

Medium polarization effects in neutron-proton pairing

Outline:

- ❑ *The puzzle of the missing neutron-proton pairing in nuclei*
- ❑ *Self-energy effects on n-p (3SD_1) pairing energy gap*
- ❑ *Induced interaction theory (Landau limit)*
- ❑ *Vertex corrections in pairing interaction in 3SD_1 two-body channel*
- ❑ *Nuclear matter vs. finite nuclei*

The puzzle of neutron-proton pairing in nuclei

○ *The neglect of the neutron-proton interaction is the major weakness of the pairing force theory. This interaction is just as strong as that between a pair of like nucleons. In fact in the $T=0$ state is stronger!* (A.M. Lane, *Nuclear Theory*, Benjamin 1964)

Later this statement received support by numerical estimates of np pairing gap in nuclear matter with realistic interactions (8-12 MeV). (for a review see U. Lombardo, *Superfluidity in nuclear matter*, World Sci. 1999, Ed. M. Baldo)

Despite the numerical predictions by A. Goodman (PRC 60,1999), no clear evidence for n-p pairing has so far been found in nuclei.

Recently, first Bertsch et al. (PRC 2010) and later Sagawa et al. (Phys.Scr. 2016) studied the competition between spin triplet and spin singlet pair correlations in nuclei. Bertsch predicted a transition from spin singlet (nn) to spin triplet (np) pairing in large $N=Z$ nuclei ($A=130-140$).

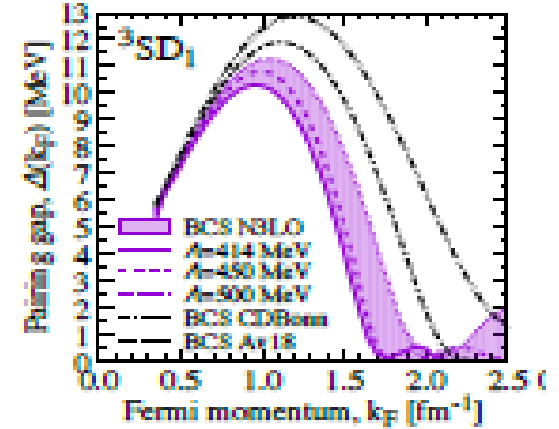
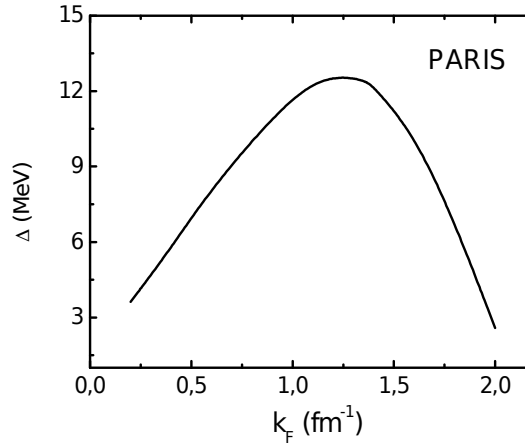
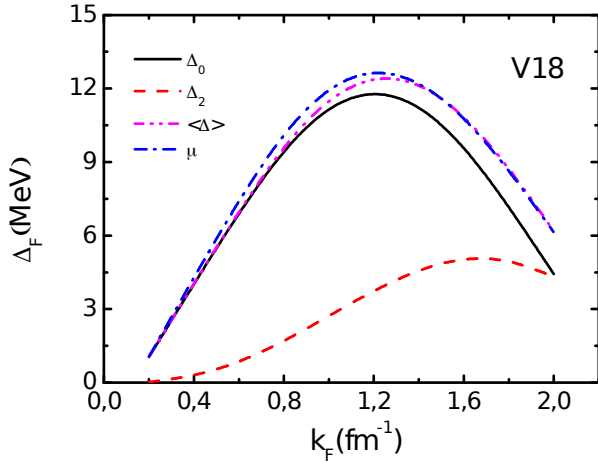
But the suppression of the np pairing in nuclei has not yet been completely understood. Possible candidates are many body effects, including self-energy and core polarization.

It is already established the role of self-energy effects (effective mass and quasi-particle strength).

The effects of core polarization have been predicted for nn and pp pairing ($S=0, T=1$) within the induced interaction theory either in nuclei (Milano group) and in nuclear

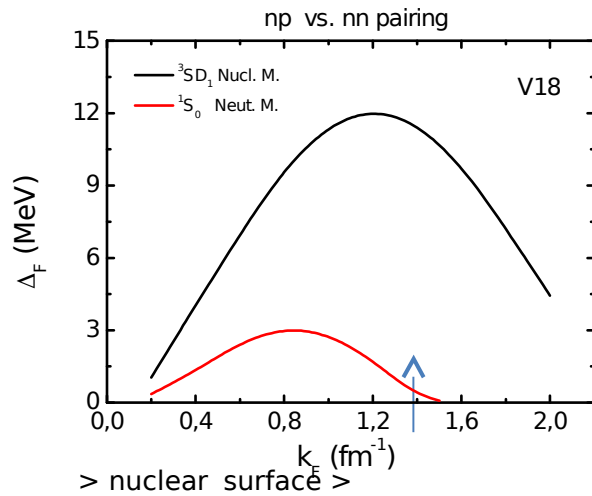
BARE FORCE

np pairing in SD-channel (spin-triplet)



