

# Xavier Vinas Meeting

- Xavier Vinas Meeting  
19-20 September 2017  
Milano



- 
- Nguyen Van Giai
- Institut de Physique Nucléaire,  
Orsay

- I have been knowing Xavier practically since he started in physics (I am a bit older ...)
- We have always shared an interest in all topics related to the **properties of atomic nuclei**, and often used the same theoretical tools.
- I highly appreciate Xavier as a **scientist**, and as a **person**.
- Here, I'll present – as a modest contribution to this event organized **for Xavier - some results recently**

# Magicity of s-d shell neutron-rich nuclei: Relativistic Hartree-Fock-Bogoliubov predictions

Jia Jie Li (1), Jérôme Margueron (2),  
Wenhui Long (1), Nguyen Van Giai (3)

1. Lanzhou University
2. IPN Lyon, Université de Lyon
3. IPN Orsay, Université Paris-Sud



# Outline of talk

- 1. Experimental facts in the Ca chain
- 2. Why do we use Relativistic Hartree-Fock Bogoliubov (RHFB) and not Relativistic Hartree Bogoliubov (RHB, usually called RMF)
- 3. Single-particle spectra and subshell closures
- Ref: Phys. Lett. B 753, 97 (2016)

# LETTER

doi:10.1038/nature12522

## Evidence for a new nuclear ‘magic number’ from the level structure of $^{54}\text{Ca}$

D. Steppenbeck<sup>1</sup>, S. Takeuchi<sup>2</sup>, N. Aoi<sup>3</sup>, P. Doornenbal<sup>2</sup>, M. Matsushita<sup>1</sup>, H. Wang<sup>2</sup>, H. Baba<sup>2</sup>, N. Fukuda<sup>2</sup>, S. Go<sup>1</sup>, M. Honma<sup>4</sup>, J. Lee<sup>2</sup>, K. Matsui<sup>5</sup>, S. Michimasa<sup>1</sup>, T. Motobayashi<sup>2</sup>, D. Nishimura<sup>6</sup>, T. Otsuka<sup>1,5</sup>, H. Sakurai<sup>2,5</sup>, Y. Shiga<sup>7</sup>, P.-A. Söderström<sup>2</sup>, T. Sumikama<sup>8</sup>, H. Suzuki<sup>2</sup>, R. Taniuchi<sup>5</sup>, Y. Utsuno<sup>9</sup>, J. J. Valiente-Dobón<sup>10</sup> & K. Yoneda<sup>2</sup>

Nature 502, 207 (2013)

- Data from proton knock out reactions
- First  $2+$  state in  $^{54}\text{Ca}$  found at about 2 MeV: indication that  $2p_{1/2}$  (neutron) is **partly** occupied while  $1f_{5/2}$  (neutron) is **partly** empty.

How much « partly »?

We show the importance of the effective tensor interaction in answering this **question**

# Recent theoretical studies

- M. Grasso, Phys. Rev. C 89, 034316 (2014)
- found magicity for  $^{52,54}\text{Ca}$  if described by Skyrme Hartree-Fock, using SLy5+Tensor
- E. Yüksel, NVG, E. Khan, K. Bozkurt, Phys. Rev. C 89, 064322 (2014)
- have performed Skyrme Hartree-Fock-Bogoliubov plus QRPA, using SLy5, SLY5+T, and T44 effective interactions

# General guidelines

- The **s.p. level scheme** of atomic nuclei determines, to a large extent, the shell and subshell closures, i.e., the magic and semi-magic numbers.
- Theoretical predictions of s.p. levels rely firstly on self-consistent (s.c.) methods: **HF, HFB**.
- We are familiar with NR s.c. mean fields, like Gogny, Skyrme, etc... where the **rank-2 irreducible tensor interaction** (we refer to it as the « **tensor interaction** ») plays a decisive role.
- How to identify this « **tensor interaction** » in **relativistic approaches** like the RMF (Relativistic Hartree), the RHF, the RHFb ? The answer is: through a **Foldy-Wouthuysen transformation** ( L.L. Foldy & S.A. Wouthuysen, Phys. Rev. 78, 29 (1950)).
- With the FW transformation, we can determine and discuss the « **tensor interaction** » of any RMF(B)/RHF(B) model

