

Density dependence of the symmetry energy from neutron skin thickness, parity violating electron scattering and electric dipole polarizability

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Together with

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M. Centelles, X. Roca-Maza, X. Viñas and M. Warda,
Phys. Rev. Lett. 102 122502 (2009), Phys. Rev. C82 054314 (2010)
M. Warda, X. Viñas, X. Roca-Maza and M. Centelles,
Phys. Rev. C80 024316 (2009) Phys. Rev. C81 054309 (2010)
X. Roca-Maza, M. Centelles, X. Viñas and M. Warda,
Phys. Rev. Lett. 106 252501 (2011)
X. Roca-Maza et al.
Phys. Rev. C88 024316 (2013), Phys. Rev. C92 064304 (2015)
X. Viñas, M. Centelles, X. Roca-Maza and M. Warda,
Eur. Phys.J A50 (2014) 27, AIP Conf. Proc. 1606 (2014) 256

Why is important the nuclear symmetry energy ?

The **nuclear symmetry energy** is a fundamental quantity in **Nuclear Physics** and **Astrophysics** because it governs, at the same time, important properties of very small entities like the atomic nucleus ($R \sim 10^{-15}$ m) and very large objects as neutron stars ($R \sim 10^4$ m)

- **Nuclear Physics:** Neutron skin thickness in finite nuclei, structure of neutron rich nuclei, Heavy-Ion collisions, Giant Resonances....
- **High-Energy Physics:** Test of the Standard Model through atomic parity non-conservation observables.
- **Astrophysics:** Supernova explosion, Neutron emission and cooling of protoneutron stars, Mass-Radius relations in neutron stars, Composition of the crust of neutron stars...

Equation of State in asymmetric matter

$$e(\rho, \delta) = e(\rho, 0) + c_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) \quad \left(\delta = \frac{\rho_n - \rho_p}{\rho} \right)$$

Around the saturation density we can write

$$e(\rho, 0) \simeq a_v + \frac{1}{2}K_v\epsilon^2 \quad \text{and} \quad c_{sym}(\rho) \simeq J - L\epsilon + \frac{1}{2}K_{sym}\epsilon^2 \quad \left(\epsilon = \frac{\rho_0 - \rho}{3\rho_0} \right)$$

$$\rho_0 \approx 0.16\text{fm}^{-3}, \quad a_v \approx -16\text{MeV}, \quad K_v \approx 230\text{MeV}, \quad J \approx 32\text{MeV}$$

However, the values of

$$L = 3\rho\partial c_{sym}(\rho)/\partial\rho|_{\rho_0} \quad \text{and} \quad K_{sym} = 9\rho^2\partial^2 c_{sym}(\rho)/\partial\rho^2|_{\rho_0}$$

which govern the density dependence of c_{sym} near ρ_0 are less certain and predictions vary largely among nuclear theories.

Constraints on the symmetry energy and its slope

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B.-A. Li, X. Han / Physics Letters B 727 (2013) 276–281

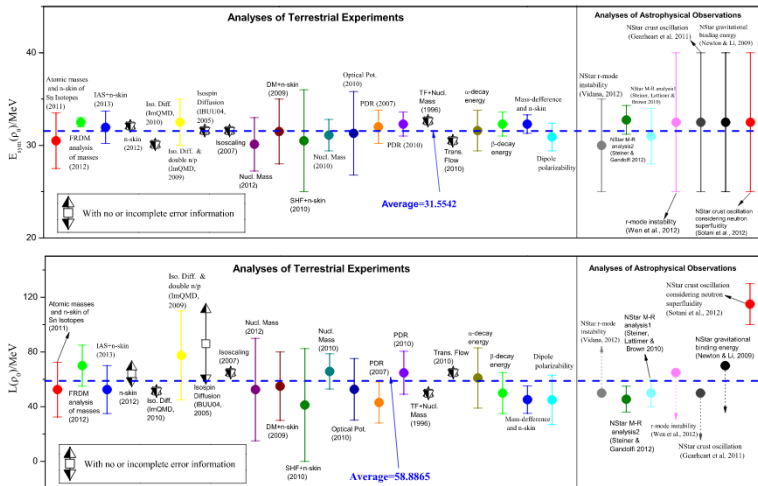


Fig. 1. (Color online.) Nuclear symmetry energy (upper) and its slope L (lower) at normal density of nuclear matter from 28 analyses of terrestrial nuclear laboratory experiments and astrophysical observations.

