NN-Correlations in the spin symmetry energy of neutron matter

Symmetry energy of nuclear matter

- Spin symmetry energy of neutron matter.
- Kinetic and potential energy contributions.

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Study of spin polarized nuclear matter and finite nuclei with finite range simple effective interaction

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He seems like a good boy !

Meeting of Young spanish physicists

Enjoying life Inside a neutron star!

70 Birthday!

Finally a serious guy!

So many collaborators!

Thanks Xavier !!!!

You Did a great Job !!!!

A great effort is being devoted to study the properties of asymmetric nuclear systems both from experimental and theoretical points of view using both effective and realistic interactions.

Large dispersion in the results ➔

"ab initio" calculations could be a safe way to study these systems. However, this procedure could mean different things …

1. Choose degrees of freedom: nucleons
2. Choose interaction: Realistic phase-shift equivalent two-body potential (CDBONN, Av18, chiral forces (N3LO,…).
3. Select three-body force

With these ingredients we build a non-relativistic Hamiltonian ===> Many-body Schrodinger equation. To solve this equation (ground or excited states) one needs a sophisticated many-body machinery.

We need as good as posible many-body theory to eliminate uncertainties!

Remember:

Nucleon-nucleon interaction is not uniquely defined.
Complicated channel structure. Tensor component of the nuclear force.
Already the deuteron is complicated.

Perturbative methods: Due to the short-range structure of a realistic potential == > infinite partial summations. Diagrammatic notation is useful.

Brueckner-Hartree-Fock. G-matrix

Self- Consistent Green's function (SCGF)

Variational methods as FHNC or VMC

$$\Psi(1,\ldots,N) = F(1,\ldots,N)\phi(1,\ldots,N)$$

$$F(1,\ldots N) = \prod_{i<j} f^{(2)}(ij)$$

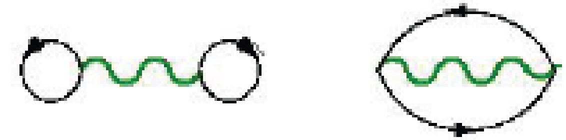
$$E = \frac{\langle \Psi \mid H \mid \Psi \rangle}{\langle \Psi \mid \Psi \rangle}$$

Quantum Monte Carlo: GFMC and AFDMC. Simulation box with a finite number of particles. Special method for sampling the operatorial correlations.

BHF approximation of ANM

Energy per particle

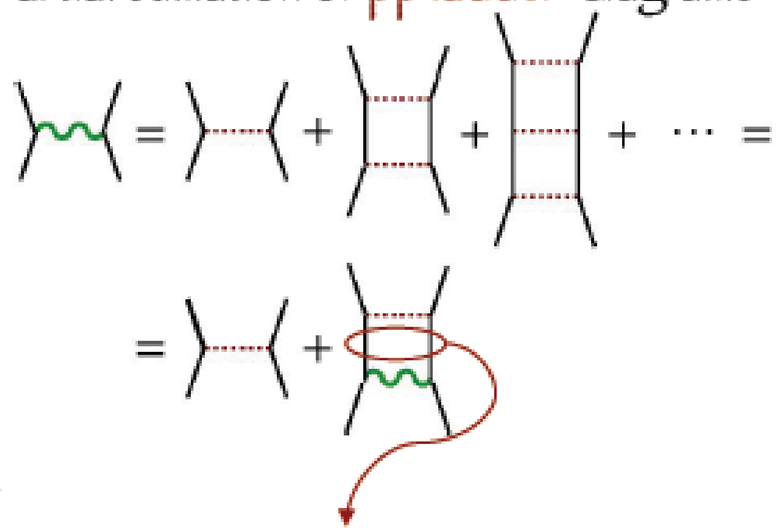
- $$\frac{E}{A}(\rho,\beta)=\frac{1}{A}\sum_{\tau}\sum_{k\le k_{F_\tau}}\left(\frac{\hbar^2k^2}{2m_\tau}+\frac{1}{2}\mathrm{Re}\left[U_\tau(\vec{k})\right]\right)$$



Infinite sumation of two-hole line diagrams

Bethe-Goldstone Equation

Partial sumation of pp ladder diagrams



- $$G(\omega)=V+V\frac{Q}{\omega-E-E'+i\eta}G(\omega)$$
- $$E_\tau(k)=\frac{\hbar^2k^2}{2m_\tau}+\mathrm{Re}\left[U_\tau(k)\right]$$
- $$U_\tau(k)=\sum_{\tau'}\sum_{k'\le k_{F_{\tau'}}}\left\langle \vec{k}\vec{k}'\left|G(\omega=E_\tau(k)+E_{\tau'}(k'))\right|\vec{k}\vec{k}'\right\rangle_A$$

✓ Pauli blocking

✓ Nucleon dressing

The microscopic study of nuclear systems requires a rigorous treatment of the nucleon-nucleon (NN) correlations.

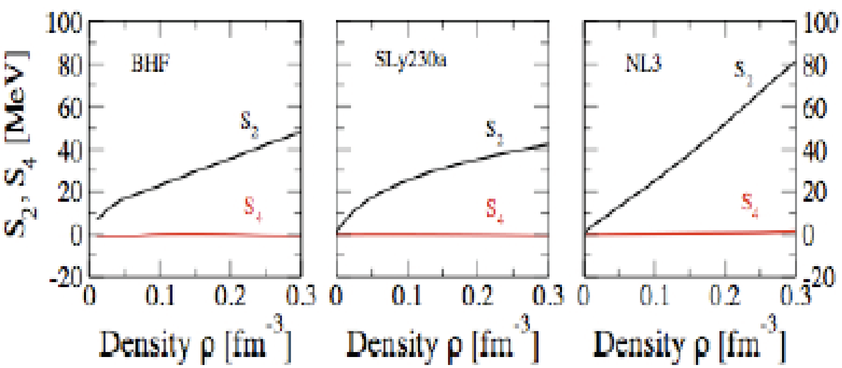
- **Strong short range repulsion and tensor components in realistic interactions, to fit NN scattering data, produce important modifications of the nuclear wave function.**
- **Simple Hartree-Fock for nuclear matter at the empirical saturation density using such realistic NN interactions provides positive energies rather than the empirical -16 MeV per nucleon.**
- **The effects of correlations appear also in the single-particle properties:Partial occupation of the single particle states which would be fully occupied in a mean field description and a wide distribution in energy of the single-particle strength. The departure of n(k) from the step function gives a measure of the importance of correlations.**

NN-interactions act differently in symmetric nuclear matter than in neutron matter. A "measure" of this isospin dependence is provided by the symmetry energy.

Charge symmetry ➔ expansion of $(E/A)_{ANM}$ on even powers of isospin asymmetry $\beta=(\rho_n-\rho_p)/(\rho_n+\rho_p)$

$$\frac{E}{A}(\rho,\beta) = E_{SNM}(\rho) + S_2(\rho)\beta^2 + S_4(\rho)\beta^4 + O(6)$$

$$E_{SNM}(\rho) = \frac{E}{A}(\rho,\beta=0), \quad S_2(\rho) = \frac{1}{2}\frac{\partial^2 E/A}{\partial\beta^2}\bigg|_{\beta=0}, \quad S_4(\rho) = \frac{1}{24}\frac{\partial^4 E/A}{\partial\beta^4}\bigg|_{\beta=0}$$



In good approximation:

$$S_2(\rho) \sim \frac{E}{A}(\rho,\beta=1) - \frac{E}{A}(\rho,\beta=0)$$

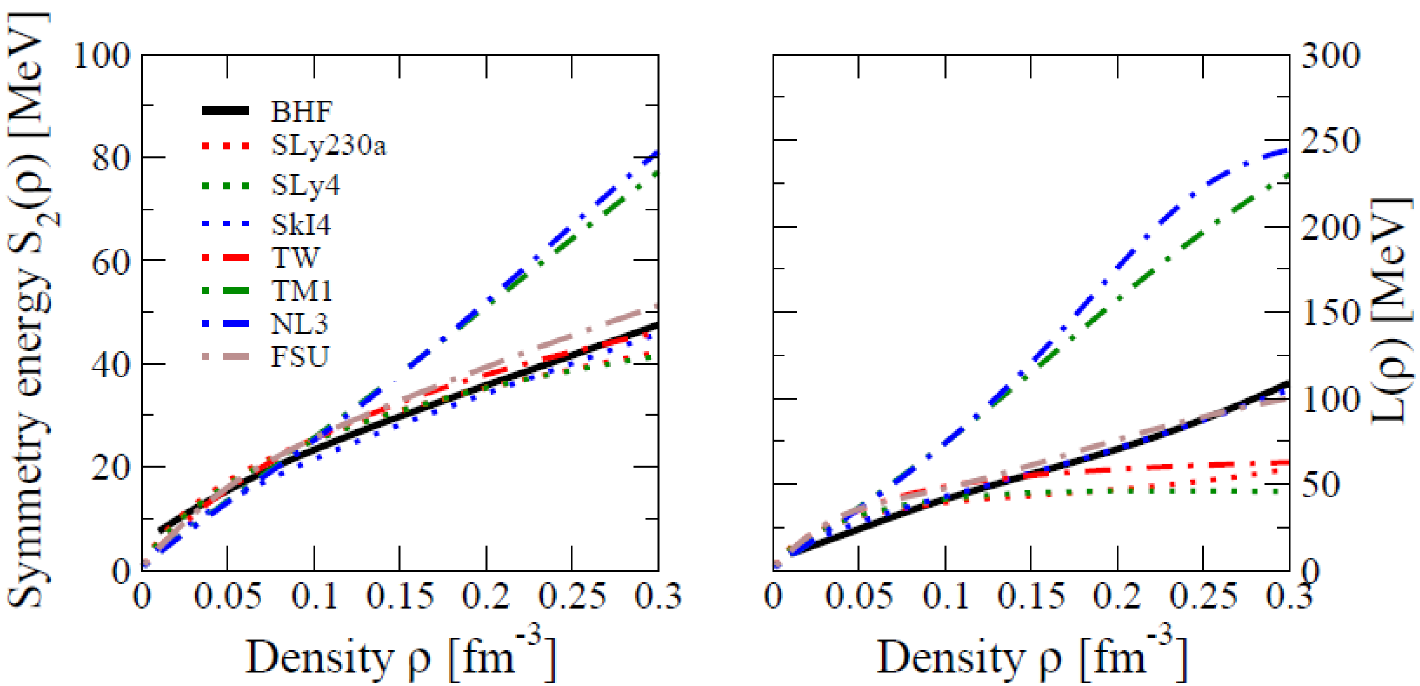
The behavior of the symmetry energy around saturation can be also characterized in terms of a few bulk parameters.