Rios et al [arXiv:1707.04144](https://arxiv.org/abs/1707.04144)

Comparison with nn pairing (spin-singlet)



free sp spectrum

Collaboration

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- Schulze H.-J. : INFN (sez . Catania)
- Schuck P. : U. Paris Sud (IPN, Orsay)

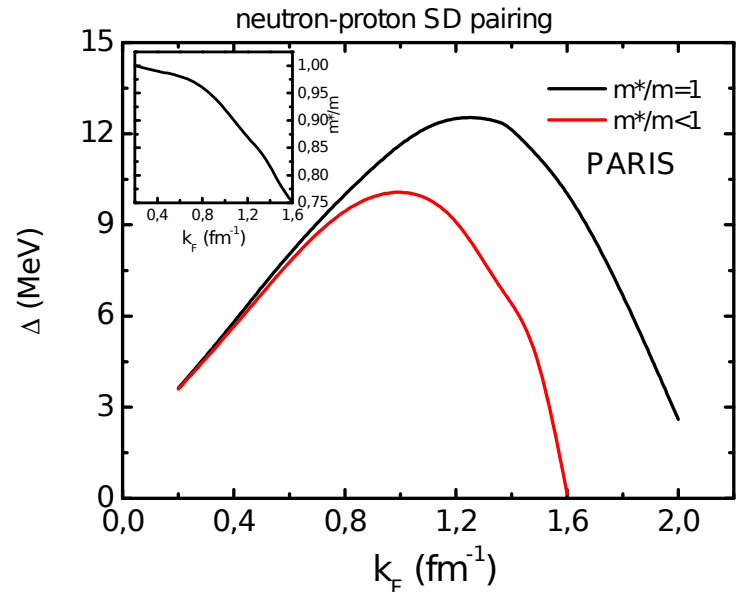
Gap Eqs with self-energy corrections

BCS model for coupled channels (SD pairing):

$$\Delta_L(k) = \sum_{L'} \int V_{LL'}(k, k') \frac{1}{2E(k')} \Delta_{L'}(k')$$

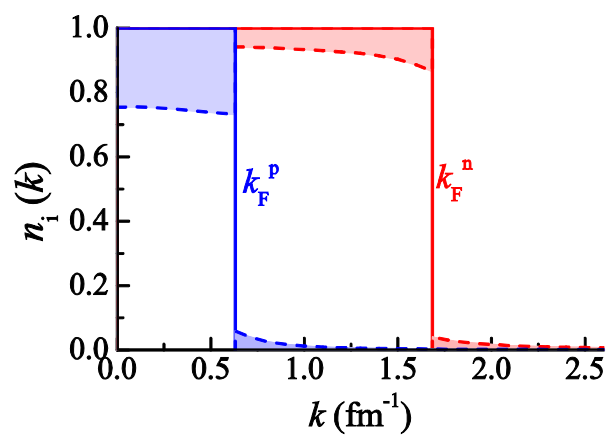
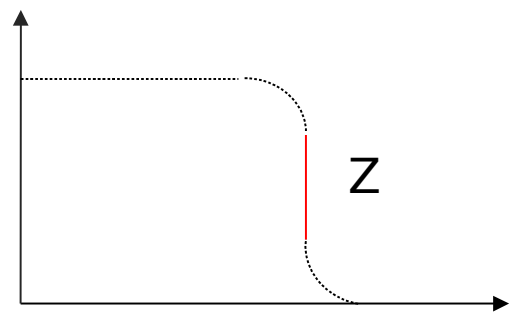
effective mass:

