Pairing in high-density neutron matter

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Nuclear Astrophysics Milan 18 September 2017

Summary

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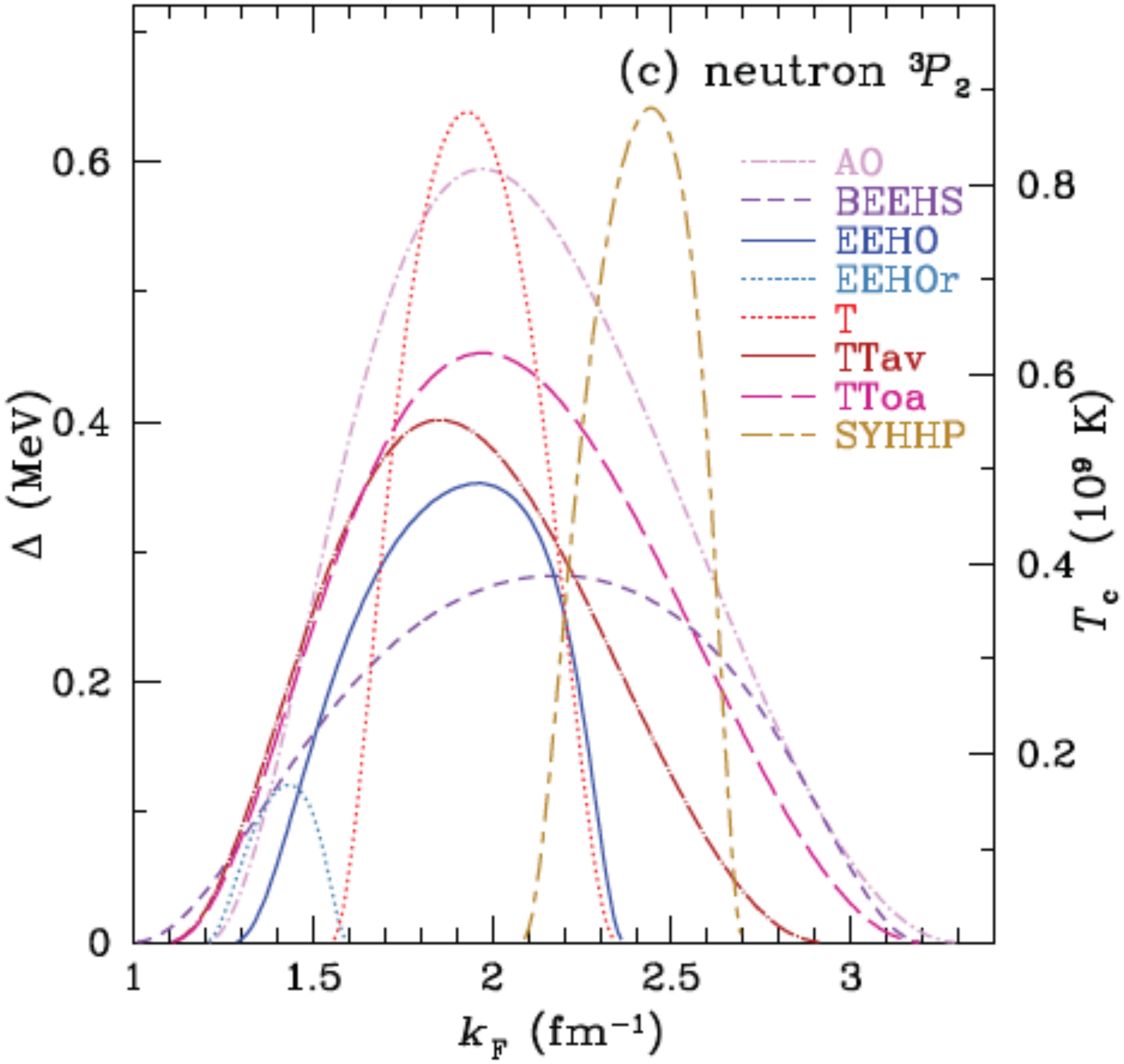
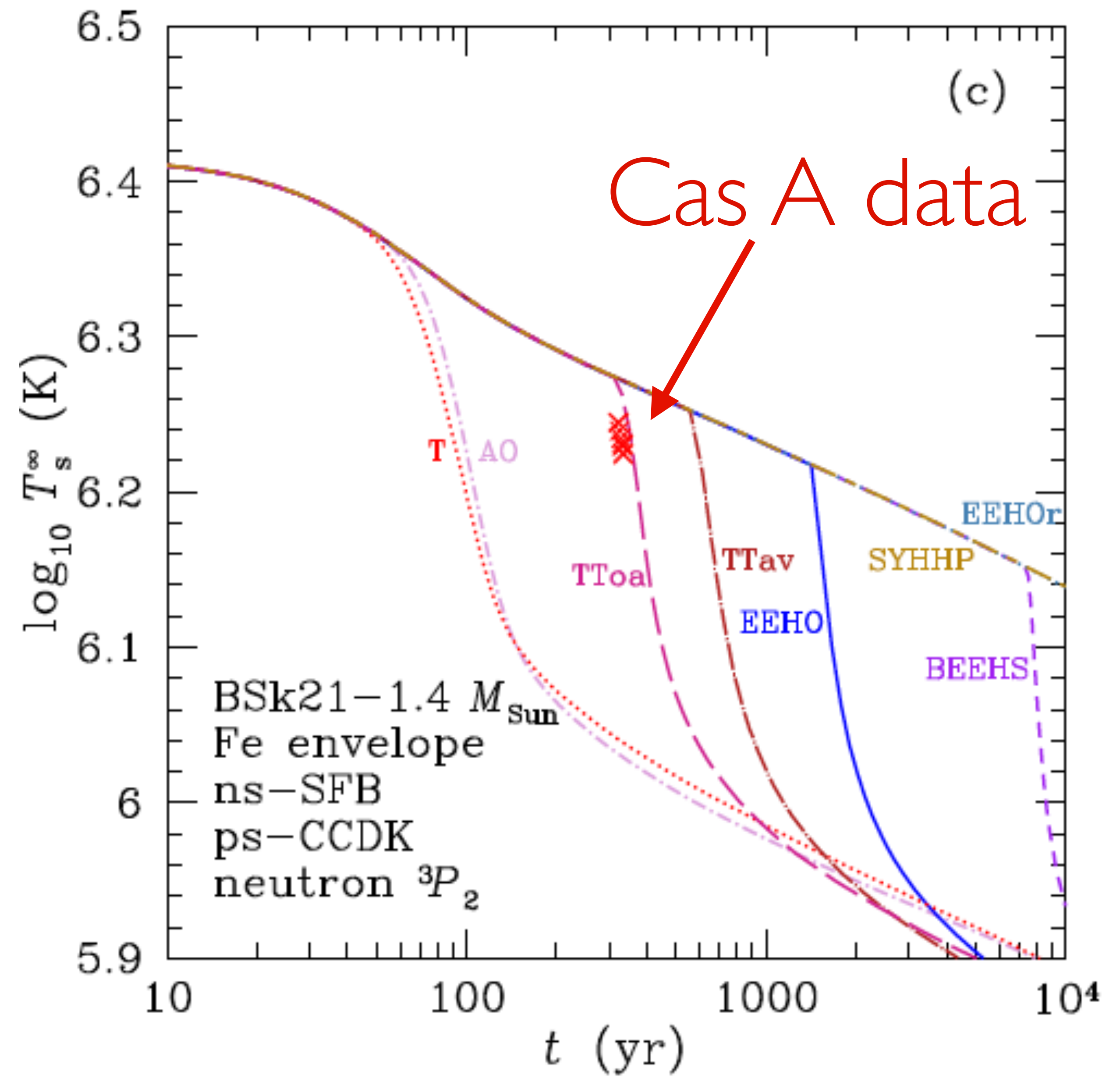
1. **Neutron star motivation**
2. Infinite matter BCS
3. Beyond-BCS with SCGF methods

Cooling of CasA



Ho, *et al.*, PRC **91** 015806 (2015)

Page, *et al.*, *PRL* **106** 081101 (2011)



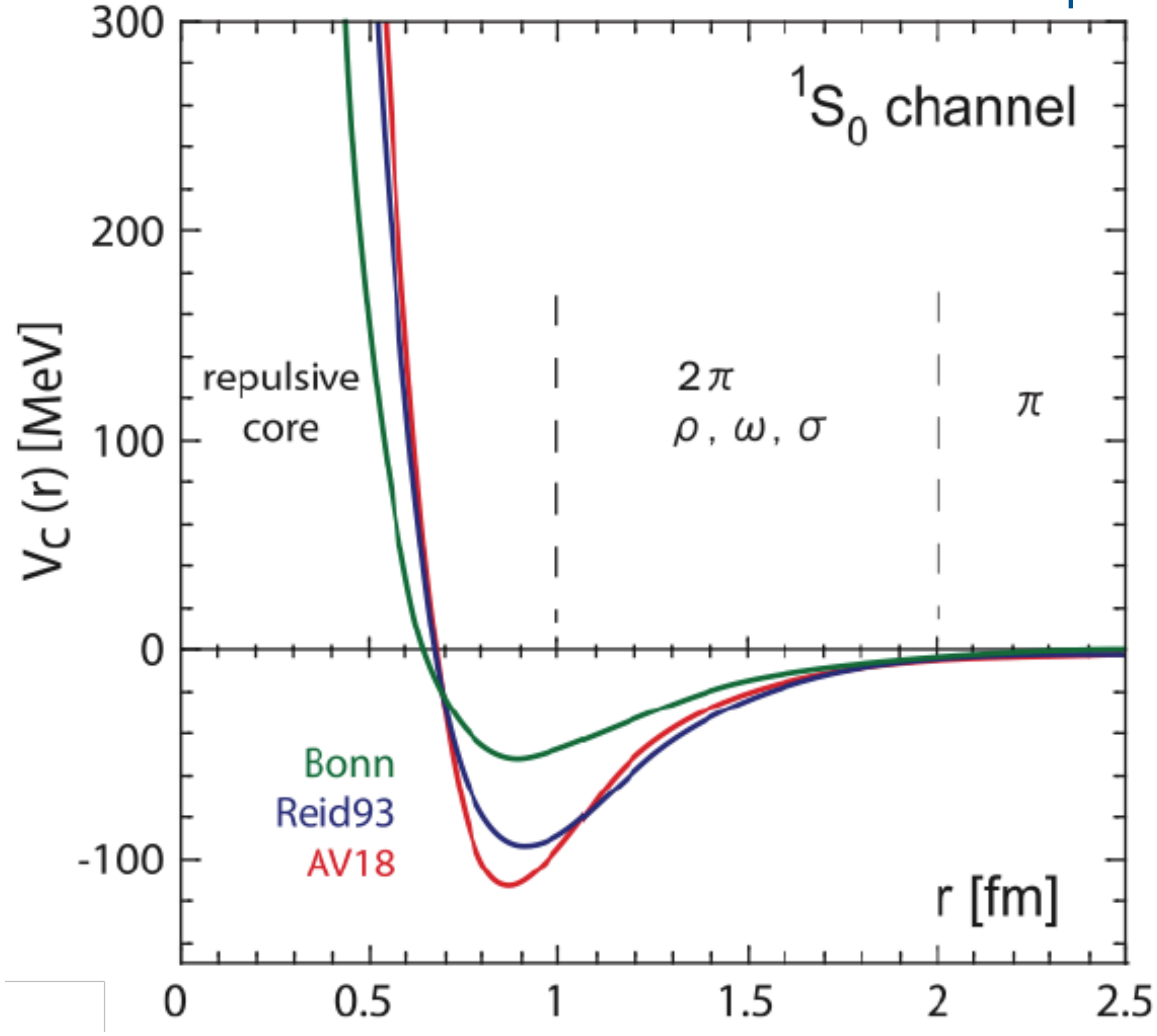
Ingredients

(a) Mass of pulsar
(b) EoS (determines radius)
(c) Internal composition
(d) **Pairing gaps** ($^1S_0$ & $^3PF_2$ channels)
(e) Atmosphere composition

Name	Process	Emissivity (erg cm^{-3} s^{-1})
Modified Urca (neutron branch)	$n+n \to n+p+e^- + \bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\sim 2\times10^{21}\, R T_9^8$
Modified Urca (proton branch)	$p+n \to p+p+e^- + \bar{\nu}_e$ $p+p+e^- \to p+n+\nu_e$	$\sim 10^{21}\, R T_9^8$
Bremsstrahlungs	$n+n \to n+n+\nu+\bar{\nu}$ $n+p \to n+p+\nu+\bar{\nu}$ $p+p \to p+p+\nu+\bar{\nu}$	$\sim 10^{19}\, R T_9^8$
Cooper pair	$n+n \to [nn]+\nu+\bar{\nu}$ $p+p \to [pp]+\nu+\bar{\nu}$	$\sim 5\times10^{21}\, R T_9^7$ $\sim 5\times10^{19}\, R T_9^7$
Direct Urca (nucleons)	$n \to p+e^- + \bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\sim 10^{27}\, R T_9^6$

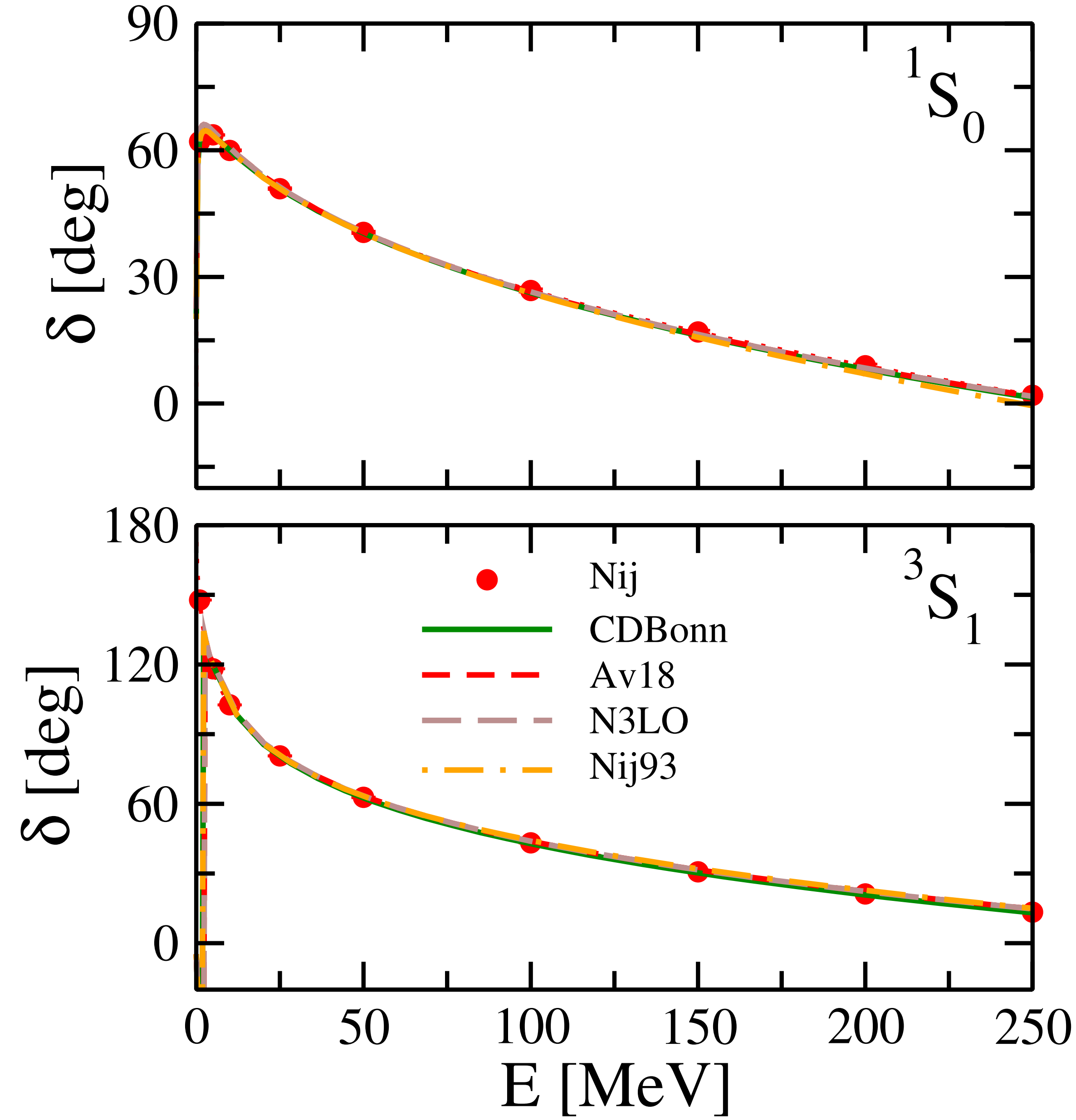
Complications

NN interaction is not unique



S. Aoki, et al. *Comput. Sci. Dis.* **1** 015009 (2008)

...but phase-shift equivalent!



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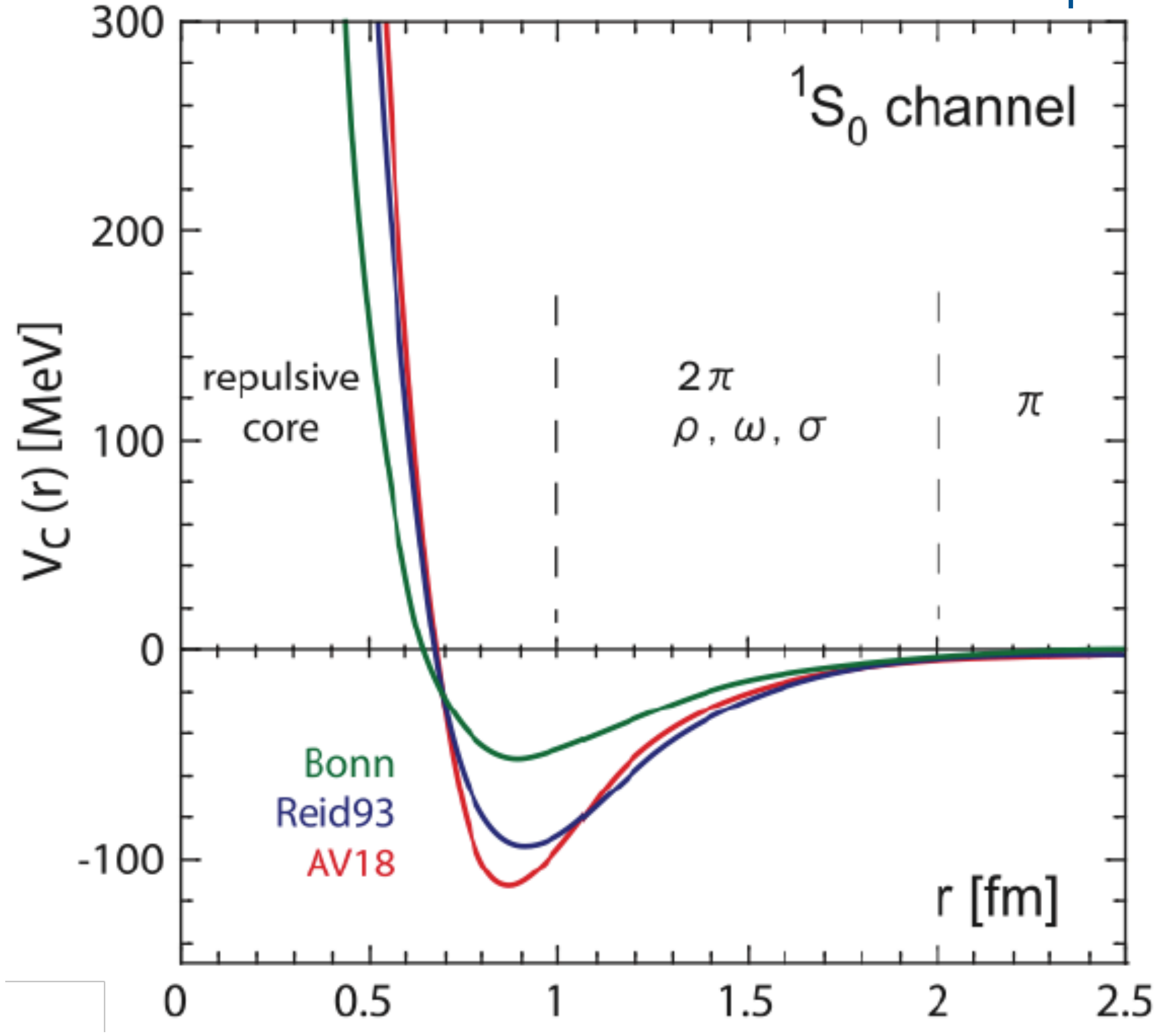

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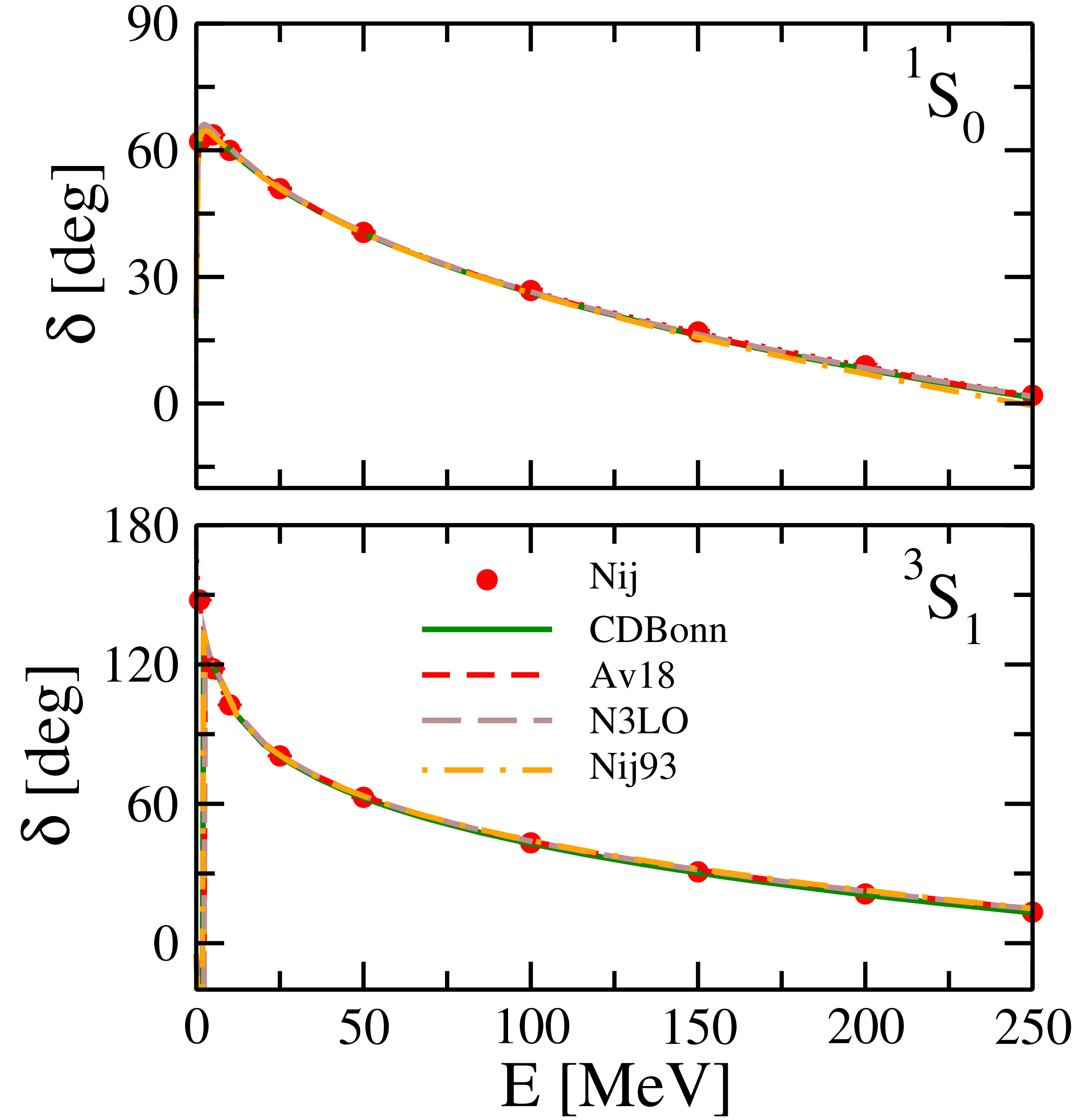

