Symmetry energy and the neutron star core-crust transition with Gogny forces

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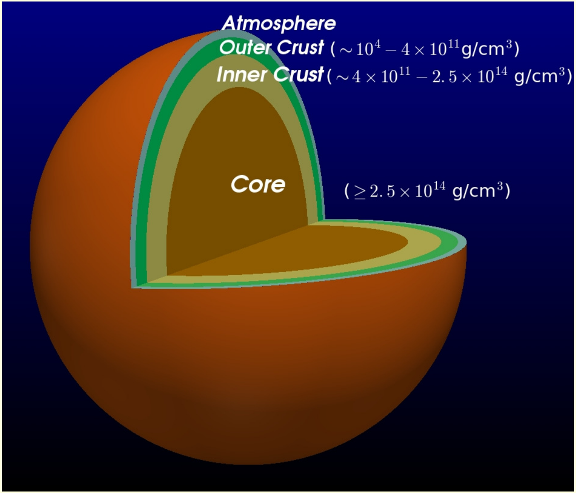
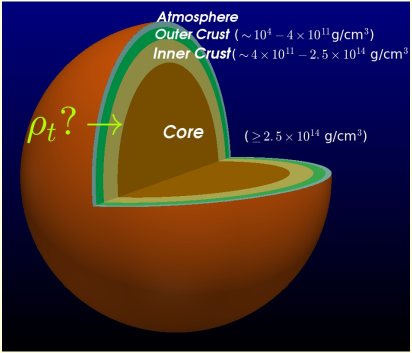
Introduction



Introduction

- **Aim of this work: Study the Core Crust Transition in NSs and their crustal properties.**
- Previous results of the core-crust transition using Skyrme interactions and RMF models, but not **with Gogny interactions**.
- Core-crust transition studied from the core to the crust.
- Different approaches: dynamical method, thermodynamical method.
- The essential ingredient is the EoS of Asymmetric Nuclear Matter. Sometimes, it is approximated by a Taylor expansion in terms of the isospin asymmetry $\delta = (\rho_n\text{-}\rho_p)/\rho$.



The Gogny Interaction

The Gogny two-body effective nuclear interaction in a homogeneous system reads as

$$V(\mathbf{r}_1, \mathbf{r}_2) = \sum_{i=1,2} \left(W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau\right) e^{-\frac{r^2}{\mu_i^2}} + t_3 \left(1 + x_3 P_\sigma\right) \rho^\alpha(\mathbf{R}) \delta(\mathbf{r})$$

We have used several representative Gogny forces: D1, D1S, D1M, D1N, D250, D260, D280 and D300.

The Asymmetric Nuclear Matter (ANM): Theory

- From $V(\mathbf{r}_1, \mathbf{r}_2)$, we can obtain the Equation of State (EoS), $E_b(\rho, \delta)$, in terms of the total nuclear density $\rho = \rho_n + \rho_p$ and the isospin asymmetry parameter, $\delta = (\rho_n - \rho_p)/\rho$.
- We have used the exact $E_b(\rho, \delta)$ to perform the calculations. Also, this EoS can be expressed as a Taylor expansion around $\delta = 0$:

$$E_b(\rho, \delta) = E_b(\rho, \delta = 0) + E_{\mathrm{sym},2}(\rho)\, \delta^2 + E_{\mathrm{sym},4}(\rho)\, \delta^4 + \ldots + E_{\mathrm{sym},2k}(\rho)\, \delta^{2k} + \mathcal{O}(\delta^{2k+2}),$$

The Asymmetric Nuclear Matter (ANM): Theory

- Energy per baryon in symmetric nuclear matter.

$$E_b(\rho,\delta) = E_b(\rho,\delta=0) + E_{\mathrm{sym},2}(\rho)\,\delta^2 + E_{\mathrm{sym},4}(\rho)\,\delta^4 + \ldots + E_{\mathrm{sym},2k}(\rho)\,\delta^{2k} + \mathcal{O}(\delta^{2k+2}),$$

The Asymmetric Nuclear Matter (ANM): Theory

- Energy per baryon in symmetric nuclear matter.

$$E_{\mathrm{sym},2}(\rho) = \frac{1}{2!}\frac{\partial^2 E_b(\rho,\delta)}{\partial \delta^2}\bigg|_{\delta=0}$$

$$E_b(\rho,\delta) = E_b(\rho,\delta=0) + E_{\mathrm{sym},2}(\rho)\,\delta^2 + E_{\mathrm{sym},4}(\rho)\,\delta^4 + \ldots + E_{\mathrm{sym},2k}(\rho)\,\delta^{2k} + \mathcal{O}(\delta^{2k+2}),$$

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- $$E_{\mathrm{sym},2k}(\rho) = \frac{1}{(2k)!}\frac{\partial^{2k} E_b(\rho,\delta)}{\partial\delta^{2k}}\bigg|_{\delta=0}$$