$$S_2(\rho) = E_{sym} + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_{sym}}{3!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \mathcal{O}(4)$$

$$E_{sym} \sim 30-32 \text{ MeV} \qquad L = 3\rho_0 \frac{\partial S_2(\rho)}{\partial \rho}\Big|_{\rho=\rho_0}$$

$$K_{sym} = 9\rho_0^2 \frac{\partial^2 S_2(\rho)}{\partial \rho^2}\Big|_{\rho=\rho_0} \qquad Q_{sym} = 27\rho_0^3 \frac{\partial^3 S_2(\rho)}{\partial \rho^3}\Big|_{\rho=\rho_0}$$

PRC80,045806 (2009)



Model	ρ_0	E_0	K_0	Q_0	E_{sym}	L	K_{sym}	Q_{sym}	K_τ	Ref.
BHF (with TBFa)	0.187	-15.23	195.5	-280.9	34.3	66.5	-31.3	-112.8	-334.7	
BHF (with TBFb)	0.176	-14.62	185.9	-224.9	33.6	66.9	-23.4	-162.8	-343.8	
BHF (without TBF)	0.240	-17.30	213.6	-225.1	35.8	63.1	-27.8	-159.8	-339.6	
SLy4	0.159	-15.97	229.8	-362.9	31.8	45.3	-119.8	520.8	-320.4	[8]
SLy10	0.155	-15.90	229.7	-358.3	32.1	39.2	-142.4	590.9	-316.7	[9]
SLy230a	0.160	-15.98	229.9	-364.2	31.8	43.9	-98.4	602.8	-292.7	[10]
SkI4	0.162	-16.15	250.3	-335.7	29.6	59.9	-43.4	358.8	-322.5	[37]
SkI5	0.156	-15.84	255.6	-301.7	36.4	128.9	159.8	11.2	-461.6	[37]
SkI6	0.159	-15.88	248.2	-326.7	34.4	82.1	-0.9	332.3	-385.8	[38]

PRC80,045806 (2009)

We have observed the isospin dependence, either of the effective interactions or of the realistic interactions looking at the results (mean field and BHF) for nuclear and neutron matter.

A new insight of the importance of correlations can be provided by analyzing the kinetic and potential energy contributions to the symmetry energy and the contribution of the different components of the potential

BHF provides the energy correction to the non-interacting system (free Fermi sea) but do not provide the separate contributions of the kinetic energy and potential energy in the correlated many-body state.

By construction, mean field calculations, with effective interactions, do not give access to the high-momentum components.
Their associated n(k) are just uncorrelated step-functions.

We study the isospin dependence of the NN interactions by comparing the results for nuclear and neutron matter.

Which components of the interaction are responsible for the symmetry energy?

How high-momentum components produced by NN-correlations affect the symmetry energy?

If correlations are measured by the departure of n(k) from the step function. Which system is more correlated nuclear matter or neutron matter ?

A new insight of the importance of correlations can be provided by analyzing the kinetic and potential energy contribution to the symmetry energy and the contribution of the different components of the potential.

BHF provides the energy correction to the non-interacting system (free Fermi sea) but do not provide the separate contributions of the kinetic and potential energies in the correlated many-body state.

The Hellmann-Feynman theorem in conjunction with BHF can be used to estimate the "real" kinetic energy.

Hellmann-Feynman theorem:
Consider a Hamiltonian depending on a paremeter

$$\frac{dE_\lambda}{d\lambda} = \frac{\left\langle \psi_\lambda \middle| \frac{d\hat{H}_\lambda}{d\lambda} \middle| \psi_\lambda \right\rangle}{\left\langle \psi_\lambda \middle| \psi_\lambda \right\rangle}$$

The nuclear Hamiltonian can be decomposed in a kinetic and a potential Energy pieces:

$$\hat{H} = \hat{T} + \hat{V}$$

Defining a λ depending Hamiltonian:

$$\hat{H}_\lambda = \hat{T} + \lambda \hat{V}$$

The expectation value of the potential energy

$$\left\langle \hat{V} \right\rangle = \frac{\left\langle \psi \middle| \hat{V} \middle| \psi \right\rangle}{\left\langle \psi \middle| \psi \right\rangle} = \left(\frac{dE_\lambda}{d\lambda} \right) \Bigg|_{\lambda=1}$$

For Av18+Urbana IX three-body force at saturation density

$$\rho_0 = 0.187 \text{ fm}^{-3}$$

	E_{NM}	E_{SM}	E_{sym}	L
<T>	53.321	54.294	-0.973	14.896
<V>	-34.251	-69.524	35.273	51.604
Total	19.070	-15.230	34.300	66.500

The main contribution to both E_{sym} and L is due to the potential energy

PRC84, 062801(R), 2011

The kinetic contribution to E_{sym} is very small and negative

In contrast, the FFG approach to the symmetry energy is ~ 14.4 MeV.
The contribution to L is smaller than the FFG which amounts ~29.2 MeV

E_{FFG}(SM) = 24.53 MeV, T(SM) - EFFG(SM) =29.76 MeV

E_{FFG}(NM) = 38.94 MeV, T(NM) - EFFG(NM) =14.38 MeV

PRC84, 062801(R), 2011

(S,T)	E_{NM}	E_{SM}	E_{sym}	L
$(0,0)$	0	5.600	−5.600	−21.457
$(0,1)$	−29.889	−23.064	−6.825	−17.950
$(1,0)$	0	−49.836	49.836	90.561
$(1,1)$	−4.362	−2.224	−2.138	0.450

TABLE III: Spin (S) and isospin (T) channel decomposition of the potential part of E_{NM}, E_{SM}, E_{sym} and L. Units are given in MeV.

✓ Largest contribution from S=1, T=0 channel

✓ Similar T=1 channel contributions to E_{NM} and E_{SM} which almost cancel out in E_{sym}

	E_{NM}	E_{SM}	E_{sym}	L
$\langle V_1\rangle$	-31.212	-32.710	1.498	-5.580
$\langle V_{\vec\tau_i\cdot\vec\tau_j}\rangle$	-4.957	3.997	-8.954	-20.383
$\langle V_{\vec\sigma_i\cdot\vec\sigma_j}\rangle$	-0.319	-0.382	0.063	2.392
$\langle V_{(\vec\sigma_i\cdot\vec\sigma_j)(\vec\tau_i\cdot\vec\tau_j)}\rangle$	-5.724	-11.388	5.664	2.521
$\langle V_{S_{ij}}\rangle$	-0.792	1.912	-2.704	-4.998
$\langle V_{S_{ij}(\vec\tau_i\cdot\vec\tau_j)}\rangle$	-4.989	-37.592	32.603	47.095
$\langle V_{\vec L\cdot\vec S}\rangle$	-7.538	-1.754	-5.784	-12.251
$\langle V_{\vec L\cdot\vec S(\vec\tau_i\cdot\vec\tau_j)}\rangle$	-2.671	-6.539	3.868	3.969
$\langle V_{L^2}\rangle$	11.850	13.610	-1.760	1.521
$\langle V_{L^2(\vec\tau_i\cdot\vec\tau_j)}\rangle$	-2.788	0.270	-3.058	-14.262
$\langle V_{L^2(\vec\sigma_i\cdot\vec\sigma_j)}\rangle$	1.265	1.383	-0.118	1.405
$\langle V_{L^2(\vec\sigma_i\cdot\vec\sigma_j)(\vec\tau_i\cdot\vec\tau_j)}\rangle$	0.051	0.008	0.043	-0.341
$\langle V_{(\vec L\cdot\vec S)^2}\rangle$	4.194	5.682	-1.488	-0.327
$\langle V_{(\vec L\cdot\vec S)^2(\vec\tau_i\cdot\vec\tau_j)}\rangle$	5.169	-6.190	11.359	31.368
$\langle V_{T_{ij}}\rangle$	0.003	0.039	-0.036	-0.022
$\langle V_{(\vec\sigma_i\cdot\vec\sigma_j)T_{ij}}\rangle$	-0.017	-0.106	0.089	0.042
$\langle V_{S_{ij}T_{ij}}\rangle$	0.004	0.079	-0.075	-0.124
$\langle V_{(\tau_{z_i}+\tau_{z_j})}\rangle$	-0.084	-0.001	-0.083	-0.331
$\langle U_1\rangle$	2.985	3.251	-0.266	-0.630
$\langle U_{(\vec\sigma_i\cdot\vec\sigma_j)(\vec\tau_i\cdot\vec\tau_j)}\rangle$	2.254	3.999	-1.745	-7.228
$\langle U_{S_{ij}(\vec\tau_i\cdot\vec\tau_j)}\rangle$	-0.935	-7.092	6.157	27.768

PRC84, 062801(R), 2011

Separate contributions from the various components of Av18 and the two-body reduced Urbana force. All energies in MeV

First Summary

❖ At the same density, neutron matter is less correlated than nuclear matter. The variation of kinetic energy respect to the FFG is smaller for neutron matter than for nuclear matter.

❖The kinetic symmetry energy is very small (compared with the FFG) and could be even negative. The potential part of the symmetry energy is very large. The main contribution coming from the tensor part of the NN interaction and the partial waves where the tensor is acting.

❖The kinetic and the potential energy have a quadratic dependence on the asymmetry paremeter.

The BHF values for the symmetry energy and L (calculated with the Av18 and a Urbana IX three-body force are compatible with the experimental determinations.

