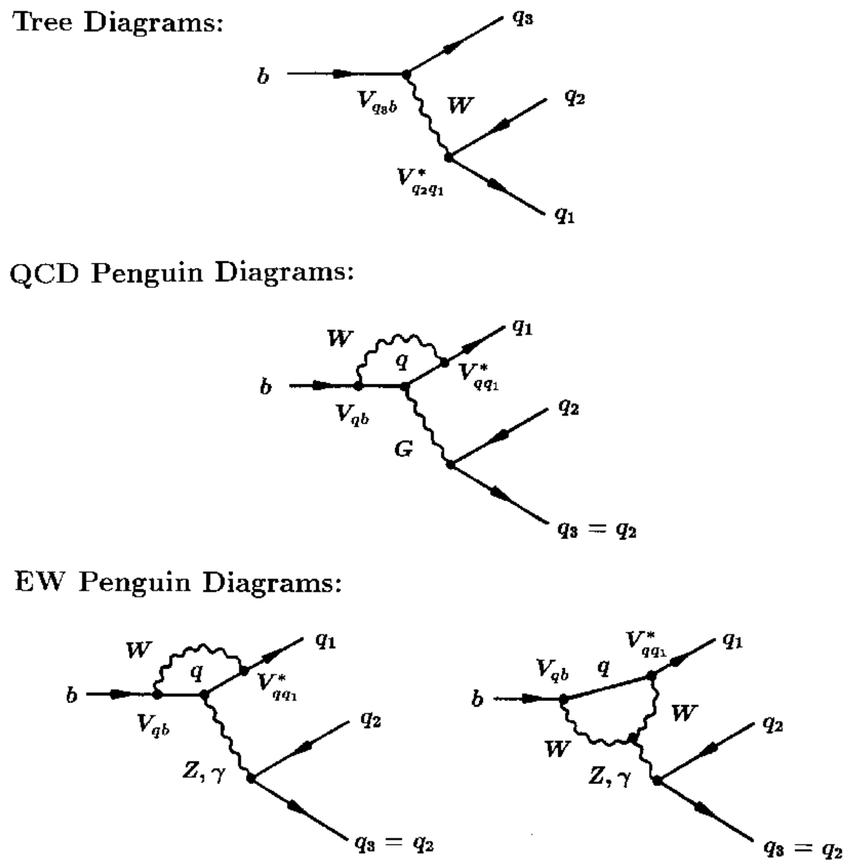

Figure 1-3. *Quark diagrams contributing to b decays.*

Figure 1-3 shows the quark diagrams for tree, penguin and electroweak penguin contributions. While quark diagrams can be easily classified in this way, the description of B decays is not so neatly divided into tree and penguin contributions once long distance physics effects are taken into account. Rescattering processes can change the quark content of the final state and confuse the identification of a contribution. There is no physical distinction between rescattered tree diagrams and long-distance contributions to the cuts of a penguin diagram. While these issues complicate estimates of various rates they can always be avoided in describing the weak-phase structure of B-decay amplitudes. The decay amplitudes for $b \to q\overline{q}q'$ can always be written as a sum of three terms with definite CKM coefficients:

$$A(q\overline{q}q') = V_{tb}V^*_{tq'}P^t_{q'} + V_{cb}V^*_{cq'}(T_{c\overline{c}q'}\delta_{qc} + P^c_{q'}) + V_{ub}V^*_{uq'}(T_{u\overline{u}q'}\delta_{qu} + P^u_{q'}). \tag{1.111}$$

Here P and T denote contributions from tree and penguin diagrams, excluding the CKM factors. As they stand, the P terms are not well defined because of the divergences of the penguin diagrams. Only differences of penguin diagrams are finite and well defined. (However, as will be seen, introduction of a common high momentum cut off in the loop diagrams does not affect the final answer, since it depends only on differences of penguin amplitudes. This can be seen by using Eqs. (1.81) and (1.82) to eliminate one of the three terms, by writing its CKM coefficient as minus the sum of the other two.

In the case of $q\overline{q}s$ decays it is convenient to remove the $V_{tb}V_{ts}^*$ term. Then

$$\begin{aligned} A(c\overline{c}s) &= V_{cb}V_{cs}^*(T_{c\overline{c}s} + P_s^c \Leftrightarrow P_s^t) + V_{ub}V_{us}^*(P_s^u \Leftrightarrow P_s^t), \\ A(u\overline{u}s) &= V_{cb}V_{cs}^*(P_s^c \Leftrightarrow P_s^t) + V_{ub}V_{us}^*(T_{u\overline{u}s} + P_s^u \Leftrightarrow P_s^t), \\ A(s\overline{s}s) &= V_{cb}V_{cs}^*(P_s^c \Leftrightarrow P_s^t) + V_{ub}V_{us}^*(P_s^u \Leftrightarrow P_s^t). \end{aligned} \tag{1.112}$$

In these expressions only differences of penguin contributions occur, which makes the cancelation of the ultraviolet divergences of these diagrams explicit. Further, the second term has a CKM coefficient that is much smaller than the first. Hence this grouping is useful in classifying the expected direct CP violations. (Note that terms $b \to d\overline{d}s$, which have only penguin contributions, mix strongly with the $u\overline{u}s$ terms and hence cannot be separated from them. Thus P terms in $A(u\overline{u}s)$ include contributions from both $d\overline{d}s$ and $u\overline{u}s$ diagrams.)