$$E(k)^2 = (k^2/2m^* - k_F^2/2m^*)^2 + \Delta^2$$

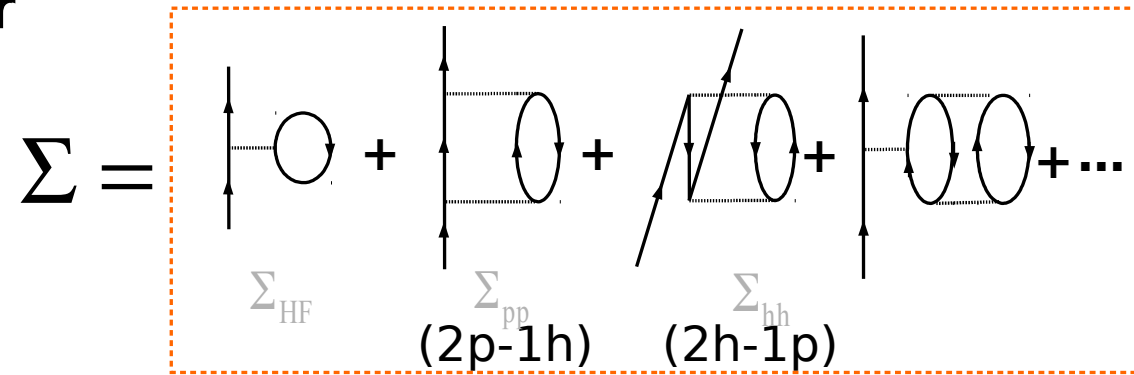


Quasi-Degenerate Fermi System

degeneracy Z-factors in asymmetric nuclear matter



Self-energy $\Sigma_k(\omega)$



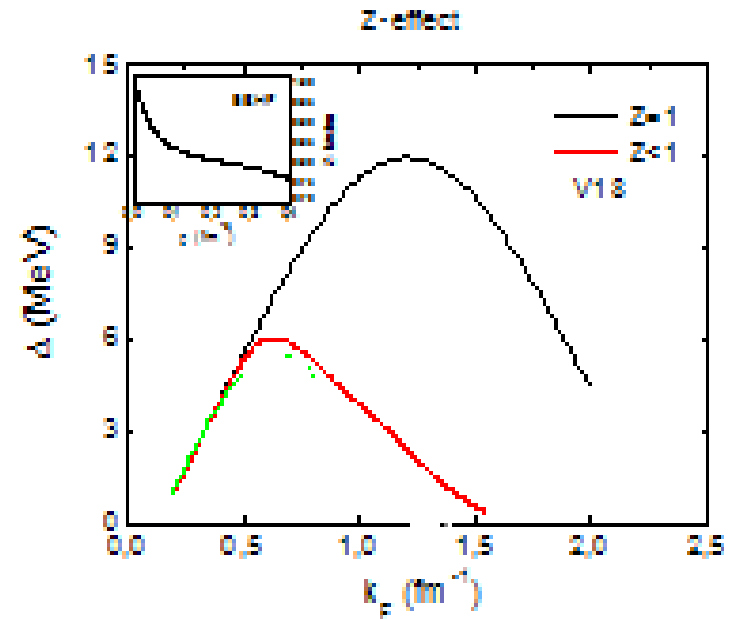
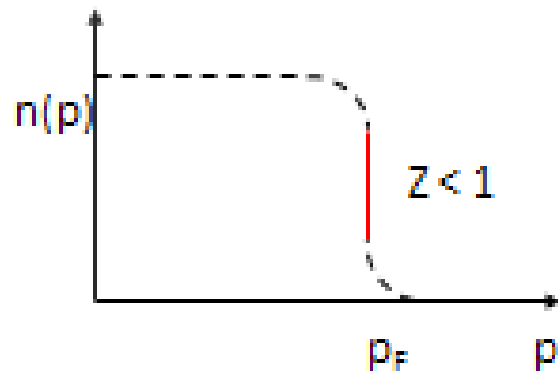
$$Z(k) = \frac{\langle \Psi(k, \omega) | \Psi(k, \omega) \rangle}{\langle \Psi(k, \omega) | \Psi(k, \omega) \rangle_{\omega = \epsilon(k)}}$$

$$Z(k_F) = 1 - \frac{\partial \Sigma_{BHF}(k, \omega)}{\partial \omega} \Big|_{\omega = \epsilon(k)}$$

Degenerate Fermi System

Self-Energy corrections

quasi-particle strength

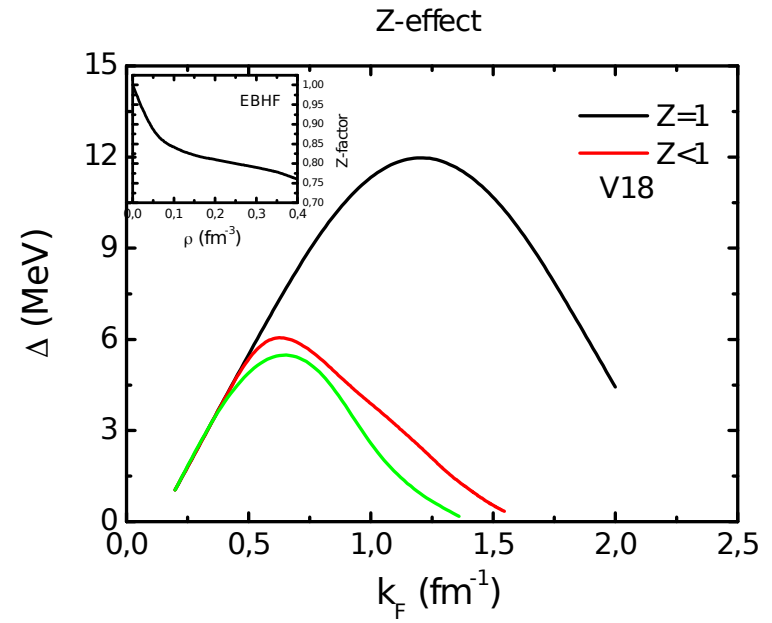


$$Z(k) = \left[1 - \frac{\partial \Sigma(k, \omega)}{\partial \omega} \right]_{\omega = \epsilon(k)}^{-1}$$