❖Three-body forces do not change the qualitative behavior.

Polarized Neutron Matter

$$\rho = \rho_\uparrow + \rho_\downarrow \qquad \Delta = \frac{\rho_\uparrow - \rho_\downarrow}{\rho}$$

Energy expanded on the spin polarization

$$E(\rho, \Delta) = E_{NP}(\rho) + S_{sym}(\rho)\Delta^2 + S_4(\rho)\Delta^4 + \mathcal{O}(6)$$

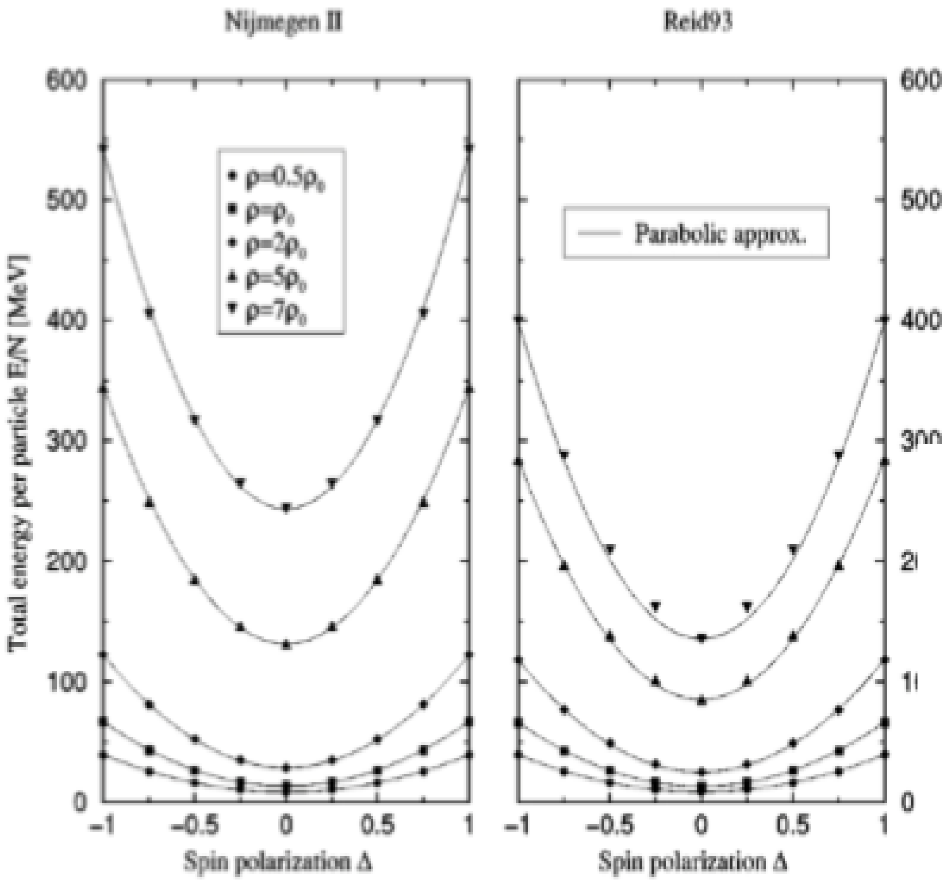
$$S_{sym}(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\Delta)}{\partial \Delta^2}\Big|_{\Delta=0}$$

Spin symmetry energy directly related to the invers of the magnetic susceptibility

$$S_4(\rho) = \frac{1}{24}\frac{\partial^4 E(\rho,\Delta)}{\partial \Delta^4}\Big|_{\Delta=0}$$

Can be neglected. Also higher terms can be neglected!

$$S_{sym}(\rho) \sim E_{TP}(\rho) - E_{NP}(\rho)$$



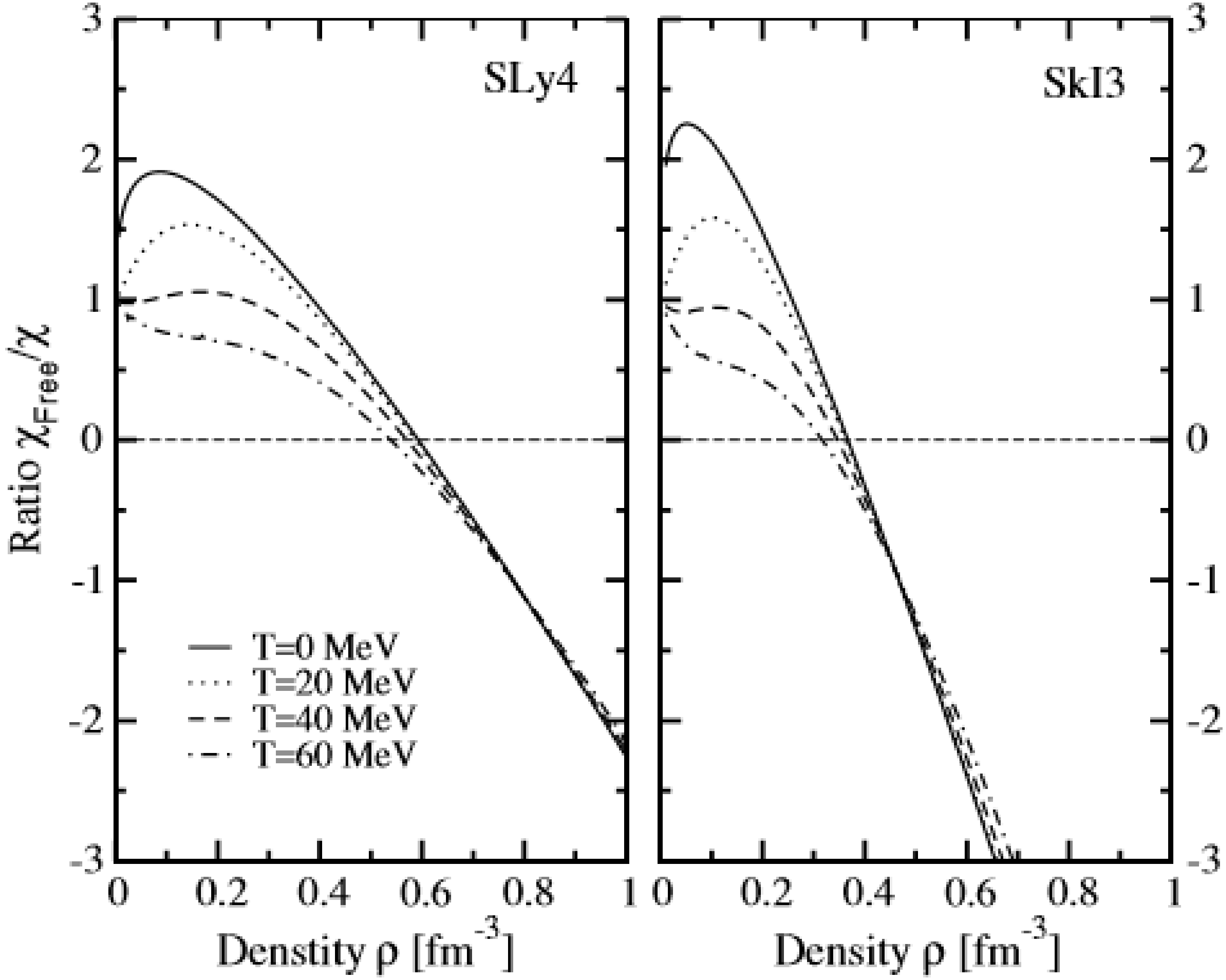
$$S_{sym}(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\Delta)}{\partial \Delta^2}\bigg|_{\Delta=0} \sim E_{TP}(\rho) - E_{NP}(\rho)$$

Magnetic Suceptibility

$$\chi(\rho) = \frac{\mu^2 \rho}{\partial^2 E(\rho,\Delta)/\partial\Delta^2\big|_{\Delta=0}} = \frac{\mu^2\rho}{2S_{sym}(\rho)}$$

In the same spirit of nuclear matter one can define

$$L_S(\rho) = 3\rho\frac{\partial S_{sym}(\rho)}{\partial\rho}$$


SLy4
SkI3
Ratio χ_{Free}/χ
Density ρ [fm^{-3}]
T=0 MeV
T=20 MeV
T=40 MeV
T=60 MeV
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1

	E_{TP}	E_{NP}	S_{sym}	L_S
$\langle T_{FS} \rangle$	55.669	35.069	20.600	41.200
$\langle T \rangle$	64.452	47.827	16.625	25.225
$\langle V \rangle$	−4.784	−31.050	26.266	75.914
Total	59.668	16.777	42.891	101.139