The Asymmetric Nuclear Matter (ANM): Theory

$$E_b(\rho,\delta) = E_b(\rho,\delta=0) + E_{\mathrm{sym},2}(\rho)\,\delta^2 + E_{\mathrm{sym},4}(\rho)\,\delta^4 + \ldots + E_{\mathrm{sym},2k}(\rho)\,\delta^{2k} + \mathcal{O}(\delta^{2k+2}),$$

The slope of the symmetry energy is defined as

$$L = 3\rho_0 \frac{\partial E_{\mathrm{sym},2}(\rho)}{\partial \rho}\bigg|_{\rho=\rho_0}$$

with $\rho_0 \simeq 0.16\ \mathrm{fm}^{-3}$ the saturation density.

The Asymmetric Nuclear Matter (ANM): Theory

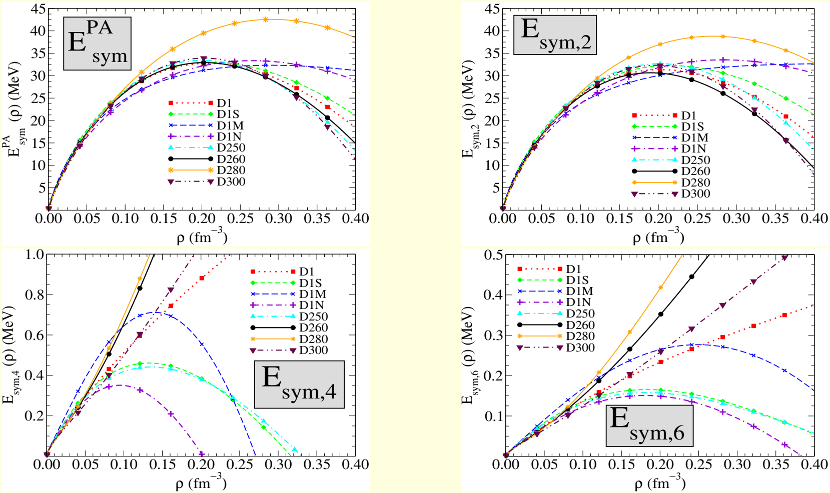
It is also common to use the empirical parabolic approximation (PA) in order to estimate the symmetry energy.

$$E_b(\rho, \delta) \simeq E_b(\rho, \delta = 0) + E_{\mathrm{sym}}^{PA}(\rho)\delta^2$$

This $E_{\mathrm{sym}}^{PA}(\rho)$ corresponds to the difference between the energy in pure neutron matter and the energy in symmetric nuclear matter:

$$E_{\mathrm{sym}}^{PA}(\rho) = E_b(\rho, \delta = 1) - E_b(\rho, \delta = 0)$$

Taylor expansion coefficients up to 6th order



β-stable Nuclear Matter

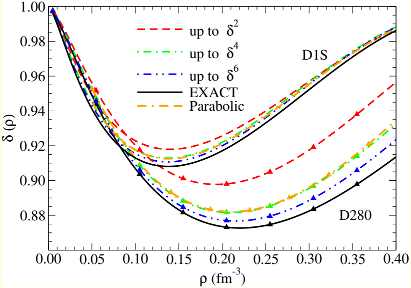
The homogeneous *npe* matter in the core is in β-equilibrium, and therefore the chemical potentials obey the condition:

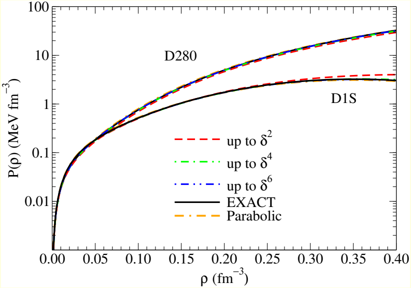
$$\boxed{\mu_{np} \equiv \mu_n - \mu_p = \mu_e} \qquad \text{with} \quad \mu_q = \frac{\partial \mathcal{H}}{\partial \rho_q}$$

We have performed the calculations with the full EoS. Moreover, if the Taylor expansion of $E_b(\rho, \delta)$ is used, we get

$$\mu_n - \mu_p = 2\frac{\partial E_b(\rho,\delta)}{\partial \delta} = 4\delta E_{\mathrm{sym},2}(\rho) + 8\delta^3 E_{\mathrm{sym},4}(\rho) + 12\delta^5 E_{\mathrm{sym},6}(\rho) + \mathcal{O}(\delta^7) = \mu_e.$$

Asymmetry and Pressure in β-stable Nuclear Matter





D1S: $L = 22.43$ MeV.
D280: $L = 46.53$ MeV.

