# Structure of neutron stars with unified equations of state

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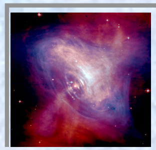
Milan (Italy), 19 – 20 September 2017



# Outline

- ❖ Astrophysical framework and motivations

- ❖ Effective nuclear models
    - ▪ Nuclear functionals and the Brussels-Montreal BSk model

- ❖ Equations of state (EoSs) of dense matter
    - ▪ *Catalysed* NS and astrophysical constraints
    - ▪ *Accreted* crust

- ❖ Conclusions & Outlook

# EoS for NS: the challenge

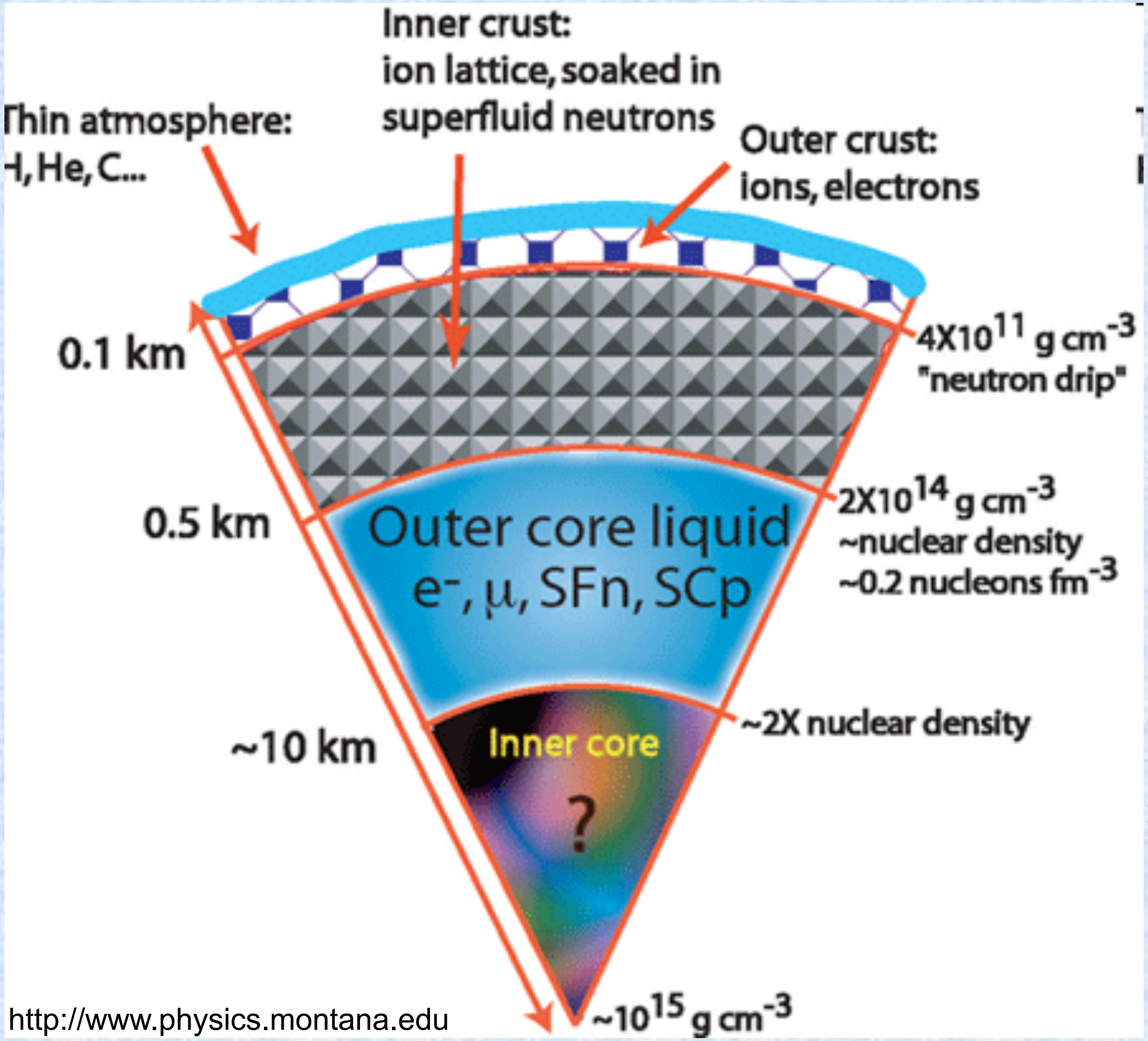
Contrarily to a normal star, in a NS:

✓ matter is highly **degenerate**!
( **T = 0** approximation )

✓ very high density!
composition uncertain

⇩

different states of matter :
inhomogeneous, homogeneous,
exotic particles ?

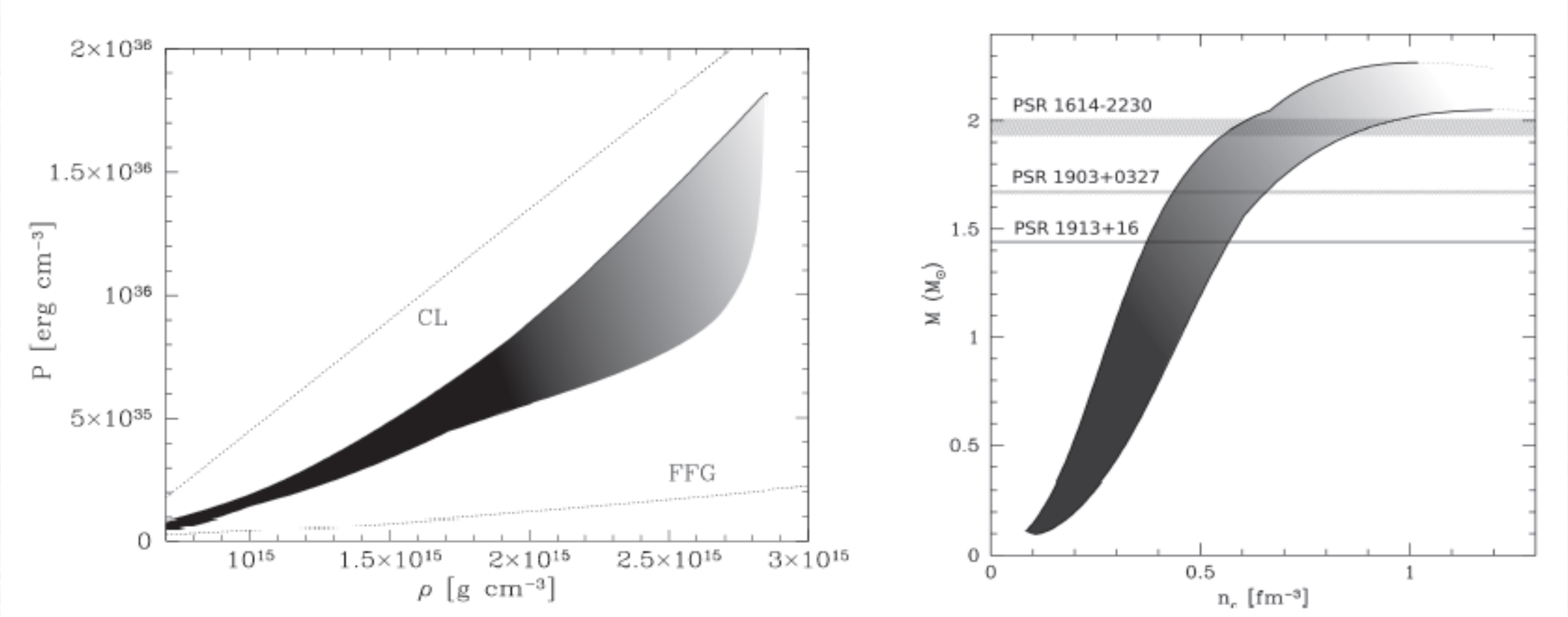


http://www.physics.montana.edu

➡ very challenging to describe in a *unified* way !