- Non-uniqueness of nucleon forces ✘

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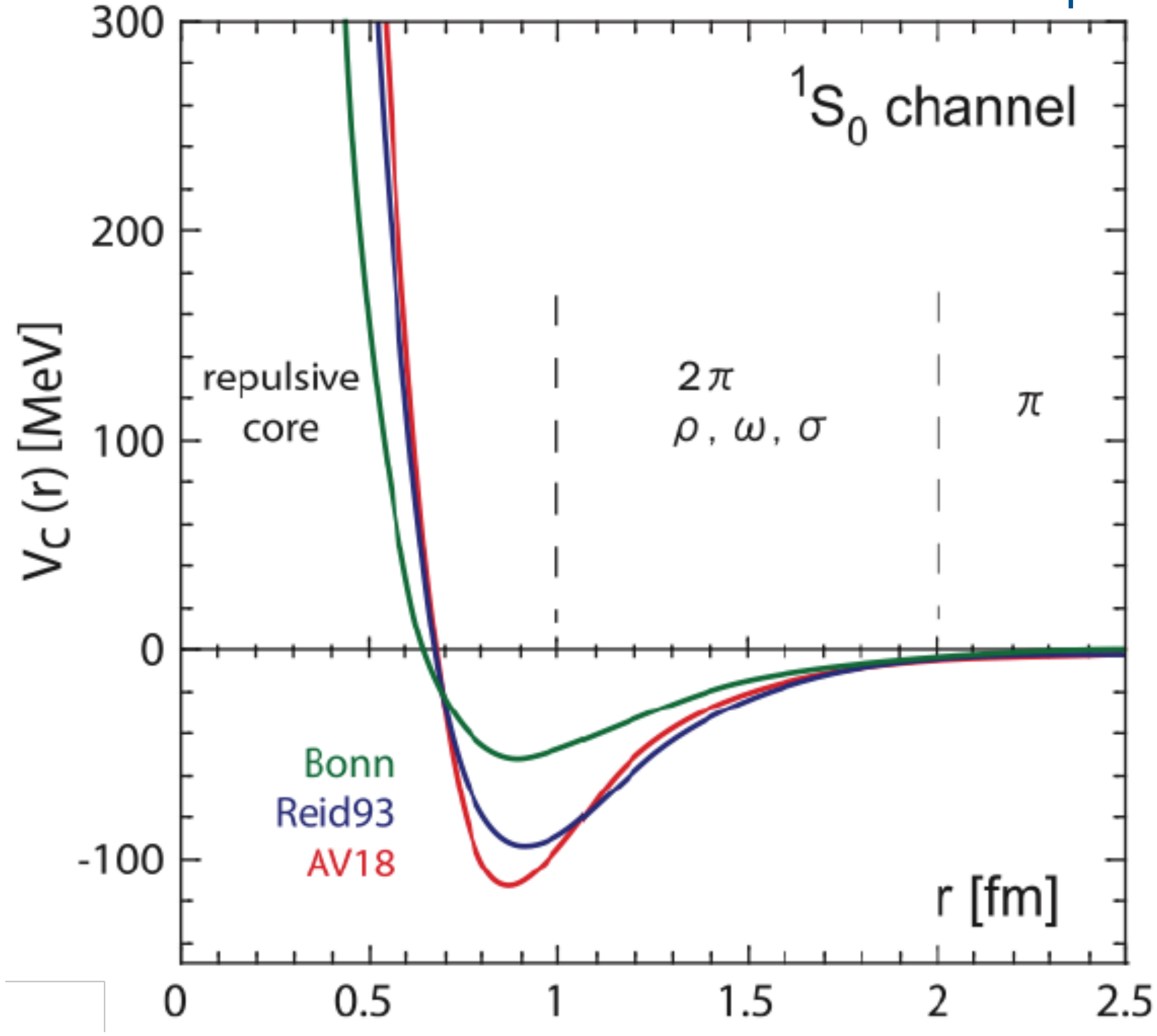
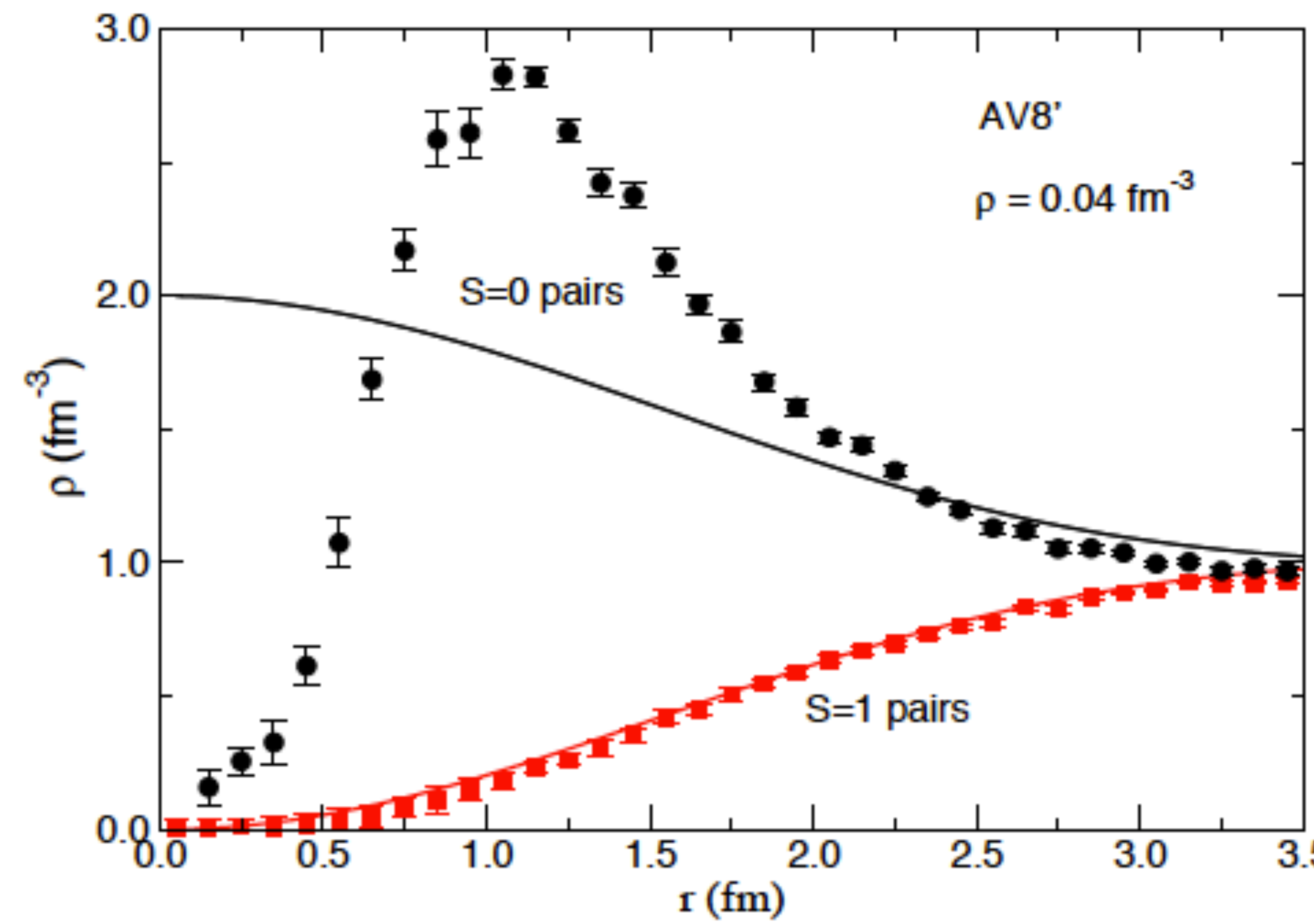


S. Aoki, et al. *Comput. Sci. Dis.* **1** 015009 (2008)

Strong short-range correlations

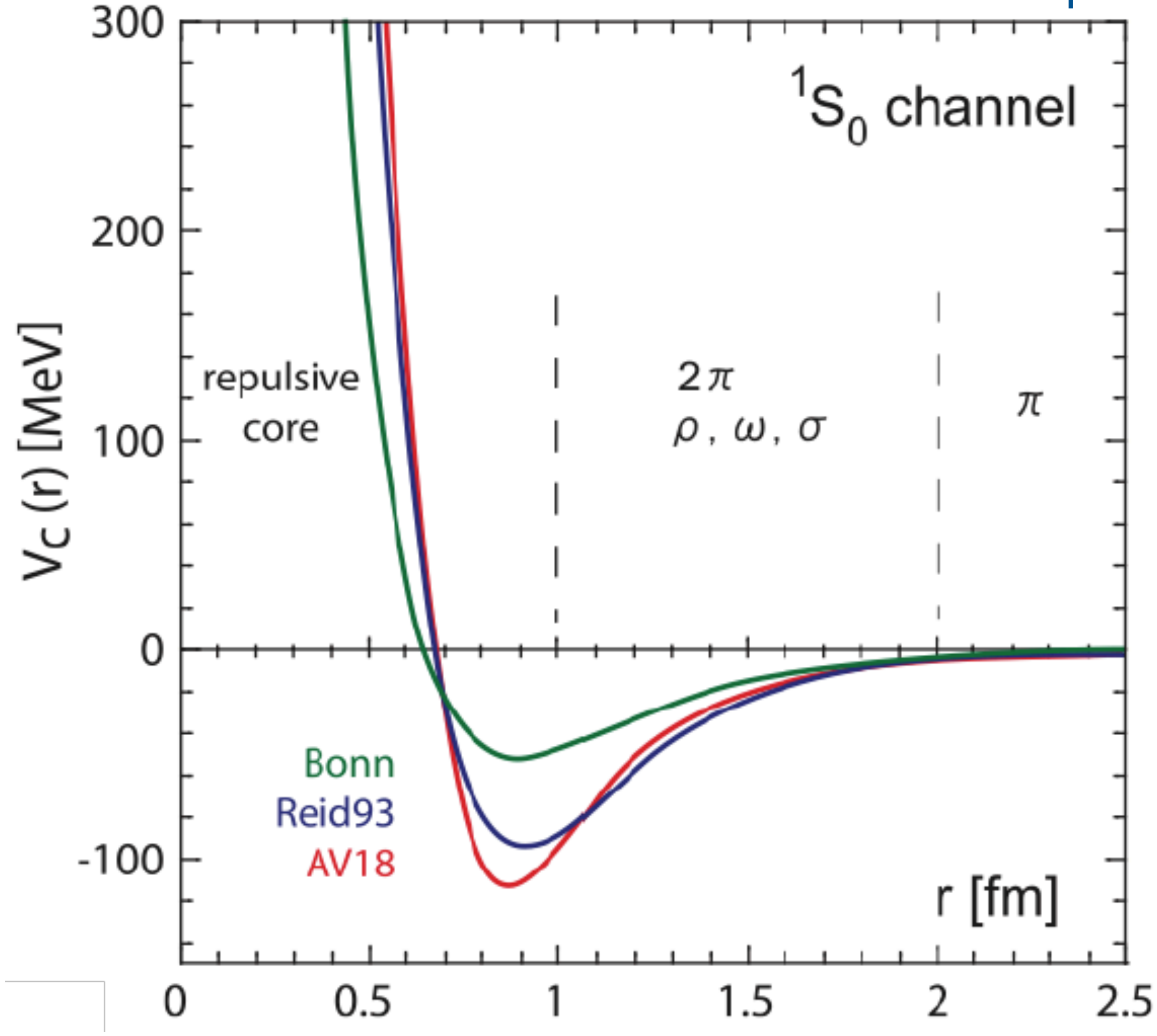


Carlson et al., *Phys. Rev. C* **68** 025802 (2003)

- Non-uniqueness of nucleon forces ✘
- Short-range core needs many-body treatment ✘

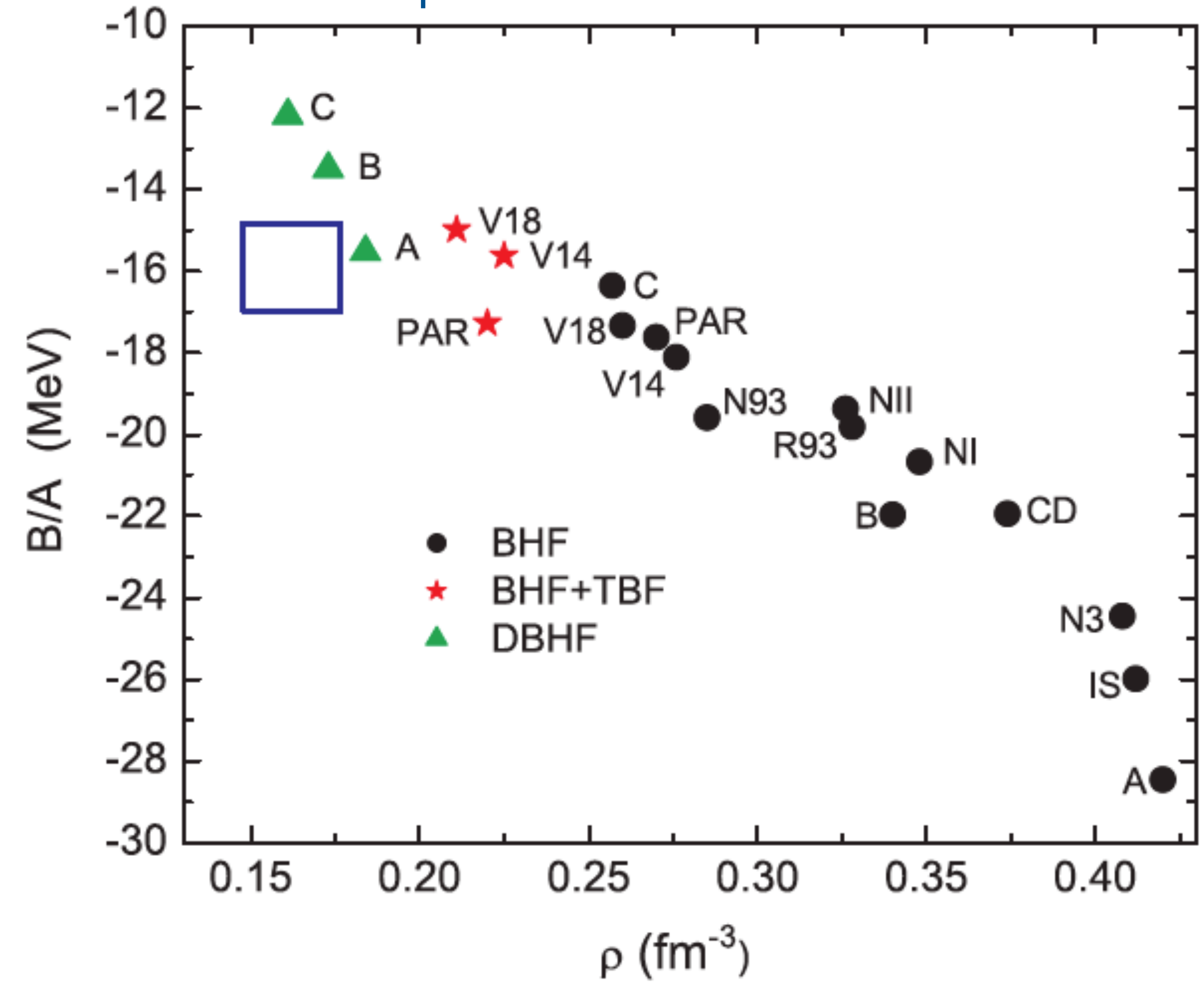
Complications

NN interaction is not unique



S. Aoki, et al. *Comput. Sci. Dis.* **1** 015009 (2008)

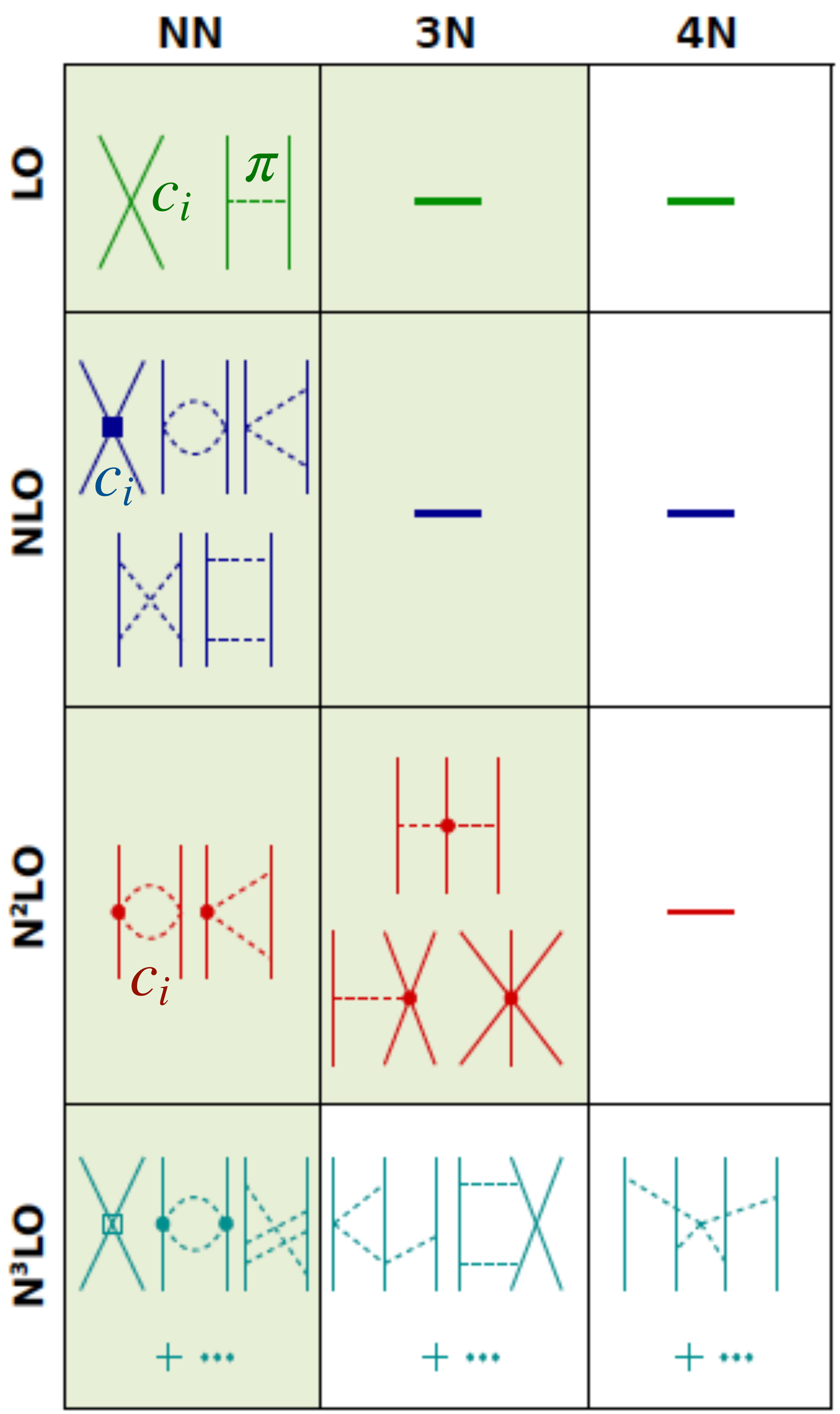
Saturation point of nuclear matter



Li, Lombardo, Schulze et al. *PRC* **74** 047304 (2006)

- Non-uniqueness of nucleon forces ✗
- Short-range core needs many-body treatment ✗
- Three-body forces needed for saturation ✗

NN forces from EFTs of QCD



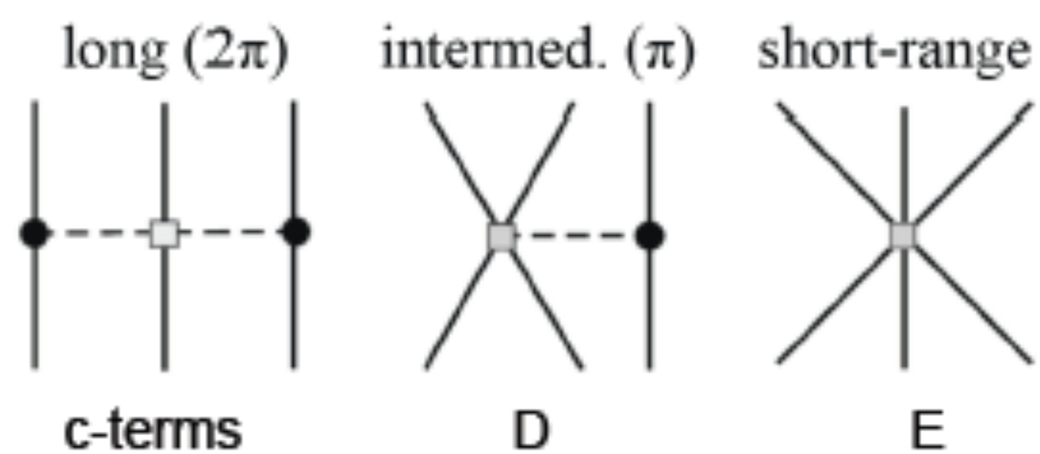
Robert Roth – TU Darmstadt – 04/2013

Chiral perturbation theory

- π and N as dof
- Systematic expansion
- 2N at N³LO - LECs from πN, NN
- 3N at N²LO - 2 more LECs
- (Often further renormalized)

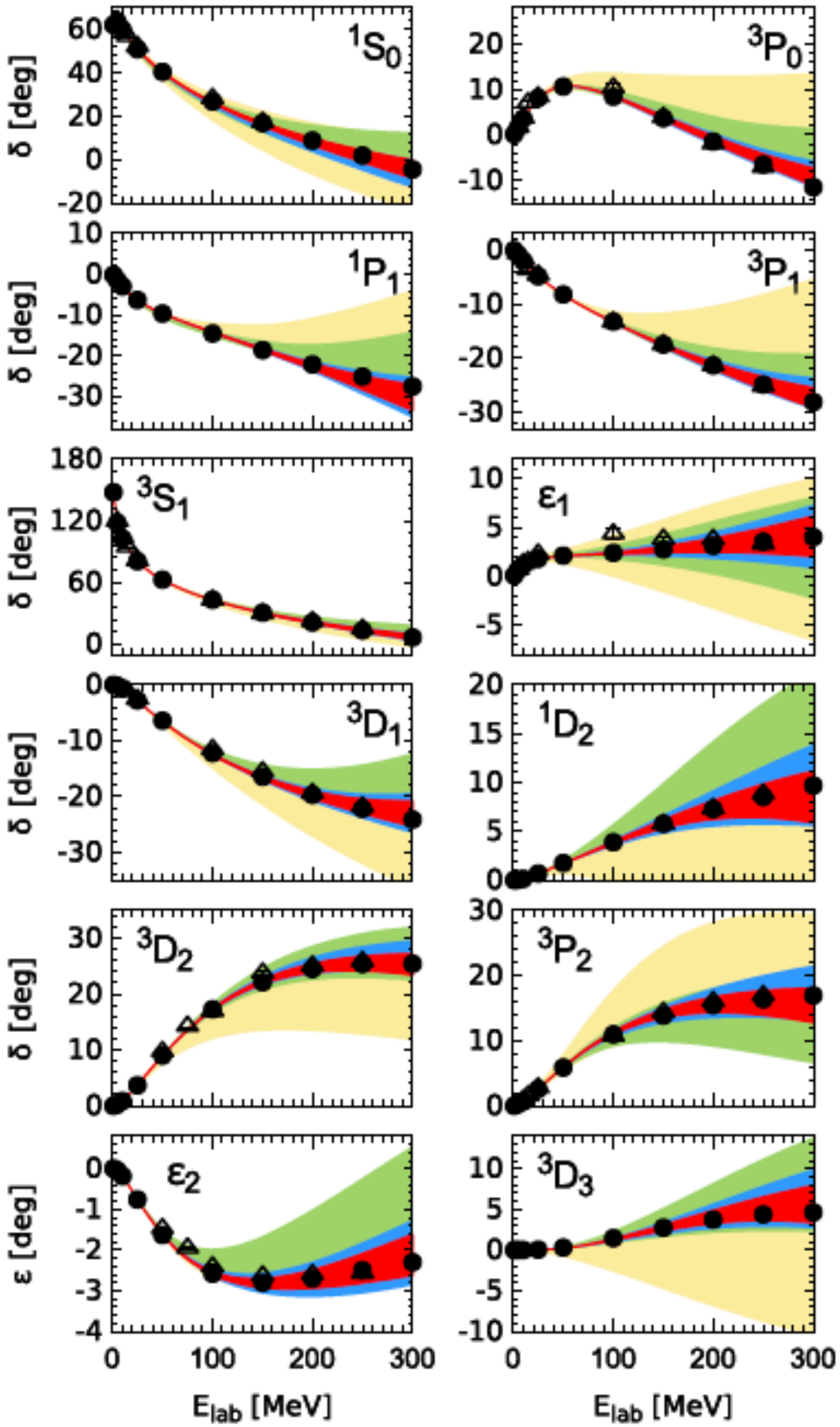
$$\mathcal{O}\left(\frac{Q}{\Lambda}\right)$$

$$\Lambda \sim 1 \text{ GeV}$$



Weinberg, *Phys. Lett. B* **251** 288 (1990), *NPB* **363** 3 (1991)
Entem & Machleidt, *PRC* **68**, 041001(R) (2003)
Tews, Schwenk *et al.*, *PRL* **110**, 032504 (2013)
Epelbaum, Frebs & Meissner, *PRL* **115**, 122301 (2015)

NN forces from EFTs of QCD



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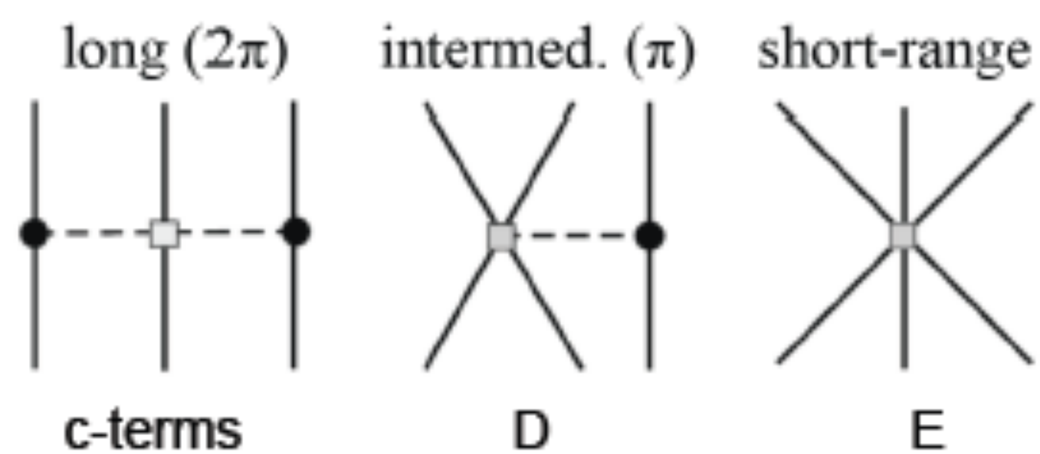