# Pseudo-Vector (PV) and Lorentz Tensor (T) couplings

$$\Gamma_{\pi}^{PV}(1, 2) \equiv -\frac{1}{m_{\pi}^2} \left( f_{\pi} \vec{\tau} \gamma_5 \gamma_{\mu} \partial^{\mu} \right)_1 \cdot \left( f_{\pi} \vec{\tau} \gamma_5 \gamma_{\nu} \partial^{\nu} \right)_2,$$

$$\Gamma_{\rho}^T(1, 2) \equiv +\frac{1}{4M^2} \left( f_{\rho} \vec{\tau} \sigma_{\lambda\mu} \partial^{\mu} \right)_1 \cdot \left( f_{\rho} \vec{\tau} \sigma^{\lambda\nu} \partial_{\nu} \right)_2,$$

These are the 2 possible terms in the Interaction Lagrangian which could give rise to a non-relativistic Tensor Interaction

**BUT:** they contribute only by their exchange term ☾ no contribution in RMF !

Connection with NR interaction: L.L. Foldy & S.A. Wouthuysen, Phys. Rev. 78, 29 (1950)

- For example, the pi-N part leads to:

$$V_{\pi}^T(r) = \frac{1}{3} f_{\pi}^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 S_{12} \left( 1 + \frac{3}{m_{\pi} r} + \frac{3}{(m_{\pi} r)^2} \right) \frac{e^{-m_{\pi} r}}{r},$$

$$V_{\pi}^C(r) = \frac{1}{3} f_{\pi}^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \frac{e^{-m_{\pi} r}}{r},$$

where  $S_{12}$  is a standard rank-2 tensor operator,

$$S_{12}(r) \equiv 3(\boldsymbol{\sigma}_1 \cdot \mathbf{e}_r)(\boldsymbol{\sigma}_2 \cdot \mathbf{e}_r) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2.$$

and similarly for the rho-N interaction.

In covariant theory with exchange interactions, the central and rank-2 tensor forces cannot be independent

# The relativistic models discussed in this talk:

- Relativistic Hartree (known as RMF)
- DD-ME2 and DD-ME(delta): no pi, no rho
- Relativistic Hartree-Fock
- PKO3: has pi-PV
- PKA1: has pi-PV and rho-Tensor
- Pairing is treated

# The RHFB Model

## Lagrangian Density

🌐 Lagrangian density: Nucleon ( $\psi$ ), Mesons ( $\sigma, \omega, \rho, \pi$ ), Photon ( $A$ )

$$\mathcal{L} = \mathcal{L}_M + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\pi + \mathcal{L}_A + \mathcal{L}_I,$$

🌐 Nucleon-Meson (Photon) interactions  $\mathcal{L}_I$ :

$$\begin{aligned} \mathcal{L}_I = \bar{\psi} \left( -g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \vec{\tau} \cdot \vec{\rho}_\mu - e \gamma^\mu \frac{1 - \tau_3}{2} A_\mu \right. \\ \left. + \frac{f_\rho}{2M} \sigma_{\mu\nu} \partial^\nu \vec{\rho}^\mu \cdot \vec{\tau} - \frac{f_\pi}{m_\pi} \gamma_5 \gamma^\mu \partial_\mu \vec{\pi} \cdot \vec{\tau} \right) \psi \quad (5) \end{aligned}$$

🌐 **Covariant** scalar ( $\sigma$ ), vector ( $\omega, \rho, A$ ), **tensor** ( $\rho$ ) and **pseudo-vector** ( $\pi$ )

# RHFB equation

W. H. Long (2010)

🌀 Bogoliubov transformation:  $\{c_\alpha, c_\alpha^\dagger\}$  to  $\{\beta_\alpha, \beta_\alpha^\dagger\}$  ( $\alpha = 1, \dots, M$ )

$$\begin{pmatrix} c_\alpha \\ c_\alpha^\dagger \end{pmatrix} = \mathcal{W} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix} = \begin{pmatrix} \psi_U & \psi_V^* \\ \psi_V & \psi_U^* \end{pmatrix} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix}, \quad \mathcal{W}^\dagger \mathcal{W} = 1. \quad (28)$$

🌀 RHFB equation: Chemical potential  $\lambda$

H. Kucharek(1991)

$$\int d\mathbf{r}' \begin{pmatrix} h(\mathbf{r}, \mathbf{r}') - \lambda & \Delta(\mathbf{r}, \mathbf{r}') \\ \Delta(\mathbf{r}, \mathbf{r}') & \lambda - h(\mathbf{r}, \mathbf{r}') \end{pmatrix} \begin{pmatrix} \psi_U(\mathbf{r}') \\ \psi_V(\mathbf{r}') \end{pmatrix} = E \begin{pmatrix} \psi_U(\mathbf{r}) \\ \psi_V(\mathbf{r}) \end{pmatrix} \quad (29)$$