The Thermodynamical Method

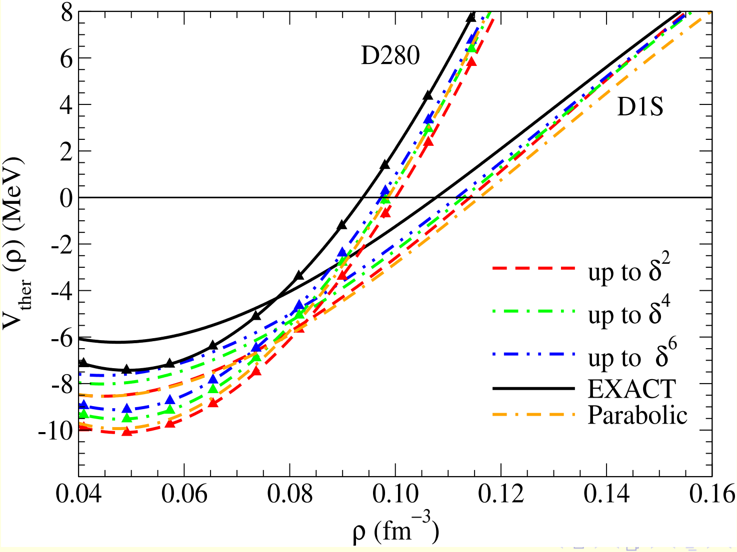
The thermodynamical method is driven by the mechanical and chemical stability conditions

$$-\left(\frac{\partial P}{\partial v}\right)_{\mu_{np}} > 0 \quad \text{and} \quad -\left(\frac{\partial \mu_{np}}{\partial q}\right)_{v} > 0.$$

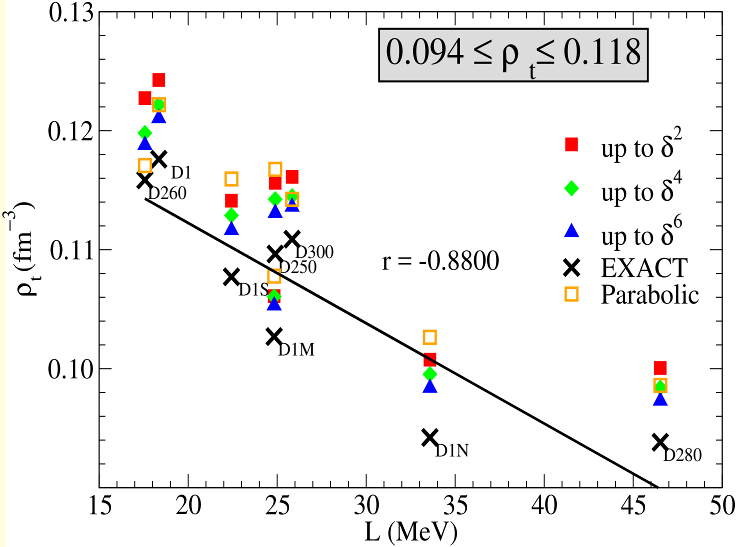
For nuclear matter in β-equilibrium, and treating the electrons as a free Fermi gas, the stability condition implies $V_{\text{ther}}(\rho) > 0$:

$$2\rho\frac{\partial E_b(\rho,x_p)}{\partial \rho} + \rho^2\frac{\partial^2 E_b(\rho,x_p)}{\partial \rho^2} - \left(\rho\frac{\partial^2 E_b(\rho,x_p)}{\partial \rho \partial x_p}\right)^2 \left(\frac{\partial^2 E_b(\rho,x_p)}{\partial x_p^2}\right)^{-1} > 0$$

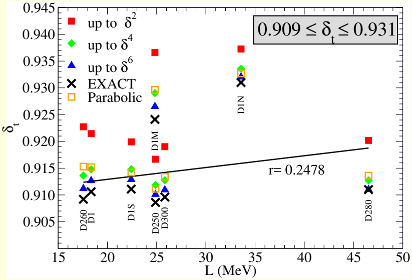
The Thermodynamical Potential

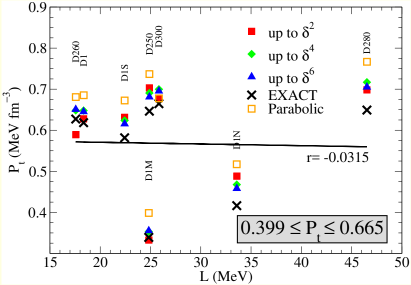


Neutron Star Core-Crust Transition: Density

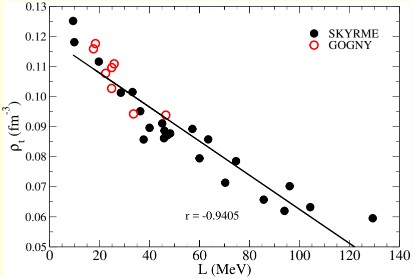


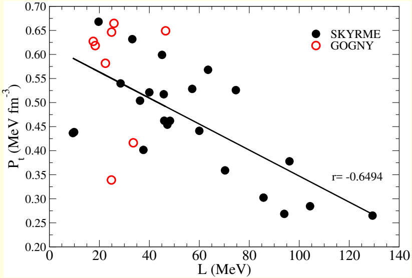
Neutron Star Core-Crust Transition: Assymetry and Pressure