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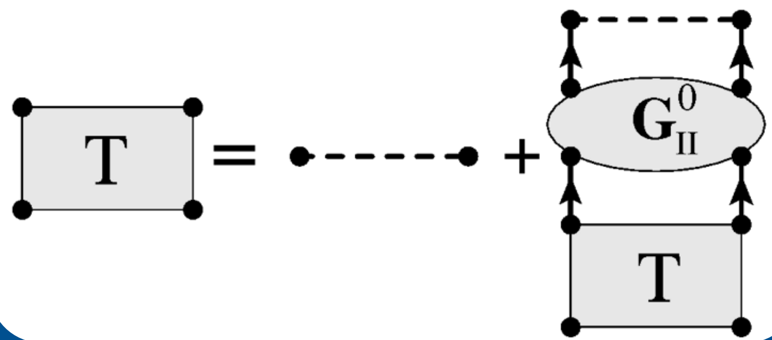
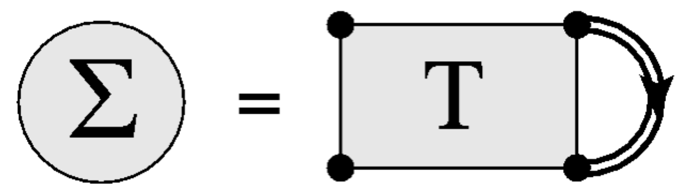
Tews, Schwenk *et al.*, *PRL* **110**, 032504 (2013)
Epelbaum, Frebs & Meissner, *PRL* **115**, 122301 (2015)

SCGF Ladder approximation

In-medium interaction

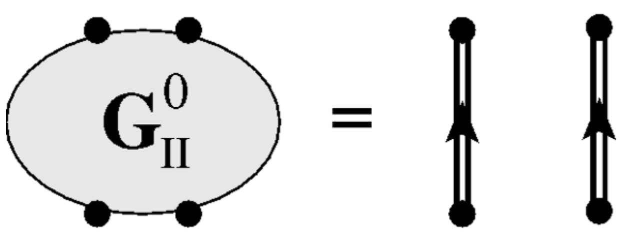


Ladder self-energy

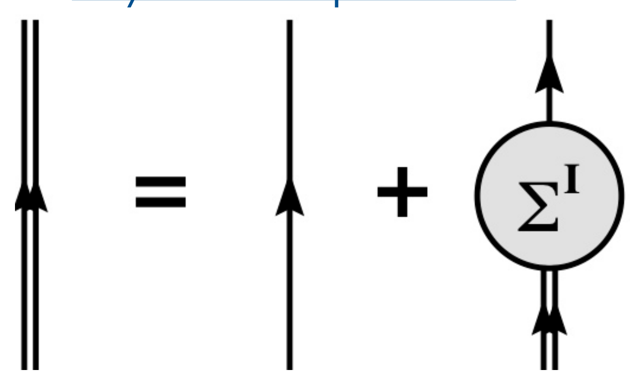


- Self-consistent resummation
- Energy and momentum integral
- @Finite T (Matsubara)

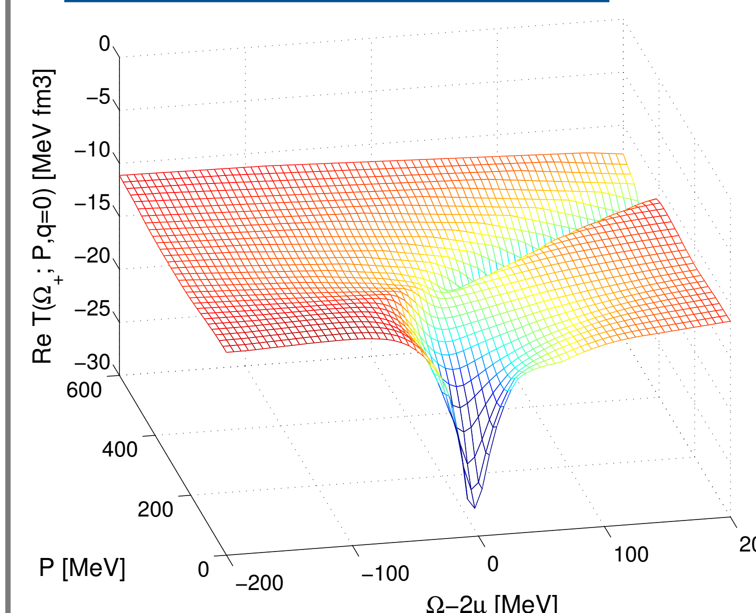
pp & hh Pauli blocking



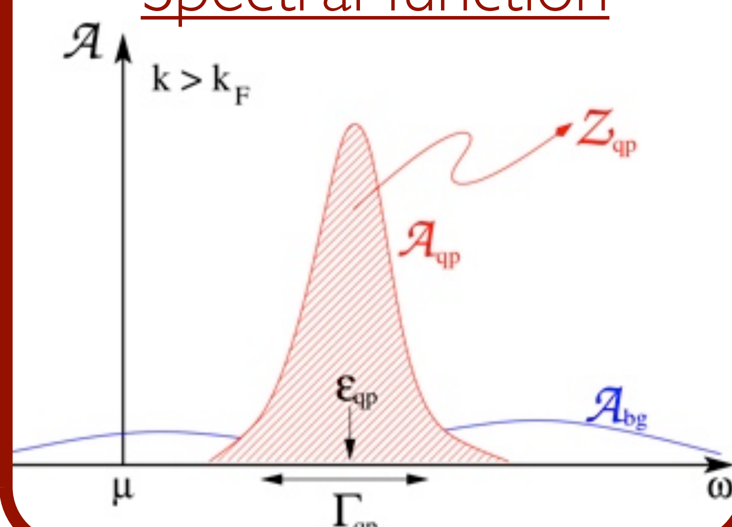
Dyson equation



T-matrix at T=5 MeV



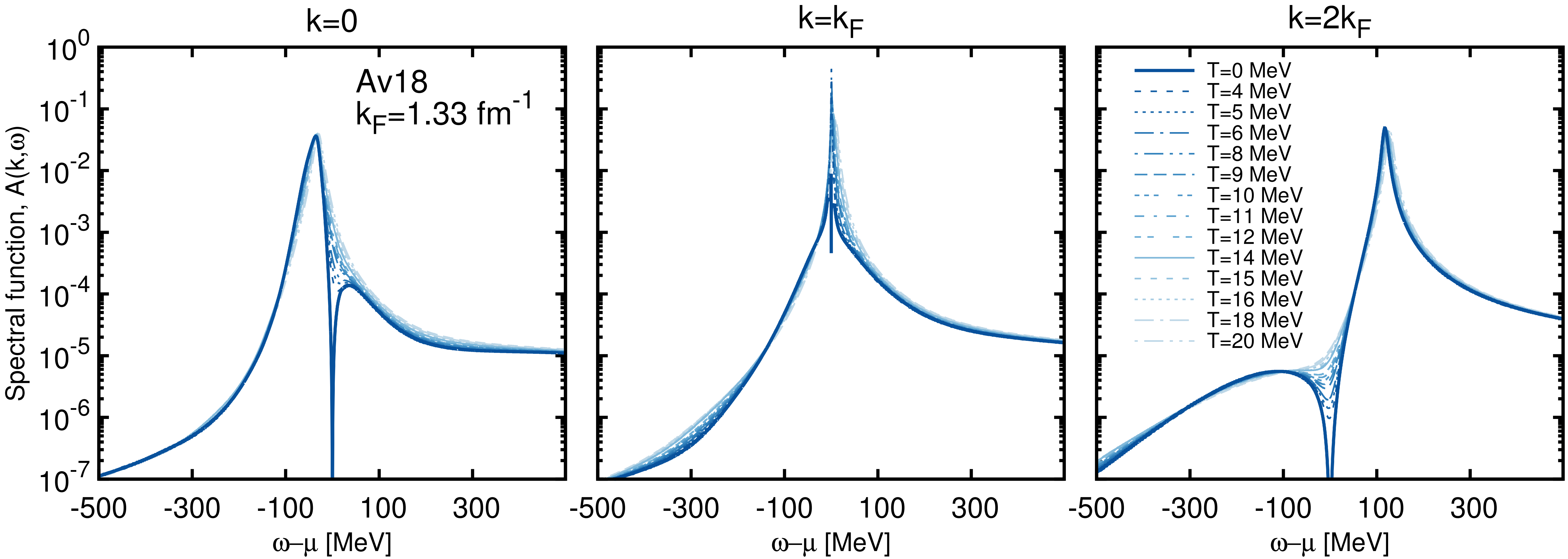
Spectral function

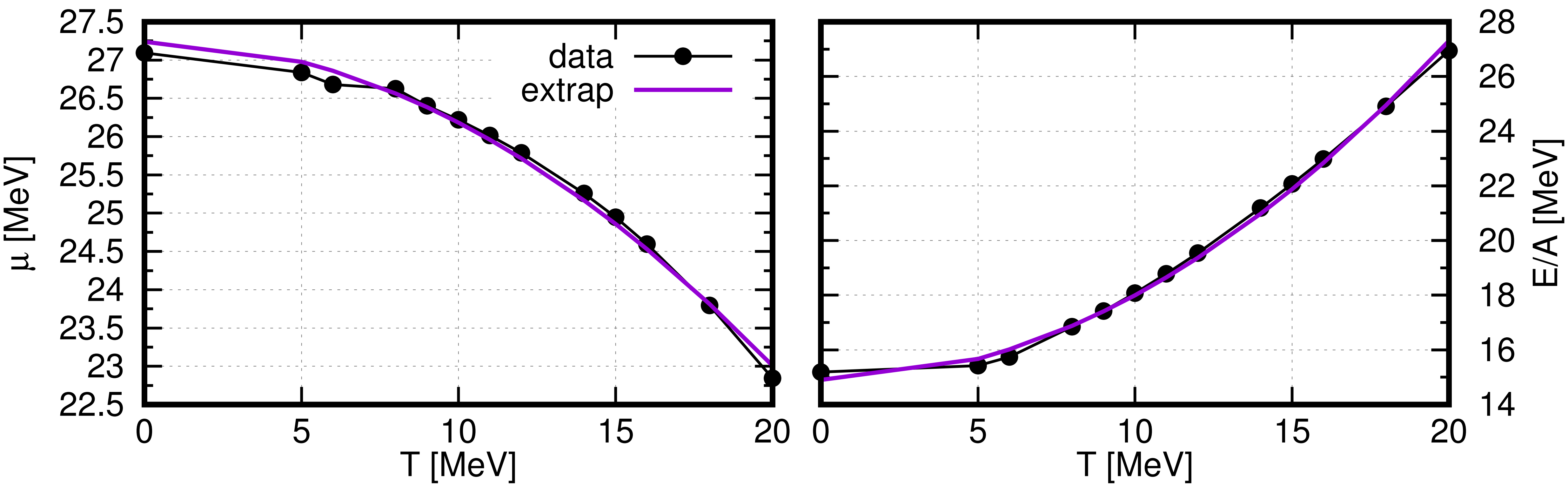


One-body properties
Momentum distribution
Thermodynamics & EoS
Transport

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

T=0 extrapolations



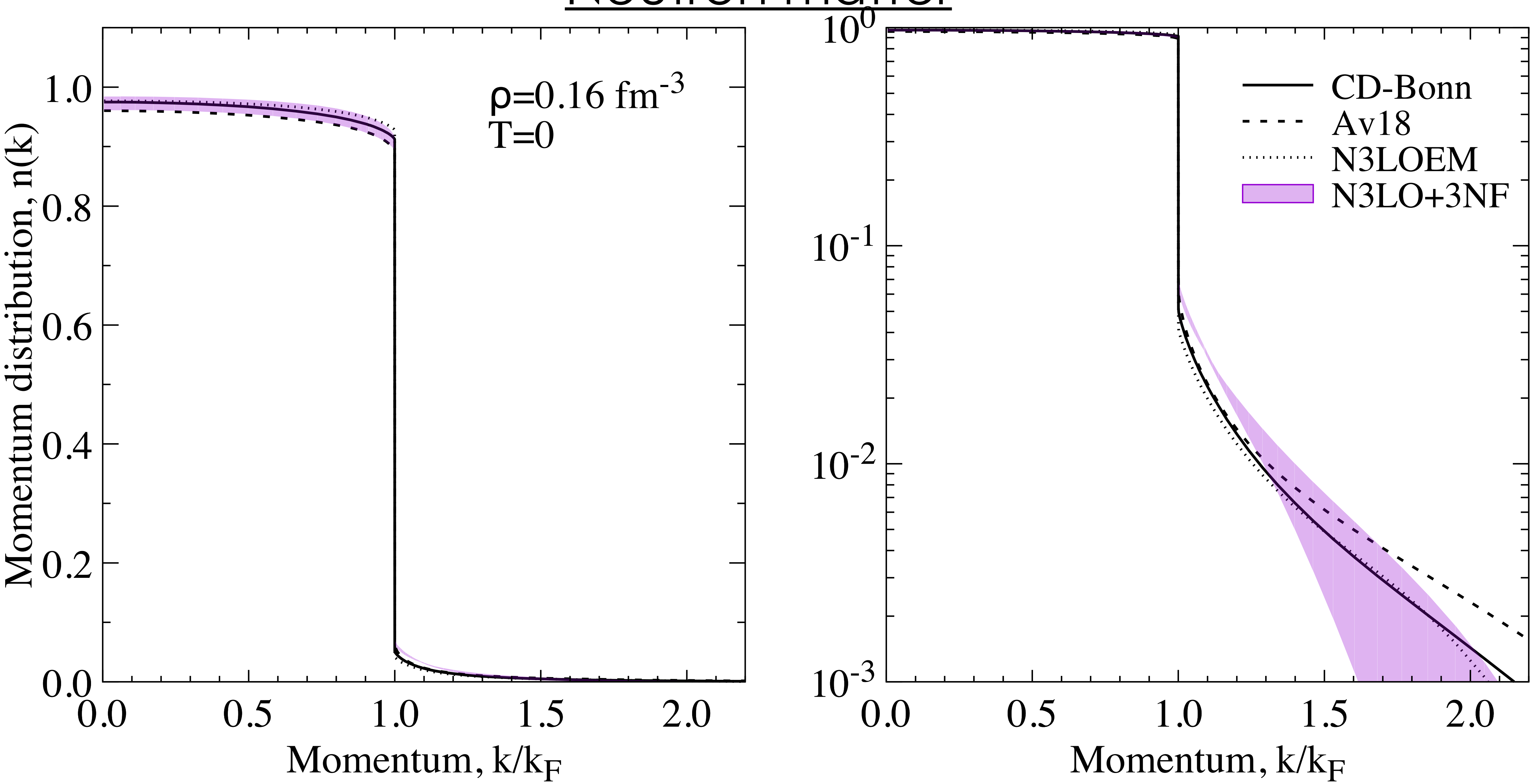


Momentum distribution

Single-particle occupation

$$n(k) = \left\langle a_k^\dagger a_k \right\rangle \qquad \nu \int \frac{d^3k}{(2\pi)^3} n(k) = \rho$$

<u>Neutron matter</u>

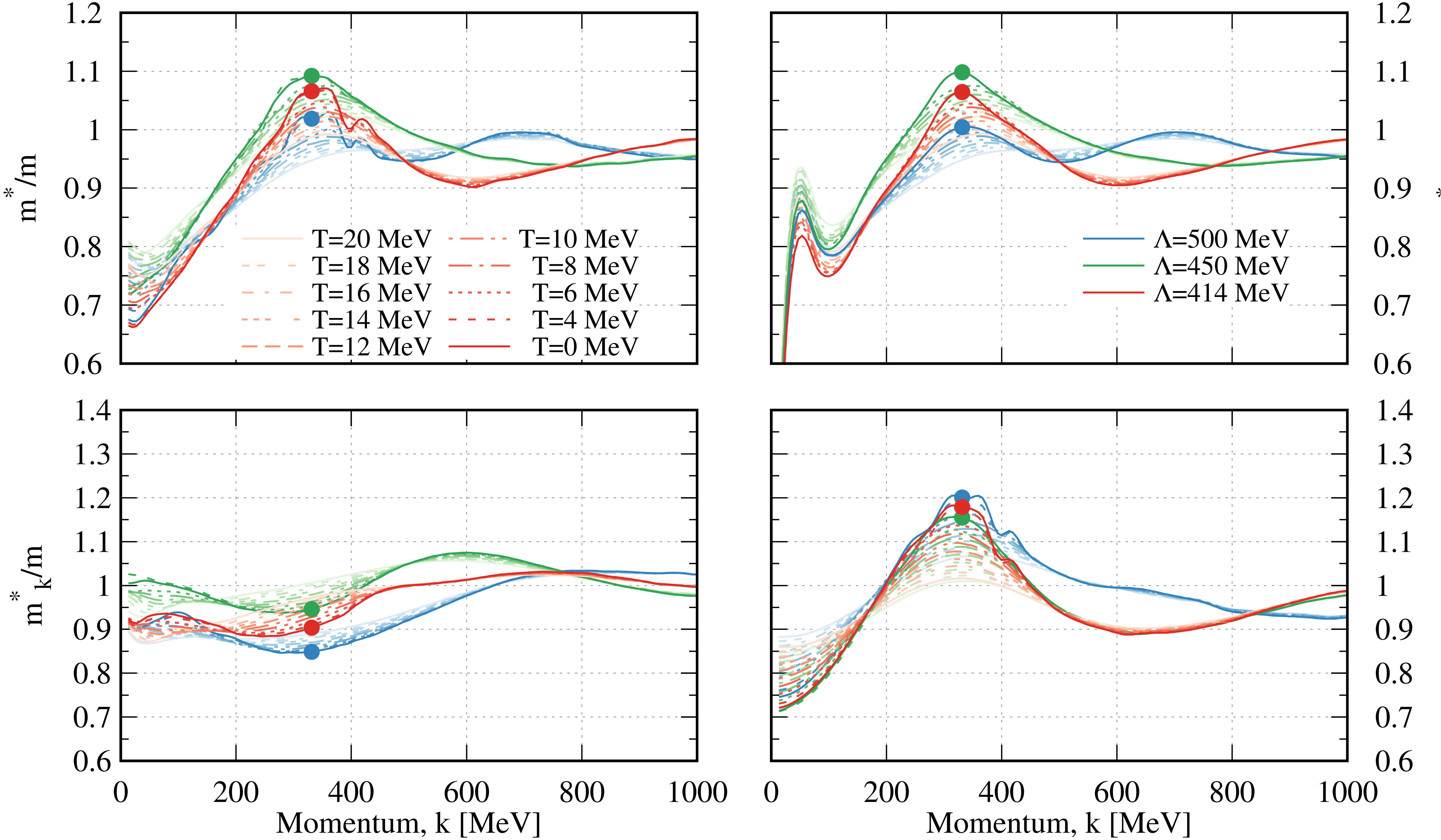


- Dependence on NN interaction under control
- PNM: 4-5% depletion at low k
- N3LO+3NF = N3LO 2NF + N2LO 3NF @ Λ=414-500 MeV (cutoff variation only)

Coraggio, Holt, et al. PRC **89** 044321 (2014)

Effective masses

Neutron matter $\rho=0.16\ fm^{-3}$



Coraggio, Holt, et al. PRC **89** 044321 (2014)

In preparation, will be publicly available

Available data

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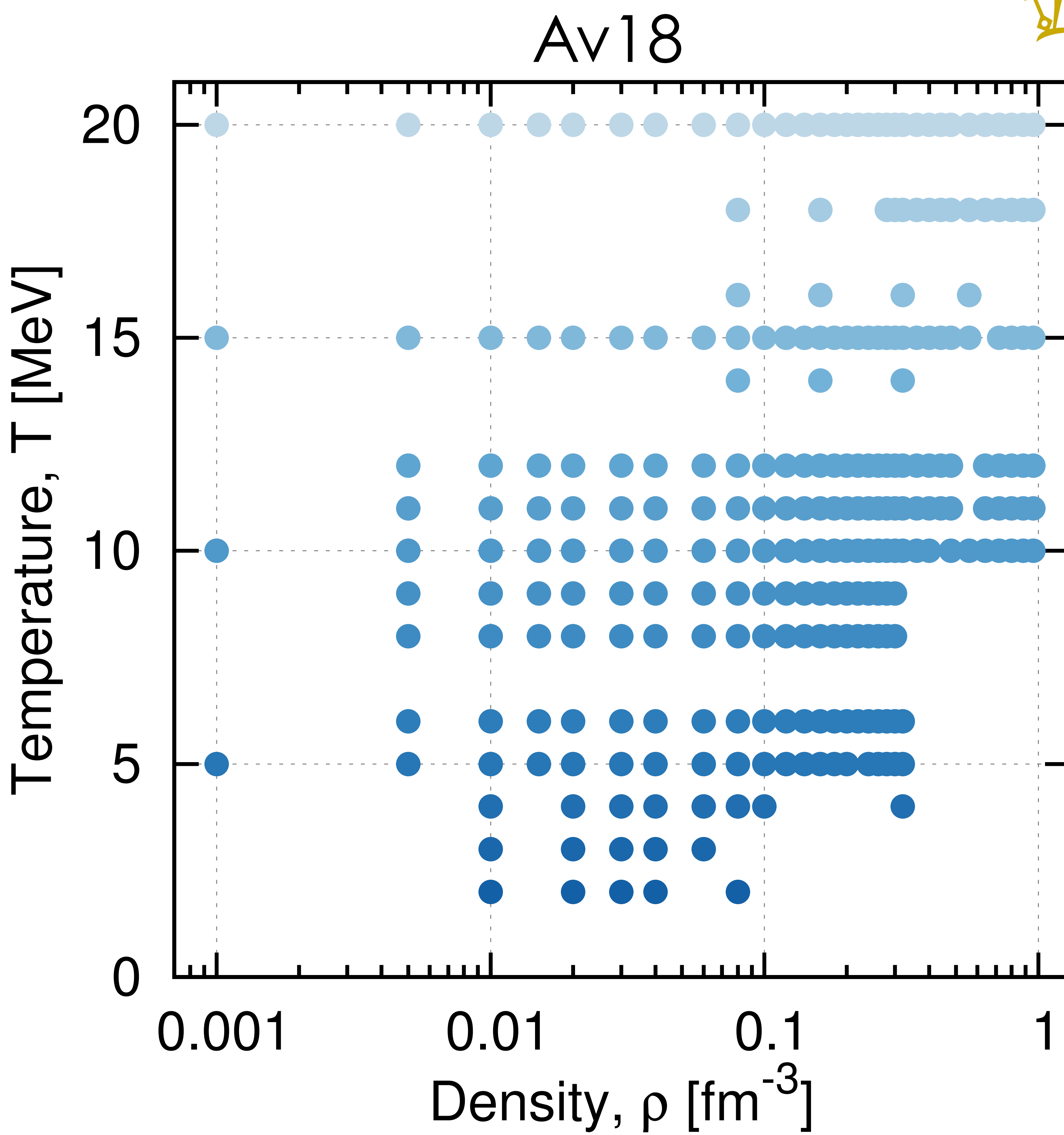
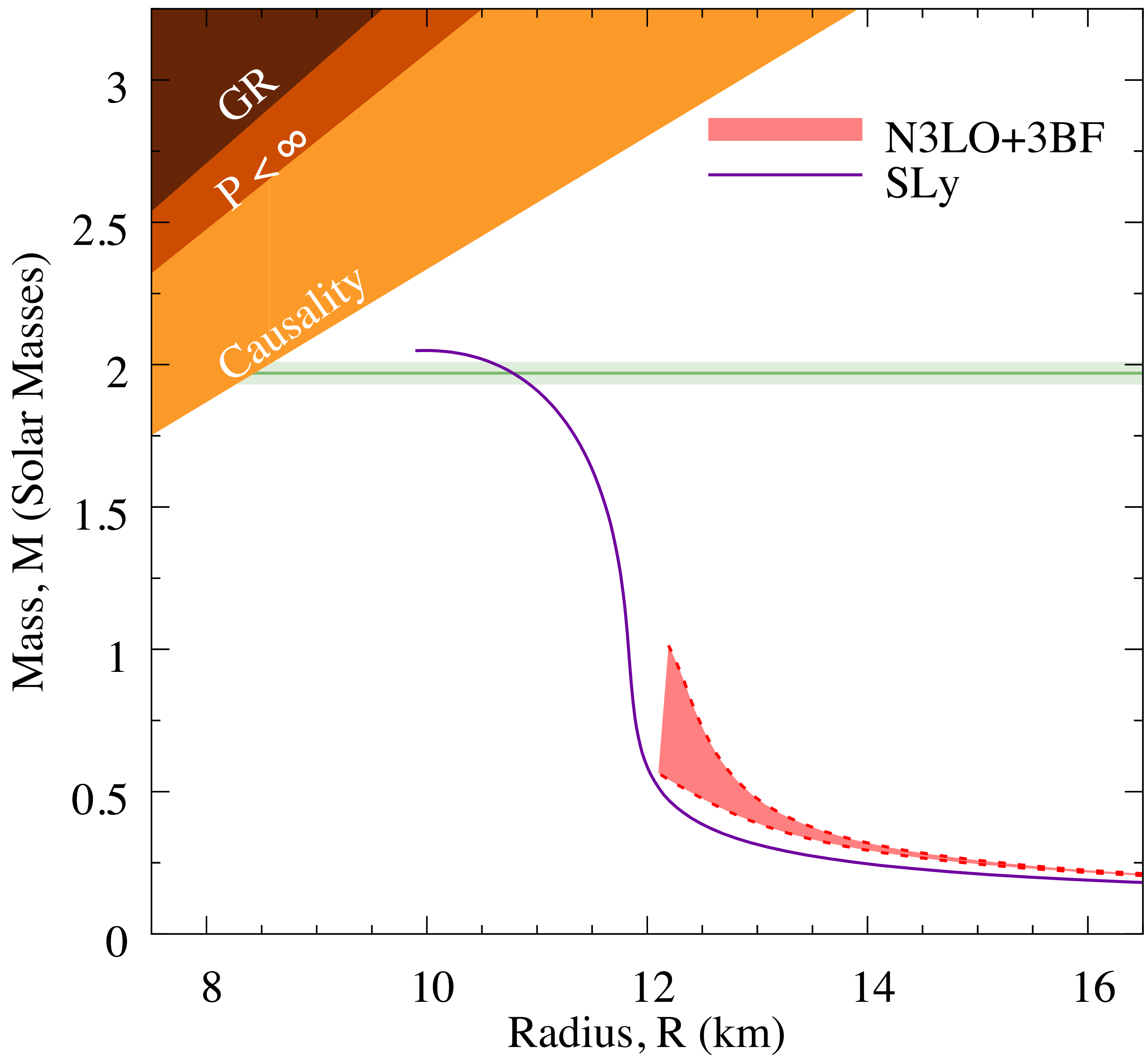