$$\rho_0 = 0.16 \text{ fm}^{-3}$$

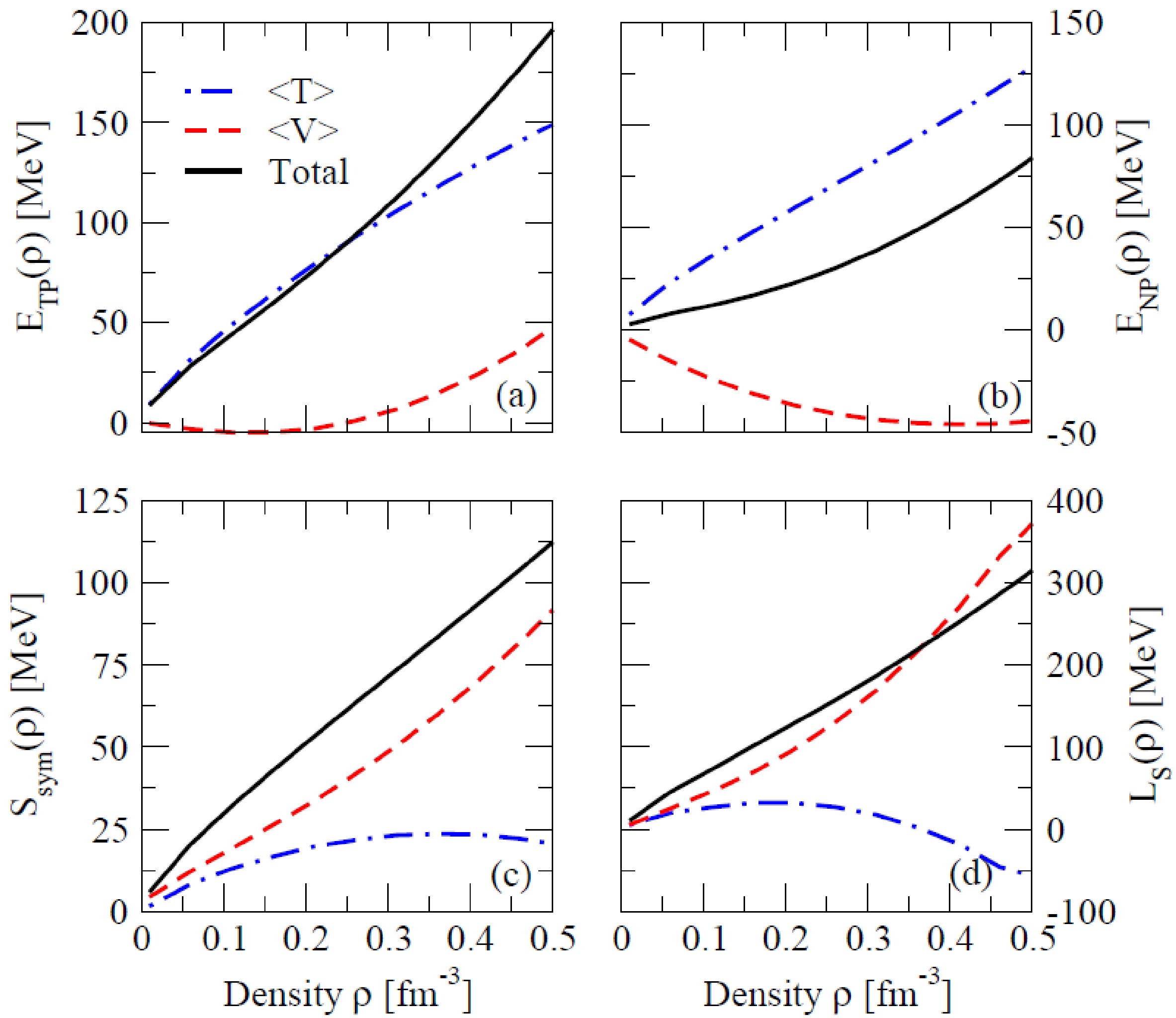
Kinetic <T>, and potential <V> contributions to the total energy per particle of totally polarized (TP) and non-polarized (NP) neutron matter at the empirical saturation density of symmetric nuclear matter.

$< T_{FS} >$ **Results of the underlying Fermi seas.**

<V> is small for TP due to the absence of S=0 channels and to the cancellations in the S=1 channel between different partial waves.

Slope of the spin symmetry energy

$$L_S(\rho) = 3\rho \frac{\partial S_{sym}(\rho)}{\partial \rho}$$



<V> for TP is small.
<V> for NP is attractive.
However, both TP and NP are not bound.

Quite linear behavior of the spin symmetry energy in a wide range of densities. One can use L to parametrize the density dependence.

Spin Channel	$\langle V\rangle_{TP}$	$\langle V\rangle_{NP}$	$S_{sym}^{(\langle V\rangle)}$	$L_S^{(\langle V\rangle)}$
$S=0$	0	-26.875	26.875	56.198
$S=1$	-4.784	-4.175	-0.609	19.716

✓ Largest contribution from S=0 (almost all $S_{sym}^{(<V>)}$ & ~ 70% of $L_s^{(<V>)}$) in particular from the $^1S_0$ and $^1D_2$ partial waves

The S=0 channel is not active in TP

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Partial wave	$\langle V \rangle_{TP}$	$\langle V \rangle_{NP}$	$S_{sym}^{(\langle V \rangle)}$	$L_S^{(\langle V \rangle)}$
$^1S_0$	0.000	−21.432	21.432	32.086
$^3P_0$	−5.499	−4.624	−0.875	3.313
$^3P_1$	19.644	13.027	6.617	30.927
$^3P_2$	−19.915	−13.299	−6.616	−14.966
$^1D_2$	0.000	−4.787	4.787	21.185
$^3F_2$	−1.263	−0.574	−0.689	−2.655
$^3F_3$	3.109	1.639	1.470	5.253
$^3F_4$	−1.726	−0.597	−1.129	−5.492
$^1G_4$	0.000	−0.607	0.607	3.055
$^3H_4$	−0.042	0.012	−0.054	−0.094
$^3H_5$	0.699	0.186	0.513	1.889
$^3H_6$	−0.028	0.024	−0.052	−0.345
$^1I_6$	0.000	−0.059	0.059	0.116
$^3J_6$	0.051	0.024	0.027	0.370
$^3J_7$	0.107	−0.025	0.132	0.476
$^3J_8$	0.050	0.020	0.030	0.332
$^1K_8$	0.000	0.011	−0.011	0.245
$^3L_8$	0.029	0.011	0.018	0.219

→ Tensor force plays a less important role for S_{sym} & L_S than for E_{sym} & L

✓ Contributions to S_{sym} from p.w. where the tensor force acts ($^3P_2$-$^3F_2$, $^3F_4$-$^3H_4$, $^3H_6$-$^3J_6$ & $^3J_8$-$^3L_8$) compensate with other p.w. (i.e., $^3P_1$ & $^3P_2$ compensate) or are small

	$\langle V\rangle_{TP}$	$\langle V\rangle_{NP}$	$S_{sym}^{\langle V\rangle}$	$L_S^{\langle V\rangle}$
$\langle V_1\rangle$	−24.856	−26.415	1.559	−3.012
$\langle V_{\vec\tau_i\cdot\vec\tau_j}\rangle$	−3.129	−4.157	1.028	0.506
$\langle V_{\vec\sigma_i\cdot\vec\sigma_j}\rangle$	3.207	−0.438	3.645	9.147
$\langle V_{(\vec\sigma_i\cdot\vec\sigma_j)(\vec\tau_i\cdot\vec\tau_j)}\rangle$	13.046	−5.470	18.516	50.328
$\langle V_{S_{ij}}\rangle$	−0.980	−0.608	−0.372	−1.075
$\langle V_{S_{ij}(\vec\tau_i\cdot\vec\tau_j)}\rangle$	−5.725	−4.219	−1.506	−3.625
$\langle V_{\vec L\cdot\vec S}\rangle$	−8.638	−6.076	−2.562	−2.855
$\langle V_{\vec L\cdot\vec S(\vec\tau_i\cdot\vec\tau_j)}\rangle$	−3.090	−2.148	−0.942	−3.303
$\langle V_{L^2}\rangle$	14.090	9.188	4.902	18.735
$\langle V_{L^2(\vec\tau_i\cdot\vec\tau_j)}\rangle$	−2.899	−2.142	−0.757	−3.238
$\langle V_{L^2(\vec\sigma_i\cdot\vec\sigma_j)}\rangle$	1.410	1.016	0.394	0.741
$\langle V_{L^2(\vec\sigma_i\cdot\vec\sigma_j)(\vec\tau_i\cdot\vec\tau_j)}\rangle$	−0.787	0.017	−0.804	−5.024
$\langle V_{(\vec L\cdot\vec S)^2}\rangle$	5.652	3.262	2.390	12.803
$\langle V_{(\vec L\cdot\vec S)^2(\vec\tau_i\cdot\vec\tau_j)}\rangle$	6.903	4.032	2.871	14.275
$\langle V_{T_{ij}}\rangle$	0.006	0.002	0.004	0.022
$\langle V_{(\vec\sigma_i\cdot\vec\sigma_j)T_{ij}}\rangle$	−0.013	−0.015	0.002	−0.010
$\langle V_{S_{ij}T_{ij}}\rangle$	0.004	0.003	0.001	−0.102
$\langle V_{(\tau_{z_i}+\tau_{z_j})}\rangle$	−0.055	−0.070	0.015	−0.054
$\langle U_1\rangle$	−0.019	1.744	−1.763	−6.967
$\langle U_{(\vec\sigma_i\cdot\vec\sigma_j)(\vec\tau_i\cdot\vec\tau_j)}\rangle$	−0.922	−0.708	−0.214	−0.872
$\langle U_{S_{ij}(\vec\tau_i\cdot\vec\tau_j)}\rangle$	2.011	2.152	−0.141	−0.506