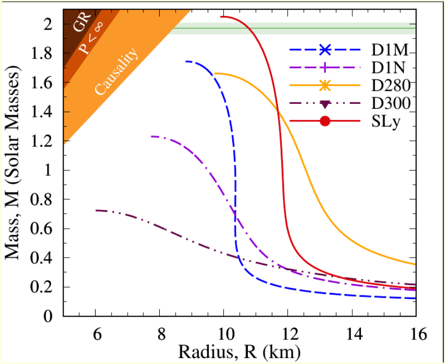
Comparison with Skyrme models





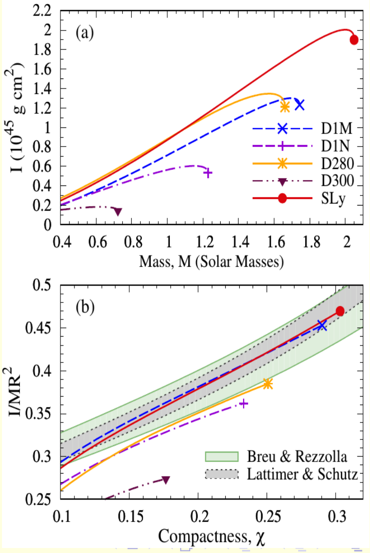
Bulk properties of the stars: Mass and Radius

- Integration of the TOV equations, using the exact β-equilibrium EoS.
- Use of a polytropic EoS for the inner crust P=a+b$\epsilon^{4/3}$.
- SLy unified EoS of DH.
- All Gogny functionals provide maximum NS masses below the observational limit ($M \approx 2M_{\odot}$): D1M: 1.76M$_{\odot}$, D280: 1.66M$_{\odot}$.
- Only D1M and D280 can generate masses above the canonical $M \approx 1.4M_{\odot}$.
- Gogny forces have not been fit to reproduce high-density matter.



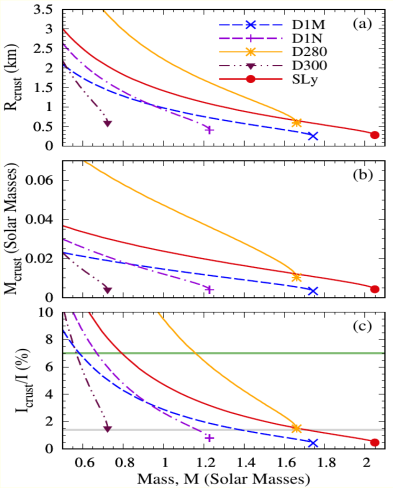
Bulk properties of the stars: Moment of Inertia

- I computed from the static mass distribution and gravitational potentials in the TOV equations.
- Moments of inertia are relatively small (below SLy).
- Case of D1M and D280, $I_{\rm max} \approx$ 1.3-1.4 $\times$ 10^{45} g cm^2, below the typical value of $I_{\rm max} \approx 2 \times 10^{45}$ g cm^2.
- Dimensionless quantity $I/(MR^2)$, scaled with the compactness $\chi = GM/R$. D1M results agree with the SLy results.

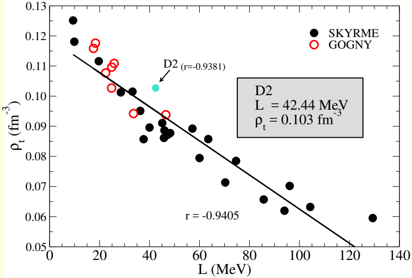


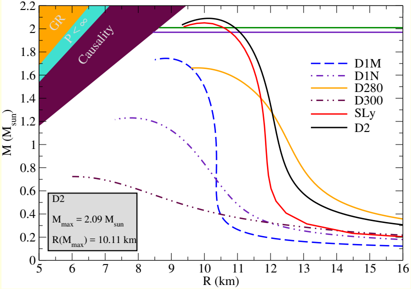
Crustal Properties

- Gogny interactions give good estimations for finite nuclei. Study of the extrapolation to neutron rich matter.
- Crust could be well described by them.
- D280 provides a thicker and heavier crust than D1M and SLy.
- D280 has higher moment of inertia than the other models.
- D1M comparable to SLy results.



Other Gogny interactions: the new D2 interaction.





