Semiclassical Approach to Pairing of Drip-Line Nuclei

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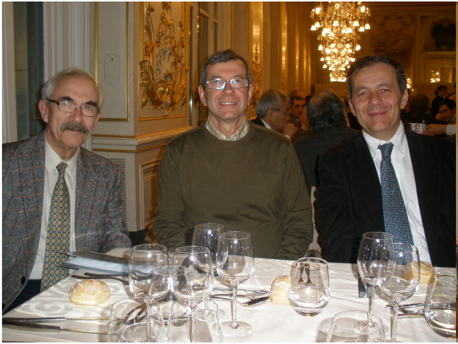
IPN Orsay and LPMMC Grenoble

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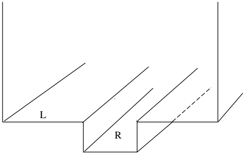
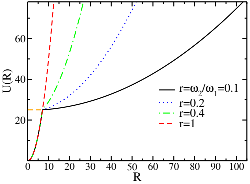
Drip-line nuclei

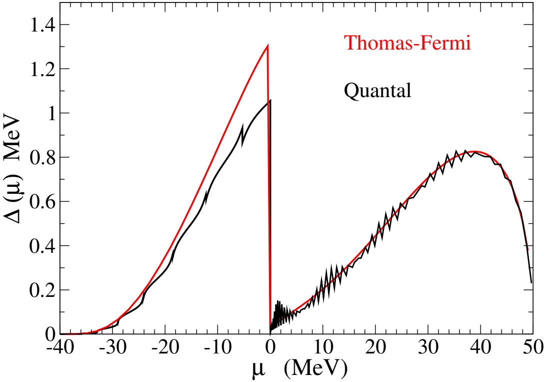




Overflow situations of superfluid fermions in finite mean field potential →
nuclei (drip line),
nuclei in Wigner Seitz cells in crust of neutron stars,
Cold atoms

etc.

: TF; black: quantal; slab with pocket, depth = - 40 MeV and cut off + 50 MeV



X.Vinyas, P.S., PRL 107

Thomas-Fermi approach to pairing.

TF-BCS

Gap equation

$$\Delta_n = \sum_{n'} \langle n|v|n'\bar{n}'\rangle \frac{\Delta_{n'}}{2\sqrt{(e_{n'}-\mu)^2+\Delta_{n'}^2}} \tag{1}$$

Time reversal invariance:

$$\langle n\bar{n}|\mathbf{r}\mathbf{r}'\rangle \equiv \langle \mathbf{r}|n\rangle\langle n|\mathbf{r}'\rangle \equiv \rho_n(\mathbf{r},\mathbf{r}')$$

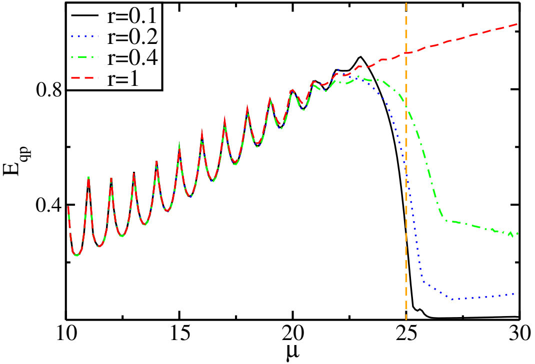
TF: $\rho_n(\mathbf{r},\mathbf{r}') \to \delta(E_n - \frac{p^2}{2m} - U(R))$

TF-gap equation

$$\Delta(E) = \int_{E'} g(E')V(E,E')\frac{\Delta(E')}{2\sqrt{(E'-\mu)^2+\Delta^2(E')}}$$

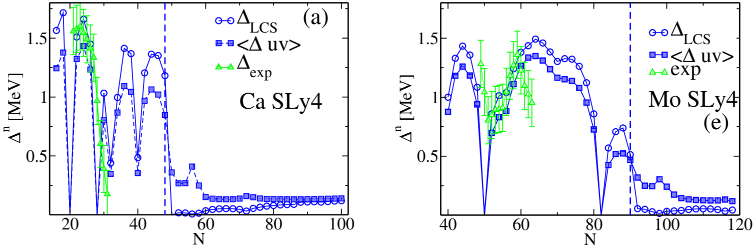
$$\kappa_n = u_n v_n \to \kappa(\mathbf{r},\mathbf{p}) = \int dE\,\kappa(E)\delta(E - \frac{p^2}{2m} - U(R)) \quad + O(\hbar^2)$$