Constraints on the slope of the symmetry energy

χ Lagrangian and

Q. Montecarlo

Neutron-Star
Observations

p & α scattering
charge ex.

Antiprotonic
Atoms

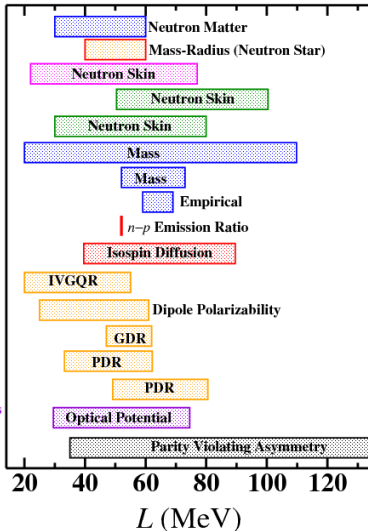
Nuclear
Model Fit

Heavy Ion
Collisions

Giant
Resonances

N - A scattering
Charge Ex. Reactions
Energy Levels

PVES



Keblers et al. PRL105 (2010) 161102

and *Gandolfi et al. PRC85 (2012) 032801 (R)*

Steiner et al. Astrophys. J. 722 (2010) 33

Lie-Wen Chen et al. PRC 82 (2010) 024321

Centelles et al. PRL 102 (2009) 122502

Warda et al. PRC 80 (2009) 024316

Kortelainen et al. PRC 82 (2010) 024313

Danielewicz NPA 727 (2003) 233

Agrawal et al. PRL109 (2012) 262501

Famiano et al. PRL 97 (2006) 052701

Tsang et al. PRL 103 (2009) 122701

Roca-Maza et al. PRC 87 (2013) 034301

Roca-Maza et al. in preparation (2013)

Trippa et al. PRC 77 (2008) 061304(R)

Klimkiewicz et al. PRC 76 (2007) 051603(R)

Carbone et al. PRC 81 (2010) 041301(R)

Xu et al. PRC 82 (2010) 054607

PREx Collab. PRL 108 112502 (2012)

Symmetry energy and neutron skin thickness in the Liquid Drop Model

- Symmetry Energy

$$a_{\text{sym}}(A) = \frac{J}{1 + x_A}, \quad x_A = \frac{9J}{4Q} A^{-1/3}$$

$$E_{\text{sym}}(A) = a_{\text{sym}}(A)(1 + x_A I_C)^2 A$$

where

$$I = (N - Z)/A, \quad I_C = e^2 Z / (20JR), \quad R = r_0 A^{1/3}$$

- Neutron skin thickness

$$S = \sqrt{3/5} \left[t - e^2 Z / (70J) + \frac{5}{2R} (b_n^2 - b_p^2) \right]$$

where

$$t = \frac{3r_0}{2} \frac{J/Q}{1 + x_A} (I - I_C) = \frac{2r_0}{3J} [J - a_{\text{sym}}(A)] A^{1/3} (I - I_C)$$

M. Centelles, M. Del Estal and X. Viñas, Nucl. Phys. **A635**, 193 (1998)

Neutron skin thickness

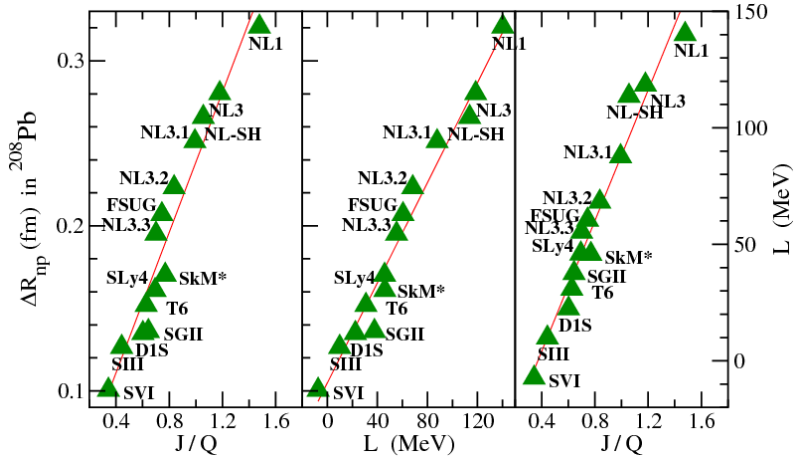


Table : Value of $a_{sym}(A)$ and density ρ that exactly fulfils $c_{sym}(\rho) = a_{sym}(A)$ for $A = 208, 116, 40$, in various nuclear models. J and a_{sym} are in MeV and ρ is in fm^{-3} .