Replacement of the density-dependent term of the original Gogny interaction

$$V_{\text{dens}}^{D1} = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2)\rho^\alpha\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right)$$

with

$$V_{\text{dens}}^{D2} = (W_3 + B_3 P_\sigma - H_3 P_\tau - M_3 P_\sigma P_\tau) \times \frac{e^{-\frac{(\mathbf{r}_1+\mathbf{r}_2)^2}{\mu_3^2}}}{(\mu_3\sqrt{\pi})^3} \frac{\rho^\alpha(\mathbf{r}_1) + \rho^\alpha(\mathbf{r}_2)}{2}$$

F. Chappert, N. Pillet, M. Girod, J.-F. Berger, PRC **91**, *034312 (2015)*

Conclusions

- If we look at the results for the transition properties using different orders of approximation, we observe that even when adding terms of higher orders the results may still be rather far from the ones calculated with the exact EoS.
- We find that Gogny interactions predict an anticorrelation in the transition density values with L, whereas the asymmetry and pressure do not present correlations with the slope parameter L.
- Only 4 Gogny forces provide numerically stable solutions of the TOV equations. Some of the bulk stellar properties predicted by the Gogny forces are incompatible with observations. In comparison with SLy results, D280 and D1M interactions seem to provide overall realistic results for the crust.
- More modern Gogny interactions, as for example the D2 interaction, can provide neutron stars with masses above the 2 solar mass limit.

Thank you for your time.

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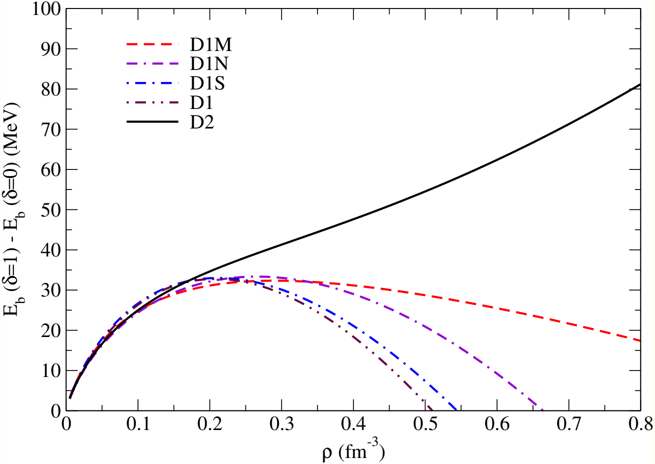
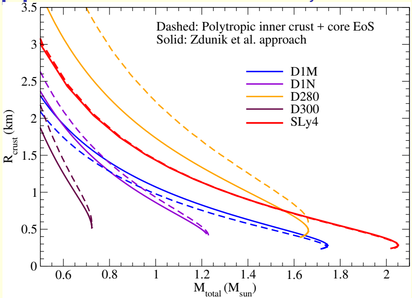
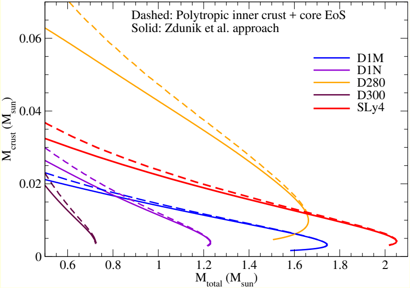
D2 Symmmetry energy



Approach of Zdunik, Fortin and Haensel





$$
\begin{aligned}
R_{\rm crust} &= \phi R_{\rm core} \frac{1 - 2GM/R_{\rm core}c^2}{1 - \phi\left(1 - 2GM/R_{\rm core}c^2\right)} \\
M_{\rm crust} &= \frac{4\pi P_t R_{\rm core}^4}{GM_{\rm core}} \left(1 - \frac{2DM_{\rm core}}{R_{\rm core}c^2}\right)
\end{aligned}
$$

with $\phi = \frac{\left((\mu_t/\mu_0)^2 - 1\right) R_{\rm core}c^2}{2GM}$

Astron. Astrophys. **599**, *A119 (2017)*

Saturation properties of nuclear matter studied using Gogny interactions. The saturation density ρ_0 has units of fm^{-3}, and all other properties have units of MeV.