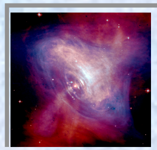
# Uncertainties in dense-matter EoS

Pressure *P* versus mass-energy density $\rho$ and corresponding NS mass *M* versus central density $n_c$ relation, as predicted by various models and consistent with the existence of massive NSs.

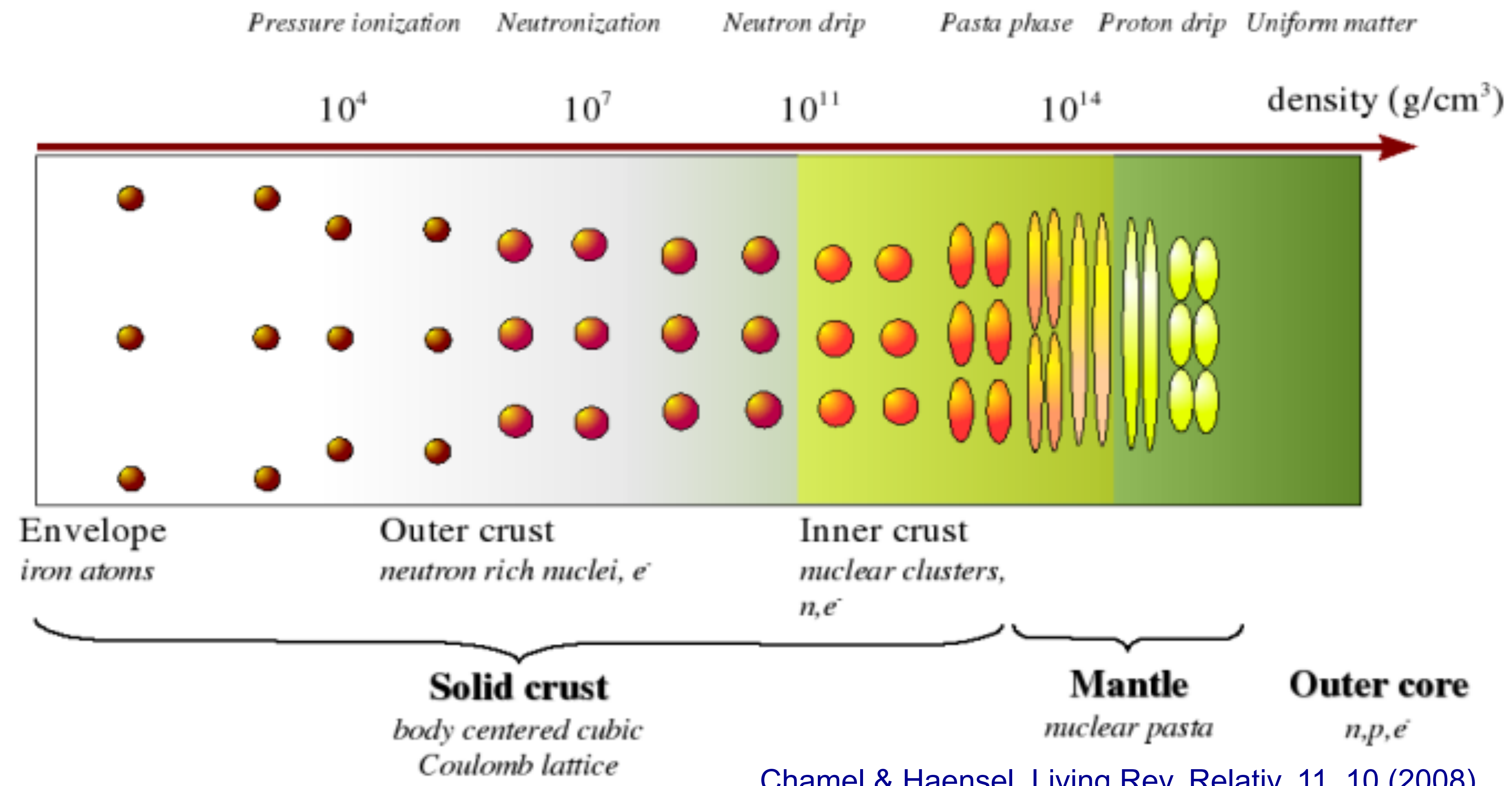


Chamel, Haensel, Zdunik, Fantina, Int. J. Mod. Phys. E 22, 1330018 (2013); E 22, 1392004 (2013)

→ uncertainties mostly at supra-saturation densities !



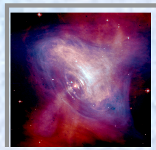
# NS crust structure



Chamel & Haensel, Living Rev. Relativ. 11, 10 (2008)

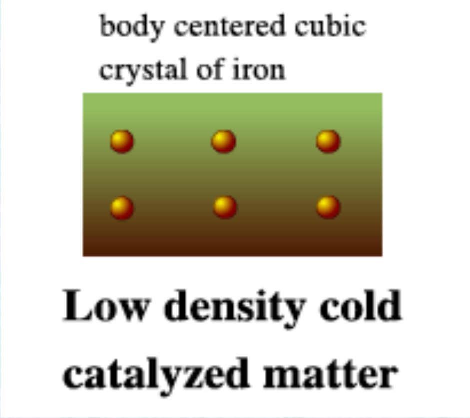
NS crust : ≈ 1% mass, ≈ 10% radius

but: related to different phenomena (e.g. glitches, X-ray bursts, deep crustal heating, etc.).



# NS crust: catalysed vs accreted

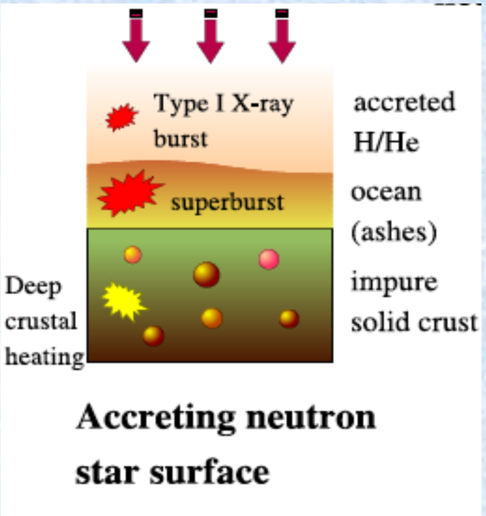
- Catalysed matter



- NS born a high T ≈ $10^{11}$ K
- **full thermodynamical equilibrium at T=0**
- ground state of matter
  → minimise Gibbs energy wrt Z,A
- no exothermic reactions possible

see e.g. Baym *et al*., ApJ 170, 299 (1971)

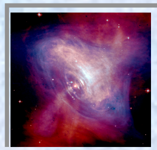
- Accreted matter



- T < $10^9$ K → T=0 is ok but:
- **matter off-equilibrium** (local min of E)
  → minimum wrt neighbours N,Z at const. A
- EC, n emission, pycnonuclear possible
- exothermic reactions possible → energy sources
  → can explain thermal radiation in SXTs in quiescence

see e.g. Haensel & Zdunik, A&A 227, 431 (1990);
A&A 229, 117 (1990); A&A 404, L33 (2003) and Refs. therein