Model	J	$A = 208$		$A = 116$		$A = 40$	
		a_{sym}	ρ	a_{sym}	ρ	a_{sym}	ρ
NL3	37.4	25.8	0.103	24.2	0.096	21.1	0.083
NL-SH	36.1	25.8	0.105	24.6	0.099	21.3	0.086
FSUGold	32.6	25.4	0.098	24.2	0.090	21.9	0.075
TF-MS	32.6	24.2	0.093	22.9	0.085	20.3	0.068
SLy4	32.0	25.3	0.100	24.2	0.091	22.0	0.075
SkX	31.1	25.7	0.102	24.8	0.096	22.8	0.082
SkM*	30.0	23.2	0.101	22.0	0.093	19.9	0.078
SIII	28.2	24.1	0.093	23.4	0.088	21.8	0.077
SGII	26.8	21.6	0.104	20.7	0.096	18.9	0.082

$$\rho \approx \rho_A = \rho_0 - \rho_0 / (1 + cA^{1/3}),$$

with c fixed by the condition $\rho_{208} = 0.1 \text{ fm}^{-3}$.

The $c_{sym}(\rho)$ - $a_{sym}(A)$ correlation

- There is a genuine relation between the symmetry energy coefficients of the EOS and of nuclei: $c_{sym}(\rho)$ equals $a_{sym}(A)$ of heavy nuclei like ^{208}Pb at a density $\rho = 0.1 \pm 0.01 \text{ fm}^{-3}$ practically independent of the mean field model used to compute them.
- A similar situation occurs down to medium mass numbers, at lower densities.
- We find that this density can be very well simulated by

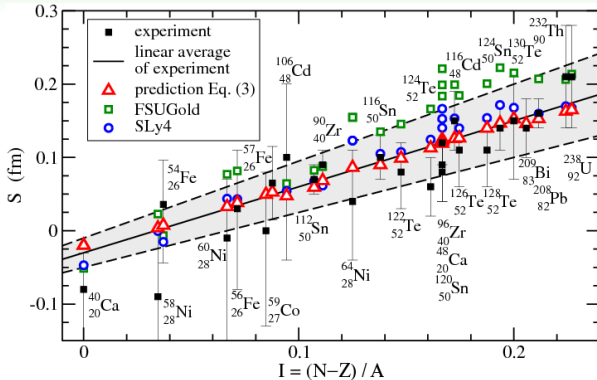
$$\rho \approx \rho_A = \rho_0 - \rho_0 / (1 + cA^{1/3}),$$

where c is fixed by the condition $\rho_{208} = 0.1 \text{ fm}^{-3}$.

- Using the equality $c_{sym}(\rho) = a_{sym}(A)$ and the LDM, the neutron skin thickness can be finally written as:

$$S = \sqrt{\frac{3}{5}} \frac{2r_0}{3} \frac{L}{J} \left(1 - \epsilon \frac{K_{sym}}{2L} \right) \epsilon A^{1/3} (I - I_C)$$

- See also Lie-Wen Chen Phys. Rev. **C83**, 044308 (2011)

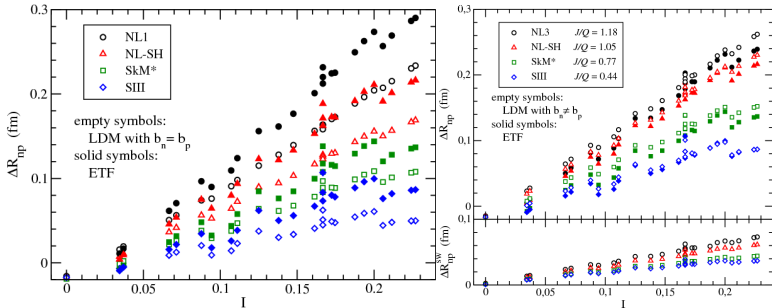


$$S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02) \text{ fm}$$

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

Assuming $c(\rho) = 31.6(\rho/\rho_0)^\gamma$ with $\rho_0 = 0.16 \text{ fm}^{-3}$ we predict
 $(b_n = b_p): L = 75 \pm 25 \text{ MeV}$