$$Z(k_F) = 1 \rightarrow \Sigma_{\text{BHF}}(k, \omega) \equiv \Sigma(k)$$

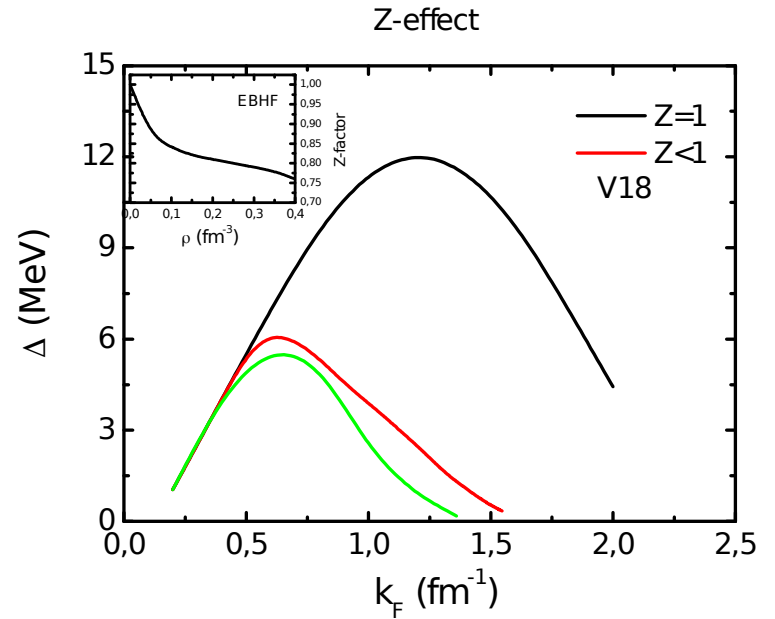
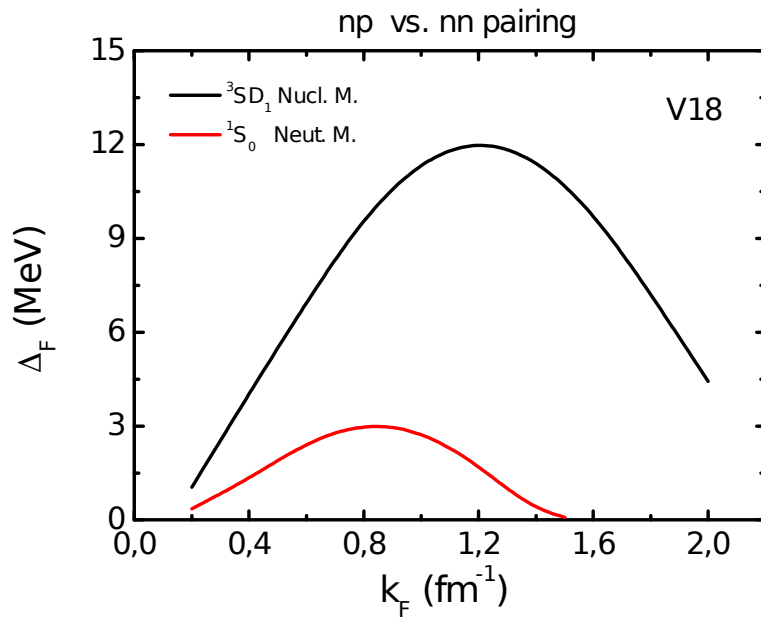
Self-Energy corrections

Full self-energy corrections



Self-Energy corrections

Full self-energy corrections



$$\Delta_{np} / \Delta_{nn} \sim 2$$

Core polarization : ph excitations
within the **BandB** induced interaction theory

$$\mathcal{J}^{\text{ph}} = G^{\text{ph}} + \mathcal{J}_i^{\text{ph}} = \begin{array}{c} \uparrow \\ | \\ \dots G \dots \\ | \\ \downarrow \end{array} + \begin{array}{c} \uparrow \\ | \\ \mathcal{J} \text{---} \text{---} \mathcal{J} \\ | \quad \text{---} \text{---} \Lambda \text{---} \text{---} \mathcal{J} \\ | \\ \downarrow \end{array}$$

G = BHF G-matrix (LNS)

\mathcal{J} = induced interaction

Λ = dressed polarization

propagator(DPP)

particle-hole coupling

$$\mathcal{J}^S(q) = G^S(q) + \mathcal{J}_i^S(q)$$

$$\mathcal{J}_i^0(q) = \frac{1}{2} \sum_{S'} (2S' + 1) \mathcal{J}^{S'}(q) \Lambda^{S'}(q) \mathcal{J}^{S'}(q)$$

$$\mathcal{J}_i^1(q) = \frac{1}{2} \sum_{S'} (-1)^{S'} \mathcal{J}^{S'}(q) \Lambda^{S'}(q) \mathcal{J}^{S'}(q),$$

particle-particle
coupling

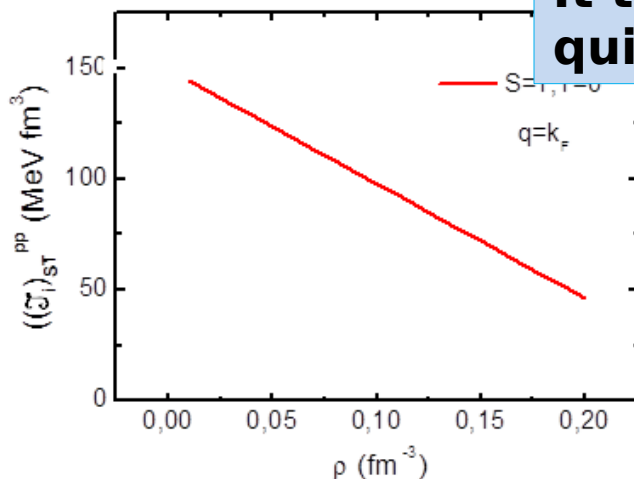
spin-singlet (e.g., neutron-neutron 1S_0 channel)