In the case of $q\overline{q}d$ decays the three CKM coefficients are all of similar magnitude. The convention is then to retain the $V_{tb}V_{td}^*$ term because, in the Standard Model, the phase difference between this weak phase and half the mixing weak phase is zero. Thus only one unknown weak phase enters the calculation of the interference between decays with and without mixing. One can choose to eliminate whichever of the other terms does not have a tree contribution. In the cases $q = s$ or d, since neither has a tree contribution either term can be removed. Thus the amplitudes can be written

$$\begin{aligned} A(c\overline{c}d) &= V_{tb}V_{td}^*(P_d^t \Leftrightarrow P_d^u) + V_{cb}V_{cd}^*(T_{c\overline{c}d} + P_d^c \Leftrightarrow P_d^u), \\ A(u\overline{u}d) &= V_{tb}V_{td}^*(P_d^t \Leftrightarrow P_d^c) + V_{ub}V_{ud}^*(T_{u\overline{u}d} + P_d^u \Leftrightarrow P_d^c), \\ A(s\overline{s}d) &= V_{tb}V_{td}^*(P_d^t \Leftrightarrow P_d^u) + V_{cb}V_{cd}^*(P_d^c \Leftrightarrow P_d^u). \end{aligned} \tag{1.113}$$

Again only differences of penguin amplitudes occur. Furthermore the difference of penguin terms that occurs in the second term would vanish if the charm and up quark masses were equal, and thus is GIM (Glashow-Illiopoulos-Maiani) suppressed. However, particularly for in modes with no tree contribution, $(s\overline{s}d)$, the interference of the two terms can still give significant direct CP violation, and thus complicate the simple predictions for the interference of decays with and without mixing [32] obtained by ignoring this term.

The penguin processes all involve the emission of a neutral boson, either a gluon (strong penguins) or a photon or Z boson (electroweak penguins). Excluding the CKM coefficients, the ratio of the contribution from the difference between a top and light quark strong penguin diagram to the contribution from a tree diagram is of order

$$r_{PT} = \frac{P^t \Leftrightarrow P^{\mathrm{light}}}{T_{q\overline{q}q'}} \approx \frac{\alpha_s}{12\pi} \ln \frac{m_t^2}{m_b^2}. \tag{1.114}$$

This is a factor of $\mathcal{O}(0.03)$. However this estimate does not include the effect of hadronic matrix elements, which are the probability factor to produce a particular final state particle content from a particular quark content. Since this probability differs for different kinematics, color flow, and spin structures, it can be different for tree and penguin contributions and may partially compensate the coupling constant suppression of the penguin term. Electroweak penguin difference terms are even more suppressed since they have an additional α_{em}/π or α_w/π compared to tree diagrams, but certain Z-contributions are enhanced by the large top quark mass and so can be non-negligible [33].

1.5.3 Low-Energy Effective Hamiltonians

The most efficient tool to analyze B decays is that of the low-energy effective Hamiltonian. The meaning and use of this tool is discussed further in the following chapter. Here the conventional notations used for the B decay Hamiltonian are simply noted. This section is based on Ref. [34], where a more detailed discussion can be found.

Low-energy effective Hamiltonians are constructed using the operator product expansion (OPE) which yields transition matrix elements of the structure

$$\langle f|\mathcal{H}_{\mathrm{eff}}|i\rangle \propto \sum_k \langle f|Q_k(\mu)|i\rangle\, C_k(\mu), \tag{1.115}$$

where μ denotes an appropriate renormalization scale. The OPE allows one to separate the "long-distance" contributions to that decay amplitude from the "short-distance" parts. Whereas the former pieces are not calculable and are relegated to the nonperturbative hadronic matrix elements $\langle f|Q_k(\mu)|i\rangle$, the latter are described by perturbatively calculable Wilson coefficient functions $C_k(\mu)$.

In the case of $|\Delta B| = 1$, $\Delta C = \Delta U = 0$ transitions one finds

$$\mathcal{H}_{\mathrm{eff}} = \mathcal{H}_{\mathrm{eff}}(\Delta B = \Leftrightarrow 1) + \mathcal{H}_{\mathrm{eff}}(\Delta B = \Leftrightarrow 1)^\dagger \tag{1.116}$$

with

$$\mathcal{H}_{\mathrm{eff}}(\Delta B = \Leftrightarrow 1) = \frac{G_{\mathrm{F}}}{\sqrt{2}} \left[\sum_{j=u,c} V_{jq}^* V_{jb} \left\{ \sum_{k=1}^{2} Q_k^{jq}\, C_k(\mu) + \sum_{k=3}^{10} Q_k^q\, C_k(\mu) \right\} \right]. \tag{1.117}$$

Here G_{F} denotes the Fermi constant, the renormalization scale μ is of $\mathcal{O}(m_b)$, the flavor label $q\,\mathrm{in}\{\mathrm{d},\mathrm{s}\}$ corresponds to $b \to d$ and $b \to s$ transitions, respectively, and Q_k^{jq} are four-quark operators that can be divided into three categories:

(i) current-current operators:

$$\begin{aligned} Q_1^{jq} &= (\overline{q}_\alpha j_\beta)_{\text{V–A}}(\overline{j}_\beta b_\alpha)_{\text{V–A}} \\ Q_2^{jq} &= (\overline{q}_\alpha j_\alpha)_{\text{V–A}}(\overline{j}_\beta b_\beta)_{\text{V–A}}. \end{aligned} \tag{1.118}$$