Influence of the surface width ($b_n \neq b_p$)



$$\sqrt{\frac{3}{5}} \frac{5}{2R} (b_n^2 - b_p^2) = 0.31I(\text{NL3}) - 0.15I(\text{SGII})$$

b_n and b_p are obtained semiclassically at ETF level

M.Centelles et al. NPA **635**, 193 (1998).

Surface contribution to the neutron skin thickness

$$\sqrt{\frac{3}{5}} \frac{5}{2R} (b_n^2 - b_p^2) = \sigma^{sw} l = \left(0.3 \frac{J}{Q} + c\right) l$$

$$c = 0.07 \text{ fm} \quad \text{and} \quad c = -0.05$$

Fit to experimental data

$$\frac{J}{Q} = 0.667 \pm 0.047 \quad c = 0.07$$

$$\frac{J}{Q} = 0.791 \pm 0.049 \quad c = -0.05$$

$$0.62 \leq \frac{J}{Q} \leq 0.84 \quad 30 \leq L \leq 80 \text{ MeV}$$

Conclusions (I)

- We have described a generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation.
- We take advantage of this relation to explore constraints on $c_{sym}(\rho)$ from neutron skins measured in antiprotonic atoms. These constraints points towards a **soft symmetry energy**.
- We discuss the L values constrained by neutron skins in comparison with most recent observations from reactions and giant resonances.
- We learn that in spite of present error bars in the data of antiprotonic atoms, the size of the final uncertainties in L is comparable to the other analyses.
- The fact that the symmetry energy takes a **similar value for a density around 0.11fm^{-3}** (average density in a a heavy nucleus) when computed with succesful mean-field models and **the correlations between its slope (L) and an observable in finite nuclei (neutron skin)** may be a more general situation relating nuclear matter properties with finite nuclei observables : see E.Khan et al Phys.Rev.Lett.**109**,092501 (2012).

What can we learn from parity-violating electron scattering ?

- See C.J. Horowitz et al, Phys. Rev. **C63**, 025501 (2001);
Shufang Ban et al, J.of Phys. **G39**,015104 (2012).

- A_{LR} is the parity-violating asymmetry

- $$A_{LR} \equiv \frac{\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}}{\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}}$$

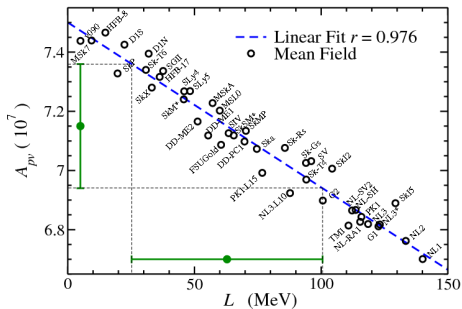
- $V_{\pm}(r) = V_{\text{Coulomb}}(r) \pm V_{\text{weak}}(r)$

- $V_{\text{weak}}(r) = \frac{G_F}{2^{3/2}} [(1 - 4 \sin^2 \theta_W) Z \rho_p(r) - N \rho_n(r)]$

- $$A_{LR}^{\text{PWBA}} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}} \left[4 \sin^2 \theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)} \right]$$

- PREX experiment $E \sim 1.05$ GeV and $\theta \sim 5^\circ$

From parity-violating electron scattering



Conclusions (II)