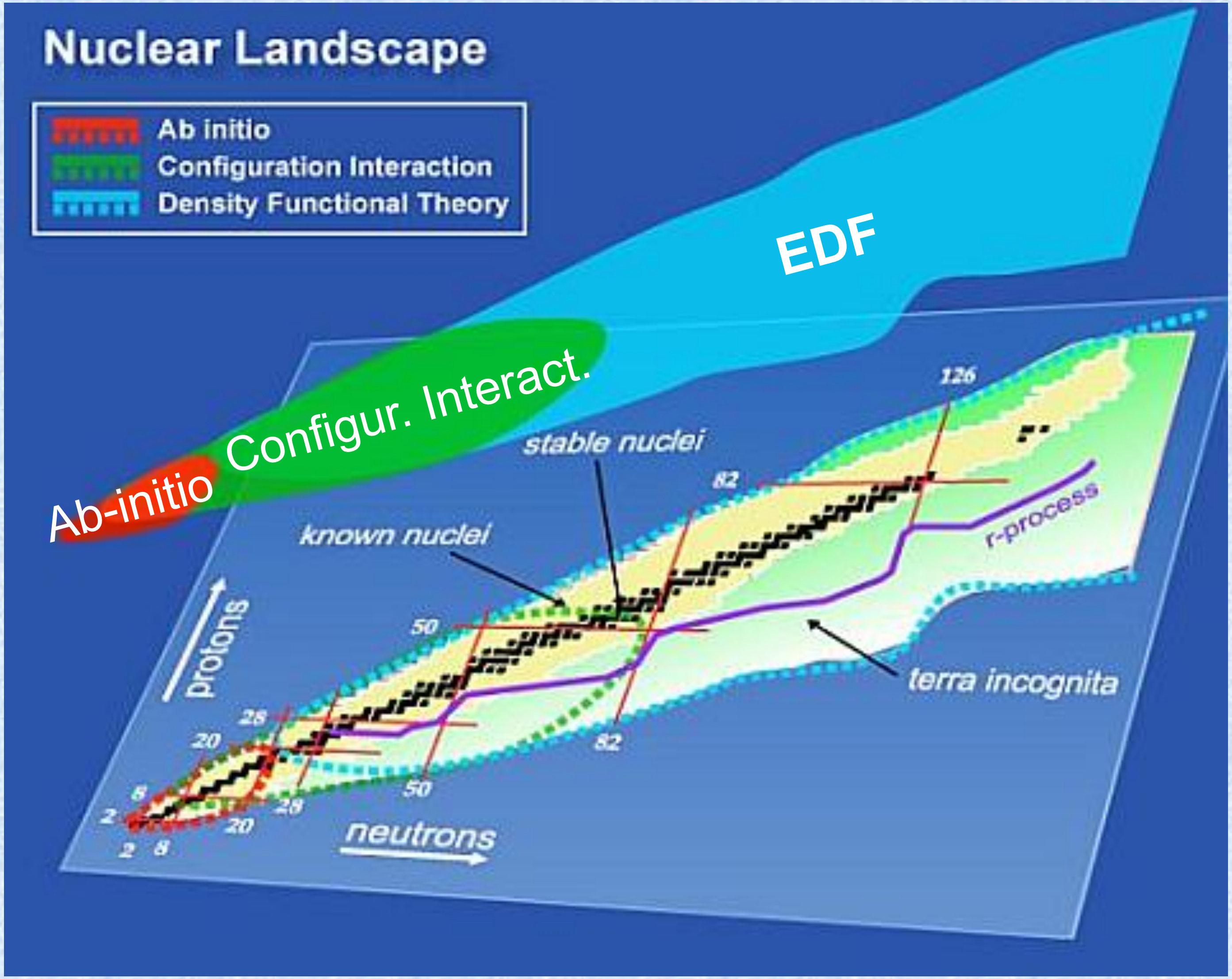
for a review: Chamel & Haensel, Living Rev. Relativ. 11, 10 (2008)



# Our goal : a unified EoS

- Our goal is to construct a ***unified*** EoS (only a few until now...)
  - → based on the *same nuclear model* from energy-density functional theory
  - → all regions of NS (and SN) interior and boundaries described consistently
- EoS both at T = 0 and finite T
  - → cold non-accreting NS (cold catalysed matter)
  - → accreting NS (off-equilibrium)
  - → SN cores
- Satisfying: - constraints from nuclear physics experiments
  - - astrophysical observations
- Direct applicable for astrophysical application

# Which theoretical framework?



Bertsch, et al., SciDAD Rev. 6 (2007)

Modelling nuclear systems under extreme conditions!

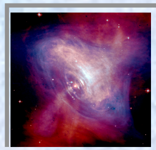
Need models to treat consistently:

→ very n-rich clusters
→ homogeneous matter

⇓

**Nuclear energy-density functional theory**

→ see Nicolas Chamel’s talk



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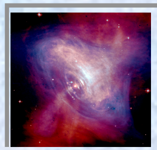


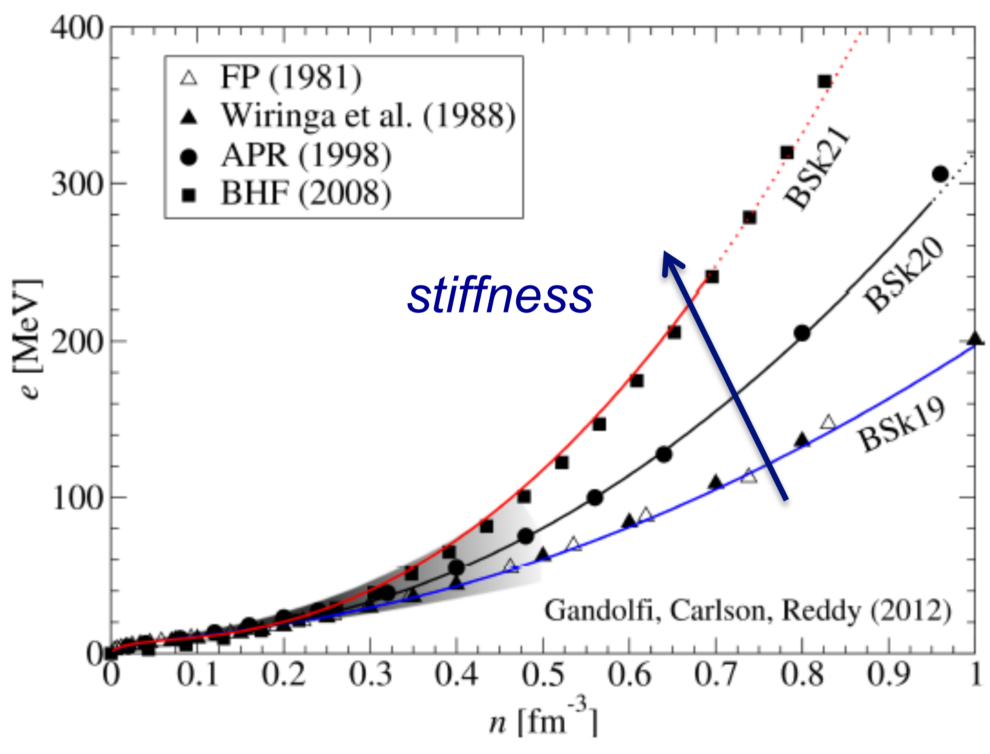
# Brussels-Montreal (BSk) functionals

Nuclear mass models based on **HFB method** with **Skyrme type energy-density functionals** (EDFs) and macroscopically deduced pairing force.
Fitted to experimental data + N-body calculations with realistic forces.

**BSk19**
**BSk20**
**BSk21**

- **fit 2010 AME data** (2149 masses, rms = 0.581 MeV)
- **different degrees of stiffness** (BSk19 softer → BSk21 stiffer)
  constrained to different microscopic neutron-matter EoSs at T = 0
- all have $J = 30$ MeV, , $K_\infty$ in experimental range (≈ 240 MeV)

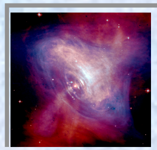
Goriely *et al.*, PRC 82, 035804 (2010)