$$4(\mathcal{J}_i)_{10}^{pp} = \sum_T \Sigma_T [(\mathcal{J}_i)_{0T}^{ph} - 3(\mathcal{J}_i)_{1T}^{ph}] = -3 \left(\frac{F_0^2 \lambda}{1+F_0 \lambda} + \frac{F'_0{}^2 \lambda}{1+F'_0 \lambda} \right) + 3 \left(\frac{G_0^2 \lambda}{1+G_0 \lambda} + \frac{G'_0{}^2 \lambda}{1+G'_0 \lambda} \right)$$

spin-triplet (e.g., neutron-proton 3S_1 channel)

$$4(\mathcal{J}_i)_{10}^{pp} = \sum_T [(4(\mathcal{J}_i)_{10}^{pp}) \Sigma_T (2T+1) (\mathcal{J}_i)_{0T}^{ph} + (\mathcal{J}_i)_{1T}^{ph}] \left(\frac{F_0^2 \lambda}{1+F_0 \lambda} - 3 \frac{F'_0{}^2 \lambda}{1+F'_0 \lambda} \right) + 3 \left(\frac{G_0^2 \lambda}{1+G_0 \lambda} - 3 \frac{G'_0{}^2 \lambda}{1+G'_0 \lambda} \right)$$

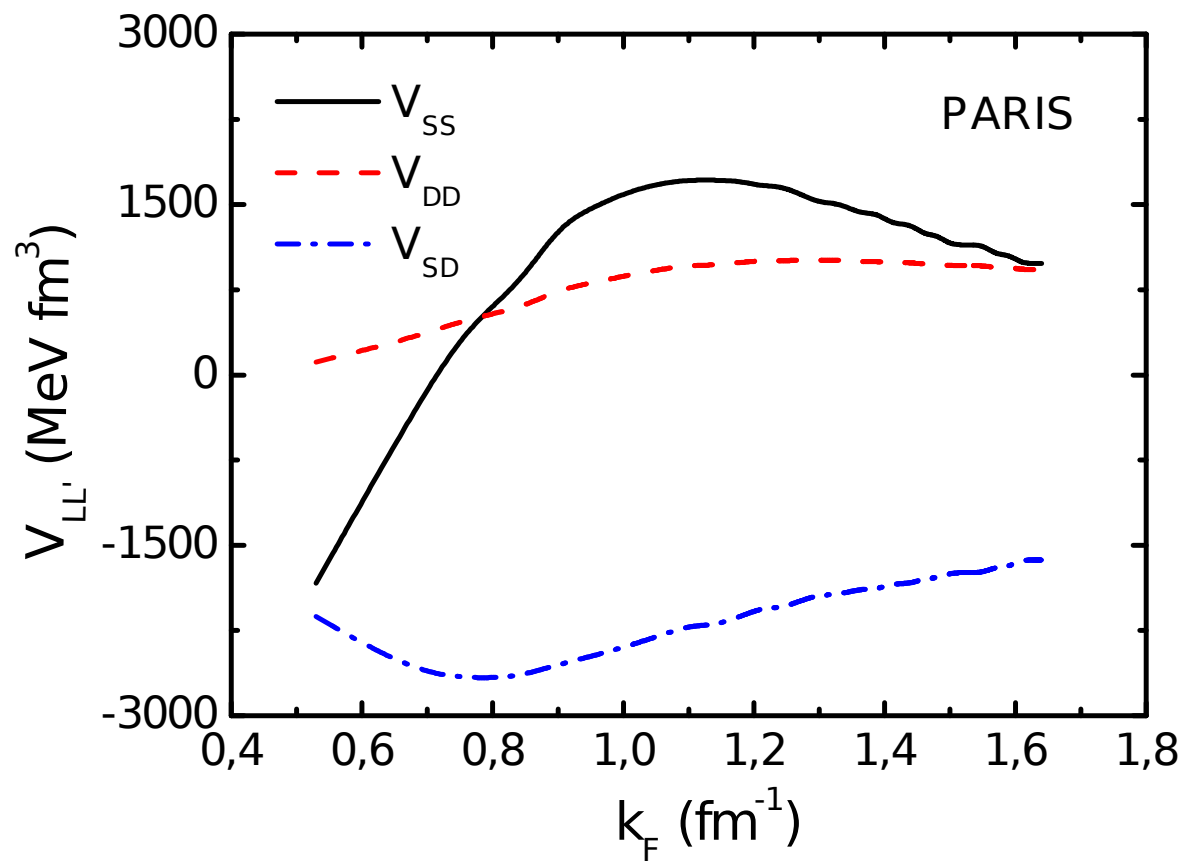
It turns out to be repulsive at any density, quite small in comparison with bare interaction



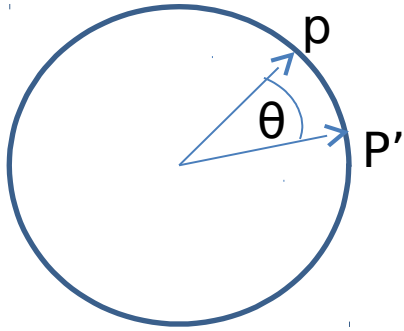
Landau limit : : $p = p' = p_F$

$(\mathcal{J}_i)_{10}^{pp}(p_F, p_F, q)$ $(0 \leq q \leq 2p_F)$

main drawback : missing off-diagonal m.e.