🌀 Pairing potential, pairing tensor  $\kappa$  and Pairing forces:

$$\Delta_\alpha(\mathbf{r}, \mathbf{r}') = -\frac{1}{2} \sum_\beta V_{\alpha\beta}^{pp}(\mathbf{r}, \mathbf{r}') \kappa_\beta(\mathbf{r}, \mathbf{r}'), \quad \kappa_\alpha(\mathbf{r}, \mathbf{r}') = \psi_{V_\alpha}^*(\mathbf{r}) \psi_{U_\alpha}(\mathbf{r}') \quad (30)$$

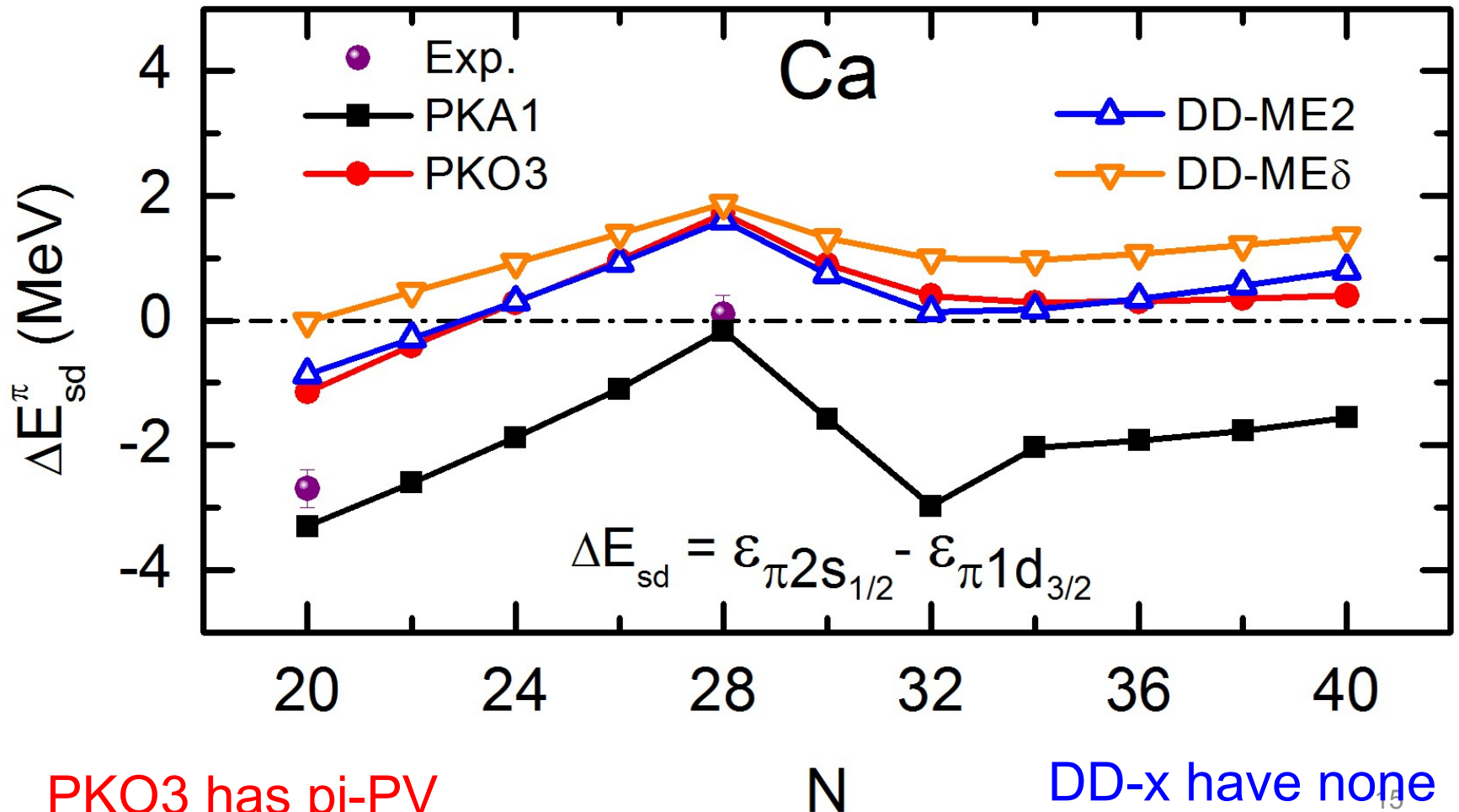
$$V(\mathbf{r}, \mathbf{r}') = V_0 \delta(\mathbf{r} - \mathbf{r}') \frac{1}{4} (1 - \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') \left( 1 - \frac{\rho(\mathbf{r})}{\rho_0} \right) \quad (31)$$

$$V(\mathbf{r}, \mathbf{r}') = \sum_{i=1,2} e^{((\mathbf{r}-\mathbf{r}')/\mu_i)^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau) \quad (32)$$