*stiffness*

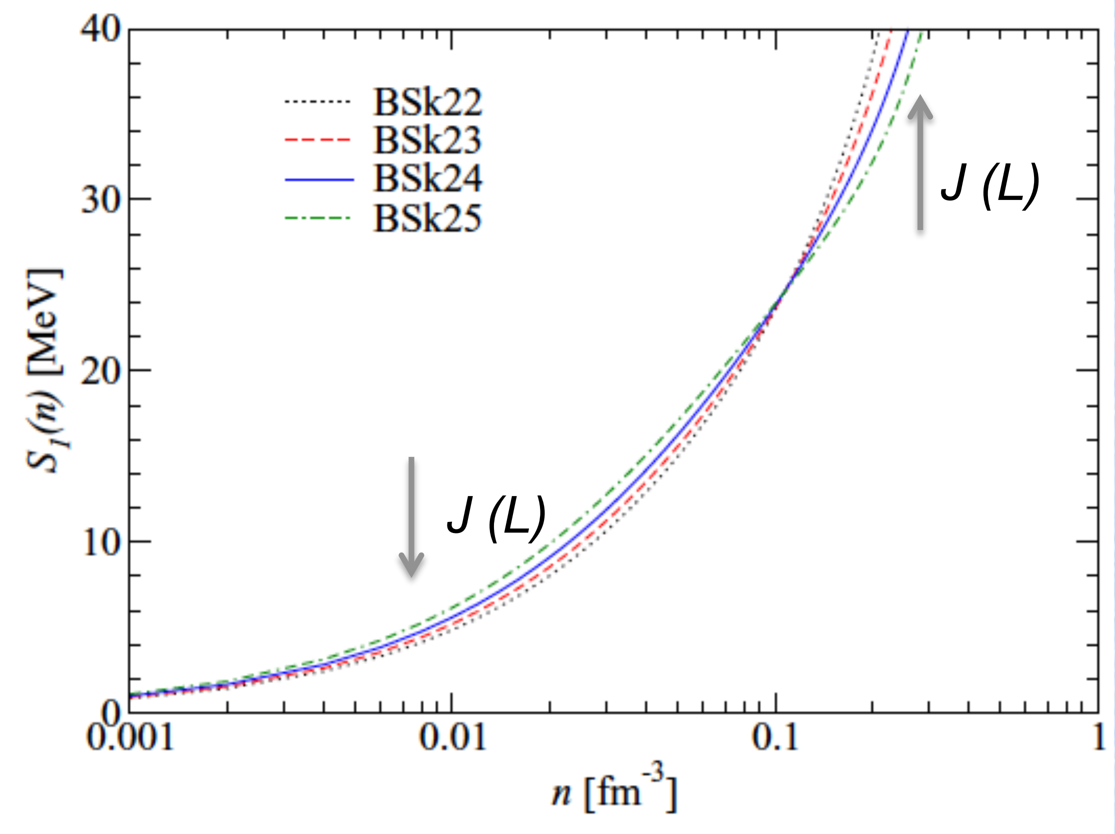
Gandolfi, Carlson, Reddy (2012)

see also: Chamel *et al.*, Acta Phys. Pol. B 46, 349 (2015)



# Brussels-Montreal (BSk) functionals

Nuclear mass models based on HFB
functionals (EDFs) and macroscopical
Fitted to experimental data + N-body c

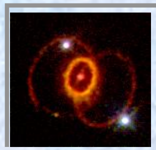


**BSk22**
**BSk23**
**BSk24**
**BSk25**
**BSk26**

- **fit 2012 AME data** (2353 masses, rms=0.5-0.6 MeV)
- constrained to microscopic neutron-matter EoSs at T = 0 (rather stiff)
- **different $E_{sym}$ coefficient** ($J$ = 32, 31, 30, 29, 30 MeV), $K_\infty$ in experimental range (≈ 240 MeV)

Goriely *et al.*, PRC 88, 024308 (2013)

**BSk** agree with constraints coming from nuclear physics**
→ see Nicolas Chamel's talk



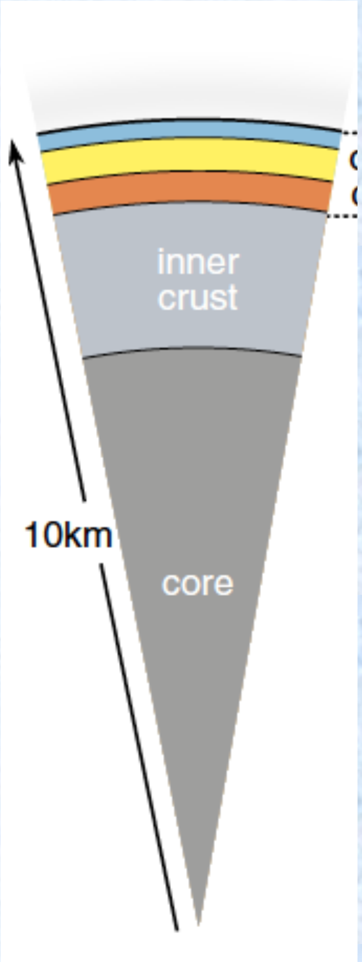
# Brussels-Montreal (BSk) functionals

✓ BSk** suitable to describe different regions of NS

↔

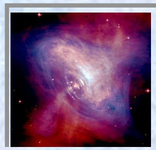
✓ BSk** also used to compute properties of infinite homogeneous nuclear matter

*possible to explore in a consistent way the role of the nuclear parameters on the NS structure and properties, and confront them with astro constraints*



To this aim: we construct ***unified EoSs*** with these functionals (until now, few unified EoSs! e.g. SLy, BCPM, Shen, LS)

- *same nuclear model* to treat different NS regions
- avoid ad-hoc matching procedures at boundaries that can cause unphysical results (see e.g. Fortin *et al*., PRC 94, 035804 (2016) )
- here: case of "$T = 0$" and only nucleons (no hyperons or quarks)



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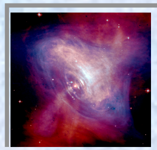
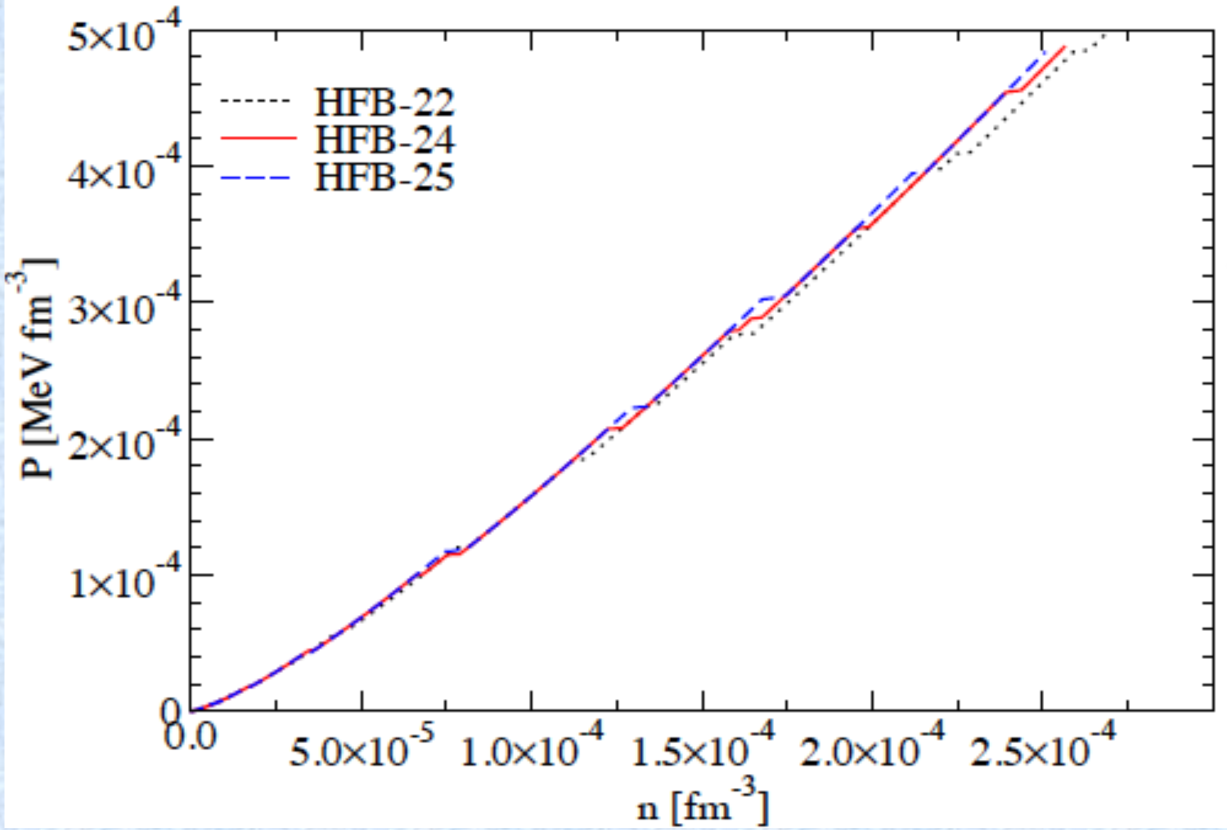


# EoS of NS: outer crust

Only microscopic inputs are nuclear masses
→ Experimental or microscopic mass models

Mass models: HFB (no approximations!)