HFB calculation in double HO-potential



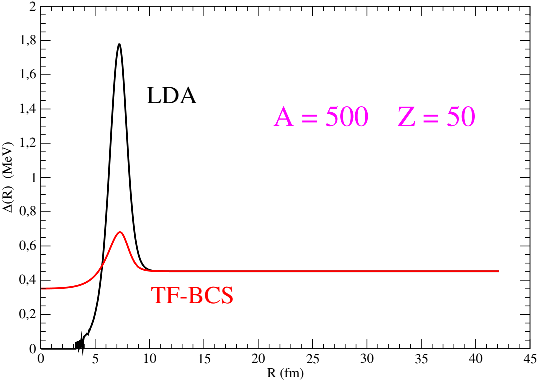
X. Vinyas, P.S. et al., PRA 90. Figure prepared by A. Pastore.
Evolution of the lowest HFB-quasi-particle energy as a function of μ.
Thomas-Fermi pour HFB later...

HFB for some drip-line nuclei



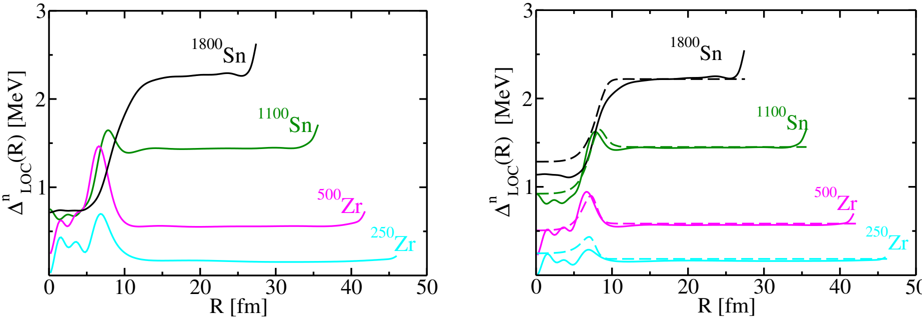
Vertical broken line corresponds to drip-line. SLy4 force is used.
Pastore, Margueron, P.S., Vinyas, PRC 88.

LDA can become very bad ...



Wigner-Seitz cell; SLy4 + pairing

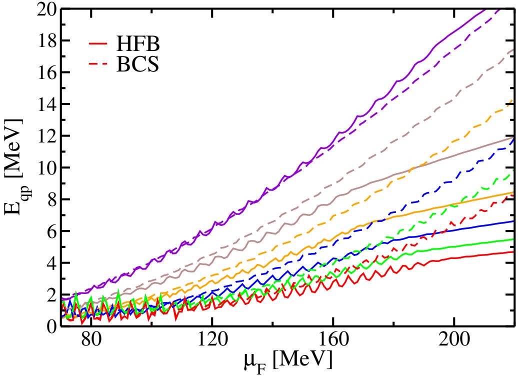
HFB BCS + TF-BCS



Wigner-Seitz cells with SLy4.

Some differences between HFB (left) and BCS (right) can be seen. TF-BCS: broken lines.

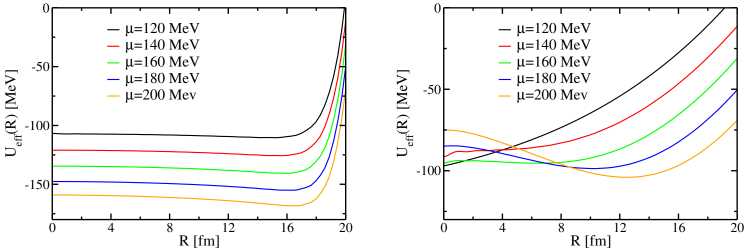
However, BCS can also become quite wrong ...