(ii) QCD penguin operators:

$$\begin{aligned} Q_3^{q} &= (\overline{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} (\overline{q}'_\beta q'_\beta)_{\text{V–A}} \\ Q_4^{q} &= (\overline{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} (\overline{q}'_\beta q'_\alpha)_{\text{V–A}} \\ Q_5^{q} &= (\overline{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} (\overline{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_6^{q} &= (\overline{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} (\overline{q}'_\beta q'_\alpha)_{\text{V+A}}. \end{aligned} \tag{1.119}$$

(iii) EW penguin operators:

$$\begin{aligned} Q_7^{q} &= \tfrac{3}{2}(\overline{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\overline{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_8^{q} &= \tfrac{3}{2}(\overline{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\overline{q}'_\beta q'_\alpha)_{\text{V+A}} \\ Q_9^{q} &= \tfrac{3}{2}(\overline{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\overline{q}'_\beta q'_\beta)_{\text{V–A}} \\ Q_{10}^{q} &= \tfrac{3}{2}(\overline{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\overline{q}'_\beta q'_\alpha)_{\text{V–A}}. \end{aligned} \tag{1.120}$$

Here α and β denote $SU(3)_{\text{C}}$ color indices, V±A refers to the Lorentz structures $\gamma_\mu(1 \pm \gamma_5)$, respectively, q' runs over the quark flavors active at the scale $\mu = \mathcal{O}(m_b)$, *i.e.*, $q' \,\text{in}\{\text{u}, \text{d}, \text{c}, \text{s}, \text{b}\}$, and $e_{q'}$ are the corresponding electrical quark charges. The current-current, QCD, and EW penguin operators are related to the tree, QCD, and EW penguin processes, depicted in Fig. 1-3.

In the case of transitions of the type $b \to q\overline{u}c$ and $b \to q\overline{c}u$ with $q \,\text{in}\{\text{d}, \text{s}\}$, only current-current operators contribute. The structure of the corresponding low-energy effective Hamiltonians is completely analogous to (1.117). To obtain it, one replaces both the CKM factors $V_{jq}^* V_{jb}$ and the flavor contents of the current-current operators (1.118) straightforwardly with the appropriate quark flavor structure, and omits the sum over penguin operators.

1.5.4 Decay Asymmetry Predictions in the Standard Model — General Patterns

As mentioned above, direct CP violations require two contributions to the decay process which differ in both their strong phases and their weak phases so that $|\overline{A}/A| \neq 1$. Purely leptonic and semileptonic decays are dominated by a single diagram and thus are unlikely to exhibit any measurable direct CP violation. Nonleptonic decays often have two terms that are comparable in magnitude and hence could have significant direct CP violations. The theoretical calculation of CP asymmetries of the type (1.50) requires knowledge of strong phase shifts and of absolute values of various amplitudes, as can be seen from (1.46). The estimates therefore necessarily have hadronic uncertainties. In contrast, a clean relationship between measured asymmetries and CKM phases is obtained when studying CP violation in the interference between decays with and without mixing for CP eigenstate modes dominated by a single term in the decay amplitude.

B decays can thus be grouped into five classes. Classes 1 and 2 are expected to have relatively small direct CP violations and hence are particularly interesting for extracting CKM parameters from interference of decays with and without mixing. In the remaining three classes, direct CP violations could be significant and the neutral decay asymmetries cannot be cleanly interpreted in terms of CKM phases.

1. Decays dominated by a single term: $b \to c\bar{c}s$ and $b \to s\bar{s}s$. The Standard Model cleanly predicts zero (or very small) direct CP violations because the second term is Cabibbo suppressed. Any observation of large direct CP-violating effects in these cases would be a clue to beyond Standard Model physics. The modes $B^+ \to \psi K^+$ and $B^+ \to \phi K^+$ are examples of this class. The corresponding neutral modes have cleanly predicted relationships between CKM parameters and the measured asymmetry from interference between decays with and without mixing.

2. Decays with a small second term: $b \to c\bar{c}d$ and $b \to u\bar{u}d$. The expectation that penguin-only contributions are suppressed compared to tree contributions suggests that these modes will have small direct CP violation effects, and an approximate prediction for the relationship between measured asymmetries in neutral decays and CKM phases can be made.

3. Decays with a suppressed tree contribution: $b \to u\bar{u}s$. The tree amplitude is suppressed by small mixing angles, $V_{ub}V_{us}^*$. The no-tree term may be comparable or even dominate and give large interference effects. An example is $B \to \rho K$.

4. Decays with no tree contribution: $b \to s\bar{s}d$. Here the interference comes from penguin contributions with different charge 2/3 quarks in the loop. An example is $B \to KK$.

5. Radiative decays: $b \to s\gamma$. The mechanism here is the same as in case 4 except that the leading contributions come from electromagnetic penguins. An example is $B \to K^*\gamma$.

Recent CLEO results on $\mathcal{B}(B \to K\pi)$ and $\mathcal{B}(B \to \pi\pi)$ [35] suggest that the matrix element of penguin operators is enhanced compared to that of tree operators. If this enhancement is significant, then some of the decay modes listed in Class 2 might actually fit better to Class 3; that is it becomes more difficult to relate a measured asymmetry to a CKM phase. For example, it is possible that $b \to u\bar{u}d$ decays have comparable contributions from tree and penguin amplitudes. On the other hand, this would also mean that some modes listed in Class 3 could be dominated by a single penguin term. For such cases an approximate relationship between measured asymmetries in neutral decays and CKM phases can be made. This is discussed in greater detail in Chapter 5.