Self-energy, spectral function & thermodynamics

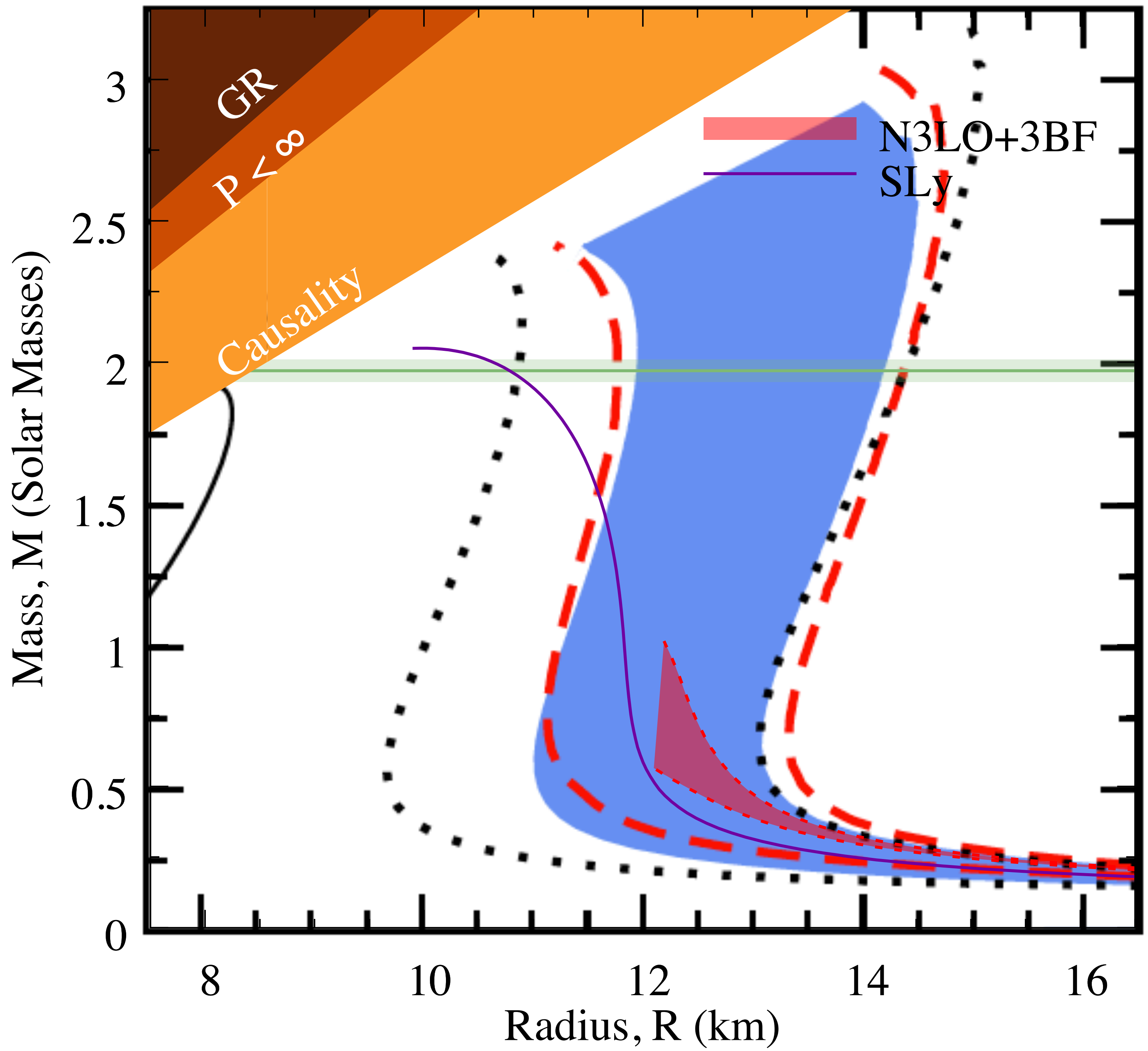
In preparation, will be publicly available

Neutron matter



- Mass-Radius relation from SCGF calculations
- Cut-off variation (N3LO) and/or SRG evolution

Neutron matter



Hebeler, Lattimer, Pethick, Schwenk *ApJ* **773** 11 (2013)

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Summary

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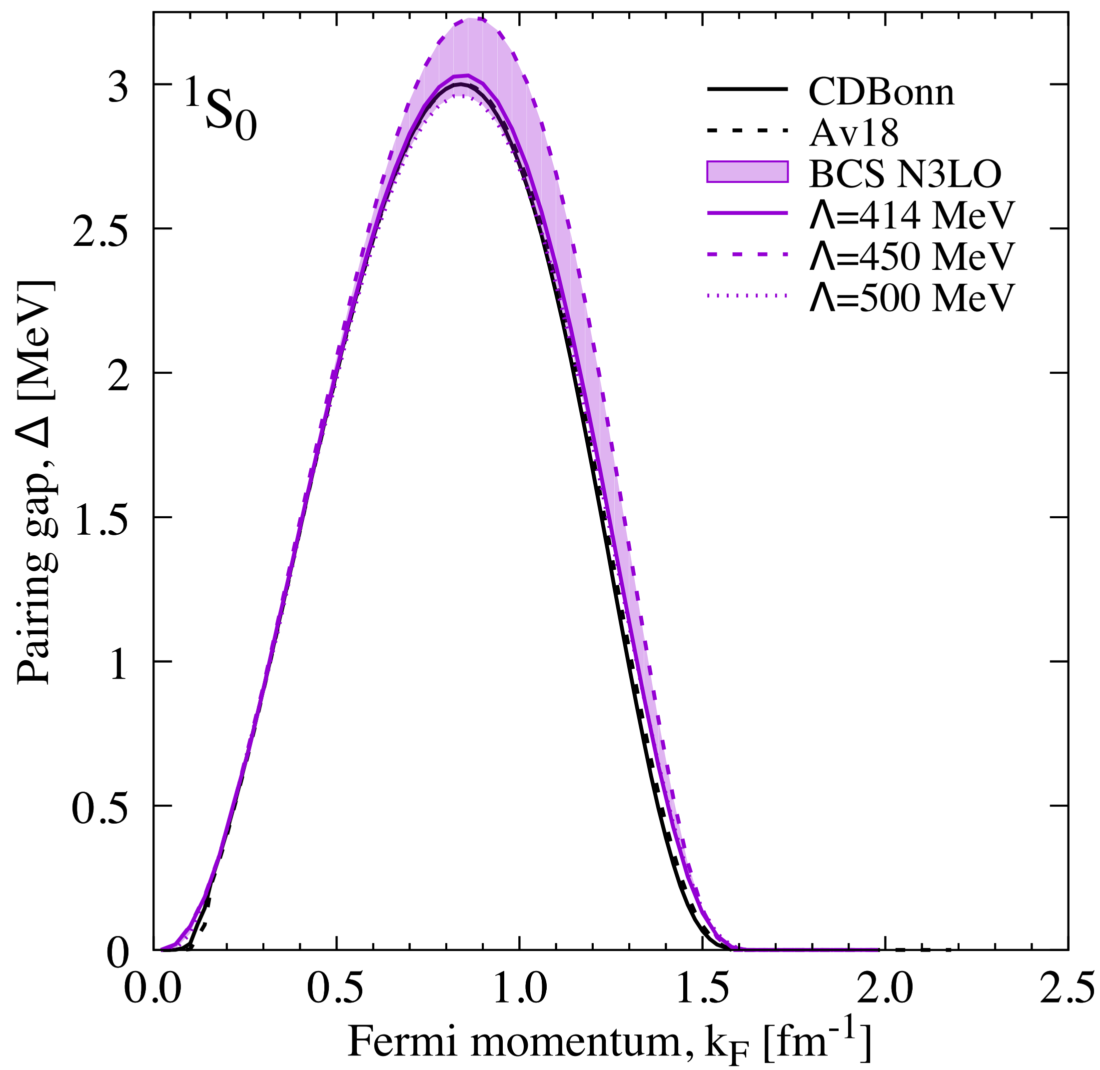
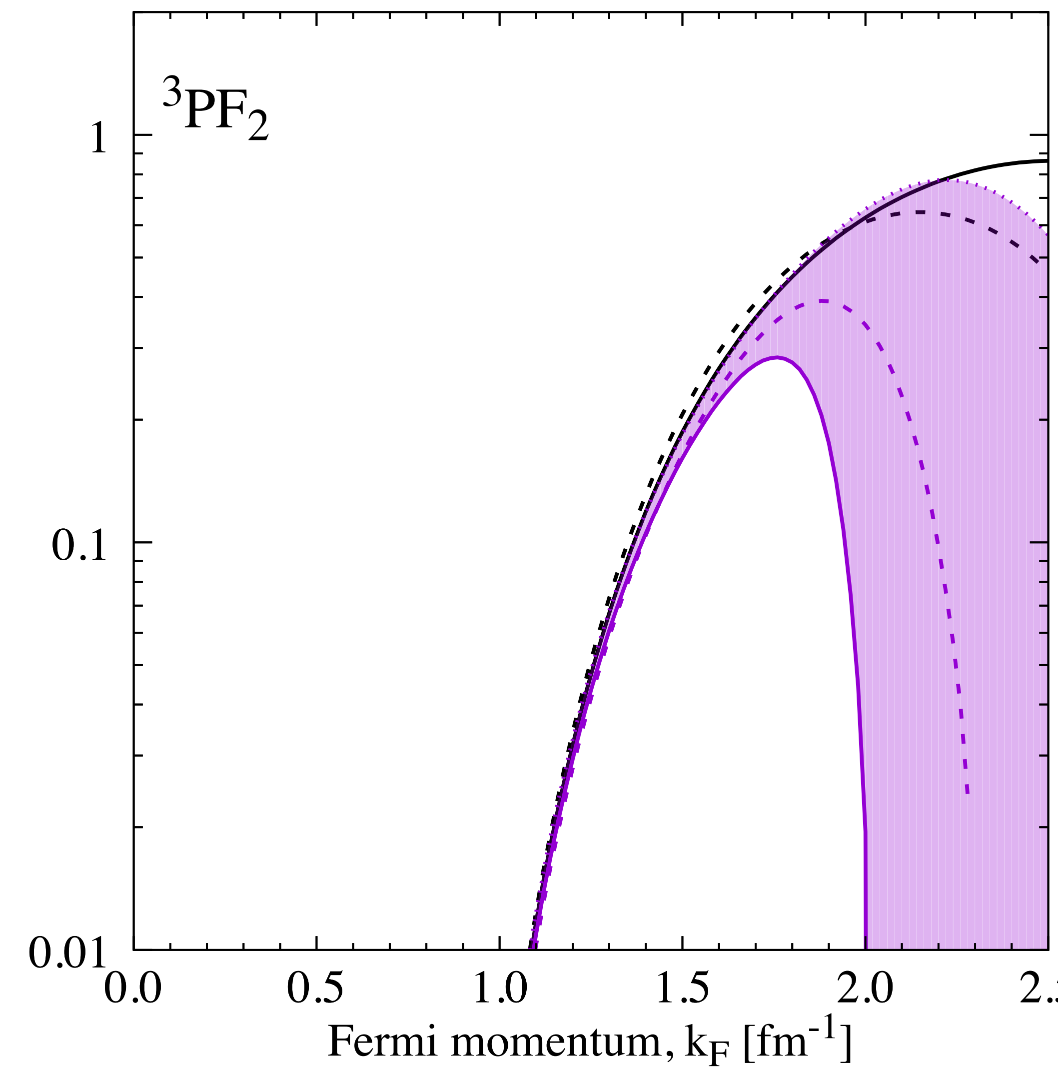
2. **Infinite matter BCS**
3. Beyond-BCS with SCGF methods

Bardeen-Cooper-Schrieffer pairing



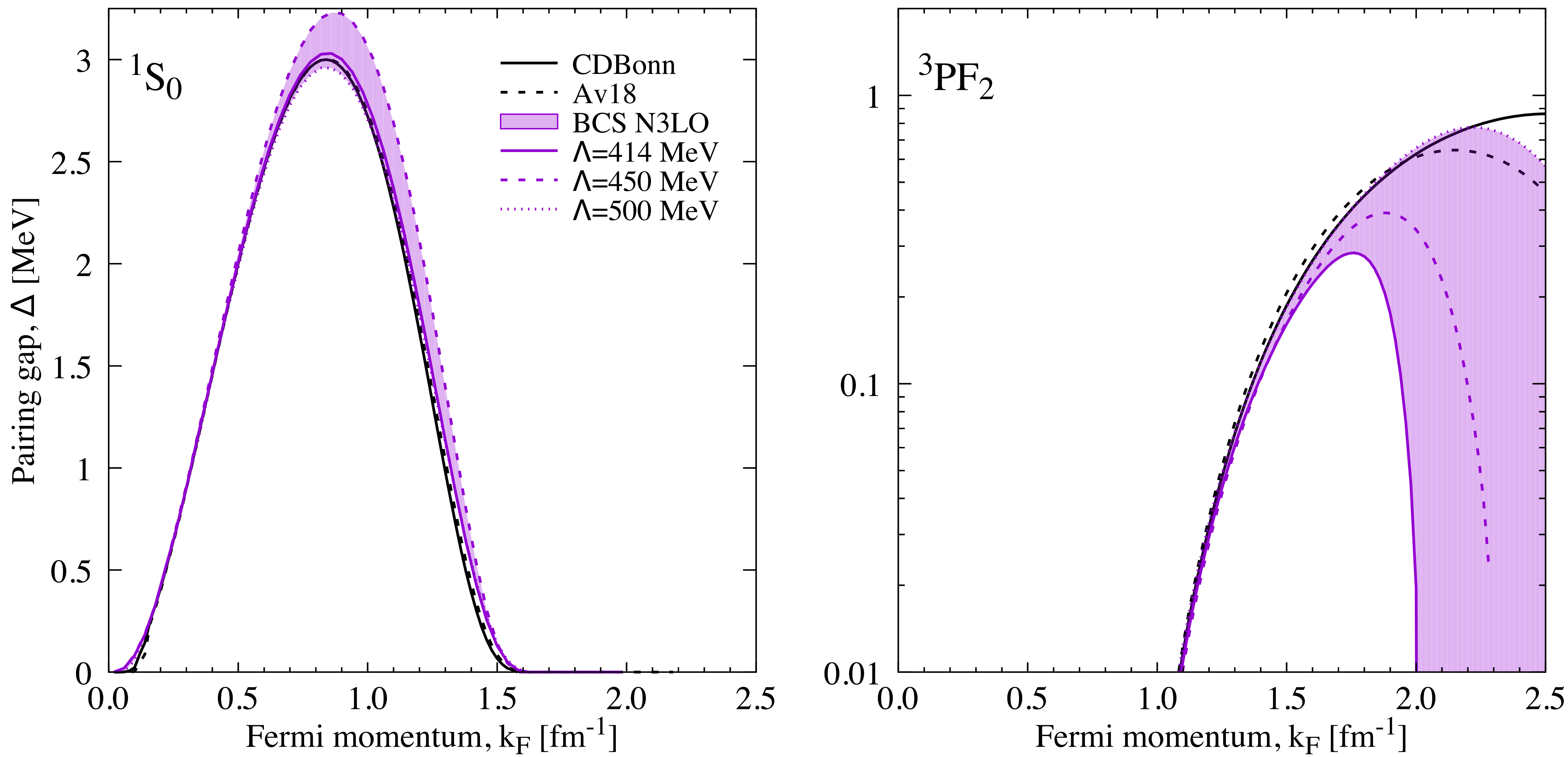


BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \chi_k = \varepsilon_k - \mu$$

- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + U(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2$

Bardeen-Cooper-Schrieffer pairing



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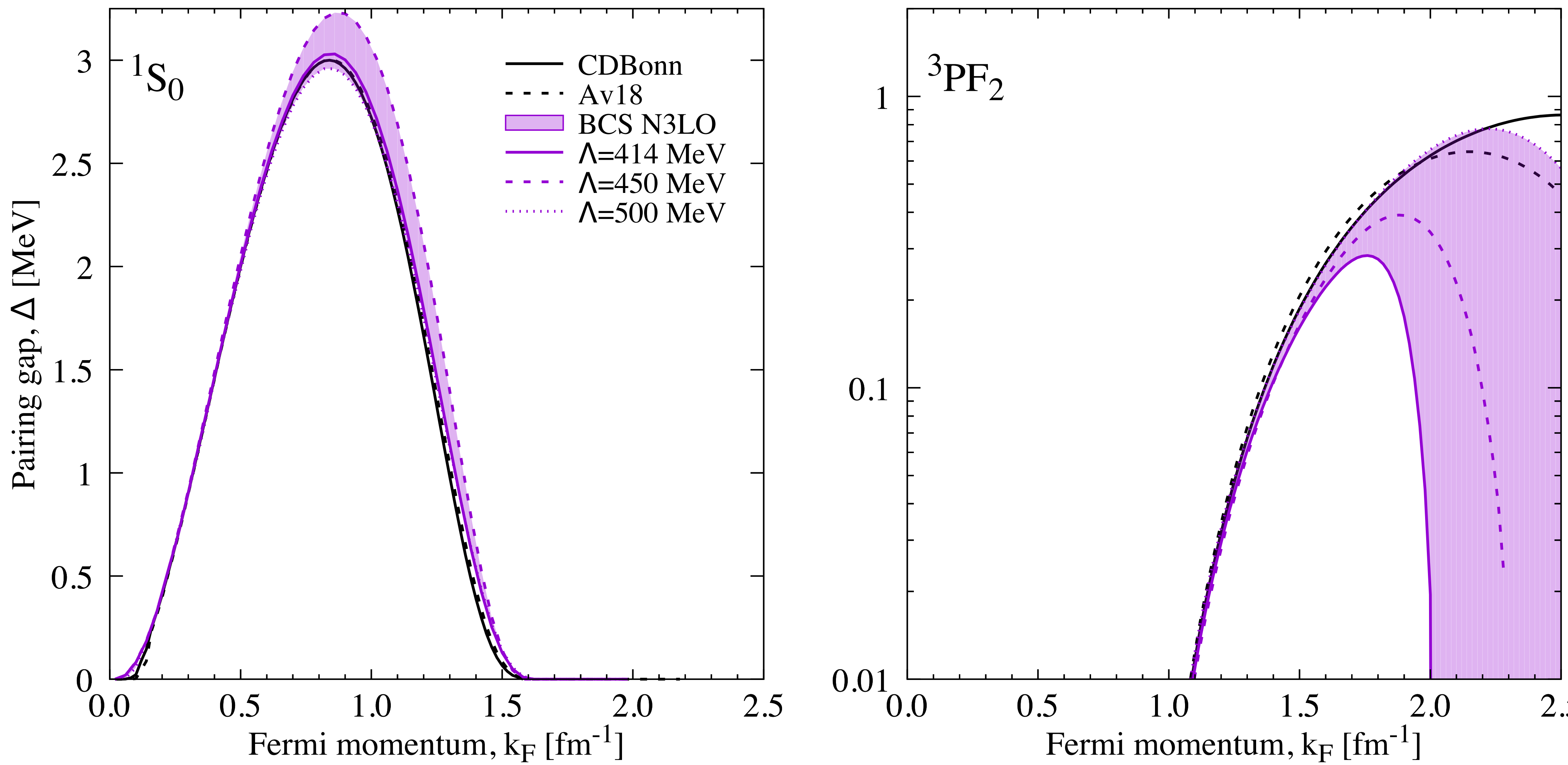

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Bardeen-Cooper-Schrieffer pairing

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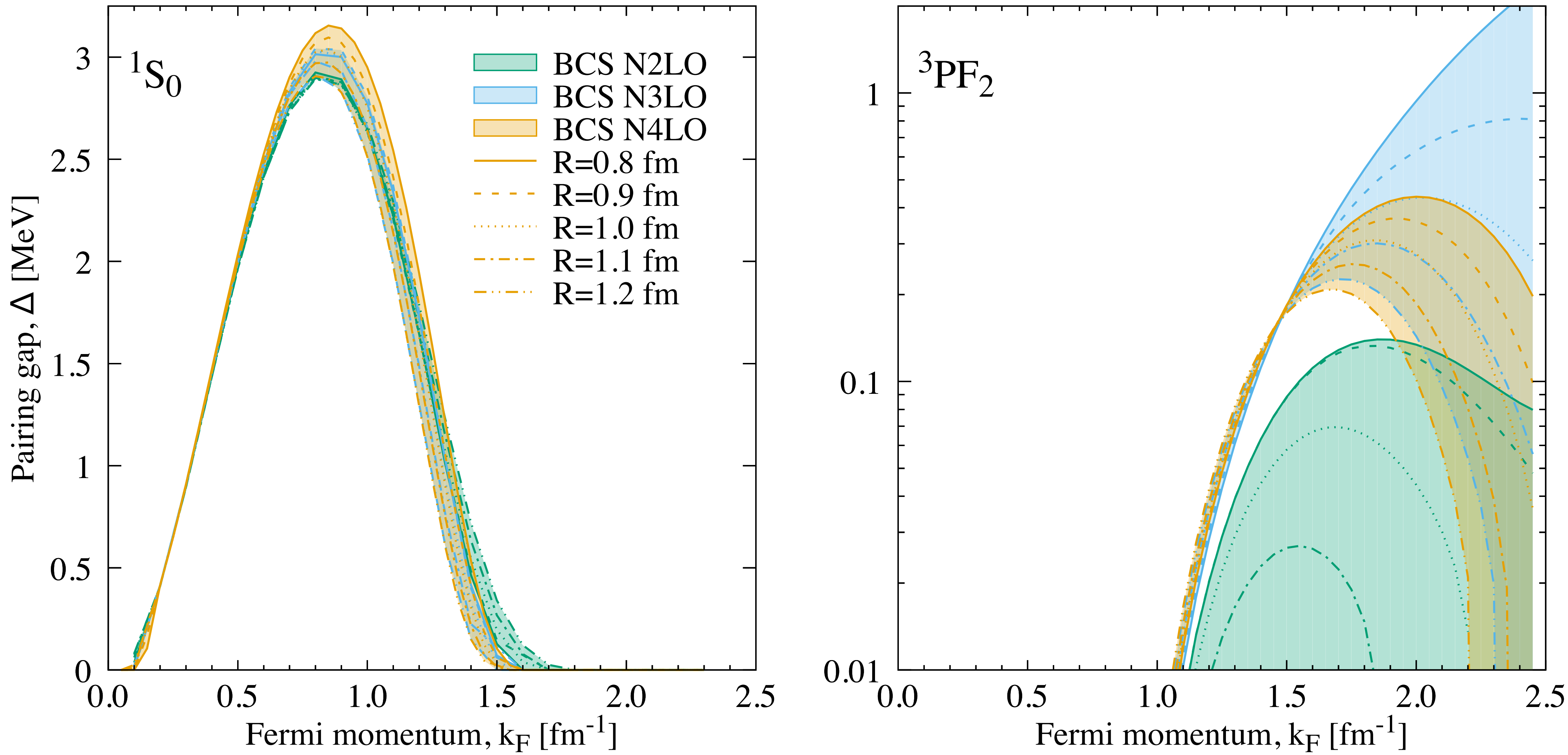