Force	D1	D1S	D1M	D1N	D250	D260	D280	D300
ρ_0	0.167	0.163	0.165	0.161	0.158	0.160	0.153	0.156
E_0	−16.31	−16.01	−16.03	−15.96	−15.80	−16.26	−16.33	−16.22
K_0	229.37	202.88	224.98	225.65	249.41	259.49	285.20	299.14
$E_{\mathrm{sym},2}\ (\rho_0)$	30.70	31.13	28.55	29.60	31.54	30.11	33.14	31.23
$E_{\mathrm{sym},4}\ (\rho_0)$	0.76	0.45	0.69	0.21	0.43	1.20	1.18	0.80
$E_{\mathrm{sym},6}\ (\rho_0)$	0.20	0.16	0.24	0.15	0.16	0.27	0.29	0.20
L	18.36	22.43	24.83	33.58	24.90	17.57	46.53	25.84
L_4	1.75	−0.52	−1.04	−1.96	−0.33	4.73	4.36	2.62
L_6	0.46	0.08	0.42	0.08	0.09	0.99	1.19	0.63
$E_{\mathrm{sym}}^{PA}\ (\rho_0)$	31.91	31.95	29.73	30.14	32.34	31.85	35.89	32.44
L_{PA}	21.16	22.28	24.67	31.95	24.94	24.33	53.25	29.80

DERIVATION OF THE THERMODYNAMICAL METHOD

In order to obtain the transition between the core and the crust of the Neutron Star, we use the thermodynamical method, which presents the following mechanical and chemical stability conditions:

$$-\left(\frac{\partial P}{\partial v}\right)_{\mu_{np}} > 0 \tag{1}$$

and

$$-\left(\frac{\partial \mu_{np}}{\partial q}\right)_{v} > 0, \tag{2}$$

This method is the long-wavelength limit of the dynamical method, which requires the convexity of the energy per particle in the single phase, when neglecting the Coulomb interaction.

DERIVATION OF THE THERMODYNAMICAL METHOD

First we study the mechanical stability condition. Considering the electrons as a free Fermi gas, the pressure given by them is $P_e = \mu_e^4/(12\pi^2)$. Therefore, when calculating the derivative of the total pressure with respect to v it gives no contribution, and Eq. (1) can be rewritten as

$$-\left(\frac{\partial P_b}{\partial v}\right)_{\mu_{np}} > 0. \tag{3}$$

DERIVATION OF THE THERMODYNAMICAL METHOD

Knowing that in β-stable matter the proton fraction is a function of the density, $x_p(\rho)$, and that $\mu_{np} = 2\partial E_b/\partial\delta = -\partial E_b/\partial x_p$, we can rewrite the mechanical stability condition as

$$-\left(\frac{\partial P_b}{\partial v}\right)_{\mu_{np}} = \rho^2\left[2\rho\frac{\partial E_b(\rho,x_p)}{\partial\rho} + \rho^2\frac{\partial^2 E_b(\rho,x_p)}{\partial\rho^2} - \frac{\left(\rho\frac{\partial^2 E_b(\rho,x_p)}{\partial\rho\partial x_p}\right)^2}{\frac{\partial^2 E_b(\rho,x_p)}{\partial x_p^2}}\right] > 0. \quad (4)$$

DERIVATION OF THE THERMODYNAMICAL METHOD

For the chemical stability condition in Eq. (2), we know that the chemical charge is $q = x_p - \rho_e/\rho$, where $\rho_e = \mu_e^3/(3\pi^2)$ is the density of electrons. Then, we can express the inequality in Eq. (2) as

$$-\left(\frac{\partial q}{\partial \mu_{np}}\right)_v = \left[\frac{\partial^2 E_b(\rho, x_p)}{\partial x_p^2}\right]^{-1} + \frac{\mu_e^2}{\pi^2 \rho} > 0. \qquad (5)$$

Given that Eq. (5) is usually valid, the stability condition for β-stable matter becomes

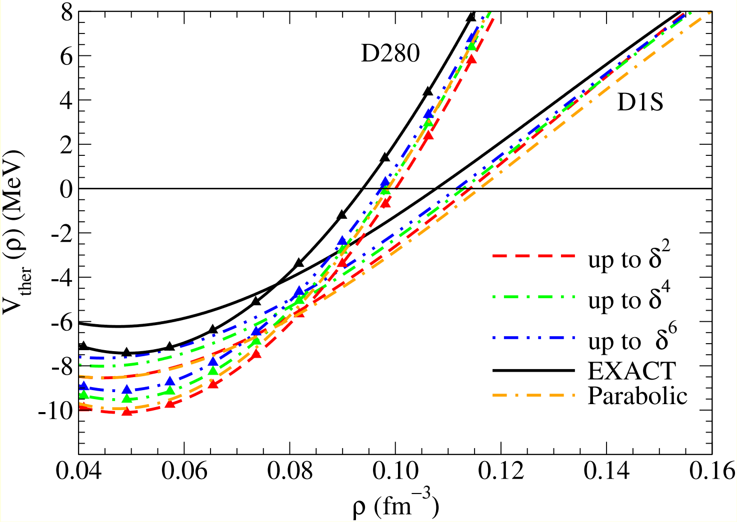
$$V_{\text{ther}} = 2\rho \frac{\partial E_b(\rho, x_p)}{\partial \rho} + \rho^2 \frac{\partial^2 E_b(\rho, x_p)}{\partial \rho^2} - \left(\rho \frac{\partial^2 E_b(\rho, x_p)}{\partial \rho \partial x_p}\right)^2 \left(\frac{\partial^2 E_b(\rho, x_p)}{\partial x_p^2}\right)^{-1} > 0. \qquad (6)$$