- We have investigated parity-violating electron scattering in nuclear models constrained by available experimental data to extract the neutron radius and skin of ^{208}Pb without specific assumptions on the shape of the nucleon densities.
- We have demonstrated a linear correlation, universal in mean field framework, between A_{pv} and Δr_{np} that has very small scatter.
- It is predicted that a 1% measurement of A_{pv} would allow to constrain the slope L of the symmetry energy to near a novel 10 MeV level.
- Experimental Results of PREX (9 % uncertainty):

$$A_{PV} = 0.665 \pm (0.060)_{stat} \pm (0.014)_{syst} ppm$$

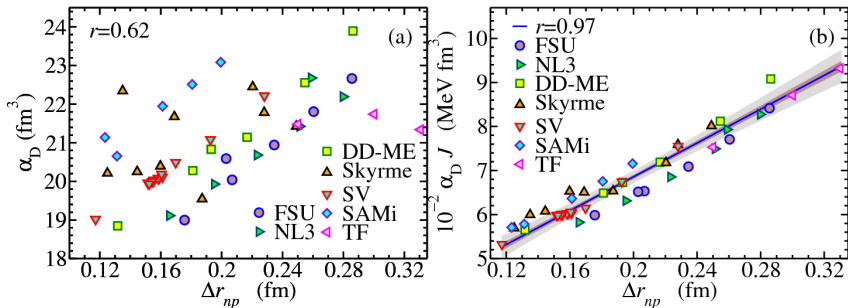
$$\Delta r_{np} = 0.302 \pm (0.175)_{expt} \pm (0.0626)_{theor} \pm (0.005)_{stange} fm$$

Dipole polarizability and symmetry energy

- Information about the symmetry energy and its density dependence is deduced by comparing available experimental data on electric dipole polarizability α_D in ^{208}Pb , ^{68}Ni and ^{120}Sn with the predictions of the RPA calculations using a representative set on nuclear energy density functionals.
- The hydrodynamical model of Lipparini and Stringari (Phys. Rep. **175**, 103 (1989) (see also W.Satula et al, Phys. Rev. **C74**, 011301 (2009)) suggest a relation between the dipole polarizability and the bulk and surface contributions to the symmetry energy:

$$\alpha_{hydro} = \frac{A \langle r^2 \rangle}{24J} \left(1 + \frac{5}{3} \frac{J - a_{sym}}{J} A^{-1/3} \right); \quad E_{-1} = \sqrt{\frac{\hbar^2 A(1 + \kappa)}{4m\alpha(D)}}$$

^{208}Pb



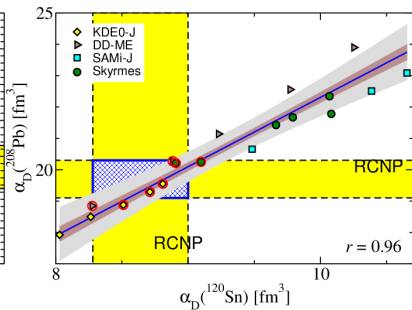
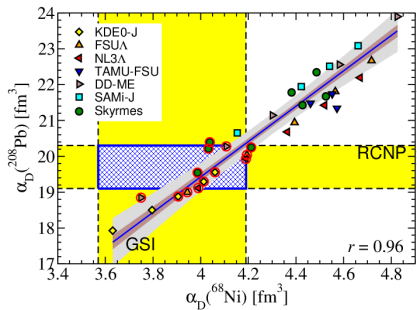
$$10^{-2} \times \alpha_D J = 3.01 \pm 0.32 + (19.22 \pm 0.73) \Delta r_{np}$$

$$\alpha_D(\text{measured}) = 20.1 \pm 0.6 \text{ fm}^3 \quad \alpha_D(\text{corrected}) = 19.6 \pm 0.6 \text{ fm}^3$$

Theory versus experiment

- 1p-1h RPA has been proven to be successful in describing E_x in many giant resonances.
- Experimental data of α_D analyzed via RPA need to include the full dipole response with low and high-energy contributions.
- If experimental data are only known in a given energy range, one may extrapolate them to high and low energies regions in order to compare with theoretical RPA calculations. The low energy part is more important than the high energy region
- Data in the high energy range shall be taken carefully due to the quasi-deuteron contributions not accounted in RPA, which produce small but sizeable corrections to α_D
- The experimental resonance width is not reproduced by the 1p-1h RPA calculations. The correction to that on the theoretical electric dipole polarizability can be estimated as $\Delta\alpha_D \lesssim -\alpha_D \frac{\Gamma^2}{4E_x^2}$