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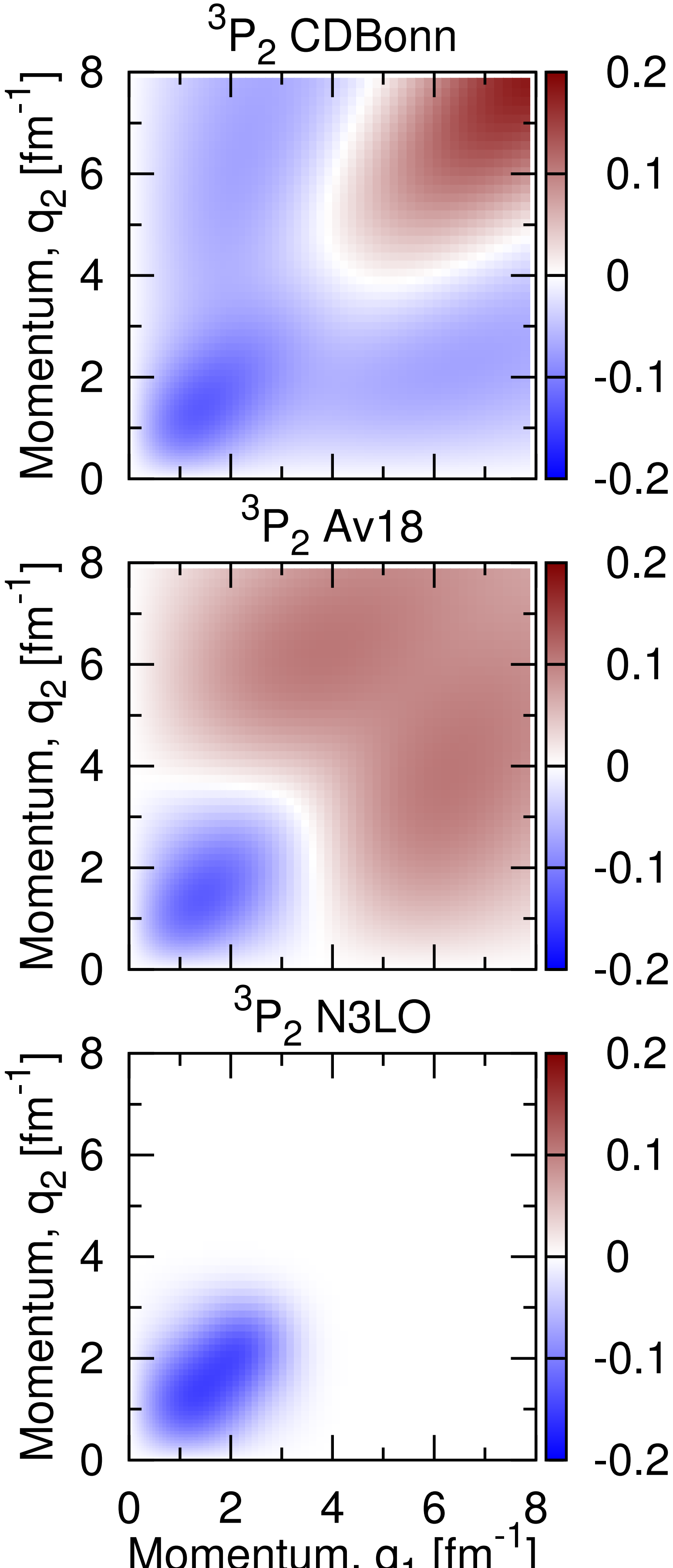
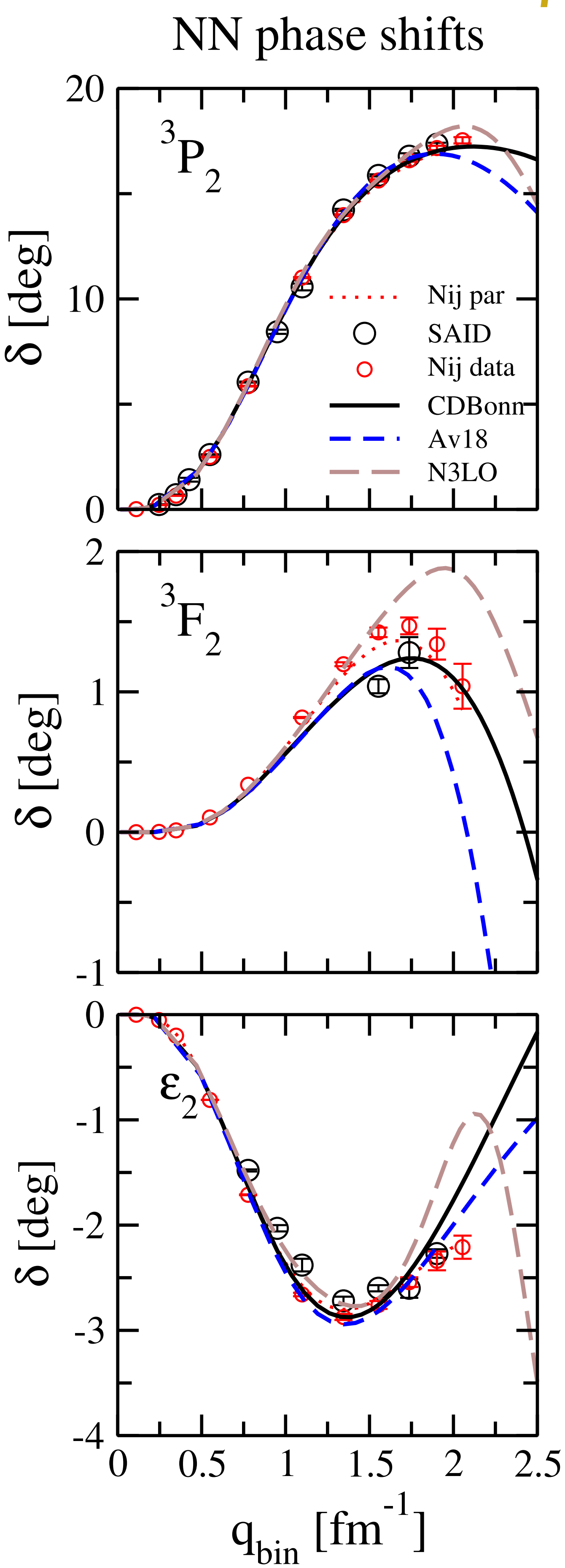
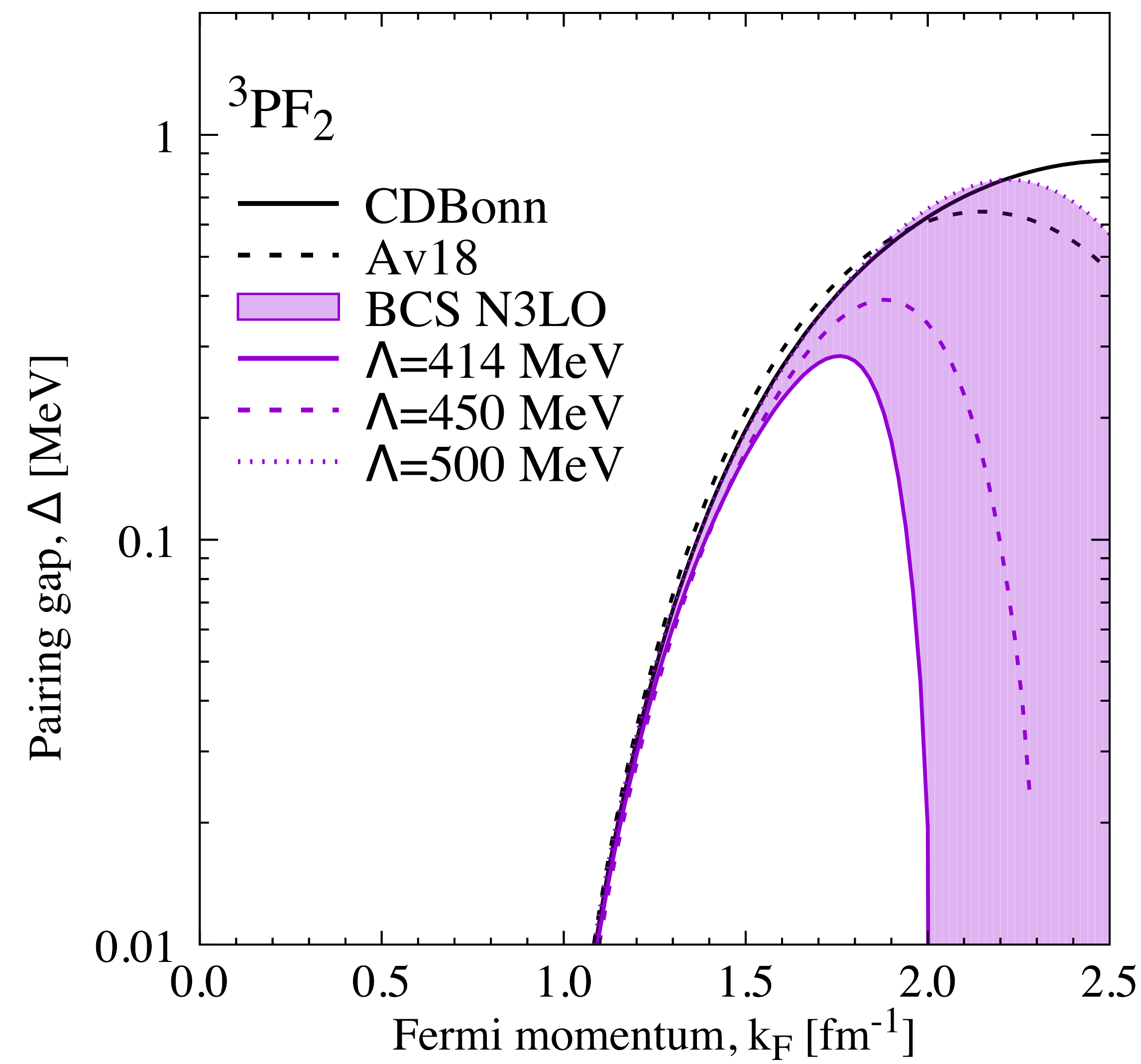
$^3PF_2$ pairing: phase shift equivalence



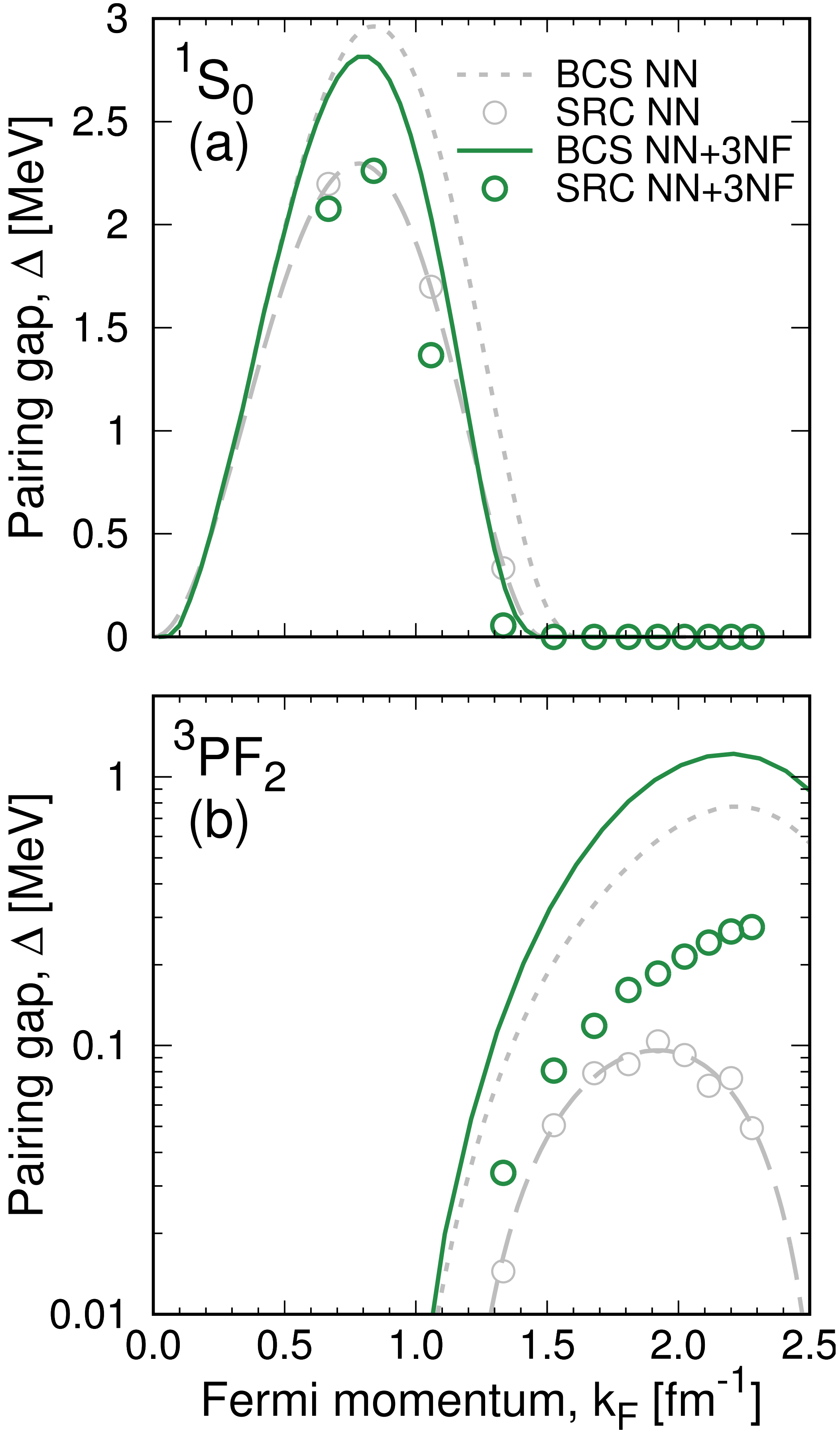


Neutron matter BCS gaps

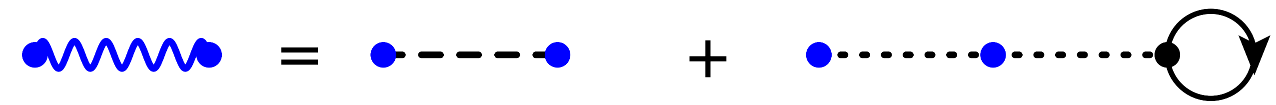


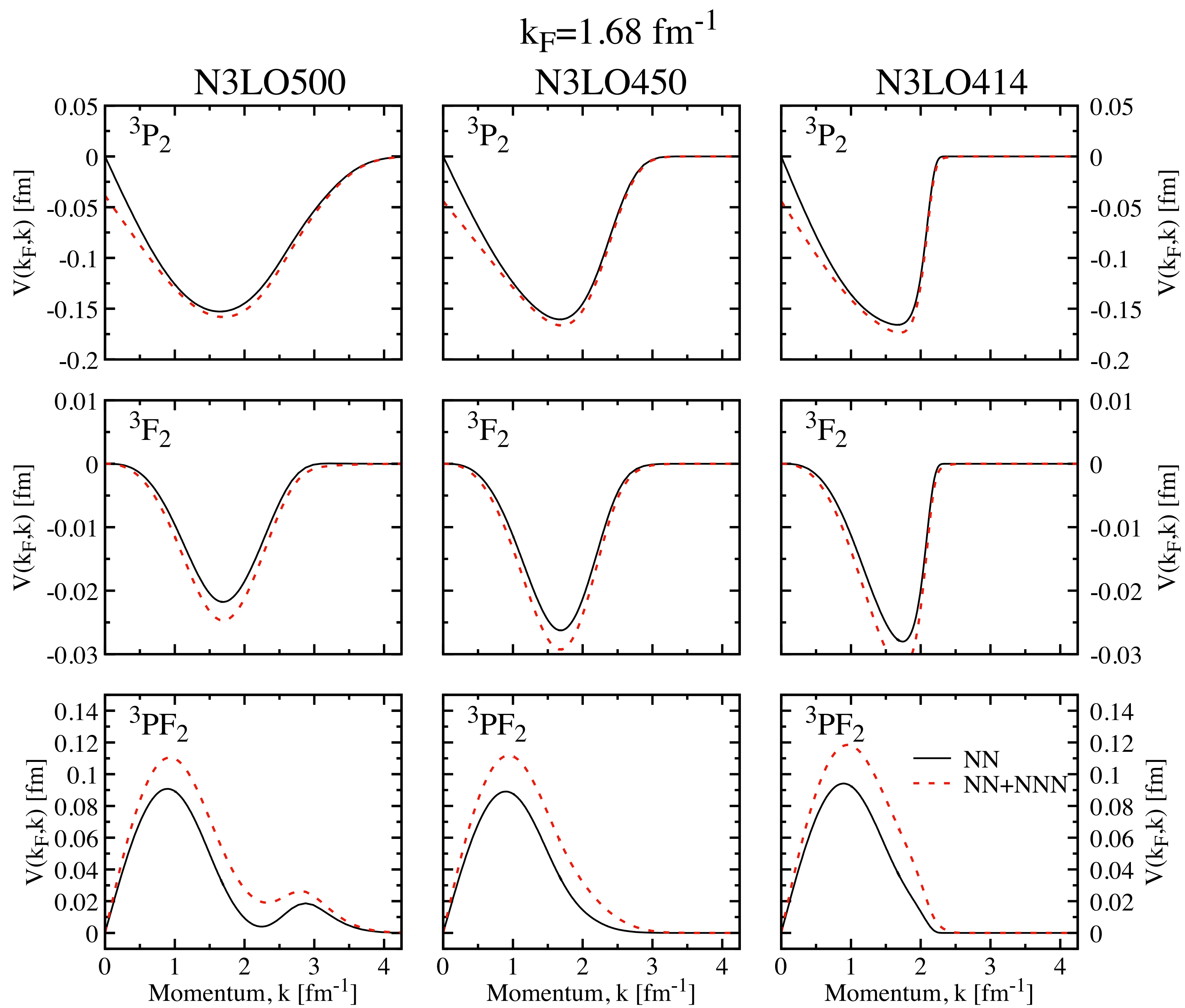
Srinivias & Ramanan, *PRC* **94** 064303 (2016)

3BF effect: estimate



Effective two-body force ⇒*NN forces*

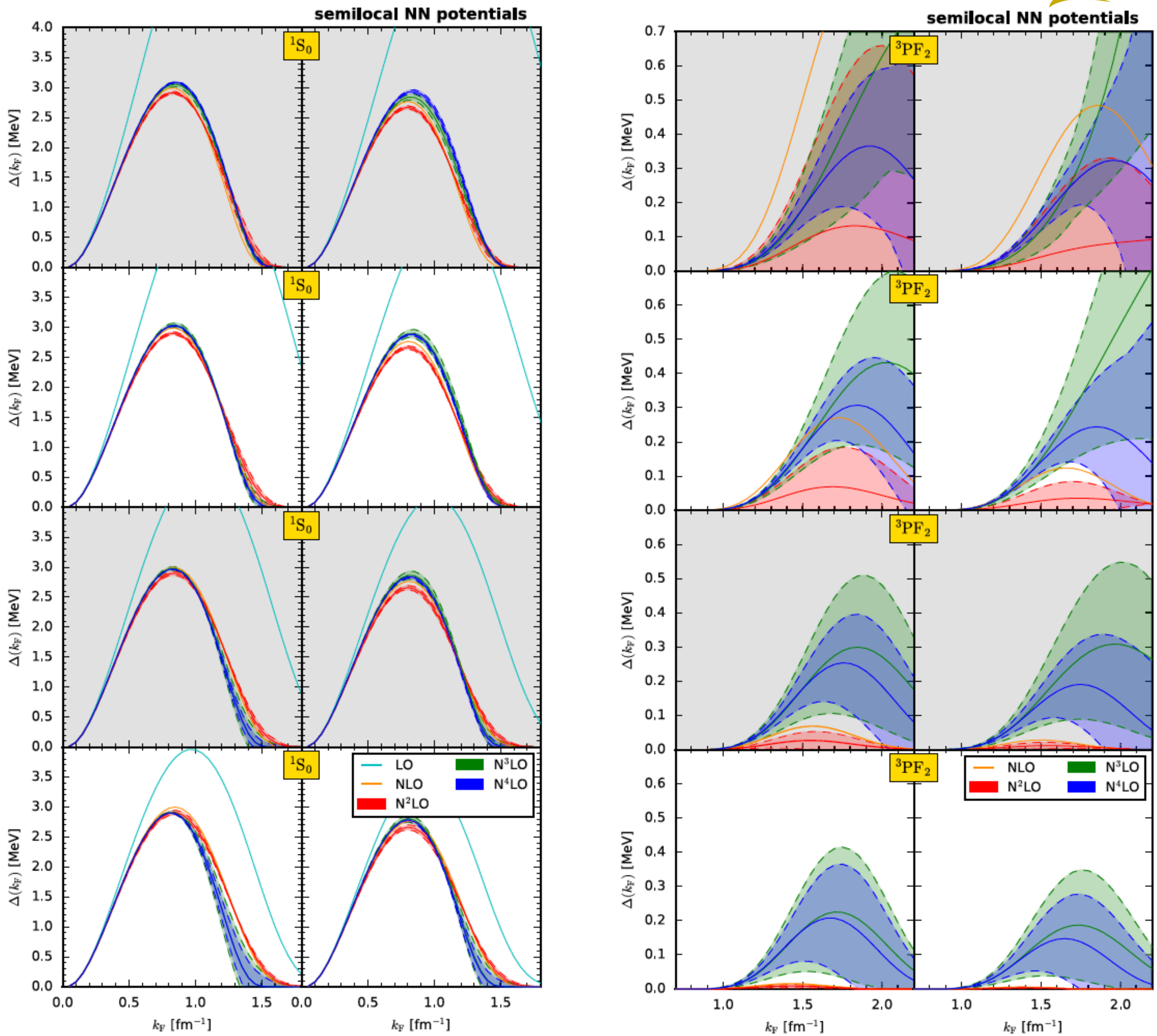




- Singlet gap: 3NF **reduce** closure
- Triplet gap: 3NF **increase** gap

Uncertainties at the BCS + HF level

"Hard"



R=0.9 fm

R=1.0 fm

R=1.1 fm

R=1.2 fm

"Soft"

Drischler, Kruger, Hebler, Schwenk, *PRC* **95** 024302 (2017)

Uncertainties at the BCS + HF level

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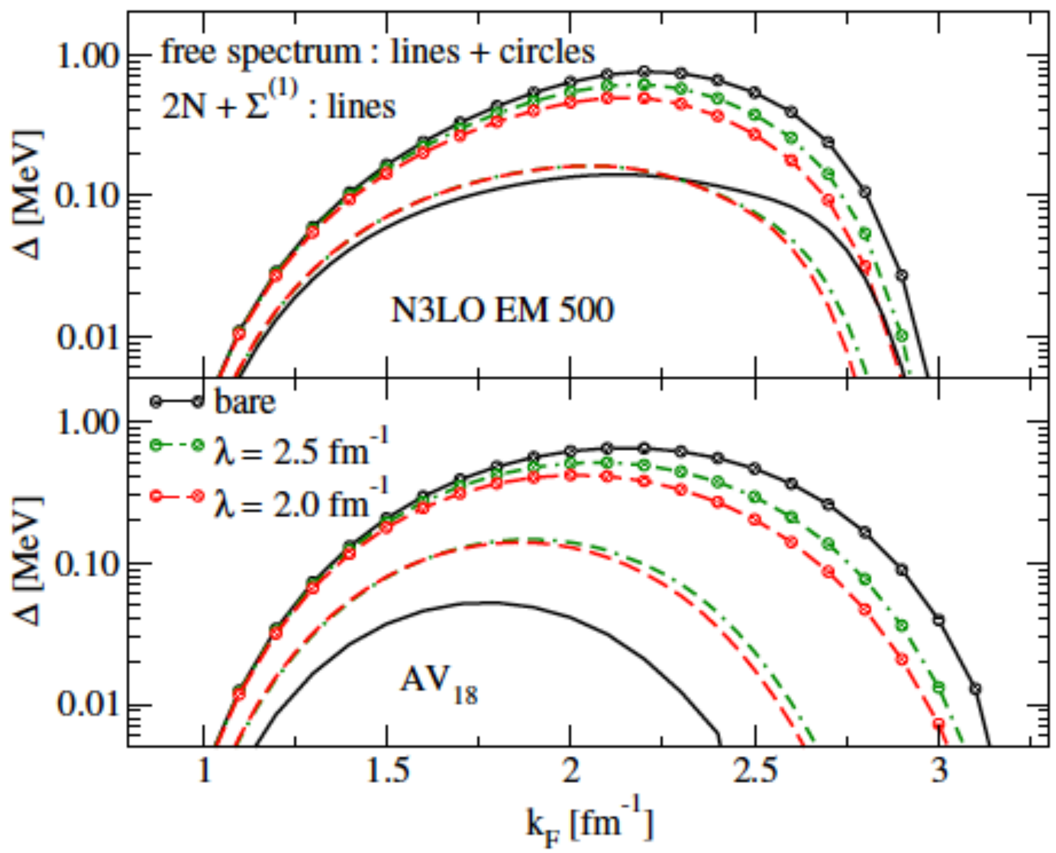
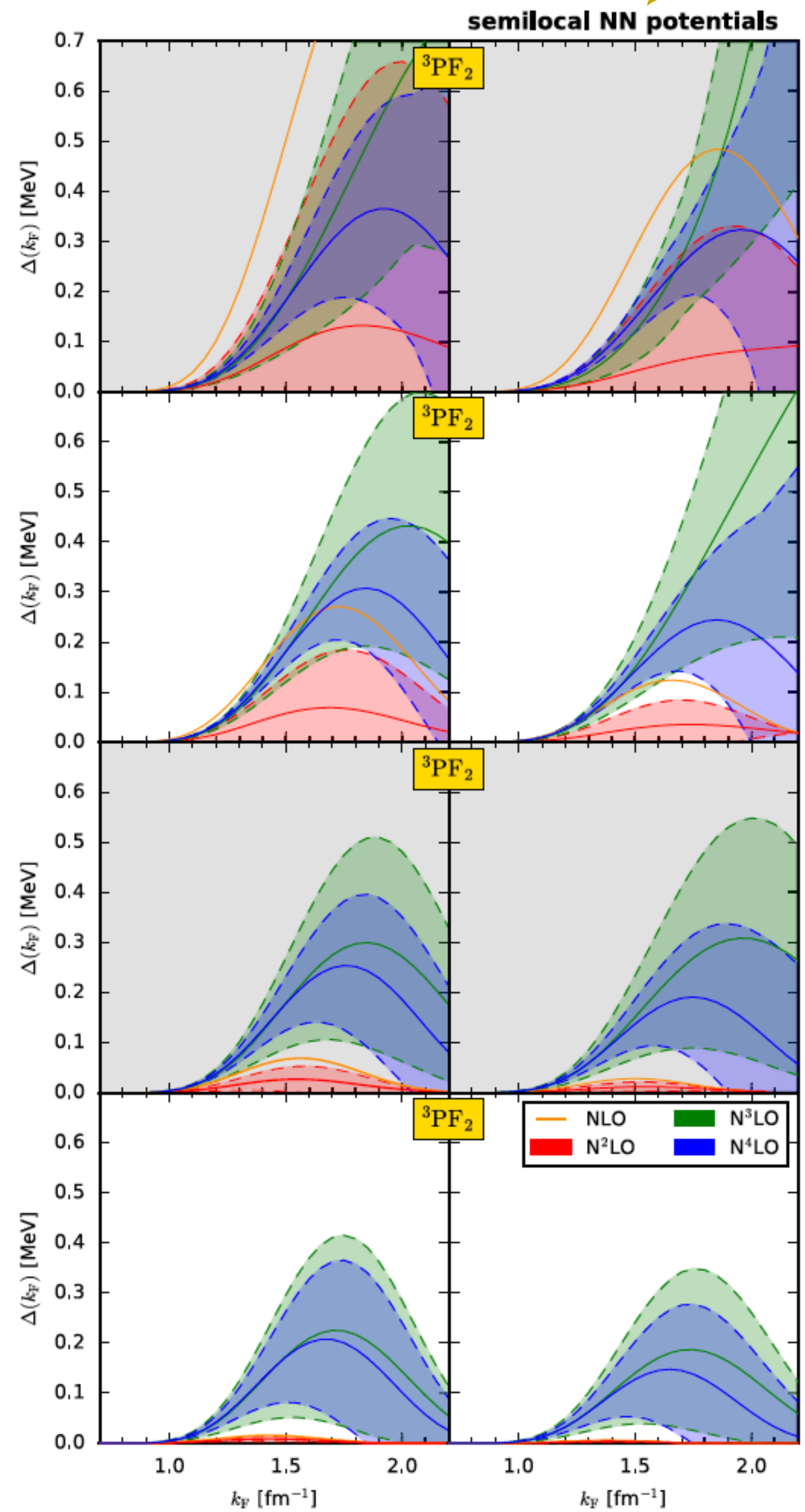