DERIVATION OF THE THERMODYNAMICAL METHOD

Rewriting the condition V_{ther} in terms of the Taylor expansion in δ one obtains

$$
\begin{aligned}
V_{\mathrm{ther}} &= \rho^2 \frac{\partial^2 E_b(\rho, \delta = 0)}{\partial \rho^2} + 2\rho \frac{\partial E_b(\rho, \delta = 0)}{\partial \rho} \\
&+ \sum_{2k} \delta^{2k} \left(\rho^2 \frac{\partial^2 E_{\mathrm{sym},2k}(\rho)}{\partial \rho^2} + 2\rho \frac{\partial E_{\mathrm{sym},2k}(\rho)}{\partial \rho} \right) \\
&- \rho^2 \left(\sum_{2k} 2k \delta^{2k-1} \frac{\partial E_{\mathrm{sym},2k}(\rho)}{\partial \rho} \right)^2 \\
&\times \left[\sum_{2k} 2k(2k-1) \delta^{2k-2} E_{\mathrm{sym},2k}(\rho) \right]^{-1} > 0.
\end{aligned} \tag{7}
$$

DERIVATION OF THE THERMODYNAMICAL METHOD



TRANSITION VALUES.

Force	D1	D1S	D1M	D1N	D250	D260	D280	D300
$\delta_t^{\delta 2}$	0.9215	0.9199	0.9366	0.9373	0.9167	0.9227	0.9202	0.9190
$\delta_t^{\delta 4}$	0.9148	0.9148	0.9290	0.9336	0.9119	0.9136	0.9127	0.9128
$\delta_t^{\delta 6}$	0.9127	0.9129	0.9265	0.9321	0.9101	0.9112	0.9110	0.9110
$\delta_t^{\mathrm{exact}}$	0.9106	0.9111	0.9241	0.9310	0.9086	0.9092	0.9110	0.9096
δ_t^{PA}	0.9152	0.9142	0.9296	0.9327	0.9111	0.9153	0.9136	0.9134
$\rho_t^{\delta 2}$	0.1243	0.1141	0.1061	0.1008	0.1156	0.1228	0.1001	0.1161
$\rho_t^{\delta 4}$	0.1222	0.1129	0.1061	0.0996	0.1143	0.1198	0.0984	0.1145
$\rho_t^{\delta 6}$	0.1211	0.1117	0.1053	0.0984	0.1131	0.1188	0.0973	0.1136
ρ_t^{exact}	0.1176	0.1077	0.1027	0.0942	0.1097	0.1159	0.0938	0.1109
ρ_t^{PA}	0.1222	0.1160	0.1078	0.1027	0.1168	0.1168	0.0986	0.1142
$P_t^{\delta 2}$	0.6279	0.6316	0.3325	0.4880	0.7032	0.5891	0.6984	0.6775
$P_t^{\delta 4}$	0.6479	0.6239	0.3531	0.4676	0.6906	0.6482	0.7169	0.6998
$P_t^{\delta 6}$	0.6452	0.6156	0.3553	0.4581	0.6810	0.6508	0.7051	0.6955
P_t^{exact}	0.6184	0.5817	0.3391	0.4164	0.6464	0.6273	0.6492	0.6647
P_t^{PA}	0.6853	0.6725	0.3985	0.5172	0.7367	0.6808	0.7667	0.6777

$E_b(\rho,\delta)$ of ANM, obtained using the Gogny interaction.

$$E_b(\rho,\delta) = E_b^{\rm kin}(\rho,\delta) + E_b^{\rm zr}(\rho,\delta) + E_b^{\rm dir}(\rho,\delta) + E_b^{\rm exch}(\rho,\delta),$$

where each one of them reads as follows

$$E_b^{\rm kin}(\rho,\delta) = \frac{3\hbar^2}{20m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3}\left[(1+\delta)^{5/3}+(1-\delta)^{5/3}\right]$$

$$E_b^{\rm zr}(\rho,\delta) = \frac{1}{8}t_3\rho^{\alpha+1}\left[2(x_3+2)-(2x_3+1)(1+\delta^2)\right]$$

$$E_b^{\rm dir}(\rho,\delta) = \frac{1}{2}\sum_{i=1,2}\mu_i^3\pi^{3/2}\left[\mathcal{A}_i\rho+\mathcal{B}_i\rho\delta^2\right]$$