It is useful to summarize all this discussion in two tables. The first table shows all cases for $b \to q\overline{q}'s$ while the second gives $b \to q\overline{q}'d$.

The last line in the first table of Table 1-1 are methods to measure the angle γ in $B_d \to DK$ decays. These modes have no penguin contributions, but can have direct CP violation due to interference of D^0 and $\overline{D}^0$ in decays to common final states. D decays to c-flavor-distinguishing states are then used to measure individual amplitude strengths. Thus the value of γ can be extracted, up to a fourfold ambiguity, via these modes if rates are high enough to make the relevant measurements accurately. These methods are discussed in more detail in Chapter 7. Again in the second table of Table 1-1 the final entry refers to direct CP violation studies in $B_d \to D\pi$ or $B_s \to DK$ decays through interference of common D^0 and $\overline{D}^0$ channels [36, 37]. Here one of the B-decay amplitudes is doubly Cabibbo suppressed, so the only hope for large interference effects is in a channel which is a doubly Cabibbo-suppressed decay of the other D state. Rates will be small, but the direct CP violation could be a large effect. Tagging via the second B is necessary to identify b-flavor. Charged $B \to D\pi$ can be similarly studied (with no tagging needed) [38].

1.5.5 Decay Asymmetry Predictions in the Standard Model — Some Sample Modes

The decay $B \to \psi K_S$ is an example of Class 1. A new ingredient in the analysis is the effect of $K \Leftrightarrow \overline{K}$ mixing. For decays with a single K_S in the final state, $K \Leftrightarrow \overline{K}$ mixing is essential because $B^0 \to K^0$ and $\overline{B}^0 \to \overline{K}^0$, and interference is possible only due to $K \Leftrightarrow \overline{K}$ mixing. This adds a factor of

$$\left(\frac{p}{q}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} e^{-2i\xi_K} \tag{1.121}$$

into $(\overline{A}/A)$. The quark subprocess in $\overline{B}^0 \to \psi\overline{K}^0$ is $b \to c\overline{c}s$ which is dominated by the W-mediated tree diagram:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = \eta_{\psi K_S} \left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right) \left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right) e^{-2i\xi_B}. \tag{1.122}$$

The CP eigenvalue of the state is $\eta_{\psi K_S} = \Leftrightarrow 1$. Combining (1.109) and (1.122), one finds

$$\lambda(B \to \psi K_S) = \Leftrightarrow \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right) \left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right) \left(\frac{V_{cs}^*V_{cb}}{V_{cs}V_{cb}^*}\right) \Longrightarrow \mathcal{I}m\,\lambda_{\psi K_S} = \sin(2\beta). \tag{1.123}$$

The second term in (1.112) is of order $r_T \sin^2\theta_C$ for this decay and thus Eq. (1.123) is clean of hadronic uncertainties to $\mathcal{O}(10^{-3})$. This measurement will thus give the theoretically cleanest

$B \to q\bar{q}s$ Decay Modes

Quark Process	Leading Term	Secondary Term	Sample B_d Modes	B_d Angle	Sample B_s Modes	B_s Angle
$b \to c\bar{c}s$	$V_{cb}V_{cs}^* = A\lambda^2$ tree + penguin $(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only $(u-t)$	$J/\psi\ K_S$	β	$\psi\eta'$ $D_s\overline{D}_s$	β_S
$b \to s\bar{s}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only $(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only $(u-t)$	ϕK_S	β	$\phi\eta'$	0
$b \to u\bar{u}s$ $b \to d\bar{d}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only $(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ tree + penguin $(u-t)$	$\pi^0\ K_S$ ρK_S	competing terms	$\phi\pi^0$ $K_S K_S$	competing terms
$b \to c\bar{u}s$ $b \to u\bar{c}s$	$V_{cb}V_{us}^* = A\lambda^3$ $V_{ub}V_{cs}^* = A\lambda^3(\rho - i\eta)$	0	$D^0K \searrow$ common $\overline{D}^0K \nearrow$ modes	γ	$D^0\phi \searrow$ common $\overline{D}^0\phi \nearrow$ modes	γ

$b \to q\bar{q}d$ Decay Modes

Quark Process	Leading Term	Secondary Term	Sample B_d Modes	B_d Angle * (leading terms only)	Sample B_s Modes	B_s Angle * (leading term)
$b \to c\bar{c}d$	$V_{cb}V_{cd}^* = -A\lambda^3$ tree + penguin $(c-u)$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only $(t-u)$	D^+D^-	$^*\beta$	ψK_S	β_S
$b \to s\bar{s}d$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only $(t-u)$	$V_{cb}V_{cd}^* = A\lambda^3$ penguin only $(c-u)$	$\phi\pi$ $K_S\overline{K}_S$	competing terms	ϕK_S	competing terms
$b \to u\bar{u}d$ $b \to d\bar{d}d$	$V_{ub}V_{ud}^* = A\lambda^3(\rho - i\eta)$ tree + penguin (uc)	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only $(t-c)$	$\pi\pi$; $\pi\rho$ πa_1	$^*\alpha$	π^0K_S ρ^0K_S	competing terms
$b \to c\bar{u}d$ $b \to u\bar{c}d$	$V_{cb}V_{ud}^* = A\lambda^2$ $V_{ub}V_{cd}^* = -A\lambda^4(\rho - i\eta)$	0	$D^0\pi^0 \searrow$ common $\overline{D}^0\pi^0 \nearrow$ modes	γ	$D^0K_S \searrow$ common $\overline{D}^0K_S \nearrow$ modes	γ

Table 1-1. *Decay modes for $b \to q\bar{q}'s$ and $b \to q\bar{q}'d$*

determination of a CKM parameter, even cleaner than the determination of $\sin\theta_C$ from $K \to \pi\ell\nu$. (If $\mathcal{B}(K_L \to \pi\nu\overline{\nu})$ were measured, it would give a comparably clean determination of $\sin\beta$ [39].)