Fantina *et al.*, Il Nuovo Cimento C **39**, 400 (2016)

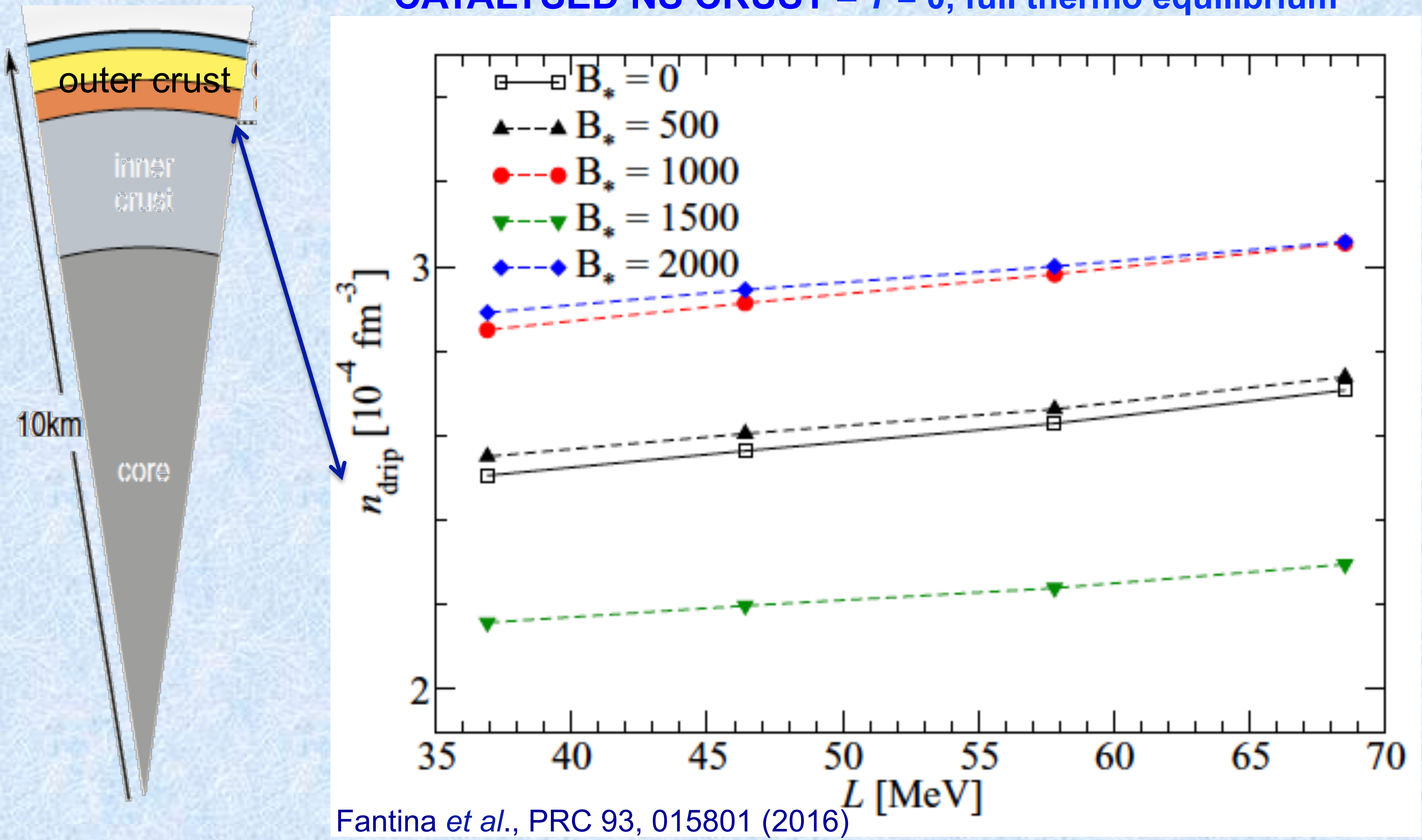
| HFB-22 | HFB-23 | HFB-24 | HFB-25 |
|---|---|---|---|
| $^{56}\mathbf{Fe}$ | $^{56}\mathbf{Fe}$ | $^{56}\mathbf{Fe}$ | $^{56}\mathbf{Fe}$ |
| $^{62}\mathbf{Ni}$ | $^{62}\mathbf{Ni}$ | $^{62}\mathbf{Ni}$ | $^{62}\mathbf{Ni}$ |
| $^{58}\mathbf{Fe}$ | $^{58}\mathbf{Fe}$ | $^{58}\mathbf{Fe}$ | $^{58}\mathbf{Fe}$ |
| $^{64}\mathbf{Ni}$ | $^{64}\mathbf{Ni}$ | $^{64}\mathbf{Ni}$ | $^{64}\mathbf{Ni}$ |
| $^{66}\mathbf{Ni}$ | $^{66}\mathbf{Ni}$ | $^{66}\mathbf{Ni}$ | $^{66}\mathbf{Ni}$ |
| $^{86}\mathbf{Kr}$ | $^{86}\mathbf{Kr}$ | $^{86}\mathbf{Kr}$ | $^{86}\mathbf{Kr}$ |
| $^{84}\mathbf{Se}$ | $^{84}\mathbf{Se}$ | $^{84}\mathbf{Se}$ | $^{84}\mathbf{Se}$ |
| $^{82}\mathbf{Ge}$ | $^{82}\mathbf{Ge}$ | $^{82}\mathbf{Ge}$ | $^{82}\mathbf{Ge}$ |
| $^{80}\mathbf{Zn}$ | $^{80}\mathbf{Zn}$ | $^{80}\mathbf{Zn}$ | $^{80}\mathbf{Zn}$ |
| $^{79}$Cu | - | $^{79}$Cu | - |
| $^{78}$Ni | $^{78}$Ni | $^{78}$Ni | $^{78}$Ni |
| $^{80}$Ni | $^{80}$Ni | $^{80}$Ni | - |
| $^{124}$Mo | $^{124}$Mo | $^{124}$Mo | $^{124}$Mo |
| $^{122}$Zr | $^{122}$Zr | $^{122}$Zr | $^{122}$Zr |
| $^{121}$Y | - | $^{121}$Y | $^{121}$Y |
| $^{120}$Sr | $^{120}$Sr | $^{120}$Sr | $^{120}$Sr |
| $^{122}$Sr | $^{122}$Sr | $^{122}$Sr | $^{122}$Sr |
| $^{124}$Sr | - | $^{124}$Sr | - |
| $^{126}$Sr | $^{126}$Sr | - | - |
| $^{122}$Kr | - | - | - |

← *J (L)*

for details, Pearson *et al.*, PRC83, 065810 (2011)