# Numerical methods

- - here, it is necessary to describe correctly the long distance behaviour of wave functions when solving RHFB equations: harmonic oscillator basis would not do.
- - we adopt the Dirac-Woods-Saxon basis (S.G. Zhou, J. Meng, P. Ring, Phys. Rev. C 68, 034323 (2003))

# Strong effects of rho-T coupling on 2s-1d proton energies



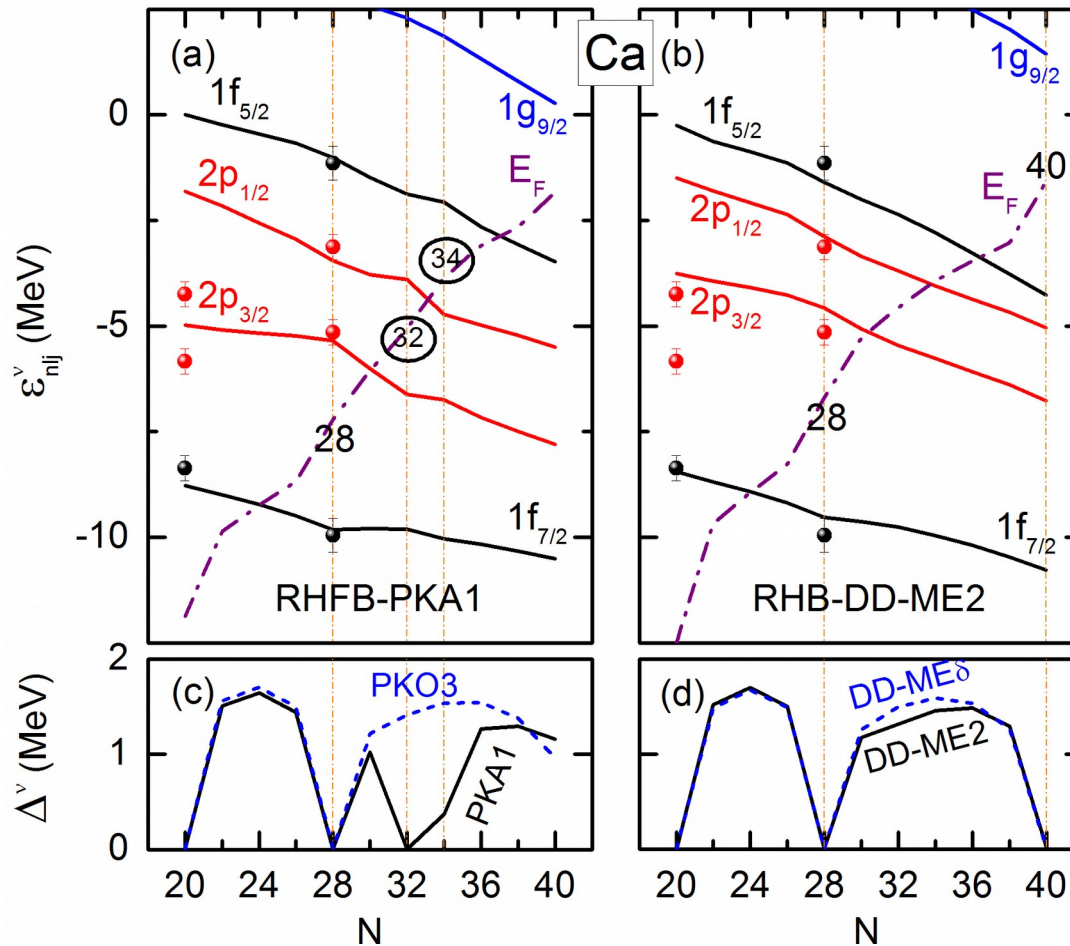
PKO3 has pi-PV

DD-x have none

PKA1 has pi-PV and rho-T (2 MeV effect!)