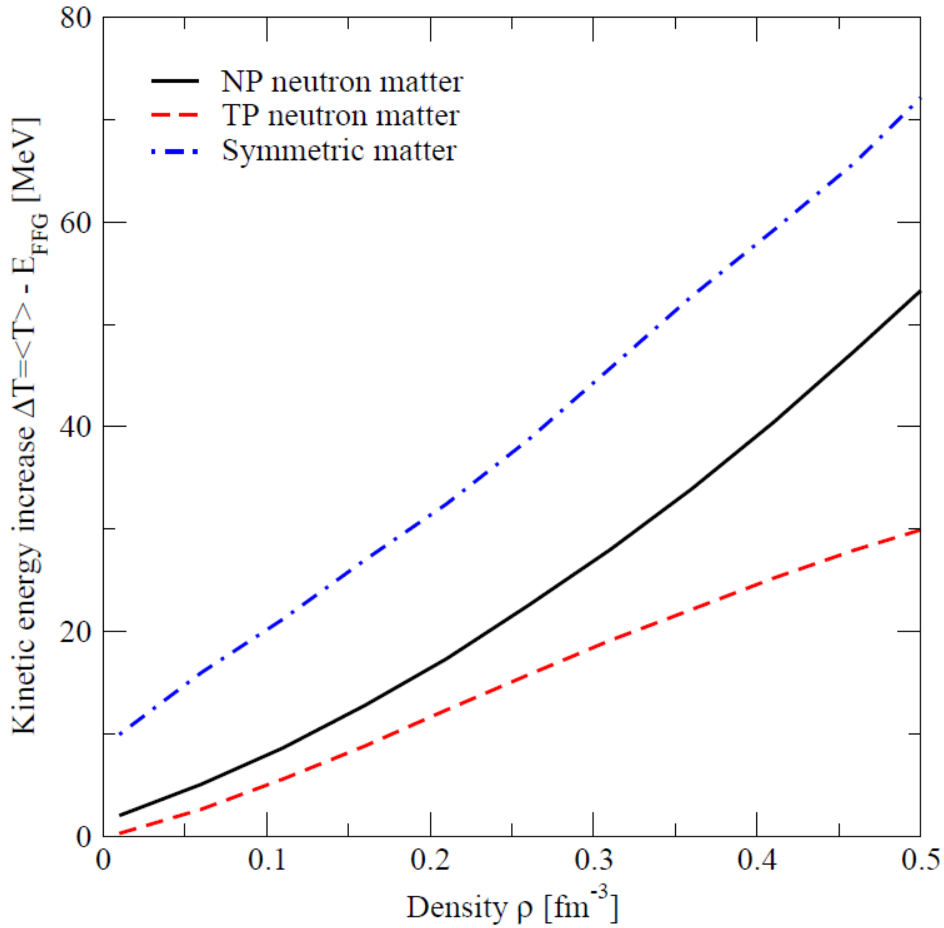
$$\rho_0 = 0.16\ \mathrm{fm}^{-3}$$

Contributions of the various components of the Av18 potential, and the reduced Urbana 3body force to the total energy per particle of TP and NP neutron matter and to the spin symmetry energy and its slope paremeter. All results in MeV.

Relevant contribution to the spin symmetry energy from $\langle V_{(\vec\sigma_i\cdot\vec\sigma_j)(\vec\tau_i\cdot\vec\tau_j)}\rangle$

Many components of the interaction give similar results for TP and NP neutron matter.

Three-body forces give small and similar contributions in both TP and NP neutrón matter and play a secondary role.



$$\Delta T = \langle T \rangle - E_{FFG}$$

As a measure of the dregree of correlation we look at the increment of the kinetic energy per particle respect to the free Fermi sea.

According to this, symmetric nuclear matter is more correlated than unpolarized neutron matter and this is more correlated than polarized neutron matter. Those differences in kinetic energies reflect the existence of larger depletion in n(k) and more high momentum components in nuclear matter.

Second Summary

❖ No ferromagnetic transition in the wide range of densities explored.

❖ NP neutron matter is more correlated than TP neutron matter and both are less correlated than symmetric nuclear matter.

❖ The main contribution of the potential energy part to the spin symmetry energy comes from $\langle V_{(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$

❖ Three-body forces give small contributions and play a secondary role in TP and NP neutron matter.

❖ Big cancellations in the different contributions of the potential energy in TP neutron matter, resulting in a very small potential energy. However the system is correlated as one can see by looking at the increment of kinetic energy respect to the Fermi sea.

Ferromagnetic transition of a two-component Fermi gas of hard spheres

PHYSICAL REVIEW A 85, 033615 (2012)

In the context of cold fermi dilute gases with repulsive short range repulsion it has been experimentally explored and theoretically argued the existence of a ferromagnetic transition

Based on the fact that for very dilute systems only the s-wave is relevant and that for fully polarized Fermi system, the s-wave scattering is forbidden and therefore the total energy equals the corresponding kinetic energy of the underlying Fermi sea, the effective opposite-spin-channel (OSC) model has been proposed:

$$H_{\mathrm{OSC}} = -\frac{\hbar^2}{2m}\left(\sum_{i=1}^{N_\uparrow}\nabla_i^2 + \sum_{i'=1}^{N_\downarrow}\nabla_{i'}^2\right) + \sum_{i,i'}^{N_\uparrow,N_\downarrow} V(r_{ii'}),$$

$$V(r) = \begin{cases} +\infty & r \leqslant R \\ 0 & \text{otherwise} \end{cases}$$

Hard-sphere potential

$$H = -\frac{\hbar^2}{2m}\left(\sum_{i=1}^{N_\uparrow} \nabla_i^2 + \sum_{i'=1}^{N_\downarrow} \nabla_{i'}^2\right) + \sum_{i<j}^{N_\uparrow} V(r_{ij}) + \sum_{i'<j'}^{N_\downarrow} V(r_{i'j'}) + \sum_{i,i'}^{N_\uparrow,N_\downarrow} V(r_{ii'}),$$

Low density expansion with s- and p-wave scattering lengths and the s-wave effective range

$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + (\nu - 1)\left(\frac{2}{3\pi}(k_F a) + \frac{4(11 - 2\ln 2)}{35\pi^2}(k_F a)^2 + \frac{1}{10\pi}(k_F r_0)(k_F a)^2 + [0.076 + 0.057(\nu - 3)](k_F a)^3\right) + \frac{1}{5\pi}(\nu + 1)(k_F a_p)^3 + (\nu - 1)(\nu - 2)\frac{16}{27\pi^3}(4\pi - 3^{3/2}) \times (k_F a)^4 \ln(k_F a) + \cdots\right], \quad (5)$$

$$\frac{E^{\mathrm{NP}}}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}x + \frac{4(11 - 2\ln 2)}{35\pi^2}x^2 + 0.231x^3\right]$$

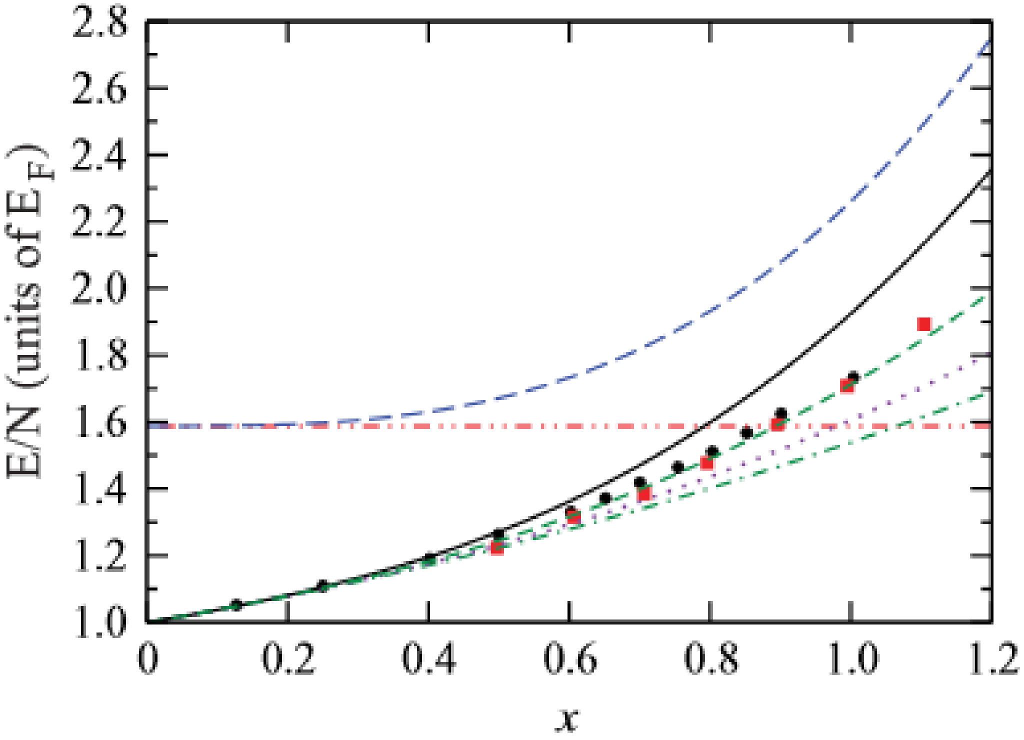
$$\frac{E^{\mathrm{P}}}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + \frac{2}{5\pi}x^3\right]$$

$$\frac{E_u^{\mathrm{NP}}}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}x + \frac{4}{35\pi^2}(11 - 2\ln 2)x^2\right]$$

$$\frac{1}{\chi} = \frac{1}{\rho}\left(\frac{\partial^2 E/N}{\partial \Delta^2}\right)_{\Delta=0}$$

$$\chi^{-1}(x_c) = 0.$$

$$E^{\mathrm{P}}(x_c') - E^{\mathrm{NP}}(x_c') = 0$$

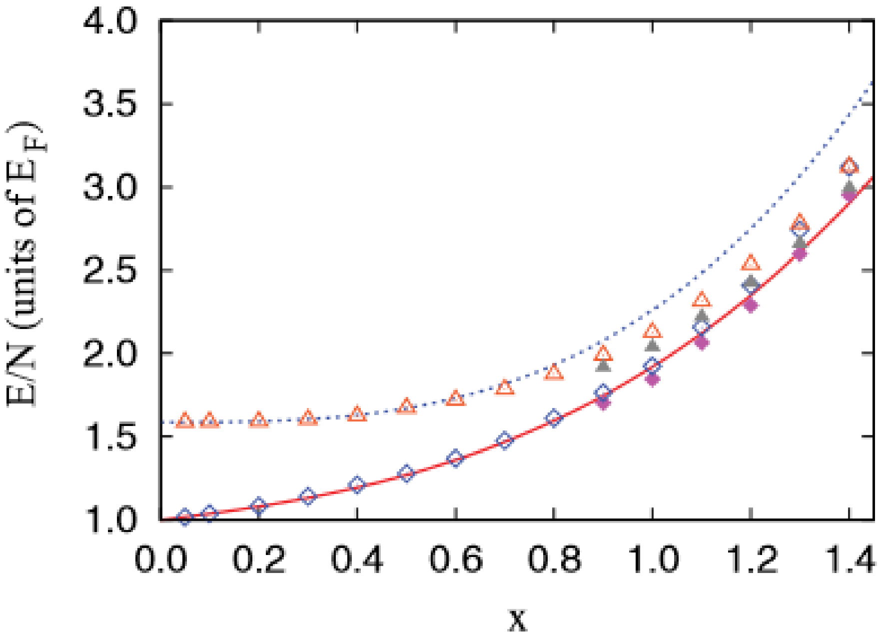


$$\Psi_B = \left[\prod_{i<j} f(r_{ij}) \right] D_\uparrow(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_{N_\uparrow}) D_\downarrow(\vec{r}'_1, \vec{r}'_2, \ldots, \vec{r}_{N_\downarrow})$$