The channel $B \to \psi K^*$ has a similar amplitude structure, but, since the two vector particles can have either even or odd relative angular momentum, the final state is not a pure CP eigenstate. The two different CP states can be separated by an analysis of the angular distribution of the decays [23]. This requires more data to get a comparable accuracy for $\sin 2\beta$, but on the other hand the branching ratio to this channel is somewhat higher and it appears to be dominated by a single CP eigenstate [40], so it may in fact give comparably accurate and equally clean results.

A second example of a theoretically clean mode in Class 1 is $B \to \phi K_S$. The quark subprocess involves flavor changing neutral current and cannot proceed via a tree-level Standard Model diagram. The leading contribution comes from penguin diagrams. The two terms in Eq. (1.112) are now both differences of penguins, but the second term is CKM suppressed and thus of order 0.02 compared to the first. Thus CP violation in the decay is at most a few percent and can be neglected in the analysis of asymmetries in this channel. The analysis is similar to the ψK_S case, and the asymmetry is proportional to $\sin(2\beta)$.

The same quark subprocesses give theoretically clean CP asymmetries also in B_s decays. The list of clean modes is given in Table 1-1.

The best known example of Class 2 is $B \to \pi\pi$. The quark subprocess is $b \to u\overline{u}d$ which is dominated by the W-mediated tree diagram. Neglecting for the moment the second, pure penguin, term in Eq. (1.113) one finds

$$\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = \eta_{\pi\pi}\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}e^{-2i\xi_B}. \tag{1.124}$$

The CP eigenvalue for two pions is $+1$. Combining (1.109) and (1.124), gives

$$\lambda(B \to \pi^+\pi^-) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{ud}^*V_{ub}}{V_{ud}V_{ub}^*}\right) \implies \mathcal{I}m\,\lambda_{\pi\pi} = \sin(2\alpha). \tag{1.125}$$

The pure penguin term in Eq. (1.113) has a weak phase, $\arg(V_{td}^*V_{tb})$, different from the term with the tree contribution, so it modifies both $\mathcal{I}m\,\lambda$ and (if there are nontrivial strong phases) $|\lambda|$. Recent results from CLEO suggest that the $B \to K\pi$ rate is comparable to or larger than the $B \to \pi\pi$ rate. This in turn indicates that the penguin contribution to $B \to \pi\pi$ channel is significant, probably 10% or more. This then introduces CP violation in decay, unless the strong phases cancel (or are zero, as suggested by factorization arguments). The resulting hadronic uncertainty can be eliminated using isospin analysis [20]. This requires a measurement of the rates for the isospin-related channels $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$ as well as the corresponding CP conjugate processes. The rate for $\pi^0\pi^0$ is expected to be small and the measurement is difficult, but even an upper bound on this rate can be used to limit the magnitude of hadronic uncertainties.

Related but slightly more complicated channels with the same underlying quark structure are $B \to \rho^0\pi^0$ and $B \to a_1^0\pi^0$. Again an analysis involving the isospin-related channels can be used to help eliminate hadronic uncertainties from CP violations in the decays. Channels such as $\rho\rho$ and $a_1\rho$ could in principle also be studied, using angular analysis to determine the mixture of CP-even and CP-odd contributions.

The analysis of $B \to D^+D^-$ proceeds along very similar lines. The quark subprocess here is $b \to c\bar{c}d$, and so the tree contribution gives

$$\lambda(B \to D^+D^-) = \eta_{D^+D^-} \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{cd}^* V_{cb}}{V_{cd} V_{cb}^*}\right) \implies \mathcal{I}m\,\lambda_{DD} = \Leftrightarrow\sin(2\beta) \qquad (1.126)$$

since $\eta_{D^+D^-} = +1$. Again, there are hadronic uncertainties due to the pure penguin term in (1.113), but they are estimated to be small. (See, however, [41].)

Now consider Class 4 decays, for example the case $B \to \phi\pi^0$. Here both terms in (1.113) are significant, though the second is GIM suppressed; that is it would vanish if charm and up quark masses were equal. Neglecting this term early studies predict no CP asymmetry in this channel in the Standard Model. However it has been shown that the presence of the second term can introduce asymmetries that may be as large as 10% [32]. Hence this channel cannot readily be used to look for violations of Standard Model predictions, unless one can reliably bound the size of the penguin effects.

In all cases the above discussions have neglected the distinction between strong penguins and electroweak penguins. The CKM phase structure of both types of penguins is the same. The only place where this distinction becomes important is when an isospin argument is used to remove hadronic uncertainties due to penguin contributions. These arguments are based on the fact that gluons have isospin zero, and hence strong penguin processes have definite ΔI. Photons and Z-bosons on the other hand contribute to more than one ΔI transition and hence cannot be separated from tree terms by isospin analysis. In most cases electroweak penguins are small, typically no more than ten percent of the corresponding strong penguins and so their effects can safely be neglected. However in Cases 3 to 5, where tree contributions are small or absent, their effects may need to be considered. A full review of the role of electroweak penguins in B decays has been given by Fleischer [34].