Pairing: D1S

# New shell closure in Ca isotopes: vanishing pairing gap at N=32?



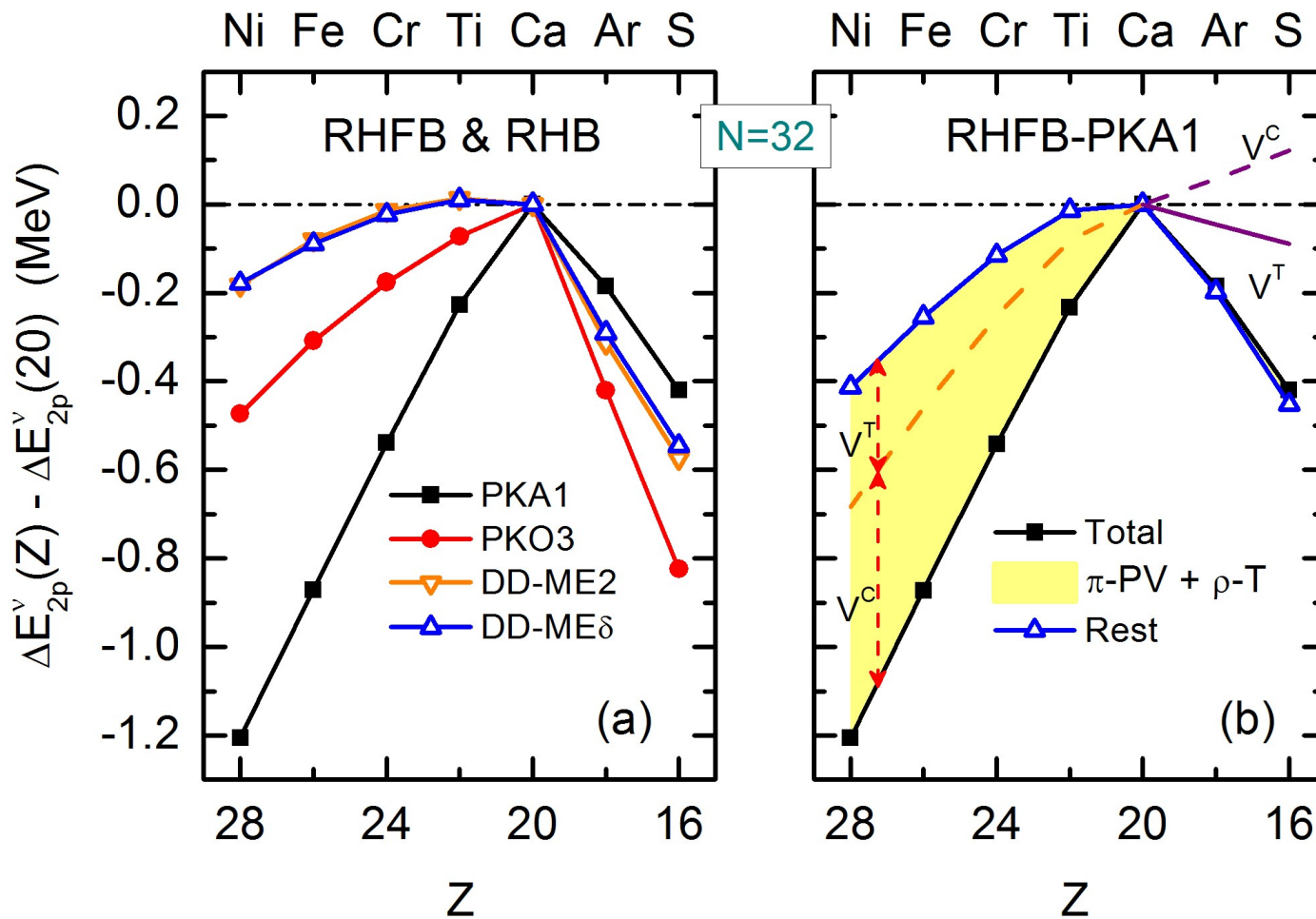
RMFB: no  
RHFB: yes  
(panels c-d)

TABLE I. Energy difference  $\Delta E(i, i') \equiv \varepsilon_i - \varepsilon_{i'}$  (in MeV) in Ni, Ca and Si at  $N = 32$  and 34. The results correspond to RHFB-PKA1, PKO3 and RHB-DD-ME2 Lagrangians. See the text for details.

Force	$\Delta E(i, i')$	$N$	Ni	Ca	Si
PKA1	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.51	<b>2.72</b>	0.81
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	1.04	<b>2.45</b>	<b>4.05</b>
PKO3	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.22	1.69	0.68
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	-1.72	0.77	2.72
DD-ME2	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.58	1.76	0.92
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	-1.23	1.21	3.18

A strong effect of the Lorentz tensor interaction (PKA1) on single-particle energies

# Evolution of $2p$ s.o. splitting along $N=32$ chain: importance of $\pi$ -PV and $\rho$ -T



# Summary

- The RHFB model confirms the  $N=34$  subshell in  $Z=20$  isotopes
- For this, one must have the contribution of
- The Lorentz tensor  $\rho$ - $N$  coupling

**THANK YOU, Xavier  
for your contributions to our field.**

**And for your stimulating and  
active presence in our nuclear  
theory community.**