Jastrow wave function with optimized two-body correlations and back-flow correlations.

$$\mathbf{r}_i \rightarrow \tilde{\mathbf{r}}_i + \lambda \sum_{j \neq i}^{N} \eta(r_{ij}) \mathbf{r}_{ij} \qquad \eta(r) = \exp\left[-\left(\frac{r - r_0}{\omega_0} \right)^2 \right]$$

Back-flow correlations decrease the energy of both P and NP systems, but is more efficient for NP. Finally, shifting the critical density up to x=1.8 which is close to the freezing density of quantum hard-spheres : xf=1.95



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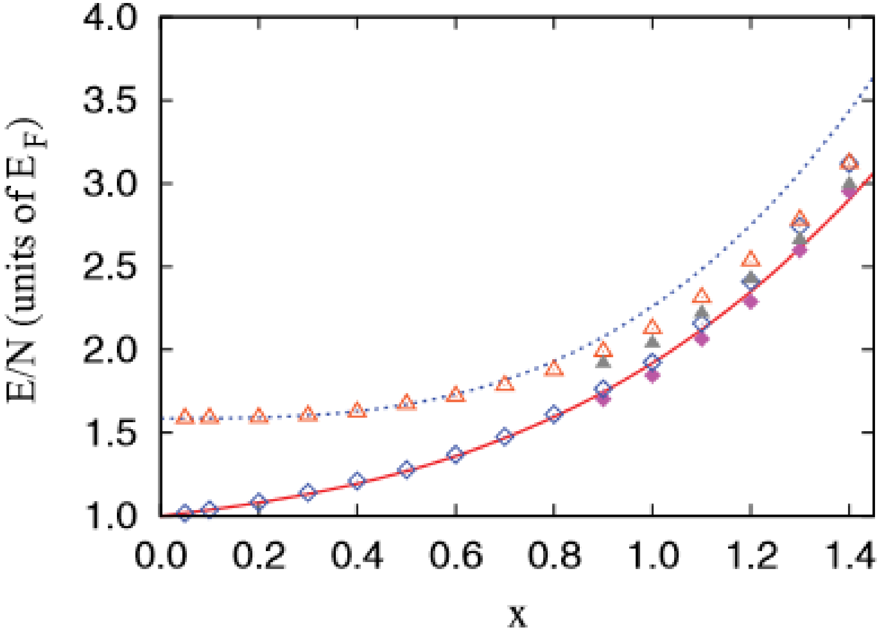

Third summary

We have shown that p-wave terms contribute significantly to the total energy of the polarized and unpolarized system at x around 1, and therefore the simplified Hamiltonian for a fully repulsive interaction can not be used in this range

Variational calculations in the framework of FHNC for a wave function including backflow correlations show that a ferromagnetic transition takes place at x around 1.8 close to the freezing density of quantum hard-spheres estimated at x around 1.95.

Possibility: To calculate the Landau parameters and do the analysis in terms of the Landau parameters. Calculate also the response.

Taylor expansion of the energy per particle of symmetric nuclear matter around the saturation density.

$$E_{SNM}(\rho) = E_0 + \frac{K_0}{2!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_0}{3!}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \mathcal{O}(4)$$

$$E_0 = E_{SNM}(\rho=\rho_0) \sim -16 \text{ MeV}$$

$$K_0 = 9\rho_0^2 \frac{\partial^2 E_{SNM}(\rho)}{\partial \rho^2}\Big|_{\rho=\rho_0} \sim 200 \text{ to } 300 \text{ MeV}$$

$$Q_0 = 27\rho_0^3 \frac{\partial^3 E_{SNM}(\rho)}{\partial \rho^3}\Big|_{\rho=\rho_0} \sim -500 \text{ to } +300 \text{ MeV}$$

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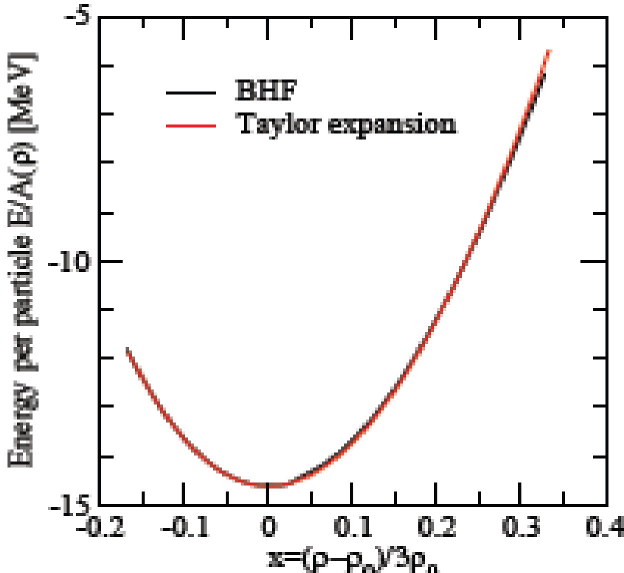


Combining the precedent expansions, one can predict the existence of a saturation density, i.e., a zero pressure condition, for a given asymmetry and rewrite the energy per particle of asymmetric matter around the new saturation density

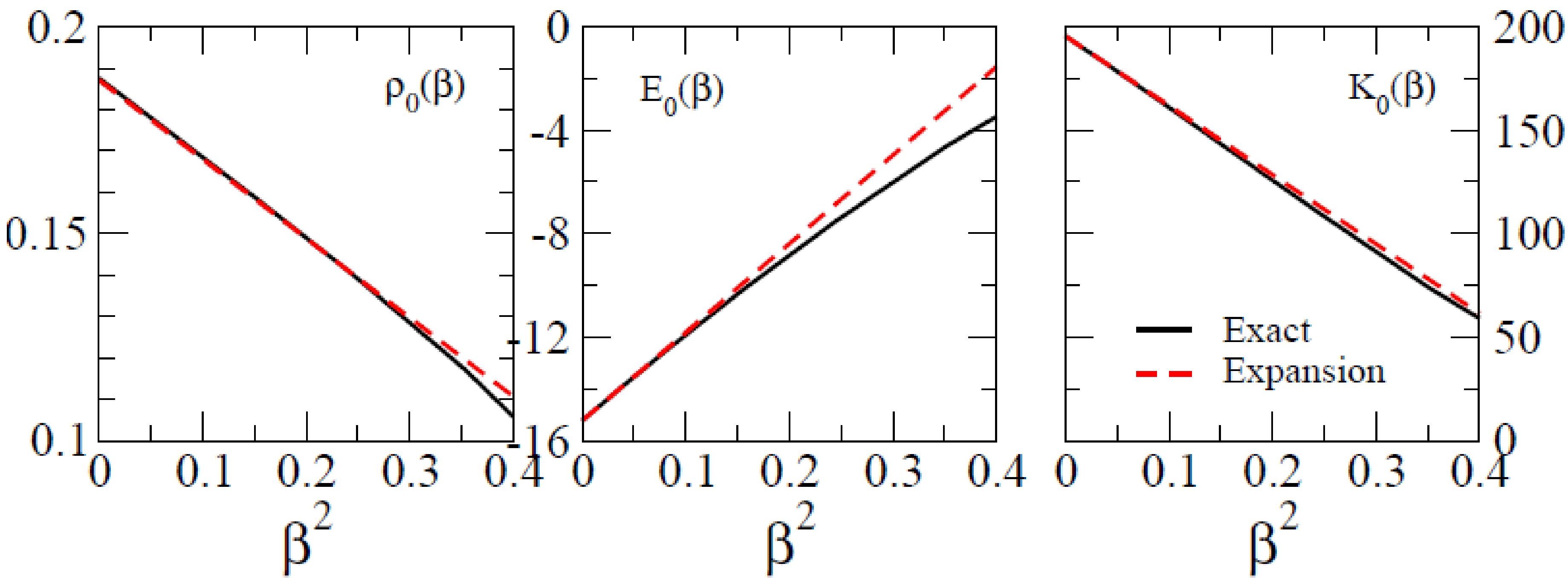
$$\frac{E}{A}(\rho,\beta) = E_0(\beta) + \frac{K_0(\beta)}{2!}\left(\frac{\rho-\rho_0(\beta)}{3\rho_0(\beta)}\right)^2 + \frac{Q_0(\beta)}{3!}\left(\frac{\rho-\rho_0(\beta)}{3\rho_0(\beta)}\right)^3 + \mathcal{O}(4)$$

$$\rho_0(\beta) = \rho_0\left(1 - 3\frac{L}{K_0}\beta^2\right) + \mathcal{O}(4)$$

$$E_0(\beta) = E_0 + E_{sym}\beta^2 + \mathcal{O}(4)$$

$$K_0(\beta) = K_0 + \underbrace{\left(K_{sym} - 6L - \frac{Q_0}{K_0}L\right)}_{K_\tau}\beta^2 + \mathcal{O}(4)$$

$$Q_0(\beta) = Q_0 + \left(Q_{sym} - 9L\frac{Q_0}{K_0}\right)\beta^2 + \mathcal{O}(4)$$



Isospin asymmetry dependence of the saturation density, energy per particle and incompressibility coefficient at the saturation point of asymmetric nuclear matter.

Solid lines show the results of the exact BHF calculations whereas dashed lines indicate the results of the previous expansion.