spin-triplet Induced Interaction



Fermi surface

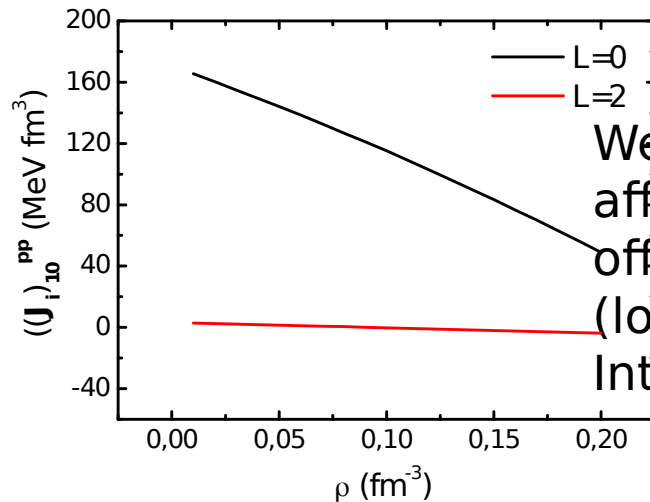
$$q = |p - p'| = 2p_F \sin \theta / 2$$

partial wave expansion

$$(\mathcal{J}_i)^{pp}_L = \frac{1}{2} \int_{-1}^1 d\cos\theta P_L(\cos\theta) (\mathcal{J}_i)^{pp}(q)$$

$$(\mathcal{J}_i)^{pp}_L = \frac{1}{2}$$

S and D partial waves



We need to understand how the induced interaction affects the pairing gap, that can be done incorporating off-diagonal m.e. of the bare interaction (low and high momentum transitions) into the pairing Interaction at the Fermi surface.

models for coupled channels (SFD, np pairing):

$$\Delta_L(k) = \sum_{L'} \int V_{LL'}(k, k') \frac{1}{2E(k')} \Delta_{L'}(k')$$

Renormalization of the pairing interaction:

$$\Delta(k) = - \int_{|k-k_f| < w} V(k, k') \frac{1}{2E(k')} \Delta(k')$$

Momentum space
 $I = P + Q$

main advantages:
$$V(k, k') = V(k, k') - \int_{|k-k_f| > w} V(k, k'') \frac{1}{2E(k'')} V(k'', k')$$

Two main advantages:
 constant gap approximation is a good approximation in the case of weak coupling ($\Delta \ll \epsilon_F$). In the strong coupling limit it can be applied if w is small enough.

The window gap equation makes it more understandable the role of off-diagonal matrix elements of the pairing interaction vs. , in particular

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Numerical procedure

if we can assume $\Delta(k) \approx \Delta(kf)$ and the gap eqs go over into a coupled system of (non-linear) algebraic equations:

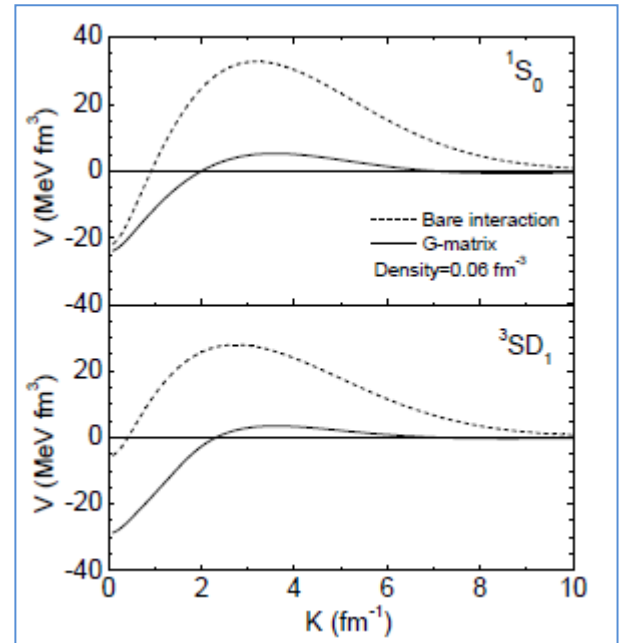
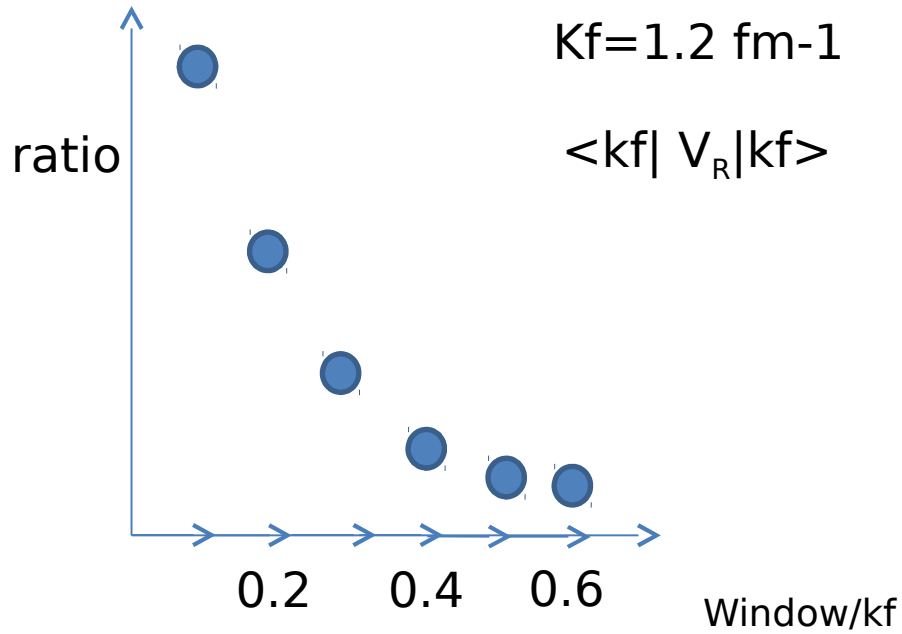
$$\Delta_0(kf) = -\Delta_0(kf) \int_{|k-kf|<w} V_{00}(k, k') \frac{1}{2E(k')} - \Delta_2(kf) \int_{|k-kf|<w} V_{02}(k, k') \frac{1}{2E(k')}$$