# Neutron drip & symmetry energy (1)

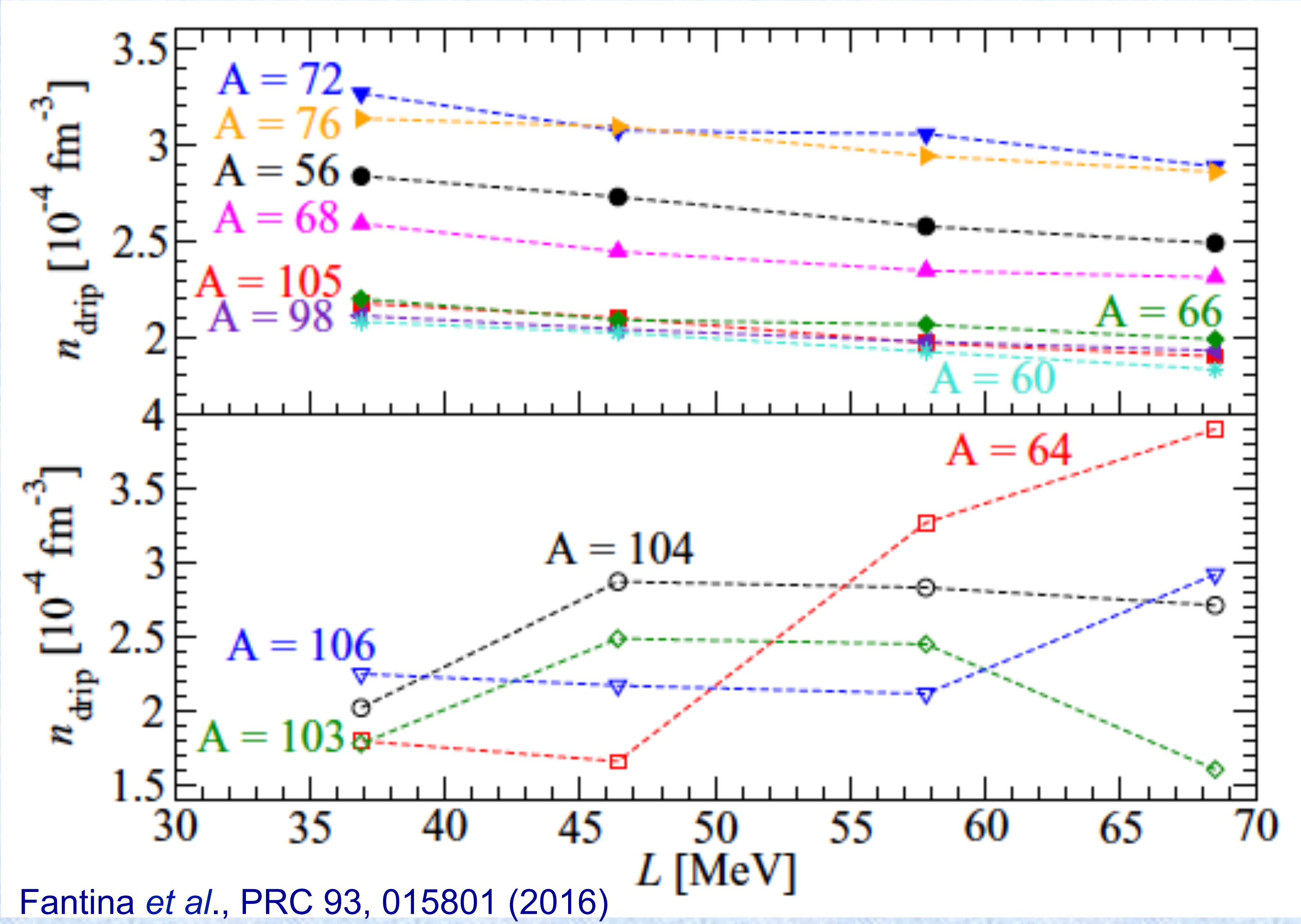
**CATALYSED NS CRUST – $T = 0$, full thermo equilibrium**



Fantina *et al.*, PRC 93, 015801 (2016)

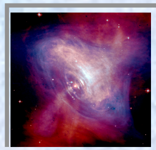
# Neutron drip & symmetry energy (2)

**ACCRETING NS CRUST – binary system, off-equilibrium**



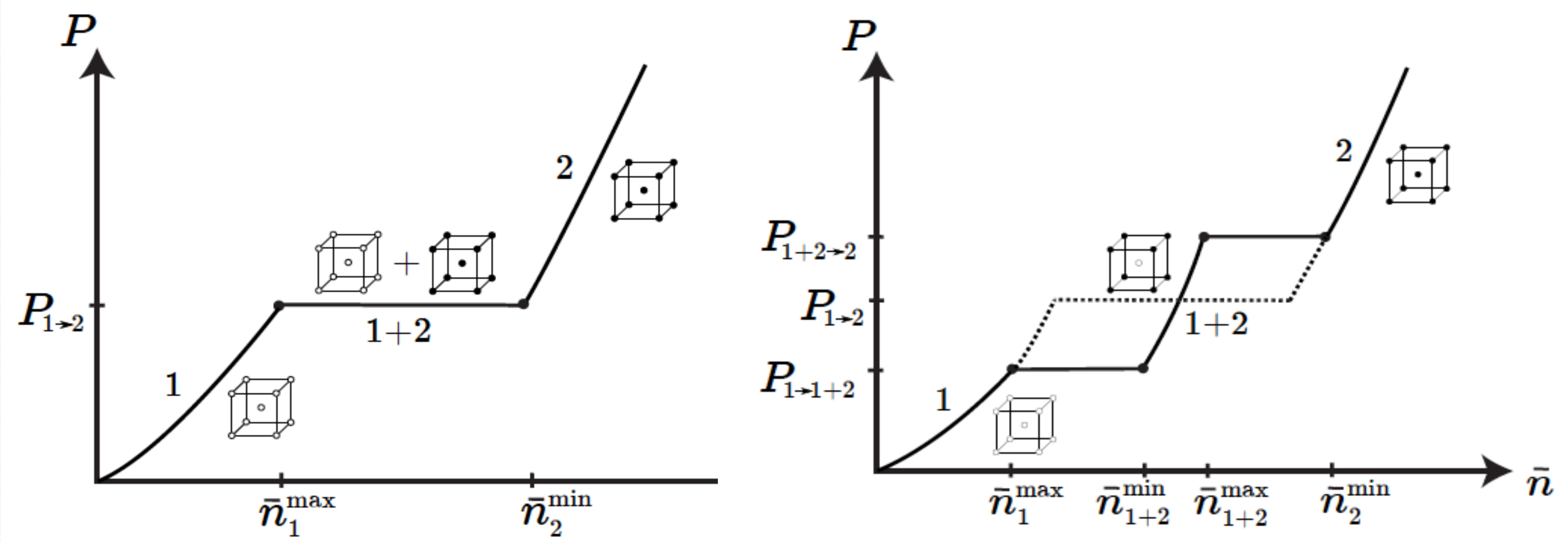
Fantina *et al.*, PRC 93, 015801 (2016)

→ sensitivity to the details of nuclear structure far from stability!
careful about *correlations* !!!



# EoS of NS: outer crust – compounds

Compounds with CsCl structure are present at interfaces ($Z_1 \neq Z_2$)



stability of multinary compounds against phase separation uniquely determined by their structure and composition

Chamel & Fantina, PRC 94, 065801 (2016)