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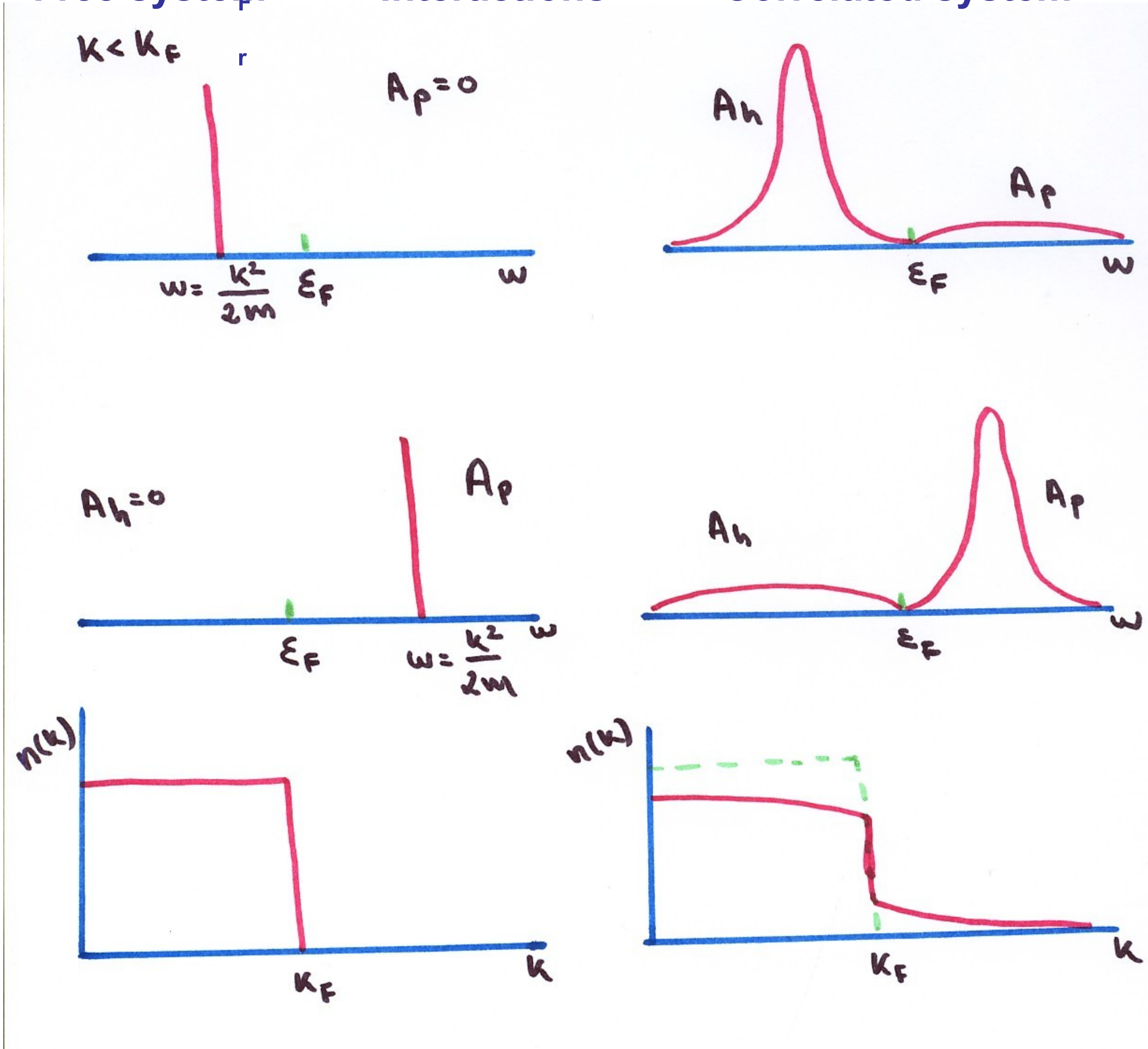
Spectral functions at zero tempearture

Free system + Interactions => Correlated system



Partial wave	E_{NM}	E_{SM}	E_{sym}	L
$^1S_0$	−23.070	−19.660	−3.410	−3.459
$^3S_1$	0	−45.810	45.810	71.855
$^1P_1$	0	4.904	−4.904	−18.601
$^3P_0$	−5.321	−4.029	−1.292	−1.898
$^3P_1$	16.110	10.720	5.390	21.949
$^3P_2$	−16.000	−9.334	−6.666	−21.168
$^1D_2$	−5.956	−3.201	−2.755	−11.033
$^3D_1$	0	0.981	−0.981	−3.739
$^3D_2$	0	−3.982	3.982	16.601
$^3D_3$	0	−0.798	0.798	4.895
$^1F_3$	0	0.694	−0.694	−3.348
$^3F_2$	−0.695	−0.229	−0.466	−1.799
$^3F_3$	2.000	0.821	1.179	4.883
$^3F_4$	−0.796	−0.194	−0.602	−3.239
$^1G_4$	−0.812	−0.247	−0.565	−3.036
$^3G_3$	0	−0.001	0.001	0.441
$^3G_4$	0	−0.213	0.213	0.449
$^3G_5$	0	−0.057	0.057	0.650
$^1H_5$	0	0.029	−0.029	0.107
$^3H_4$	0.033	0.040	−0.007	0.232
$^3H_5$	0.225	−0.033	0.258	0.968
$^3H_6$	0.043	0.034	0.009	0.144
$^1I_6$	−0.082	0.023	−0.105	−0.591
$^3I_5$	0	−0.029	0.029	0.342
$^3I_6$	0	0.067	−0.067	−0.819
$^3I_7$	0	−0.021	0.021	0.239
$^1J_7$	0	−0.027	0.027	0.385
$^3J_6$	0.044	0.020	0.024	0.283
$^3J_7$	−0.062	−0.060	−0.002	−0.313
$^3J_8$	0.036	0.014	0.022	0.242
$^1K_8$	0.031	0.021	0.010	0.169
$^3K_7$	0	−0.011	0.011	0.138
$^3K_8$	0	0.038	−0.038	−0.491
$^3L_8$	0.021	0.006	0.015	0.166

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$^3S_1$ gives the larger contribution to the symmetry energy and L

Large cancellations between the other partial waves.

Effective interactions. Skyrme type

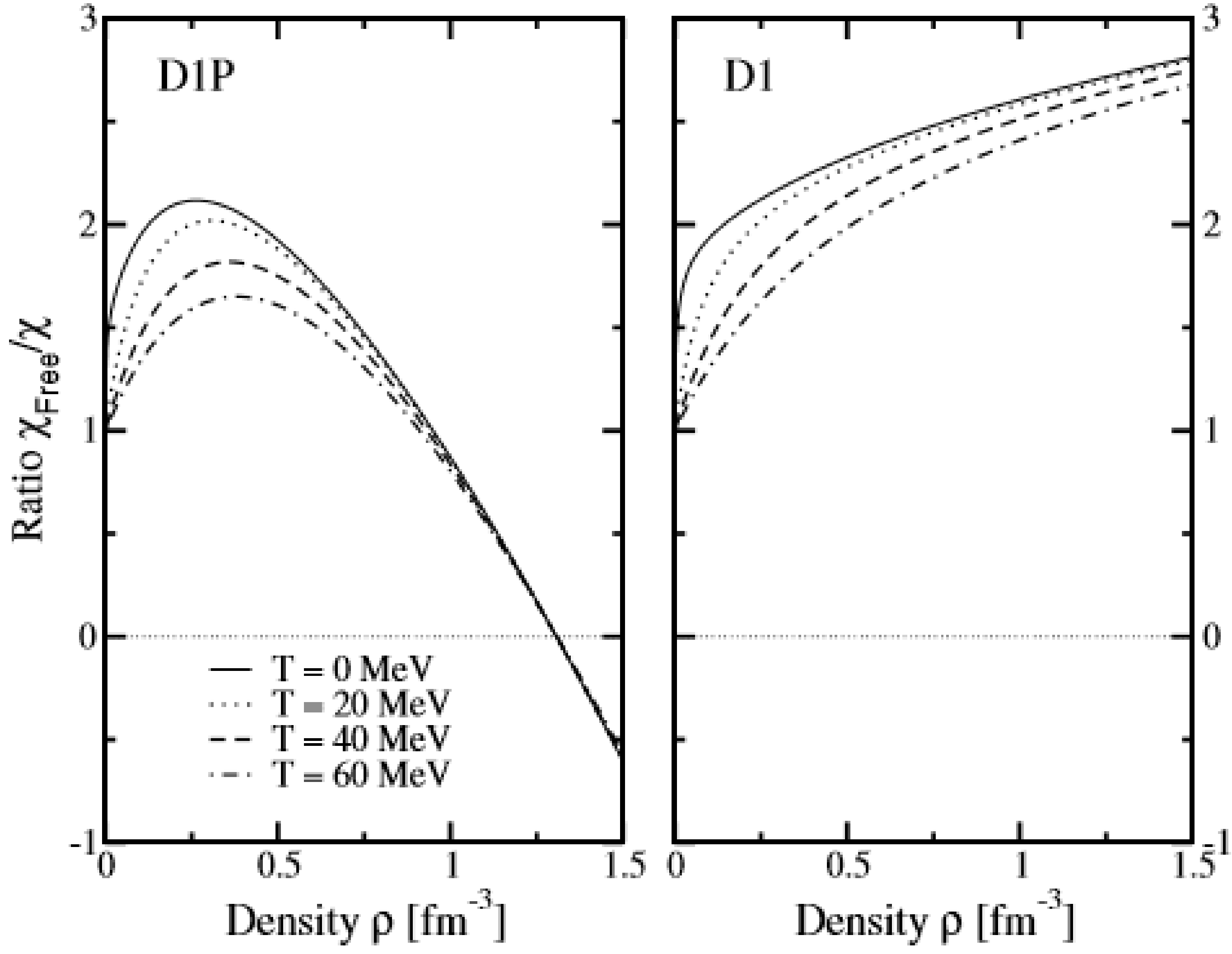
$$V(\vec{r}) = t_0(1 + x_0 P^\sigma)\delta(\vec{r}) + \frac{1}{6} t_3(1 + x_3 P^\sigma)\rho^\alpha \delta(\vec{r})$$
$$+ \frac{1}{2} t_1(1 + x_1 P^\sigma)\left[\overleftarrow{k}^2 \delta(\vec{r}) + \delta(\vec{r})\overrightarrow{k}^2\right] + t_2(1 + x_2 P^\sigma)\left[\overleftarrow{k} \cdot \delta(\vec{r})\overrightarrow{k}\right]$$

$$U_n(\vec{k}) = \sum_{|\vec{k}'| < k_{F_n}} \langle \vec{k}\vec{k}' | V_{nn;nn} \mid \vec{k}\vec{k}' \rangle_A + \sum_{|\vec{k}'| < k_{F_p}} \langle \vec{k}\vec{k}' | V_{np;np} \mid \vec{k}\vec{k}' \rangle_A$$

$$\frac{E}{A}(\rho, \beta) = \frac{1}{A} \sum_\tau \sum_{|\vec{k}| < k_{F_\tau}} \left(\frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} Re[U_\tau(\vec{k})] \right)$$