1.5.6 Effects of Physics Beyond the Standard Model

A more detailed examination of the effects in a variety of theories beyond the Standard Model is given in Chapter 13 of this book and in various reviews [42]. Here only some very general observations are in order.

By now the Standard Model and its particle content are so well established that any future theory will certainly contain them. However extensions that go beyond the Standard Model inevitably introduce additional fields. Along with them there often come additional coupling constants and hence the possibility of additional CP-violating phases. Even if no new phases occur there can be changes in the relationship between various physical quantities and CKM matrix element magnitudes and phases. Effects of physics beyond the Standard Model can manifest themselves in two ways, as additional contributions to the mixing of B^0 and $\overline{B}^0$ states, and/or as additional contributions to some set of decays.

An additional contribution to the mixing would have two effects: a change in the relationship between x_d and $|V_{td}V_{tb}|$ which led to Eq. (1.106) and a change in the relationship between the phase of q/p and the phase of $V_{tb}V_{td}^*$. However, since all λ_f have a common factor q/p, it would not change the relative phases between various λ_f.

Additional contributions to the decays can only be unambiguously and model-independently observed in cases where an amplitude is dominated by a single weak-phase term in the Standard Model. Then such terms destroy the relationship between the asymmetry and a CKM matrix phase and so lead to inconsistencies. For example, various modes that have the same Standard Model asymmetry may actually give different asymmetries [43]. In cases where two competing terms with different weak phases occur in the Standard Model expression, any additional term, whatever its phase, can always be absorbed into these two terms, appearing simply as changes in their magnitudes. Since these magnitudes cannot as yet be calculated in a model-independent and reliable fashion, this makes it quite difficult to identify changes from the Standard Model in these cases. However by a systematic study of expected patterns and improved theoretical calculations of matrix elements, one may be able to identify the impact of contributions beyond the Standard Model in these cases as well.

1.6 Some Comments about the K System

This section briefly reviews the K system in order to understand (a) the similarities and differences between neutral K and neutral B mesons and (b) the implications of CP violation as measured in K decays for future measurements of B decays.

1.6.1 The Neutral K System

In marked difference from the neutral B mesons, the neutral K meson states differ significantly in their lifetimes:

$$\tau_S = (0.8927 \pm 0.0009) \times 10^{-10}\ \mathrm{s}, \quad \tau_L = (5.17 \pm 0.04) \times 10^{-8}\ \mathrm{s}, \tag{1.127}$$

where the sub-indices S and L stand for the short-lived and long-lived mass eigenstates, respectively. Indeed, for the K system it is more useful to define the eigenstates by the lifetimes,

$$|K_S\rangle \ = \ p|K^0\rangle + q|\overline{K}^0\rangle, \tag{1.128}$$

$$|K_L\rangle \ = \ p|K^0\rangle \Leftrightarrow q|\overline{K}^0\rangle, \tag{1.129}$$

namely $\Delta\Gamma_K \ < \ 0$ by definition. (The p, q coefficients are of course different in the B and K systems. The notation $(q/p)_K$ for the ratio in the K system is used wherever the distinction is necessary.) The mass difference in the K system is measured to be

$$\Delta m_K \equiv M_L \Leftrightarrow M_S = (3.491 \pm 0.009) \times 10^{-15}\ GeV. \tag{1.130}$$

Equations (1.127) and (1.130) provide a convenient empirical approximation:

$$\Delta\Gamma_K \approx \Leftrightarrow 2\Delta m_K, \tag{1.131}$$

which is quite different from the B_d system (1.27).

The calculation of $(q/p)_K$ according to (1.15) proceeds a little differently than for the B_d case. To understand the situation in the K system, it is useful to define a phase ϕ_{12} according to

$$\frac{M_{12}}{\Gamma_{12}} \equiv \Leftrightarrow \left|\frac{M_{12}}{\Gamma_{12}}\right| e^{i\phi_{12}}. \tag{1.132}$$

Since CP-violating effects in the K system are known to be small, $\phi_{12} \ll 1$, so that ϕ_{12} can be used as a small expansion parameter. To leading order in ϕ_{12}, Eqs. (1.13) and (1.14) give

$$\Delta m_K = 2|M_{12}|, \quad \Delta\Gamma_K = \Leftrightarrow 2|\Gamma_{12}|. \tag{1.133}$$

Consequently, Eq. (1.132) can be rewritten, to first order in ϕ_{12}, as

$$\frac{M_{12}}{\Gamma_{12}} = \frac{\Delta m_K}{\Delta\Gamma_K}(1 + i\phi_{12}). \tag{1.134}$$

In some arbitrary phase convention,

$$\Gamma_{12} = |\Gamma_{12}| e^{-2i\xi_K}. \tag{1.135}$$

Using (1.134) and (1.135), gives from (1.15):

$$\left(\frac{q}{p}\right)_K = e^{2i\xi_K}\left[1 \Leftrightarrow i\phi_{12}\frac{1 + i\frac{\Delta\Gamma_K}{2\Delta m_K}}{1 + \left(\frac{\Delta\Gamma_K}{2\Delta m_K}\right)^2}\right]. \tag{1.136}$$

Thus $(q/p)_K$ is, to a good approximation, a pure phase. Actually, (1.136) implies that in the CP limit ($\phi_{12} = 0$), the CP transformation law is $CP|K^0\rangle = e^{2i\xi_K}|\overline{K}^0\rangle$. The K_S and K_L states are CP eigenstates to $\mathcal{O}(\phi_{12}) \sim 10^{-3}$ approximation.