Srinivias & Ramanan, *PRC* **94** 064303 (2016)



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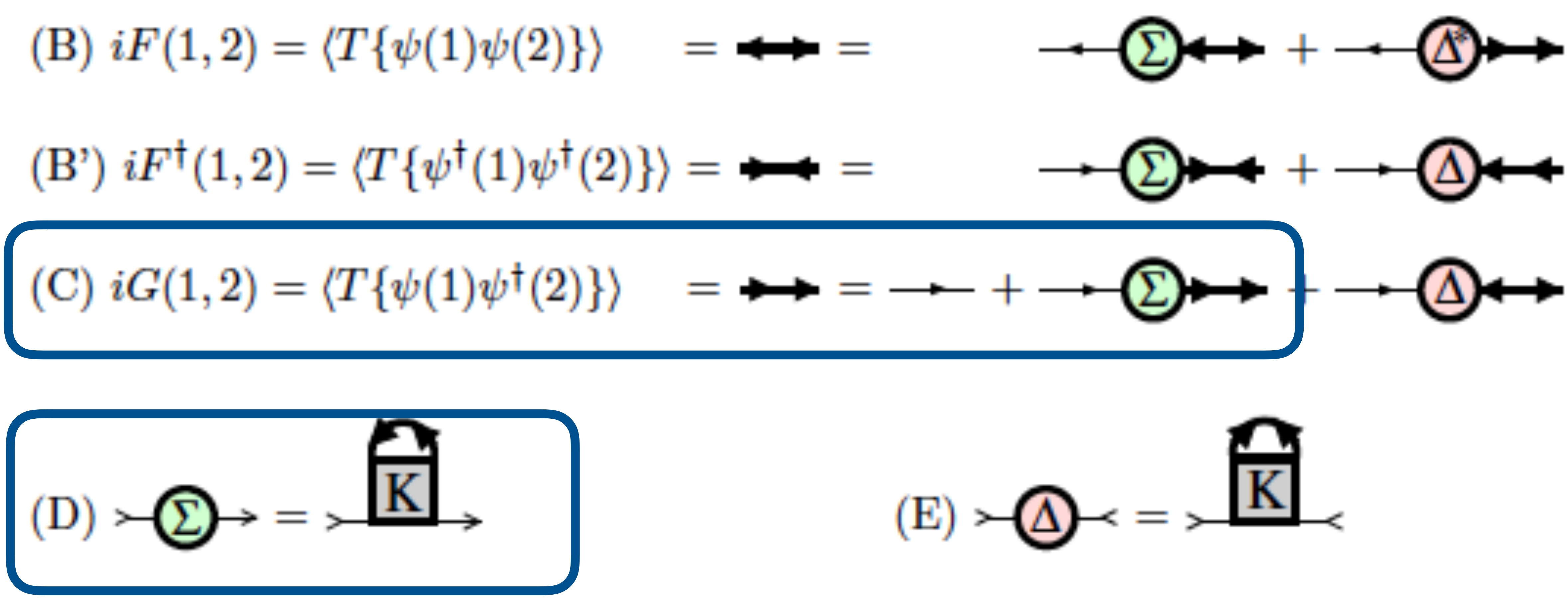
Beyond BCS 101: SRC



Normal state

- **BCS** is *lowest order* in Gorkov Green's function expansion
- T-matrix can be extended to paired systems

Bozek, *Phys. Rev. C* **62** 054316 (2000); Muether & Dickhoff, *Phys. Rev. C* **72** 054313 (2005)

Beyond BCS 101: SRC

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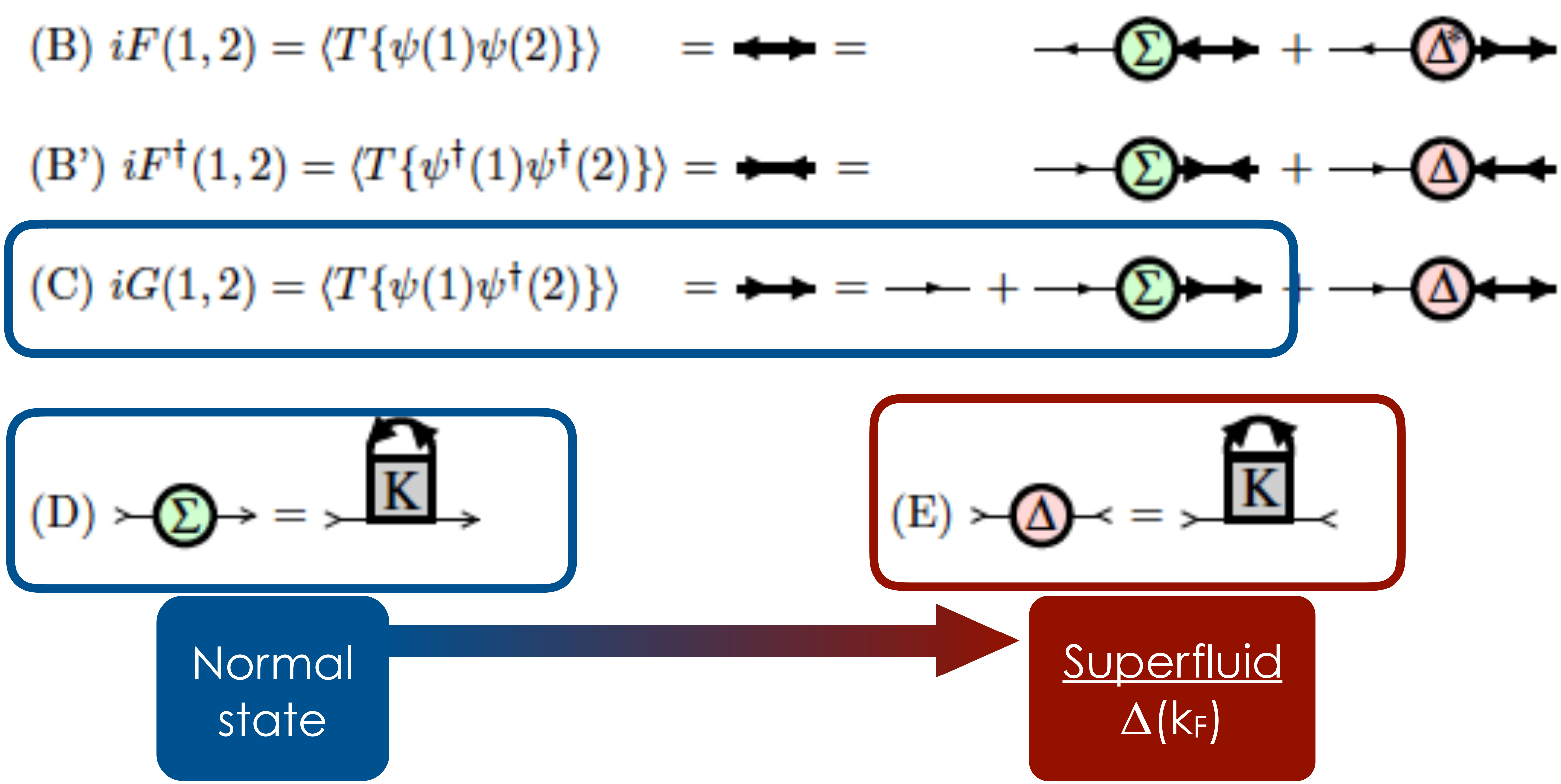

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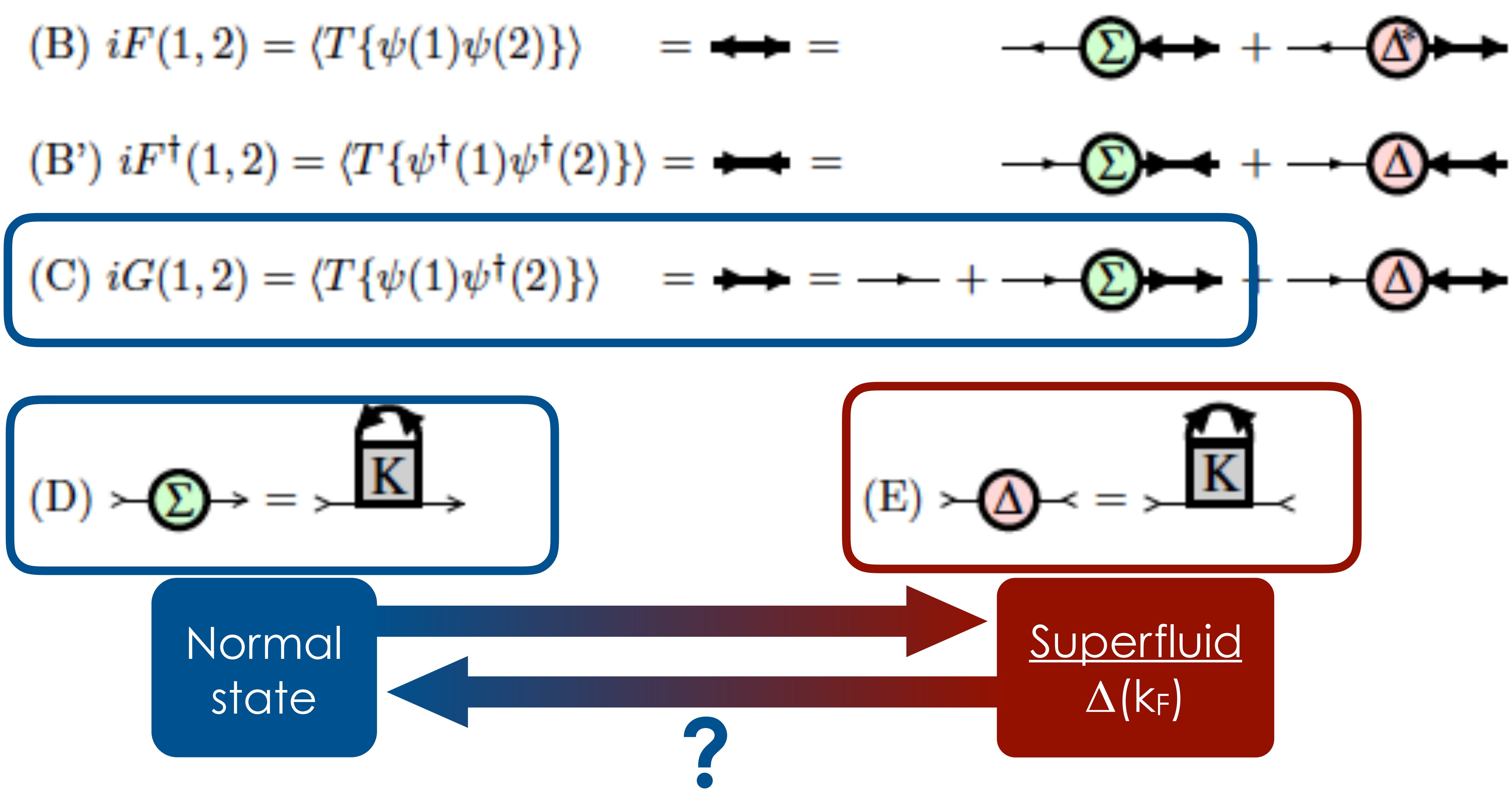

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Bozek, *Phys. Rev. C* **62** 054316 (2000); Muether & Dickhoff, *Phys. Rev. C* **72** 054313 (2005)

Gorkov gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\xi_{k'}} \Delta_{k'}^{L'} \quad +$$

$$\frac{1}{2\xi_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_s(k,\omega')$$

BCS+SRC gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad +$$

$$\frac{1}{2\bar{\chi}_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_{\cancel{s}}(k,\omega')$$

BCS+Z-factor equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{Z_k \langle k|V_{nn}^{LL'}|k'\rangle Z_{k'}}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

$$+ \quad \chi_k = \varepsilon_k - \mu$$

$$+ \quad \varepsilon_k = \frac{k^2}{2m} + U(k)$$

BCS gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

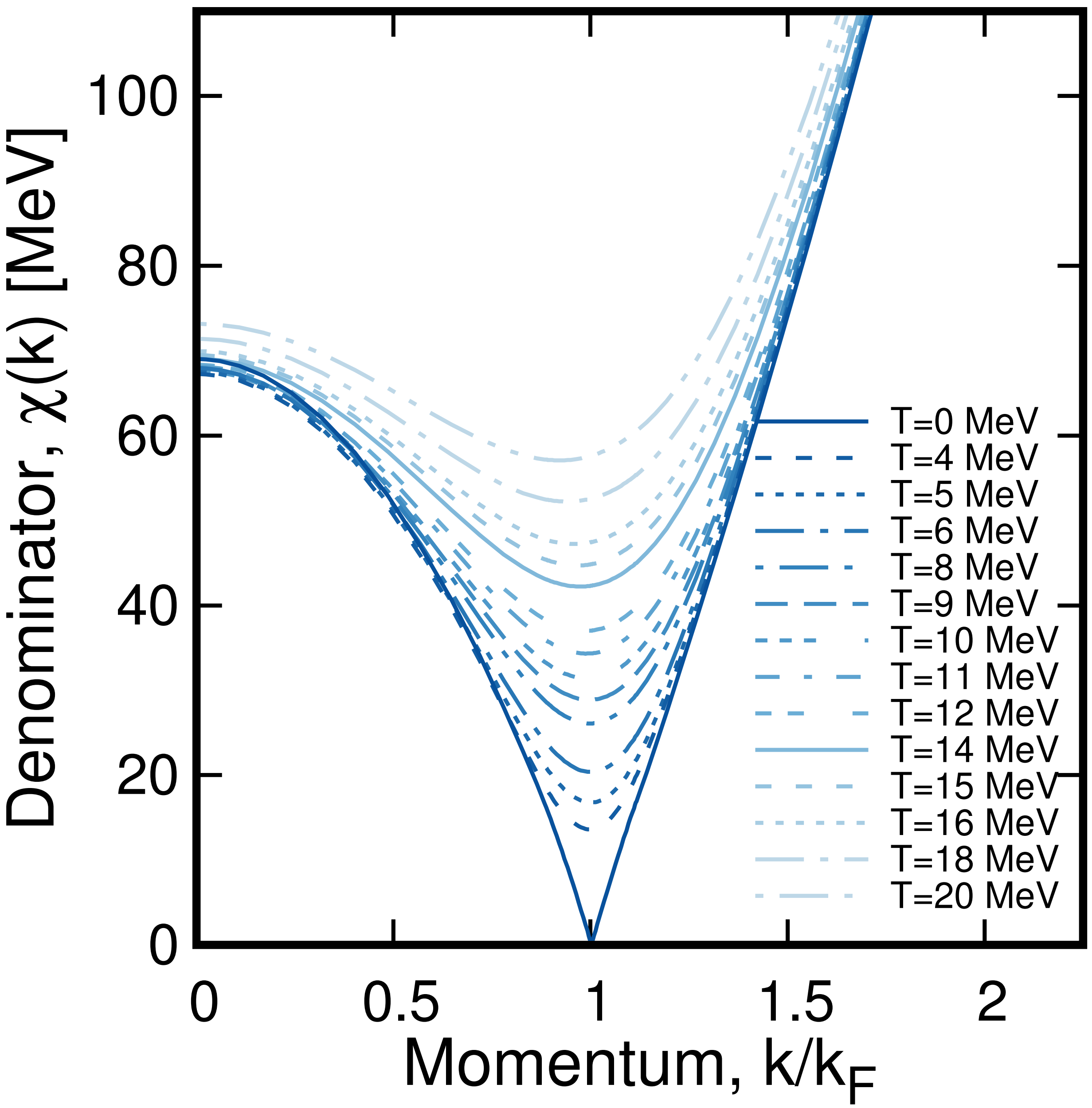
$$+ \quad \chi_k = \varepsilon_k - \mu$$

$$+ \quad \varepsilon_k = \frac{k^2}{2m}$$

Beyond BCS 101: SRC

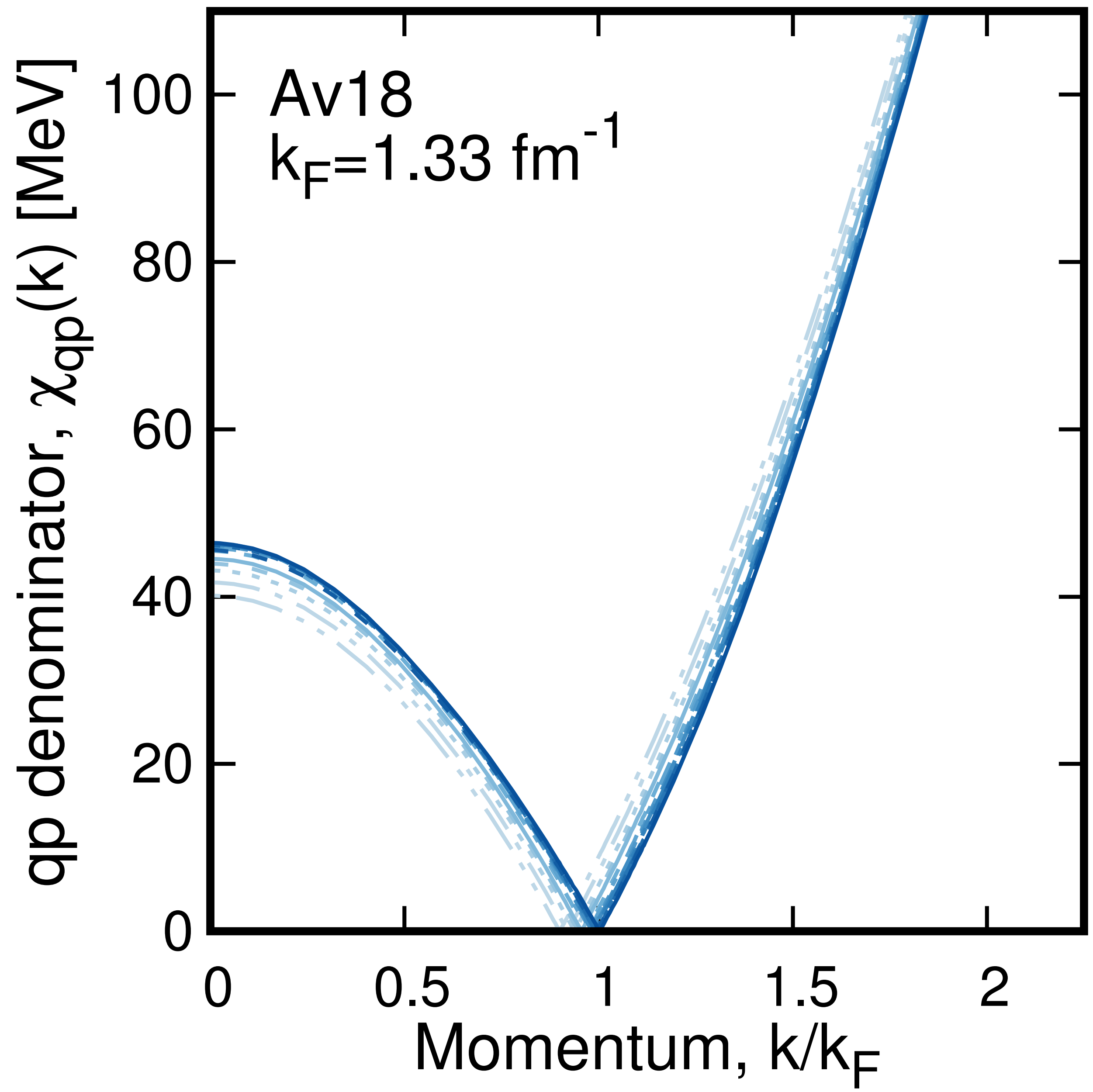
Full off-shell

$$\frac{1}{2\bar{\chi}_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A(k,\omega')$$