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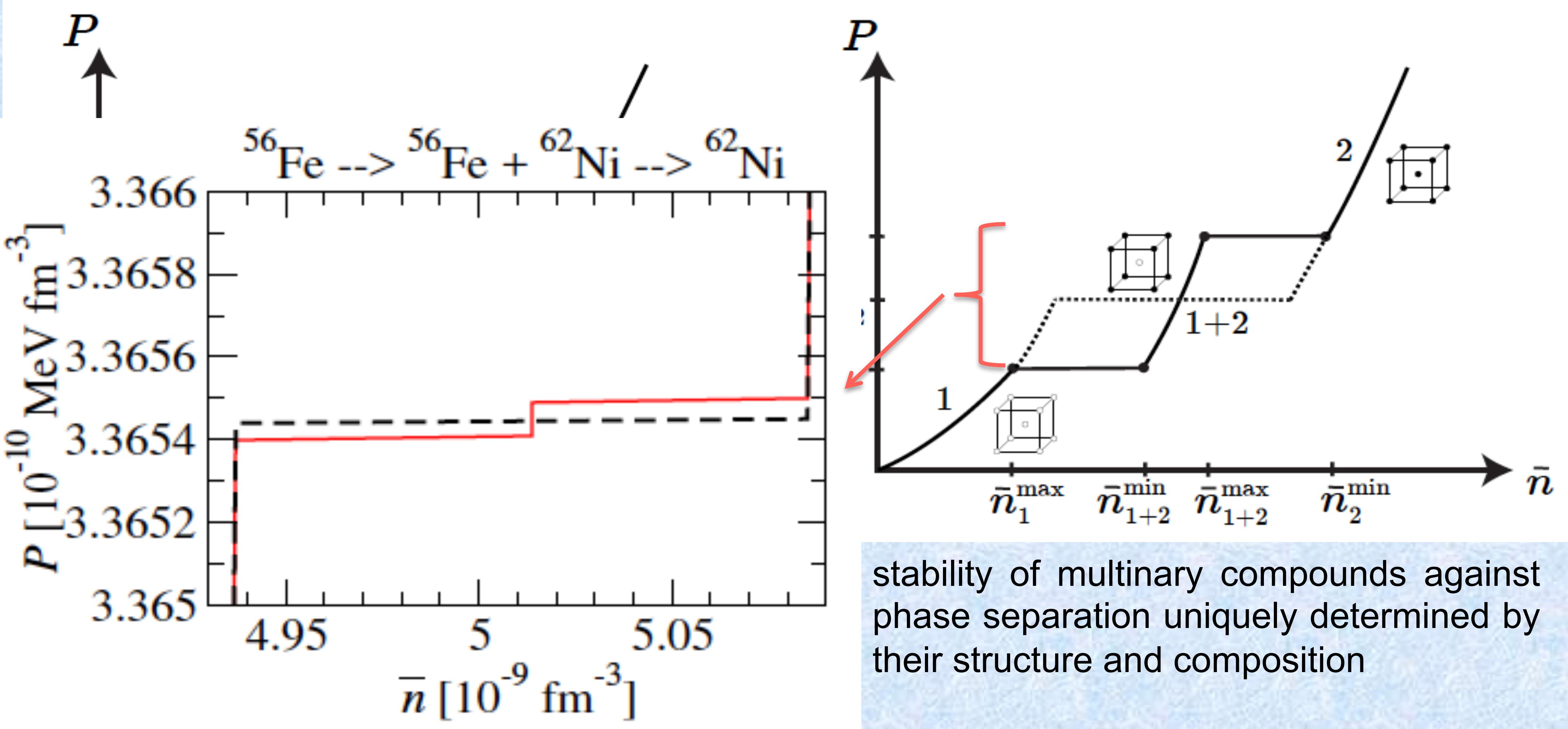

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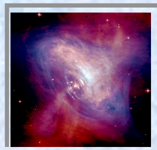
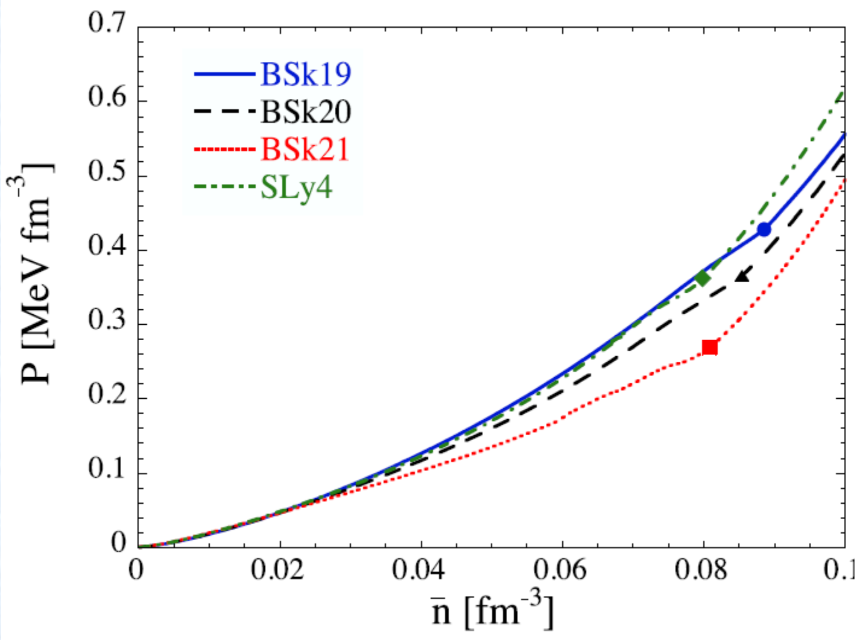
Chamel & Fantina, PRC 94, 065801 (2016)



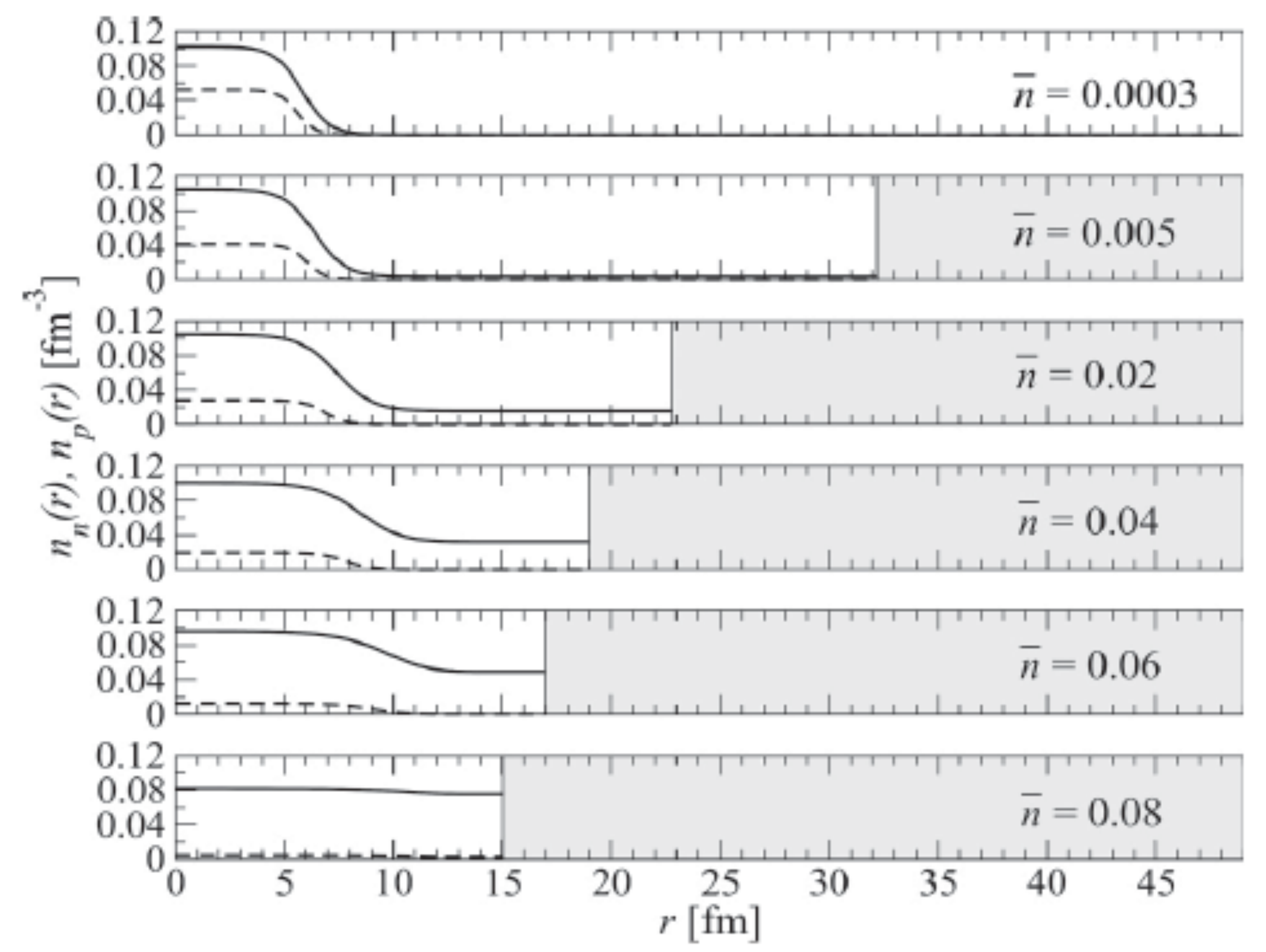
# EoS of NS: inner crust (BSk19-20-21)

Semi-classical model : Extended Thomas Fermi (4th order in ħ)
+ proton shell corrections (Strutinski Integral theorem)