As a result of the large lifetime difference between the neutral kaons, kaon experiments can easily separate the mass eigenstates and investigate K_L and K_S decays independently. This is impossible in B experiments, so there one will follow the decays of $B^0_{\rm phys}(t)$ and $\overline{B}^0_{\rm phys}(t)$ instead.

To compare the effects of CP violation in mixing, note that Eqs. (1.133) and (1.28) imply that $|q/p|_K$ and $|q/p|_{B_d}$ are both very close to 1. CP violation in mixing is then small in both systems. However, while for the B_d system the reason for that is the small lifetime difference, in the K system the reason is the smallness of the relevant CP-violating phase.

Finally, consider CP violation in the interference of mixing and decay. This could give a theoretically clean observable, provided that the decay is dominated by a single weak phase or a single strong phase. It is not difficult to find K decays into final CP eigenstates where $|\overline{A}/A| = 1$ to a good approximation: for example, the $\Delta I = 1/2$ rule implies that $K \to \pi^0\pi^0$ and $K \to \pi^+\pi^-$ are both dominated by a single strong phase. The difference in width, Γ_{12}, is completely dominated by the two pion intermediate state and therefore

$$\arg(\Gamma_{12}) = \arg(A^*_{2\pi}\overline{A}_{2\pi}) = \arg(\overline{A}_{2\pi}/A_{2\pi}). \tag{1.137}$$

In the approximation that $(\overline{A}_{2\pi}/A_{2\pi})$ is a pure phase, thus

$$\frac{\overline{A}_{2\pi}}{A_{2\pi}} = \Leftrightarrow \frac{\Delta\Gamma_K}{2\Gamma^*_{12}} = e^{-2i\xi_K}. \tag{1.138}$$

(See (1.135) for the last equation.) However, Eq. (1.136) shows that in the approximation where q/p is a pure phase, it is given by $q/p = e^{2i\xi_K}$. Thus, the prediction for CP asymmetry in $K \to 2\pi$ which is clean of hadronic uncertainties is simply zero:

$$\lambda(K \to \pi\pi) = 1 \implies \mathcal{I}m\,\lambda_{\pi\pi} = 0. \tag{1.139}$$

It should hold (as it does!) to $\mathcal{O}(10^{-3})$. To learn something about CP violation it is necessary to go beyond this approximation and use

$$\frac{q}{p}\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = 1 \Leftrightarrow i\phi_{12}\frac{1 + i\frac{\Delta\Gamma_K}{2\Delta m_K}}{1 + \left(\frac{\Delta\Gamma_K}{2\Delta m_K}\right)^2}. \tag{1.140}$$

Thus a value of ϕ_{12} can be cleanly extracted from measurements of CP violation in $K \to \pi\pi$. However, the translation of ϕ_{12} into electroweak parameters requires the knowledge of either the long distance contribution to M_{12} or the matrix element of the relevant four quark operator between K^0 and $\overline{K}^0$ states. This introduces large hadronic uncertainties into the calculation.

1.6.2 Measuring *CP* Violation in the *K* System

CP violation was first (and so far only) measured in K decays [1]. A number of complementary measurements have been made. CP asymmetries in the semileptonic K decays,

$$\delta(\ell) = \frac{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) \Leftrightarrow \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) + \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}, \tag{1.141}$$

have been measured, giving

$$\delta(\mu) = (3.04 \pm 0.25) \times 10^{-3}, \quad \delta(e) = (3.33 \pm 0.14) \times 10^{-3}. \tag{1.142}$$

These asymmetries are manifestations of CP violation in mixing:

$$\delta = \frac{1 \Leftrightarrow |q/p|_K^2}{1 + |q/p|_K^2}, \tag{1.143}$$

hence the statement above that $|q/p|_K$ is very close to unity.

The asymmetries in the two-pion channels,

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \quad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}. \tag{1.144}$$

have been measured:

$$|\eta_{00}| = (2.275 \pm 0.019) \times 10^{-3}, \quad \phi_{00} = 43.5 \pm 1.0^o, \tag{1.145}$$

$$|\eta_{+-}| = (2.285 \pm 0.019) \times 10^{-3}, \quad \phi_{+-} = 43.7 \pm 0.6^o. \tag{1.146}$$

A straightforward evaluation gives

$$\eta_{00} = \frac{pA_{00} \Leftrightarrow q\overline{A}_{00}}{pA_{00} + q\overline{A}_{00}} = \frac{1 \Leftrightarrow \lambda_{00}}{1 + \lambda_{00}}, \quad \eta_{+-} = \frac{pA_{+-} \Leftrightarrow q\overline{A}_{+-}}{pA_{+-} + q\overline{A}_{+-}} = \frac{1 \Leftrightarrow \lambda_{+-}}{1 + \lambda_{+-}}. \tag{1.147}$$

As shown below, η_{00} and η_{+-} are affected by all three types of CP violation: $|q/p| \neq 1$ and $\mathcal{I}m\,\lambda \neq 0$ give $\mathcal{O}(10^{-3})$ effects, while $|\overline{A}/A| \neq 1$ gives an $\mathcal{O}(10^{-6})$ effect.