$$\Delta_2(kf) = -\Delta_0(kf) \int_{|k-kf|<w} V_{20}(k, k') \frac{1}{2E(k')} - \Delta_2(kf) \int_{|k-kf|<w} V_{22}(k, k') \frac{1}{2E(k')}$$

$E^2(k) = (\epsilon_k - \epsilon_f)^2 + \Delta^2(k) \equiv (\epsilon_k - \epsilon_f)^2 + \Delta^2(kf)$. It can be numerically

solved along with equations for the renormalized interaction by means of linearization and iteration.

Off-diagonal matrix elements $V(k_f, k)$



$$(\mathcal{J}_i)_{10}^{pp} \ll \langle k_f | V_R | k_f \rangle$$

No appreciable effect expected!

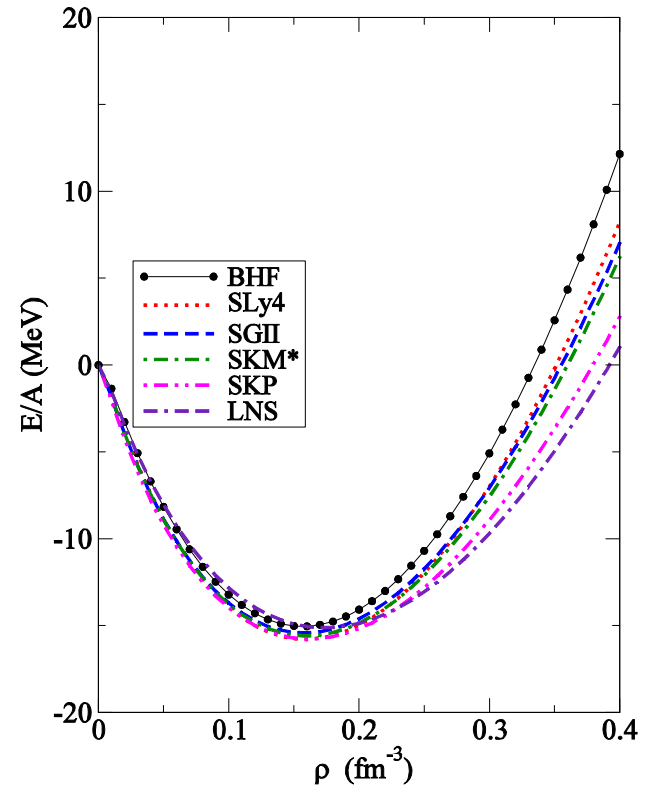
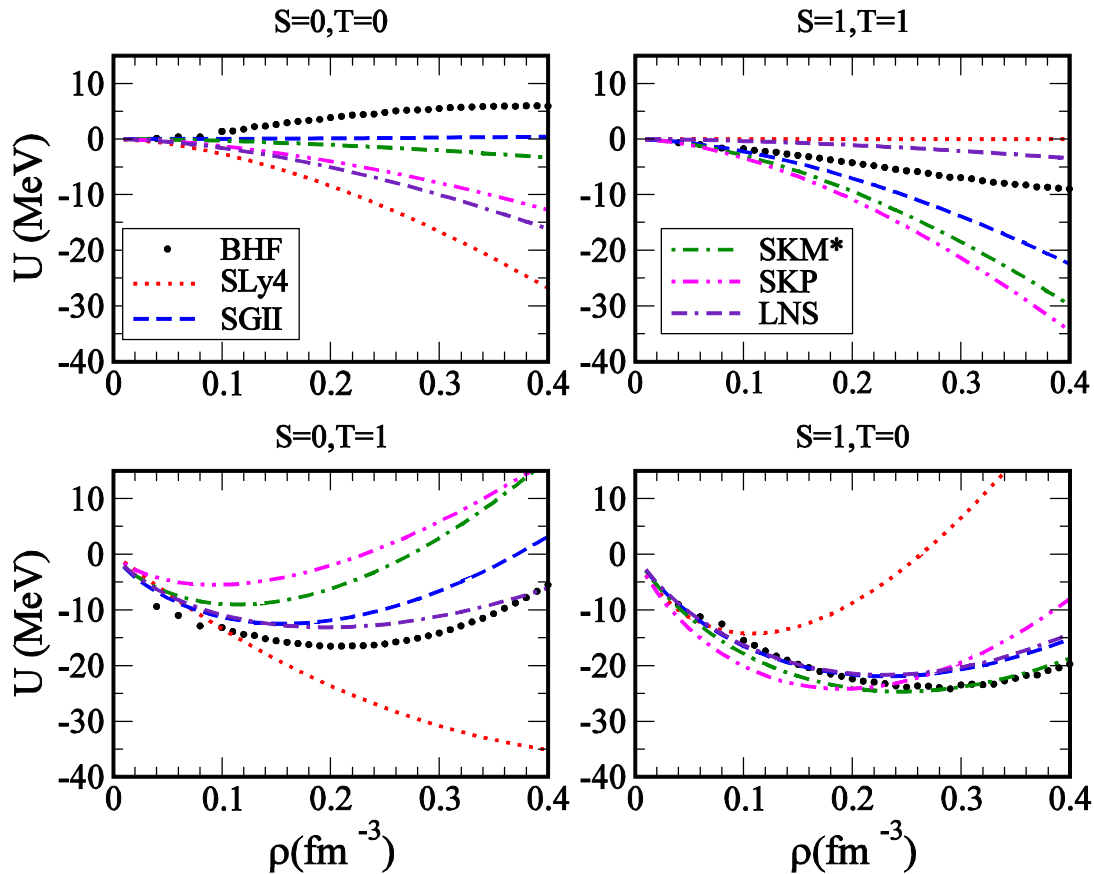
conclusions:

There is a general consensus that self-energy corrections conspire against all kinds of pair correlations, including the neutron-proton $3S_1$ pairing. In nuclear matter they severely reduce the pairing gaps and shrink the pairing domain to lower densities, typical of nuclear surface.

Vertex corrections could provide a screening to the pair interaction and could be good candidates to solve the puzzle of the missing neutron-proton pairs in nuclei. We found in fact that, in the deuteron channel, they have a repulsive effect in contrast with what happens in the case of $1S_0$ channel. However we found a negligible effect in the approximation adopted in the present calcs.

Approaches in finite nuclei attribute the missing np pairing to the spin-orbit splitting. For $N=Z$ very heavy nuclei a crossover from nn (pp) pairing to np pairing is predicted. This result is consistent with the present calculation.

Isospin splitting of potential energy (BHF: Bonn+3BF)



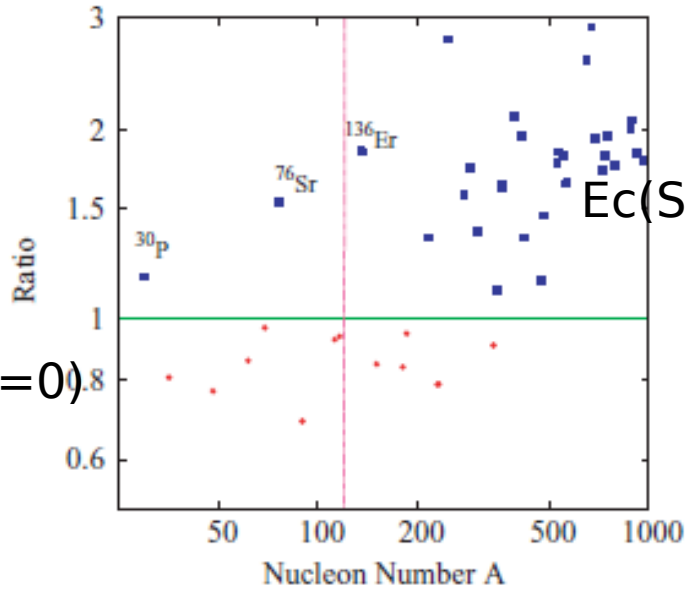
$$E/A = K + U$$

$$a_s(\rho) = U(\rho, \beta=1) - U(\rho, \beta=0)$$

Strong compensation for SLy4 Tensor component U_{10} the largest

ChEFT: $U_{10} \approx U_{SD} \approx -17 \text{ MeV}$ (Kohno, 2013)

Correlation energies $E_{c,S=1}/E_{c,S=0}$

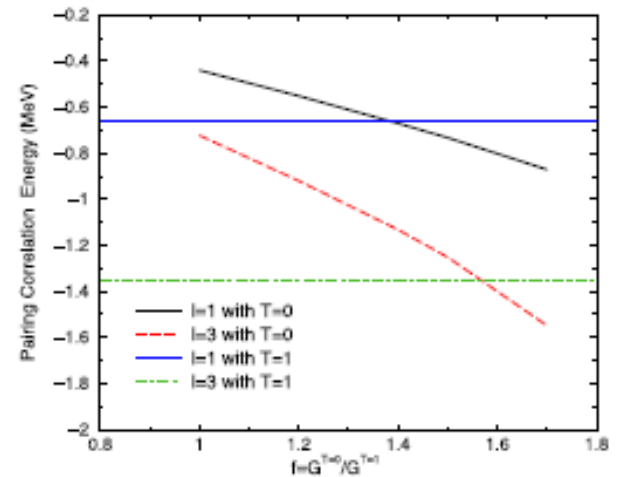


$E_{c,S=1} > E_{c,S=0}$

$E_{c,S=1} < E_{c,S=0}$

Sagawa et al. Phys.Scr.91,2016

Bertsch et al. PRC 81 (2010)





Xavier,
Welcome into the club of die-hard
physicists!