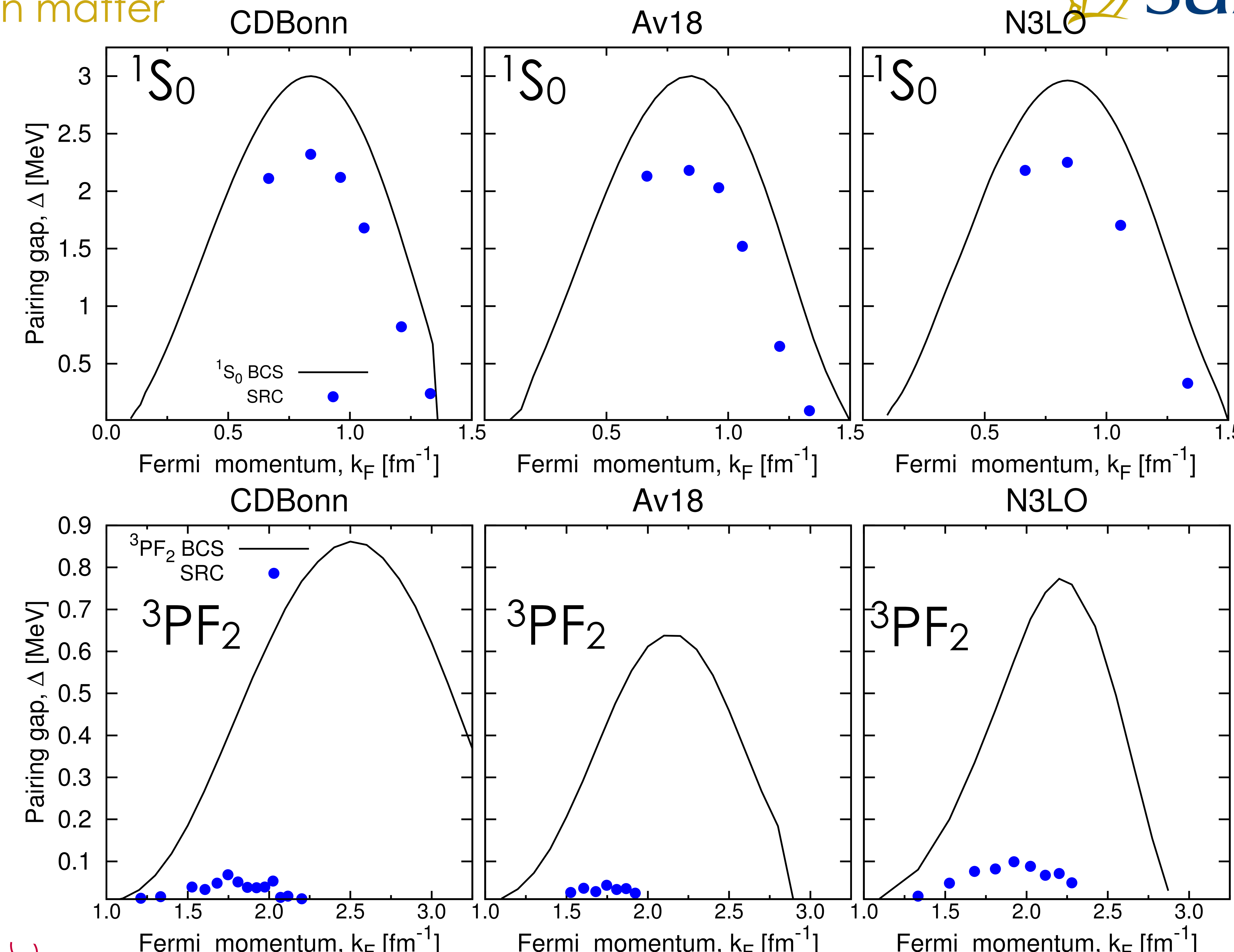
Quasi-particle ⇔ BCS

$$\frac{1}{2\bar{\chi}_k} = \frac{1}{2|\varepsilon_k - \mu|}$$



Beyond BCS 101: SRC

Neutron matter

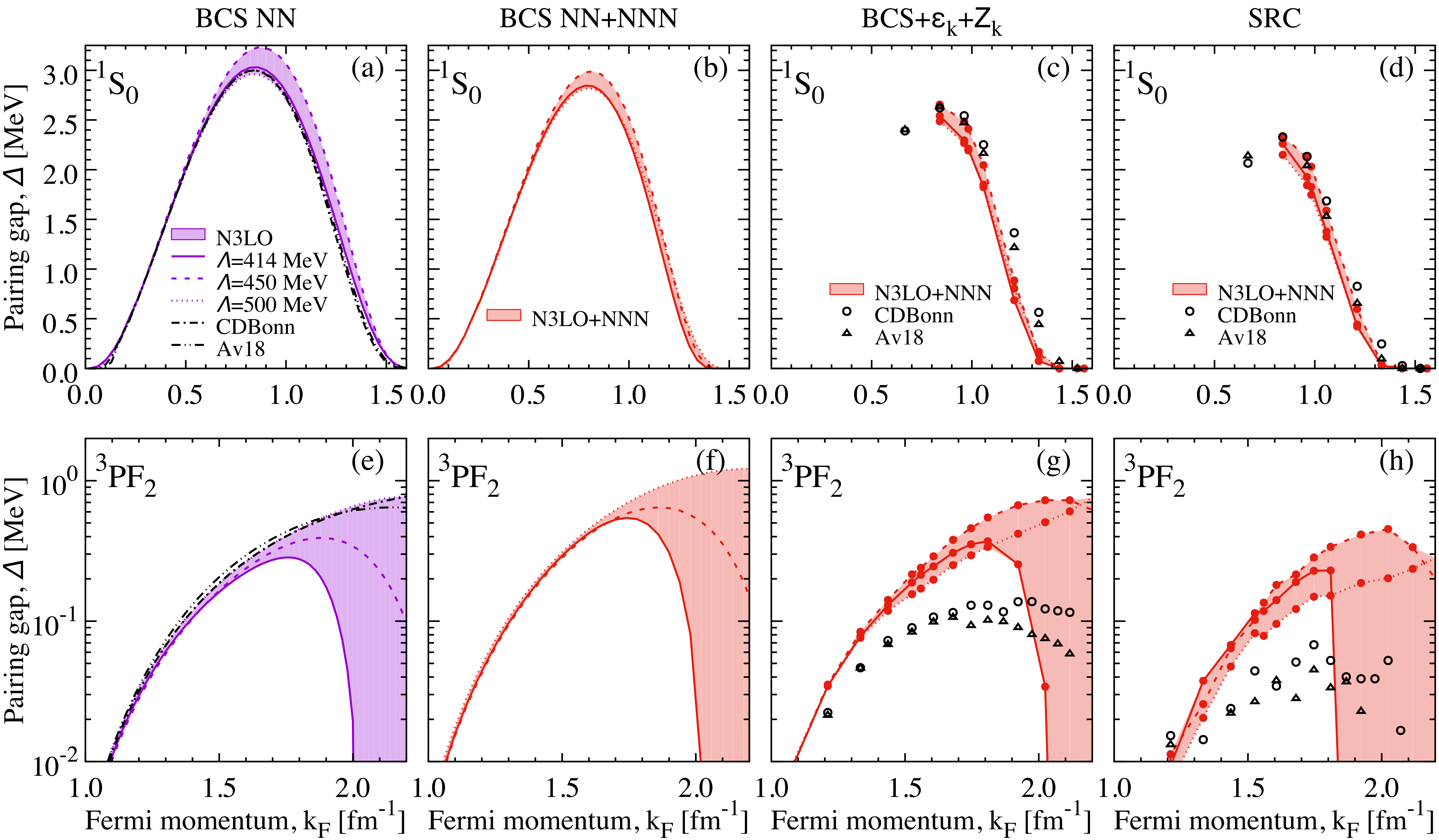


BCS+SRC

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \frac{1}{2\chi_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_{\cancel{s}}(k,\omega')$$

Beyond BCS 101: SRC

Neutron matter

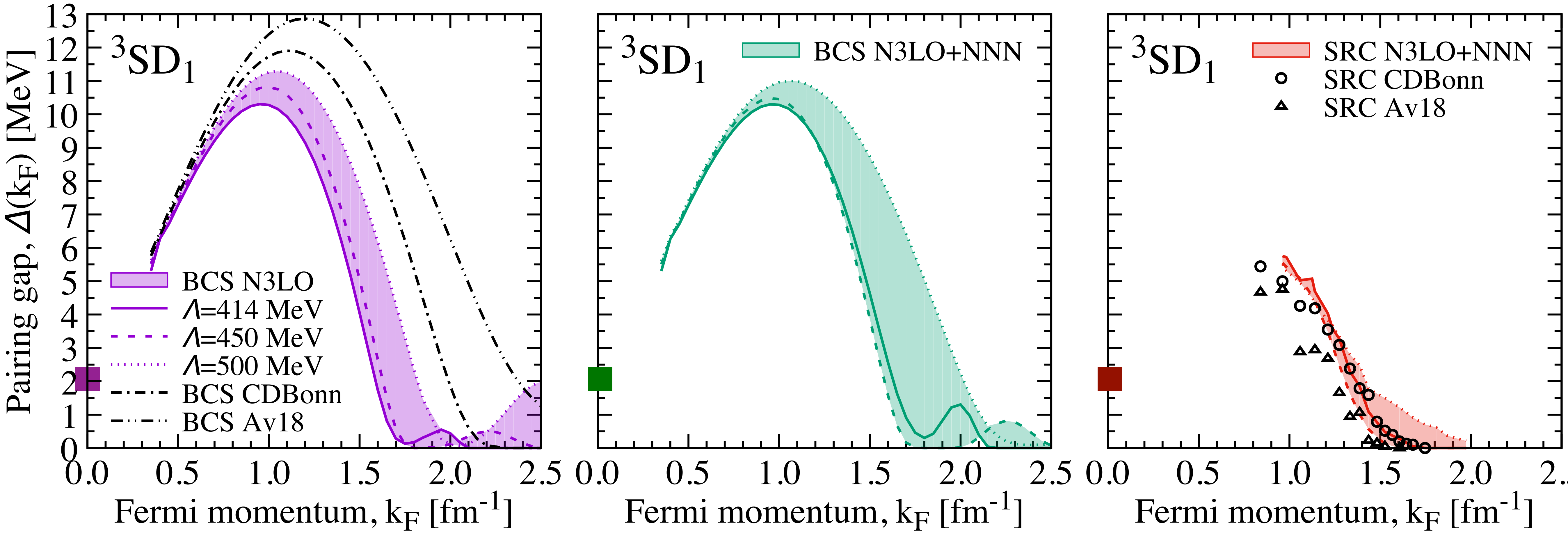


BCS+SRC

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \frac{1}{2\chi_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_{\cancel{s}}(k,\omega')$$

Beyond BCS 101: $^3SD_1$

Symmetric matter

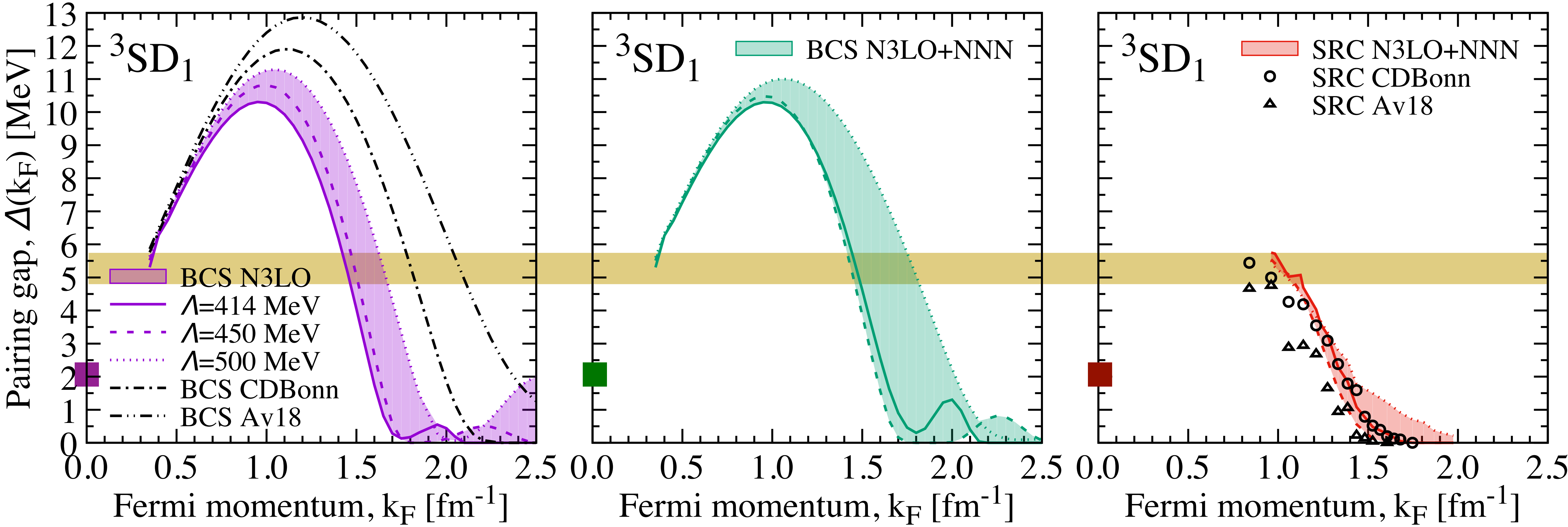


Muether & Dickhoff, *PRC* **72** 054313 (2005)
Rios, Polls & Dickhoff, arXiv:1707.04140
Maurizio, Holt & Finelli, *PRC* **90**, *044003* (2014)
U. Lombardo's talk

- **Massive** gaps ^{3}**SD**$_1$ channel but…
- **No** evidence of strong *np* nuclear pairing
- 3NF do **not** alter picture **significantly**
- Short-range correlations **deplete** gap

Beyond BCS 101: $^3SD_1$

Symmetric matter



Muether & Dickhoff, *PRC* **72** 054313 (2005)
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- **Massive** gaps ^{3}**SD$_1$** channel but…
- **No** evidence of strong *np* nuclear pairing
- 3NF do **not** alter picture **significantly**
- Short-range correlations **deplete** gap

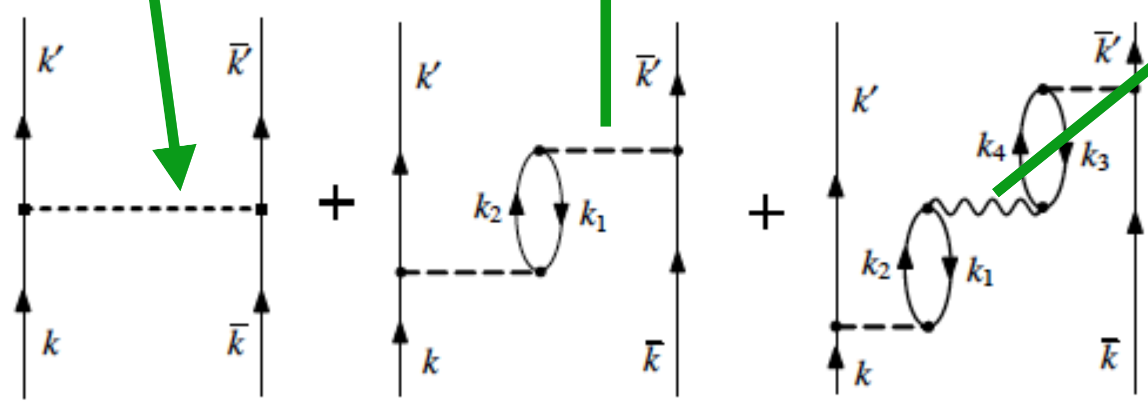
Beyond BCS 201: LRC

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\boxed{\langle k | V_{nn}^{LL'} | k' \rangle}}{2\sqrt{\boxed{\chi_{k'}^2} + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

ph recoupled G-matrix

Effective Landau parameters

$\mathcal{V}_{\text{pair}} =$



(a) (b) (c)

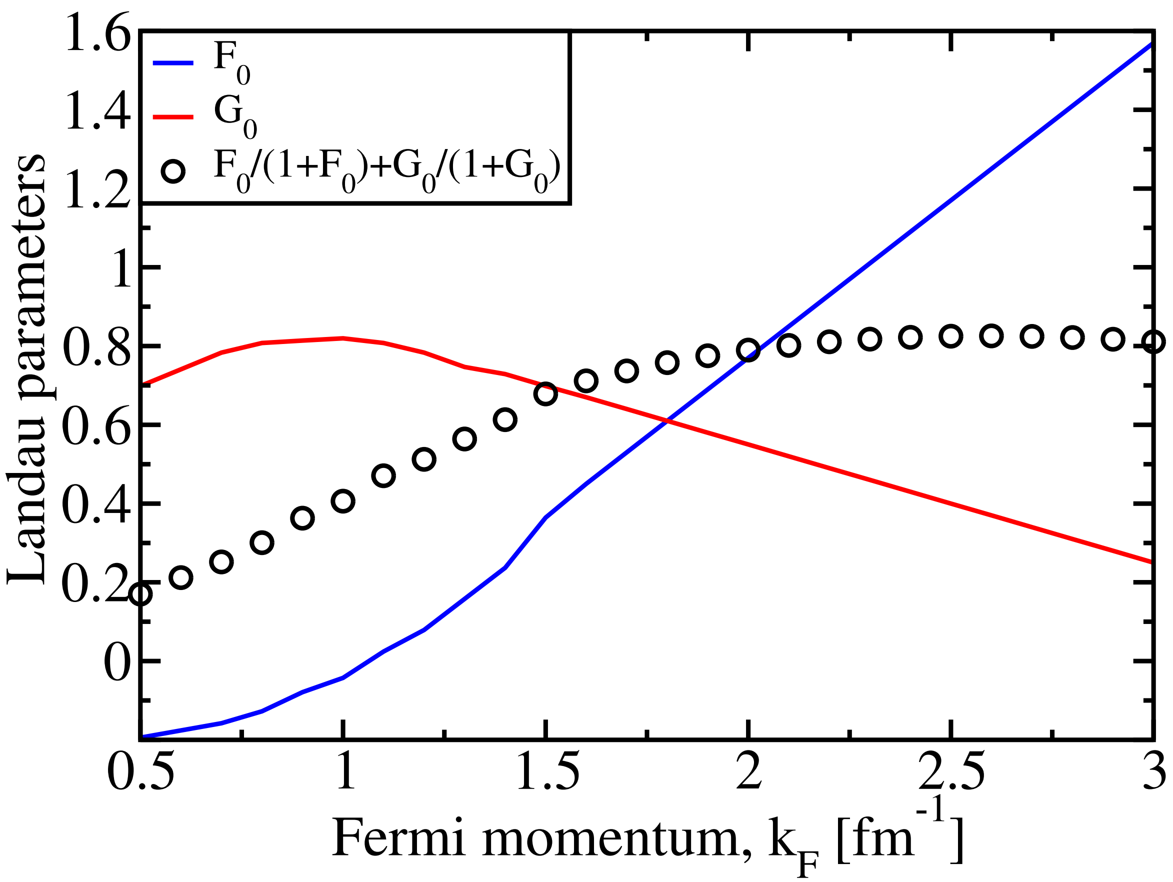
$$\langle 1\bar{1} | \mathcal{V} | 1\bar{1} \rangle = \frac{1}{4} \sum_{2,2'} \sum_{S,T} (-)^S (2S+1) \langle 12 | G_{ST}^{\text{ph}} | 1'2' \rangle_A \langle 2'\bar{1} | G_{ST}^{\text{ph}} | 2\bar{1}' \rangle_A \Lambda(22')$$

$$\Lambda_{ST}(q) = \frac{\Lambda_{ST}^0(q)}{1 - \Lambda_{ST}^0(q) \times F_{ST}}$$

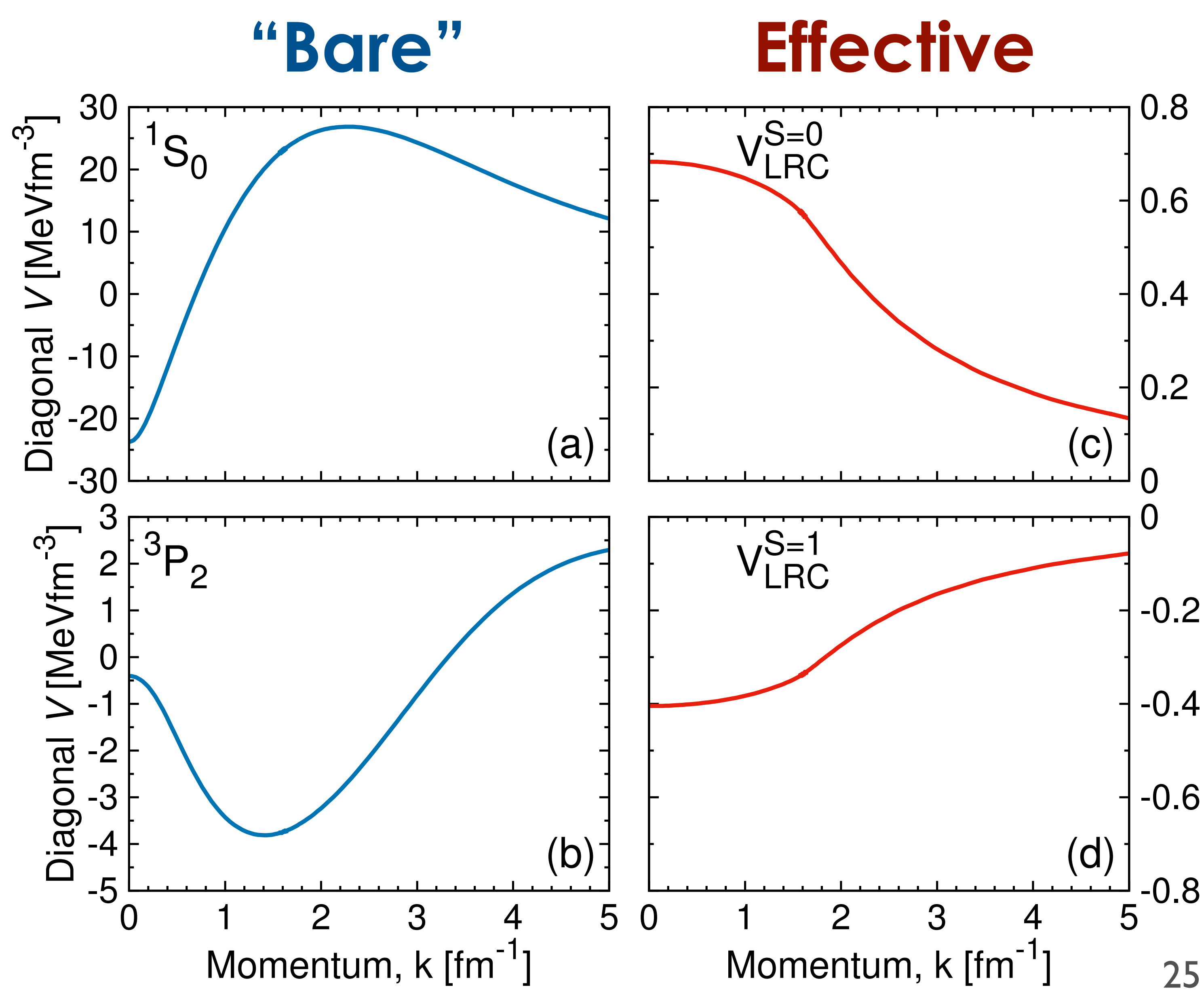
- Bare NN potential only is not the only possible interaction
- Diagram (a): nuclear interaction
- Diagram (b): in-medium interaction, density and spin fluctuations
- Diagram (c): included by Landau parameters

Cao, Lombardo & Schuck, *Phys Rev C* **74** 064301 (2006)

Landau parameters



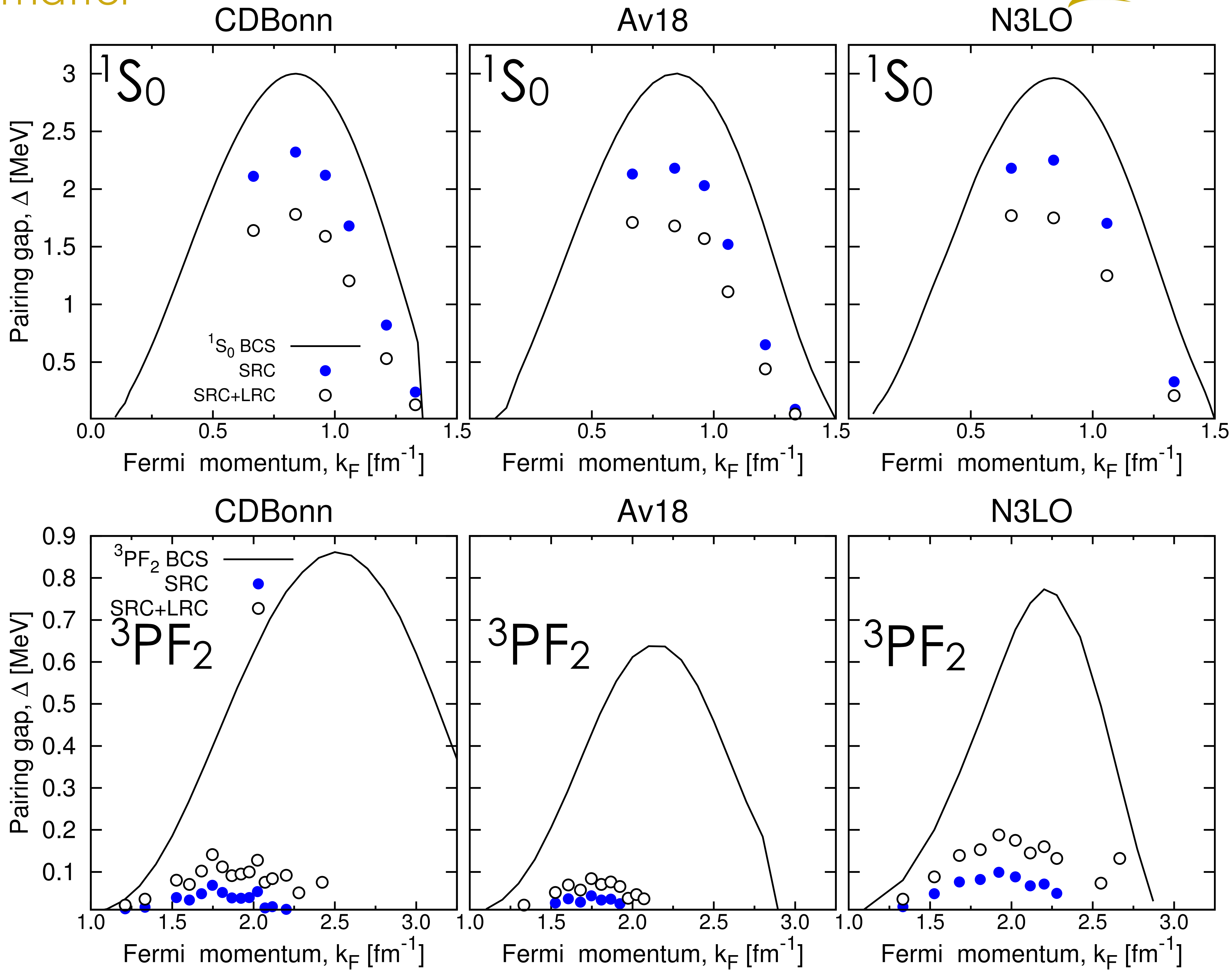
- **Effective** Landau parameters
- **Not consistent** (yet) with NN force
- LRC in **nuclei** will be **different**