1.6.3 The ε_K and ε'_K Parameters

There is a possible contribution to (1.147) from direct CP violation. This is due to the fact that there are two isospin channels, leading to final $(2\pi)_{I=0}$ and $(2\pi)_{I=2}$ states:

$$\langle \pi^0\pi^0| = \sqrt{\frac{1}{3}}\langle(\pi\pi)_{I=0}| \Leftrightarrow \sqrt{\frac{2}{3}}\langle(\pi\pi)_{I=2}|, \tag{1.148}$$

$$\langle \pi^+\pi^-| = \sqrt{\frac{2}{3}}\langle(\pi\pi)_{I=0}| + \sqrt{\frac{1}{3}}\langle(\pi\pi)_{I=2}|. \tag{1.149}$$

However, the possible interference effects are small because (on top of the smallness of the relevant CP-violating phases) the final $I = 0$ state is dominant (this is the $\Delta I = 1/2$ rule). Isospin amplitudes can be defined by

$$A_I = \langle(\pi\pi)_I|H|K^0\rangle, \quad \overline{A}_I = \langle(\pi\pi)_I|H|\overline{K}^0\rangle. \tag{1.150}$$

Experimentally, $|A_2/A_0| \approx 1/20$. Instead of η_{00} and η_{+-} one defines two combinations, ε_K and ε'_K, in such a way that the possible direct CP-violating effects are isolated into ε'_K.

The definition of ε_K is

$$\varepsilon_K \equiv \frac{1}{3}(\eta_{00} + 2\eta_{+-}) = \frac{1 \Leftrightarrow \lambda_0}{1 + \lambda_0}, \tag{1.151}$$

where $\lambda_0 \equiv (q/p)_K(\overline{A}_0/A_0)$ and the equation holds to first order in A_2/A_0 (at zeroth order $\eta_{00} = \eta_{+-} = \varepsilon_K$). As, by definition, only one strong channel contributes to λ_0, there is indeed no direct CP violation in (1.151).

Note that ε_K is a manifestation of CP violation in both mixing and the interference between decays with and without mixing (to see this explicitly, examine Eqs. (1.136) and (1.140)):

$$\left|\frac{q}{p}\right|_K \Leftrightarrow 1 = \phi_{12}\frac{\frac{\Delta\Gamma_K}{2\Delta m_K}}{1 + \left(\frac{\Delta\Gamma_K}{2\Delta m_K}\right)^2}, \tag{1.152}$$

$$2\varepsilon_K \approx 1 \Leftrightarrow \left(\frac{q}{p}\right)_K \frac{\overline{A}_0}{A_0} = \phi_{12}\frac{i \Leftrightarrow \frac{\Delta\Gamma_K}{2\Delta m_K}}{1 + \left(\frac{\Delta\Gamma_K}{2\Delta m_K}\right)^2}. \tag{1.153}$$

As $\Delta\Gamma_K \approx \Leftrightarrow 2\Delta m_K$, the deviation of $|q/p|_K$ from unity (CP violation in mixing) and the deviation of $\mathcal{I}m[(q/p)_K(\overline{A}_0/A_0)]$ from zero (CP violation in the interference between decays with and without mixing) are both $\mathcal{O}(\phi_{12})$ and thus contribute to ε_K at the same order. One can interpret Eqs. (1.152) and (1.153) to imply that $\mathcal{R}e(\varepsilon_K)$ is a manifestation of CP violation in mixing while $\mathcal{I}m(\varepsilon_K)$ is a manifestation of CP violation in the interference between decays with and without mixing. As (1.153) predicts $\arg(\varepsilon_K) \approx \pi/4$, the magnitudes of the two phenomena are similar.

One can define ε'_K by

$$\varepsilon'_K \equiv \frac{1}{3}(\eta_{+-} \Leftrightarrow \eta_{00}) = \frac{2(\lambda_{00} \Leftrightarrow \lambda_{+-})}{3(1+\lambda_{00})(1+\lambda_{+-})} \approx \frac{1}{6}\frac{q}{p}\left(\frac{\overline{A}_{00}}{A_{00}} \Leftrightarrow \frac{\overline{A}_{+-}}{A_{+-}}\right), \tag{1.154}$$

where the last equality used (1.145) which gives $\lambda_{00} \approx \lambda_{+-} \approx 1$. One can further evaluate (1.154) in terms of A_0 and A_2 with the help of the relationships given in Eqs. (1.148) and (1.149). The approximations $(q/p)(\overline{A}_0/A_0) \approx 1$ and $|A_2/A_0| \ll 1$ give

$$\varepsilon'_K = \frac{i}{\sqrt{2}}\left|\frac{A_2}{A_0}\right| e^{i(\delta_2 - \delta_0)} \sin(\phi_2 \Leftrightarrow \phi_0). \tag{1.155}$$

In the derivation of (1.155), since it is a good approximation to replace q/p with a pure phase, one sees that there is no CP violation in mixing in ε'_K. Equations (1.154) and (1.155) imply that $\mathcal{R}e(\varepsilon'_K)$ is a manifestation of CP violation in decay while $\mathcal{I}m(\varepsilon'_K)$ is a manifestation of CP violation in the interference between decays with and without mixing. For recent experimental results, see [44].

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