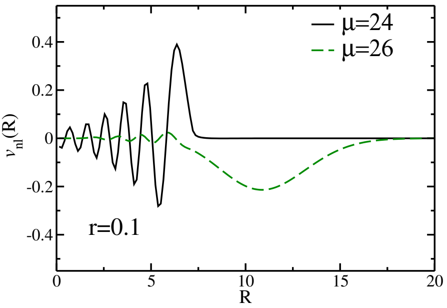
Deep Woods-Saxon potential from steep violet to soft (HO-like) red

What is reason for failure?

$$U_{\mathrm{eff.}} = U(R) - \mu + \frac{\Delta^2(R)}{E_{\mathrm{qp}}}$$



Pockets of different Woods-Saxon potentials with varying width parameters a1 = 1, 11



μ=24
μ=26
r=0.1
v_{nl}(R)
R
0.4
0.2
0
-0.2
-0.4
0
5
10
15
20

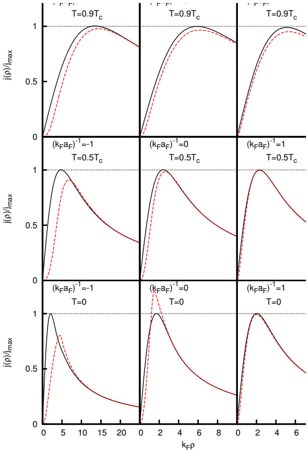
Anomalous density. Wigner-Kirkwood $\hbar$-expansion:

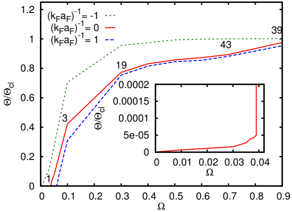
$$\kappa = \kappa_0 \equiv \kappa_{\mathrm{LDA}} + \kappa_2 \tag{2}$$

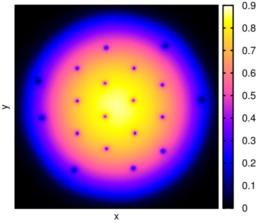
$$\begin{aligned}
\kappa_2 \quad = \quad & -\left(\frac{h}{4E^3} - \frac{3\Delta^2}{4E^5} + \frac{5h^2\Delta^2}{12E^7}\frac{\hbar^2 p^2}{m}\right)\frac{\hbar^2\nabla^2\Delta}{4m} \\
+ \quad & \left(\frac{3h\Delta}{4E^5} - \frac{5h\Delta^3}{4E^7} + \frac{\Delta}{12E^5}\frac{\hbar^2 p^2}{m}\right)\frac{\hbar^2(\nabla\Delta)^2}{4m} \\
+ \quad & \left(\frac{\Delta}{4E^3} - \frac{3\Delta^3}{8E^5}\right)\frac{\hbar^2}{4m}2\nabla^2 U \\
- \quad & \left(\frac{h\Delta}{2E^5} - \frac{5h\Delta^3}{4E^7}\right)\frac{\hbar^2}{4m}\left[(\nabla U)^2 + + \frac{1}{3}\frac{\hbar^2 p^2}{m}\nabla^2 U\right] \\
+ \quad & \left(\frac{1}{2E^3} - \frac{5h^2\Delta^2}{2E^7}\right)\frac{\hbar^2}{4m}\left[\nabla U \cdot \nabla\Delta\right],
\end{aligned} \tag{3}$$

Strinati

$$-\frac{1}{g}\Delta(\mathbf{r}) = \int \frac{\mathbf{dk}}{(2\pi)^3}\frac{\Delta(\mathbf{r})}{2E(r,0,k)} + \frac{1}{2}\int \frac{\mathbf{dk}}{(2\pi)^3}\left[\frac{e_k - \mu(\mathbf{r})}{2E^3(r,0,k)}\nabla^2_{\mathbf{r}}\frac{\hbar^2\Delta(\mathbf{r})}{4m}\right]. \qquad (4)$$







Conclusions:

BCS good approximation for containers with steep surface.

TF-BCS very good approximation to quantal BCS.

BCS not applicable for wide HO.

Pairing strongly quenched at the drip.

$\hbar$-expansion for HFB.

Applications of TF-BCS and TF-HFB

DEAR XAVIER: ALL THE BEST FOR RETIREMENT!

THAT OUR NICE COLLABORATION STILL WILL GO ON...... AND ON!!

THANK YOU!