$$
\begin{aligned}
E_b^{\mathrm{exch}}(\rho,\delta) &= -\sum_{\mathrm{i}=1,2} \frac{1}{\mu_i^3 k_F^3} \left\{ \frac{\mathcal{C}_i}{2} \left[F(\mu_i k_{Fn}) + F(\mu_i k_{Fp}) \right] \right. \\
& -\mathcal{D}_i \sum_{s=\pm 1} s \left[\frac{\sqrt{\pi}}{4} \mu_i^3 (k_{Fn} + s k_{Fp}) \right. \\
& \times (k_{Fn}^2 + k_{Fp}^2 - s k_{Fn} k_{Fp}) \mathrm{erf}\left(\frac{\mu_i}{2} (k_{Fn} + s k_{Fp}) \right) \\
& \left. \left. + \left(\frac{\mu_i^2}{2} \left(k_{Fn}^2 + k_{Fp}^2 - s k_{Fn} k_{Fp} \right) - 1 \right) e^{-\mu_i^2 (k_{Fn} + s k_{Fp})^2 / 4} \right] \right\},
\end{aligned}
$$

with

$$
F(\eta) = \frac{\sqrt{\pi}}{2} \eta^3 \mathrm{erf}(\eta) + \left(\frac{\eta^2}{2} - 1 \right) e^{-\eta^2} - \frac{3\eta^2}{2} + 1
$$

and the error function defined as

$$
\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt e^{-t^2}.
$$

The constants $\mathcal{A}_i$ and $\mathcal{B}_i$ define, respectively, the isoscalar and isovector part of the direct term, and in the exchange part of the energy $E_b(\rho, \delta)$, $\mathcal{C}_i$ relates to the interaction between particles with the same isospin (neutron-neutron or proton-proton) whereas the constant $\mathcal{D}_i$ considers interactions between particles with different isospin (neutron-proton, proton-neutron).

$$\begin{aligned}
\mathcal{A}_i &= \frac{1}{4}\left(4W_i + 2B_i - 2H_i - M_i\right) \\
\mathcal{B}_i &= -\frac{1}{4}\left(2H_i + M_i\right) \\
\mathcal{C}_i &= \frac{1}{\sqrt{\pi}}\left(W_i + 2B_i - H_i - 2M_i\right) \\
\mathcal{D}_i &= \frac{1}{\sqrt{\pi}}\left(H_i + 2M_i\right).
\end{aligned}$$

SYMMETRY ENERGIES UP TO SIXTH ORDER

$$
\begin{aligned}
E_{\mathrm{sym},2}(\rho) &= \frac{\hbar^2}{6m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3} - \frac{1}{8}t_3\rho^{\alpha+1}(2x_3+1) + \frac{1}{2}\sum_{i=1,2}\mu_i^3\pi^{3/2}\mathcal{B}_i\rho \\
&+ \frac{1}{6}\sum_{i=1,2}\left[-\mathcal{C}_i G_1(\mu_i k_F) + \mathcal{D}_i G_2(\mu_i k_F)\right] \\
E_{\mathrm{sym},4}(\rho) &= \frac{\hbar^2}{162m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3} \\
&+ \sum_{i=1,2}\frac{1}{324}\left[\mathcal{C}_i G_3(\mu_i k_F) + \mathcal{D}_i G_4(\mu_i k_F)\right] \\
E_{\mathrm{sym},6}(\rho) &= \frac{7\hbar^2}{4374m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3} \\
&+ \sum_{i=1,2}\frac{1}{43740}\left[\mathcal{C}_i G_5(\mu_i k_F) - \mathcal{D}_i G_6(\mu_i k_F)\right]
\end{aligned}
$$

SYMMETRY ENERGIES UP TO SIXTH ORDER

Splitting the contributions given by the like or unlike isospin interactions, we have defined the following functions $G_j(\eta)$:

$$
\begin{aligned}
G_1(\eta) &= \frac{1}{\eta} - \left(\eta + \frac{1}{\eta}\right) e^{-\eta^2} \\
G_2(\eta) &= \frac{1}{\eta} - \frac{1}{\eta} e^{-\eta^2} - \eta \\
G_3(\eta) &= -\frac{14}{\eta} + e^{-\eta^2} \left(\frac{14}{\eta} + 14\eta + 7\eta^3 + 2\eta^5\right) \\
G_4(\eta) &= \frac{14}{\eta} - 8\eta + \eta^3 - 2e^{-\eta^2} \left(\frac{7}{\eta} + 3\eta\right) \\
G_5(\eta) &= -\frac{910}{\eta} + e^{-\eta^2} \left(\frac{910}{\eta} + 910\eta + 455\eta^3 + 147\eta^5 + 32\eta^7 + 4\eta^9\right) \\
G_6(\eta) &= -\frac{910}{\eta} + 460\eta - 65\eta^3 + 3\eta^5 + e^{-\eta^2} \left(\frac{910}{\eta} + 450\eta + 60\eta^3\right).
\end{aligned}
$$