^{68}Ni and ^{120}Sn



$$\alpha_D(^{68}\text{Ni}) = 0.063 \pm 0.048 + (0.20 \pm 0.01)\alpha_D(^{208}\text{Pb})$$

$$\alpha_D(^{120}\text{Sn}) = 0.22 \pm 0.45 + (2.21 \pm 0.14)\alpha_D(^{208}\text{Pb})$$

Results

- Neutron skin thickness

^{208}Pb **0.187 - 0.125^a** **0.159 ± 0.028^b** fm

^{120}Sn **0.158 - 0.108^a** **0.122 ± 0.033^b** fm

^{68}Ni **0.193 - 0.146^a** **0.163 ± 0.034^b** fm

- Symmetry energy J and slope L at saturation

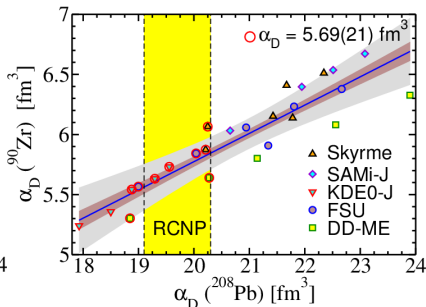
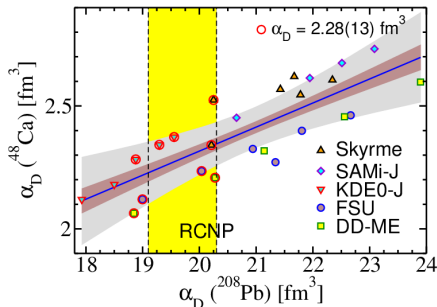
$$J = 35 - 28^a \quad L = 66 - 10^a \text{ MeV}$$

- Comments

a) From models that reproduce simultaneously the experimental polarizability in ^{208}Pb , ^{120}Sn and ^{68}Ni .

b) From the $\alpha_D J - \Delta r_{np}$ correlation with $J = 31 \pm 2$ MeV

^{48}Ca and ^{90}Zr



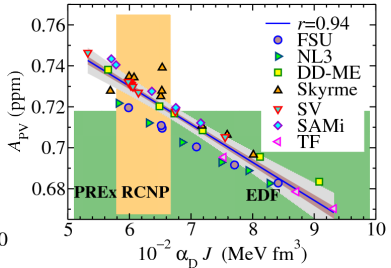
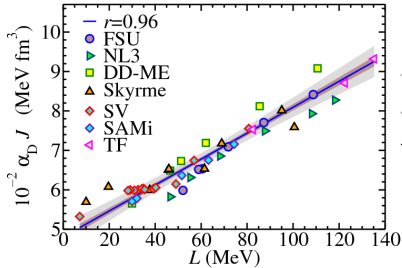
$$\alpha_D(^{48}\text{Ca}) = 0.36 \pm 0.06 + (0.098 \pm 0.013)\alpha_D(^{208}\text{Pb})$$

$$\alpha_D(^{90}\text{Zr}) = 1.07 \pm 0.10 + (0.23 \pm 0.02)\alpha_D(^{208}\text{Pb})$$

With the selected models

$$\alpha_D(^{48}\text{Ca}) = 2.28 \pm 0.13 \text{ fm}^3 \quad \alpha_D(^{90}\text{Zr}) = 5.69 \pm 0.21 \text{ fm}^3$$

Dipole polarizability + parity-violating electron scattering



$$L = -146 \pm (1)_{\text{theor}} + [6.11 \pm (0.18)_{\text{expt}} \pm (0.26)_{\text{theor}}] \times J$$

with $J = 31 \pm 2 \text{ MeV}$ $L = 43 \pm 16 \text{ MeV}$

Conclusions (III)

- We use insights from the Droplet Model to understand correlations between the electric polarizability, the neutron skin thickness and the properties of symmetry energy at saturation.
- The product $\alpha_D J$ is as far a better isovector indicator than α_D alone.
- Adopting $J = 31 \pm 2$ MeV one obtains $\Delta r_{np} \simeq 0.165$ fm for ^{208}Pb and $L \simeq 43$ MeV.
- A_{pv} is strongly correlated not only with Δr_{np} but also with $\alpha_D J$.