- **Small** correction
- Repulsive in **singlet**
- Attractive in **triple**

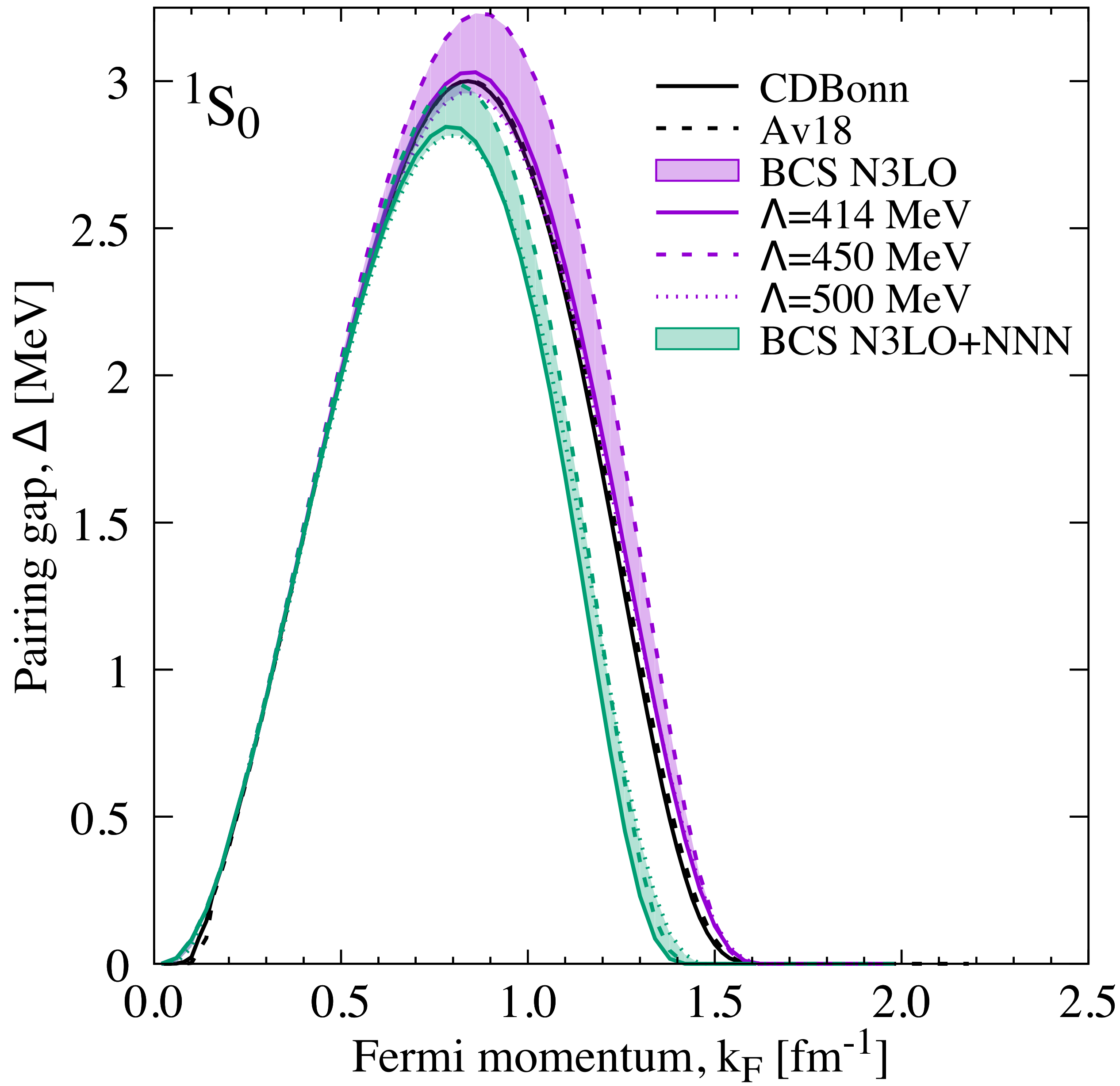
Beyond BCS 201: results $^1S_0$

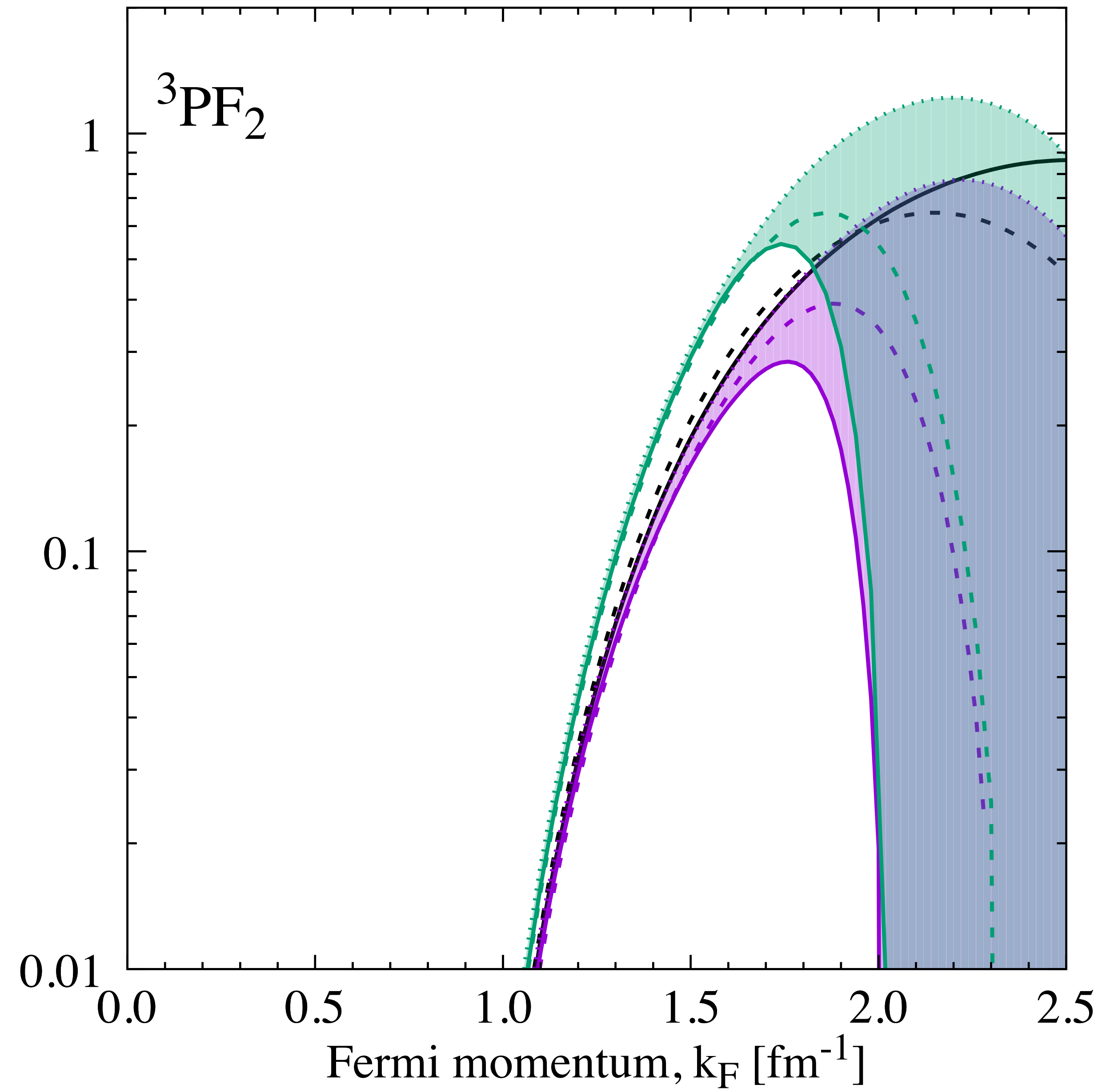
Neutron matter



- LRC $^1S_0$ ($^3PF_2$) produces (anti-)screening

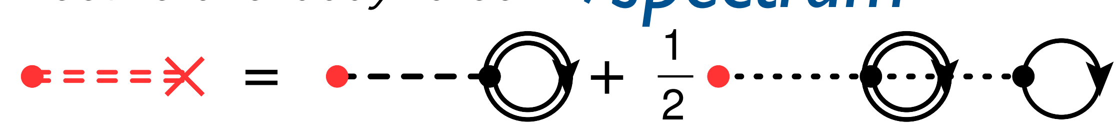
3BF effect: uncertainty estimate





- Singlet gap: 3NF **reduce** closure
- Triplet gap: 3NF **increase** gap
- **LRC** effect **to be explored**

Effective one-body force ⇒*spectrum*



Effective two-body force ⇒*NN forces*

3BF effect: uncertainty estimate

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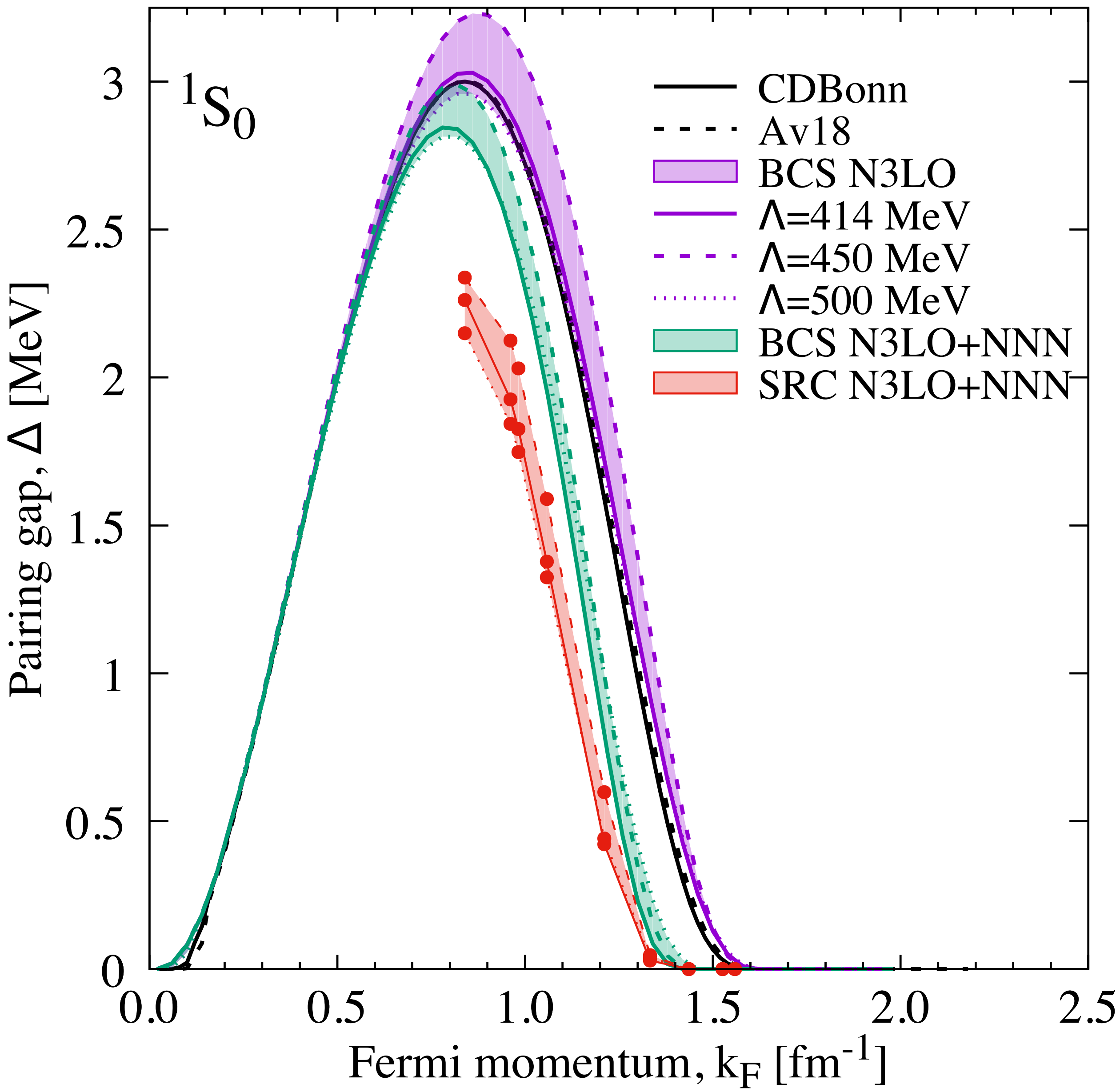
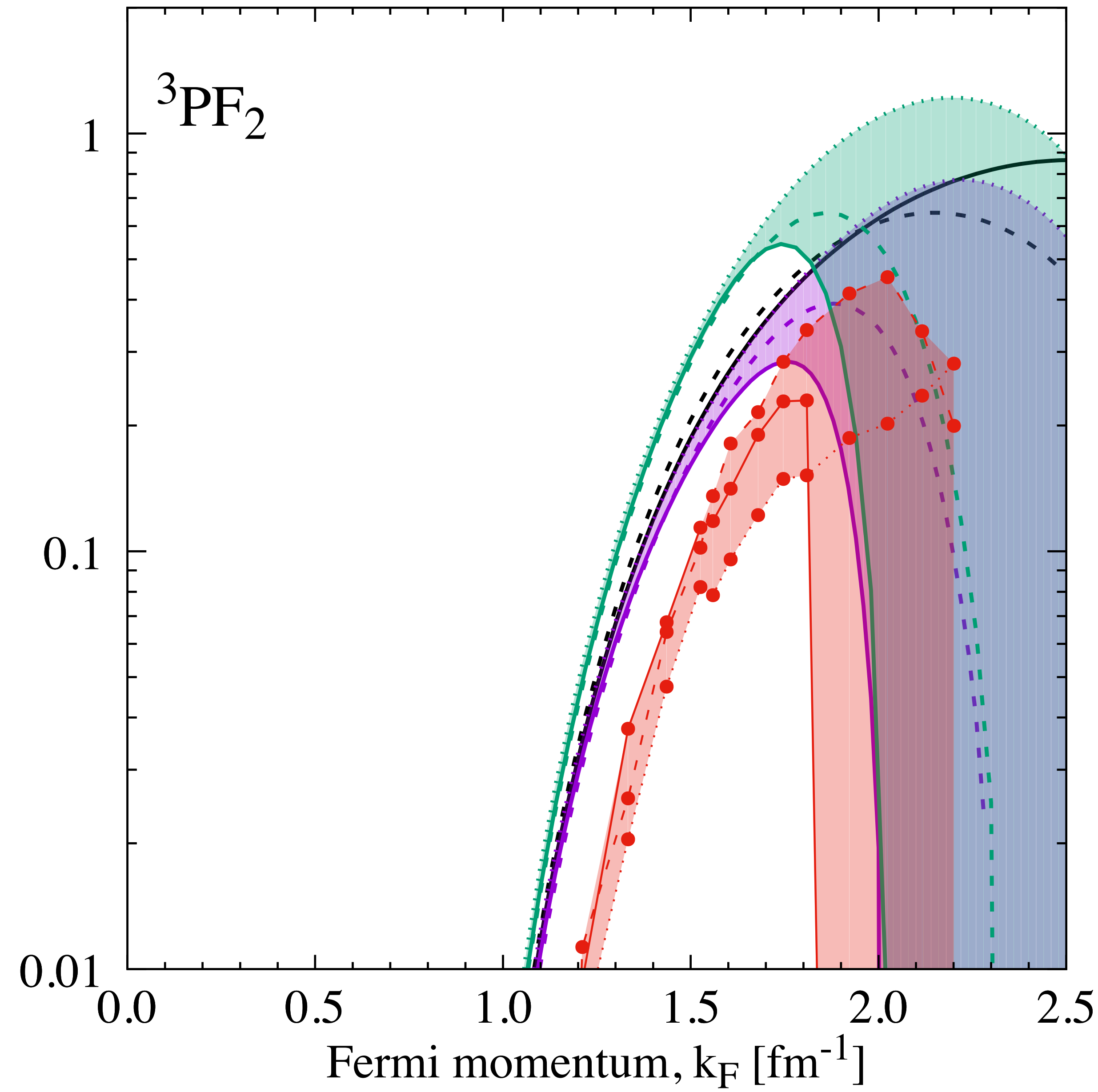
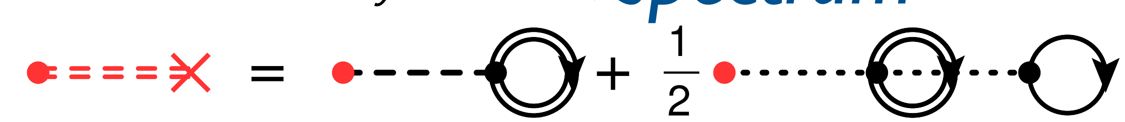
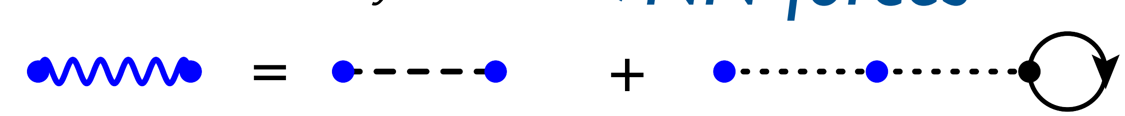




Effective one-body force ⇒*spectrum*



Effective two-body force ⇒*NN forces*



- Singlet gap: 3NF **reduce** closure
- Triplet gap: 3NF **increase** gap
- **LRC** effect **to be explored**

Superfluid in the core

RESEARCH ARTICLE

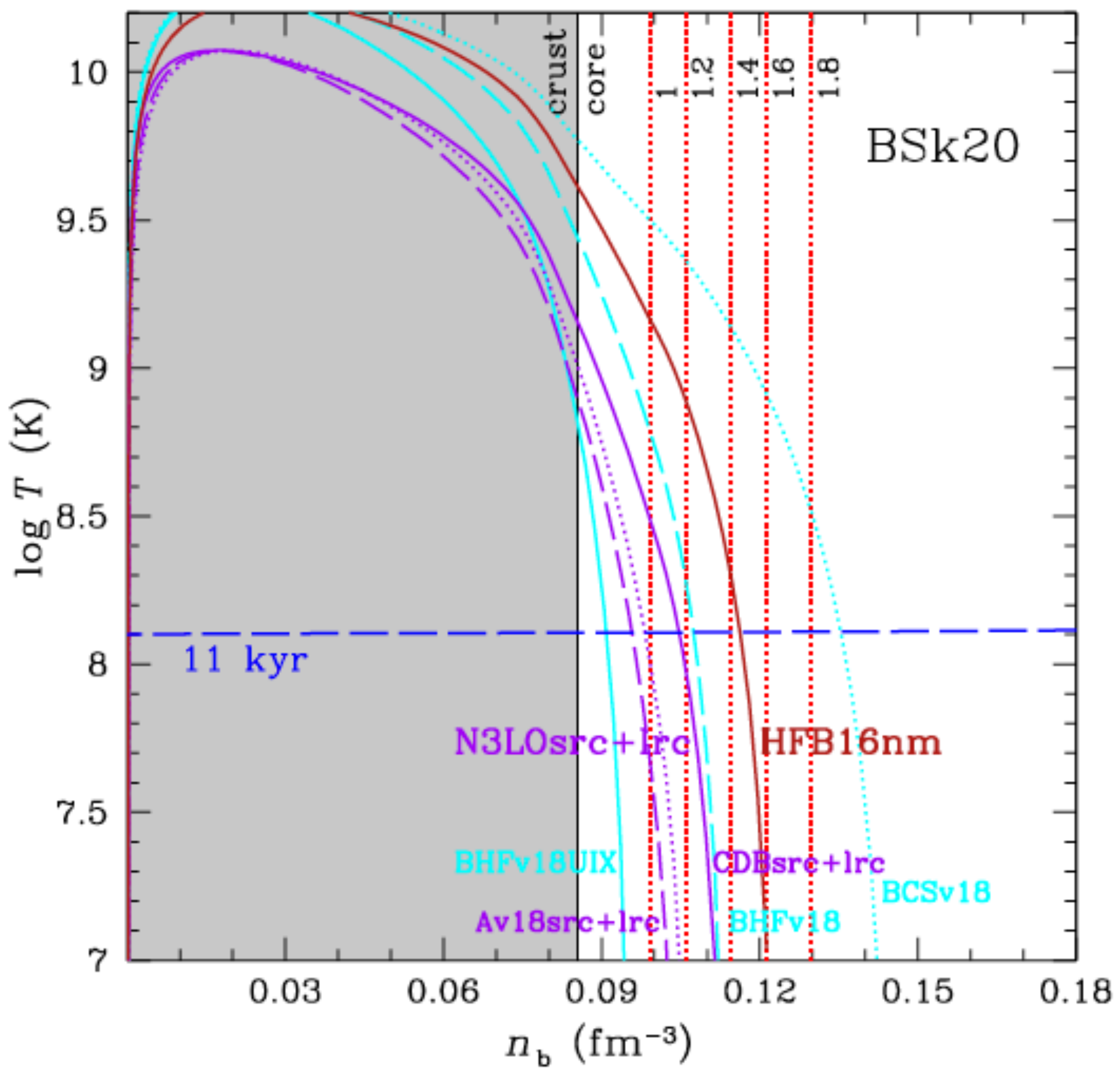


Fig. 2. Temperature dependence of neutron superfluid models as a function of baryon number density for the BSk20 nuclear equation of state. Thick curved lines are the superfluid critical temperature for the (labeled) models from (23). The vertical solid line indicates the separation between the crust (shaded region) and the core. Vertical dotted lines denote the density at which the superfluid moment of inertia (using the SFB superfluid model) is 1.6% of the total stellar moment of inertia for neutron stars of different mass (labeled in units of solar mass). The (nearly horizontal) dashed line is the temperature of a 1.4 M_{Sun} neutron star at an age of 11,000 years.

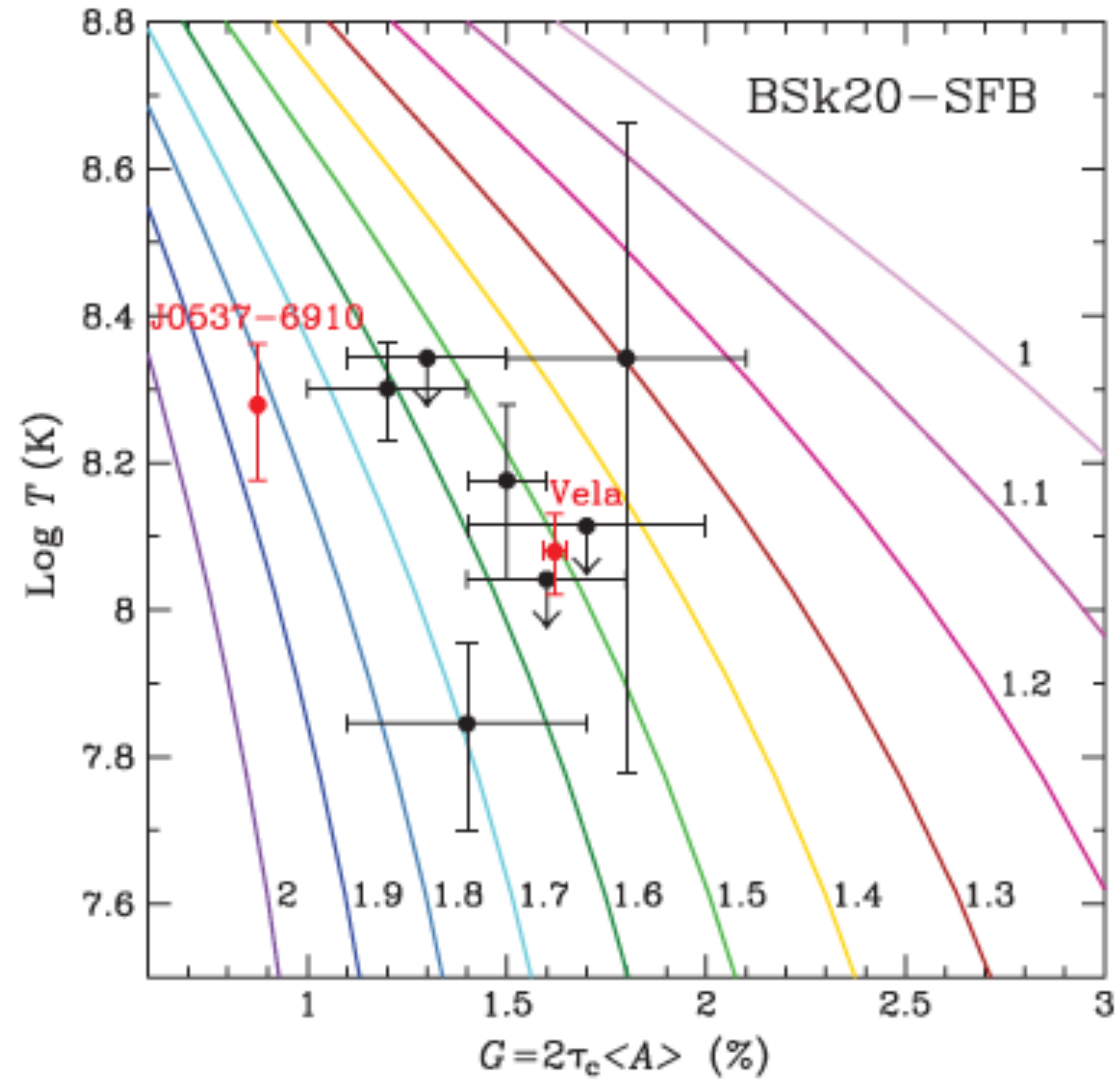


Fig. 3. Neutron star mass from pulsar observables *G* and interior temperature *T*. Data points are for pulsars with measured *G* from glitches and *T* from an age or surface temperature observation (see Table 1). Lines (labeled by neutron star mass, in units of solar mass) are the theoretical prediction for *G* and *T* using the BSk20 nuclear equation of state and SFB neutron superfluid models.

Ho et al., *Sci. Adv.* 2015;1:e1500578
Ho et al., 0.7566/JPSCP.14.010805

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Conclusions

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- Ab initio nuclear **theory** to treat beyond-BCS **correlations**
- **Different** NN forces provide robust predictions
- Challenges ahead
  - Full self-consistent **Gorkov**
  - **Consistent** treatment of **LRC**
  - **Pairing** in isospin **asymmetric** matter

Postdoc position opening imminently

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