Gogny type

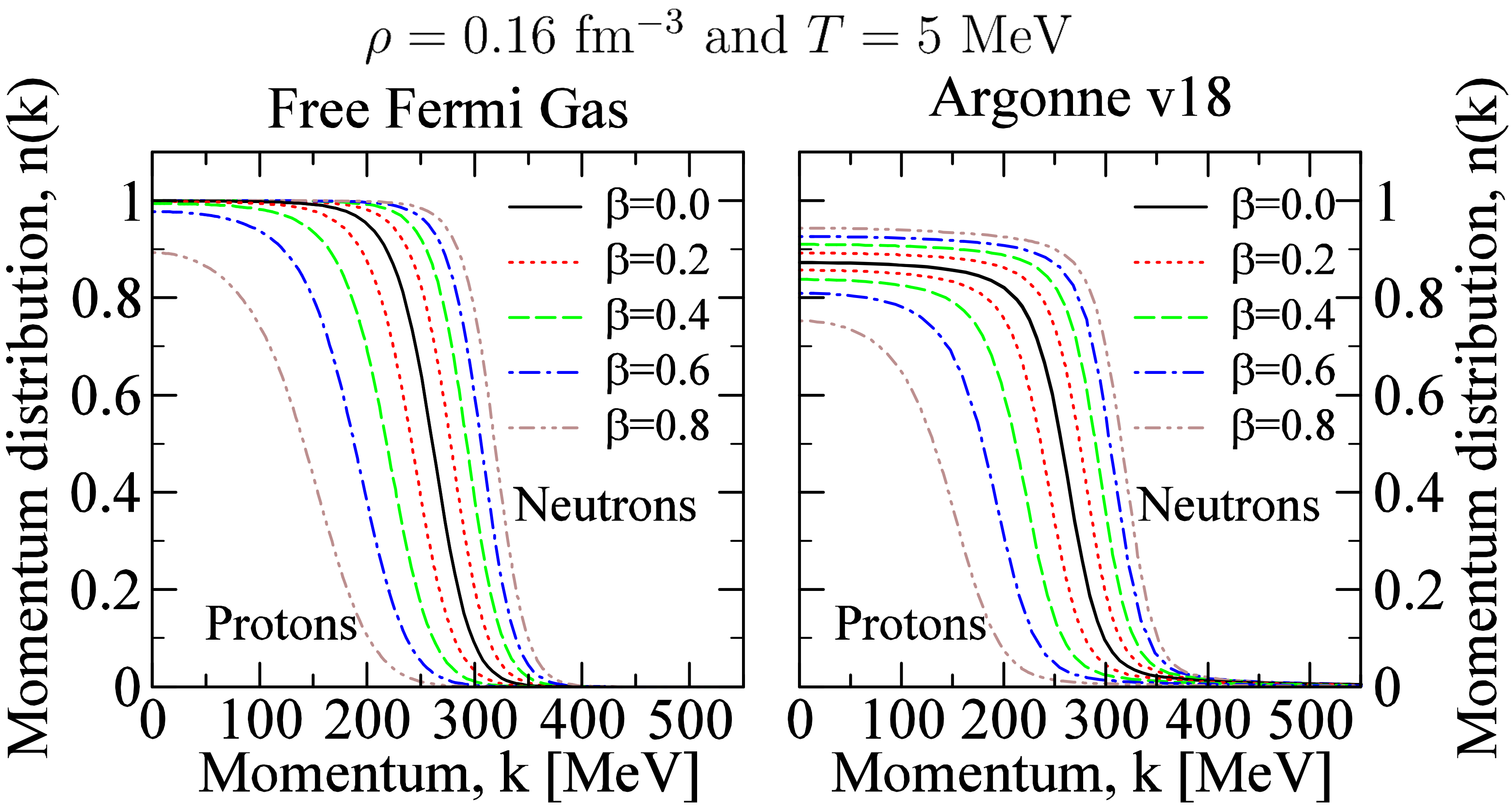
$$V_{NN}(\vec{r}) = \sum_{i=1}^{2} e^{-\frac{r^2}{\mu_i}} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)$$
$$+ t_0 (1 + x_0 P_\sigma)\rho_N^\alpha \, \delta(\vec{r}),$$

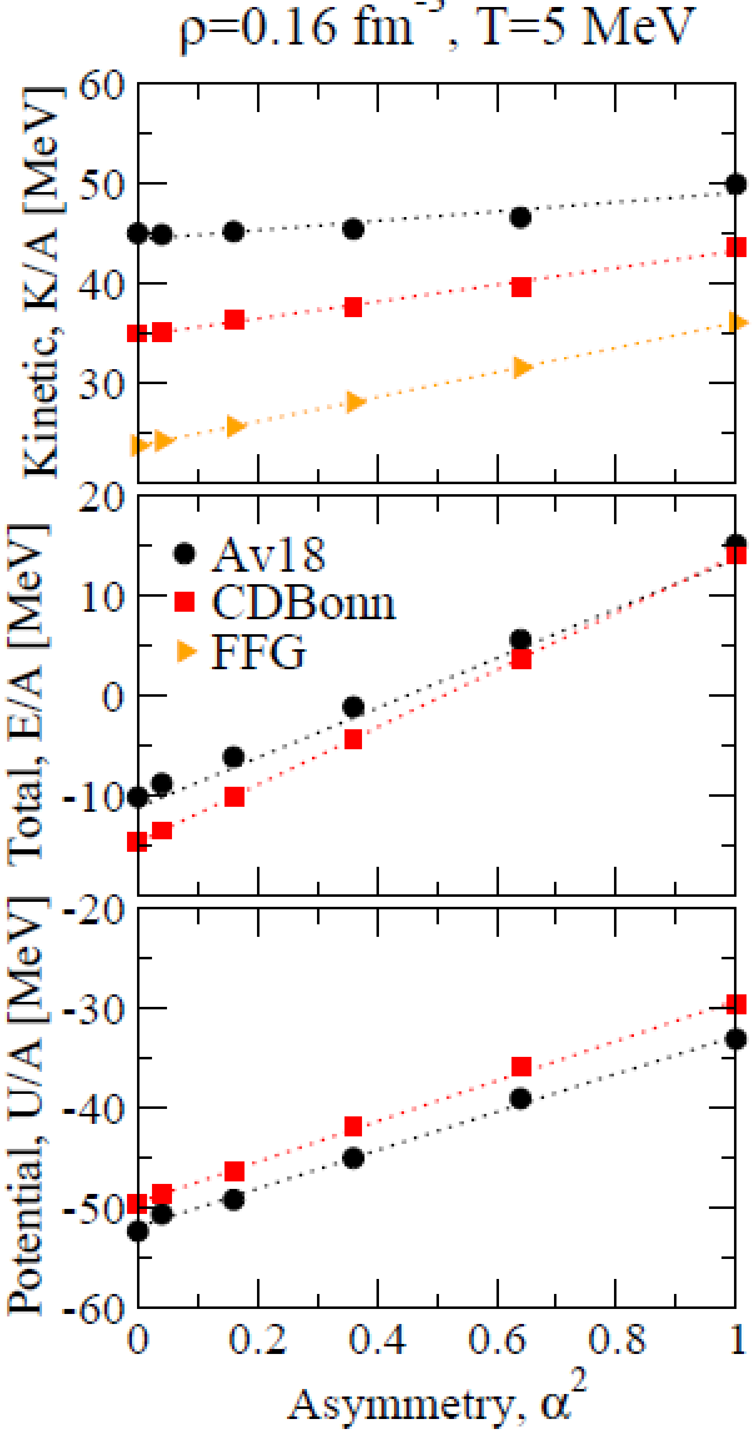


Phys. Rev C74(2006),068801

Neutron and proton momentum distributions for different asymmetries

The less abundant component (the protons) are very much affected by thermal effects.





Isospin asymmetry dependence of the kinetic and potential energy contributions to the total energy. For the CDBonn (circles) and Av18 (squares) potentials. The triangles of the upper panel give the energy of the FFG in the same conditions, rho= 0.16 fm^{-3} and T=5 MeV.

Almost linear dependence ===> quadratic dependence on the asymmetry paremeter.
Different slopes.
The variation of the kinetic energy is much smaller tan the variation of the potential energy.

EPL, 97 (2012)22001

Argonne v18

$$v(NN) = v^{EM}(NN) + v^{\pi}(NN) + v^{R}(NN)$$

is the sum of 18 operators that respect some symmetries.

$$v_{ij} = \sum_{p=1,18} v_p(r_{ij}) O^p_{ij}$$

$$O^{p=1,14}_{ij} = 1, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\ \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j, (\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j),\ S_{ij}, S_{ij}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j),\ \mathbf{L}\cdot\mathbf{S}, \mathbf{L}\cdot\mathbf{S}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j),$$
$$L^2, L^2(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j),\ L^2(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j), L^2(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j),\ (\mathbf{L}\cdot\mathbf{S})^2, (\mathbf{L}\cdot\mathbf{S})^2(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)$$

$$O^{p=15,18}_{ij} = T_{ij},\ (\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)T_{ij},\ S_{ij}T_{ij},\ (\tau_{zi} + \tau_{zj})$$