Pearson *et al.*, PRC85, 065803 (2012)



very smooth
crust-core transition

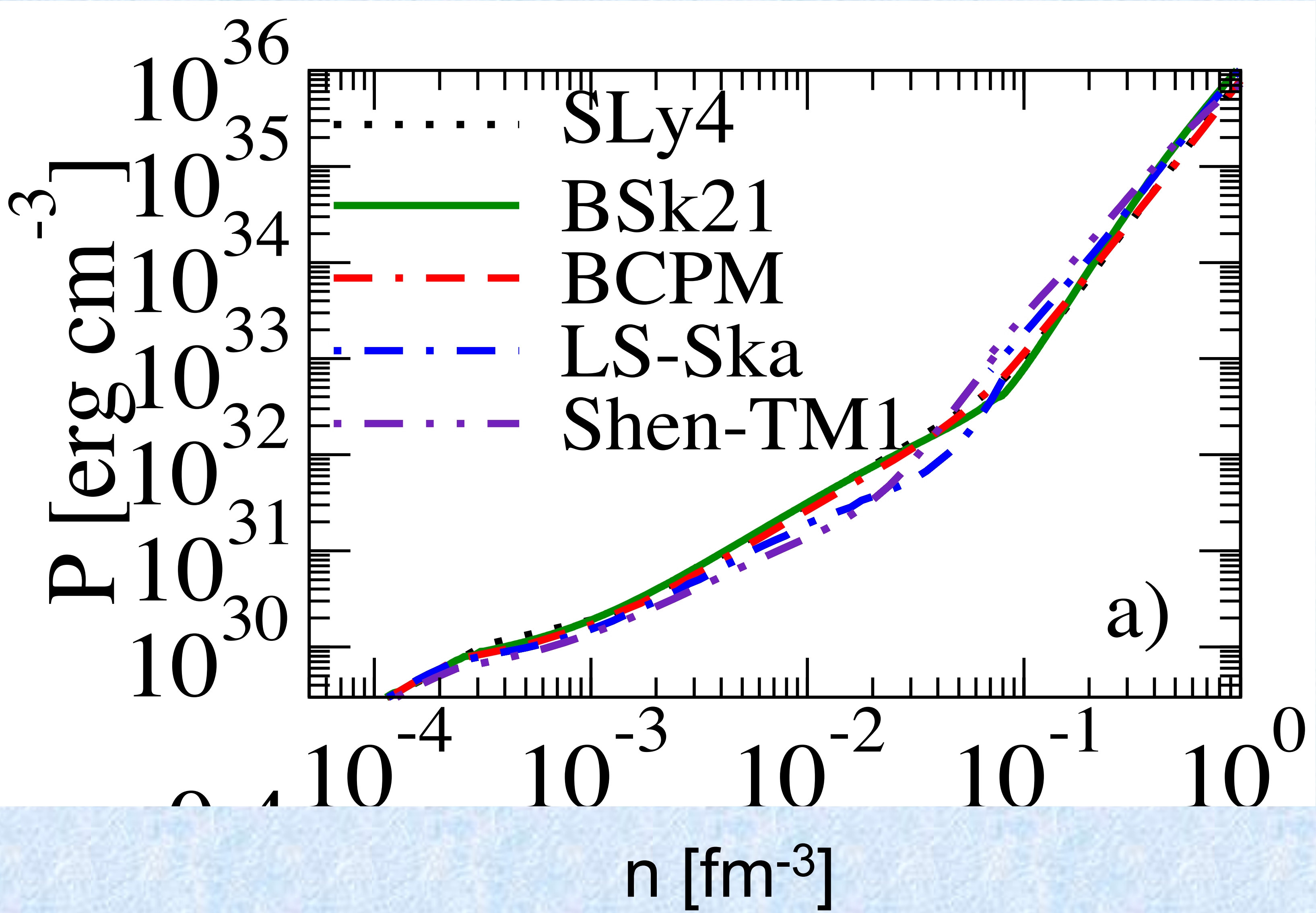


Impact of proton pairing (BCS approximation): Pearson *et al.*, PRC 91, 018801 (2015)

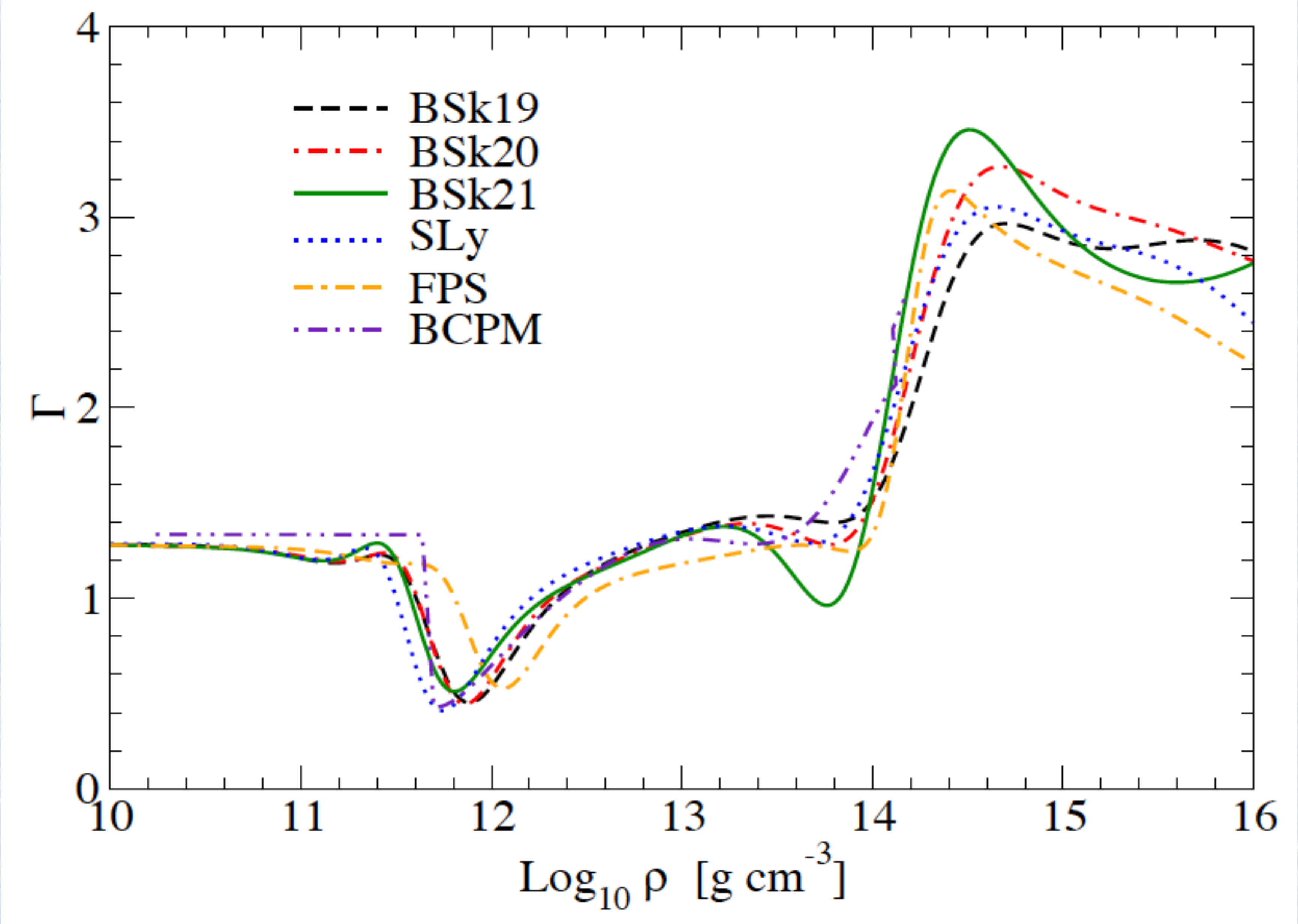
Here we do not consider possible phase transition in the core !

→ see discussion in : Chamel, Fantina, Pearson, Goriely, A&A 553, A22 (2013)

# (Some) unified EoS for NS in β equil

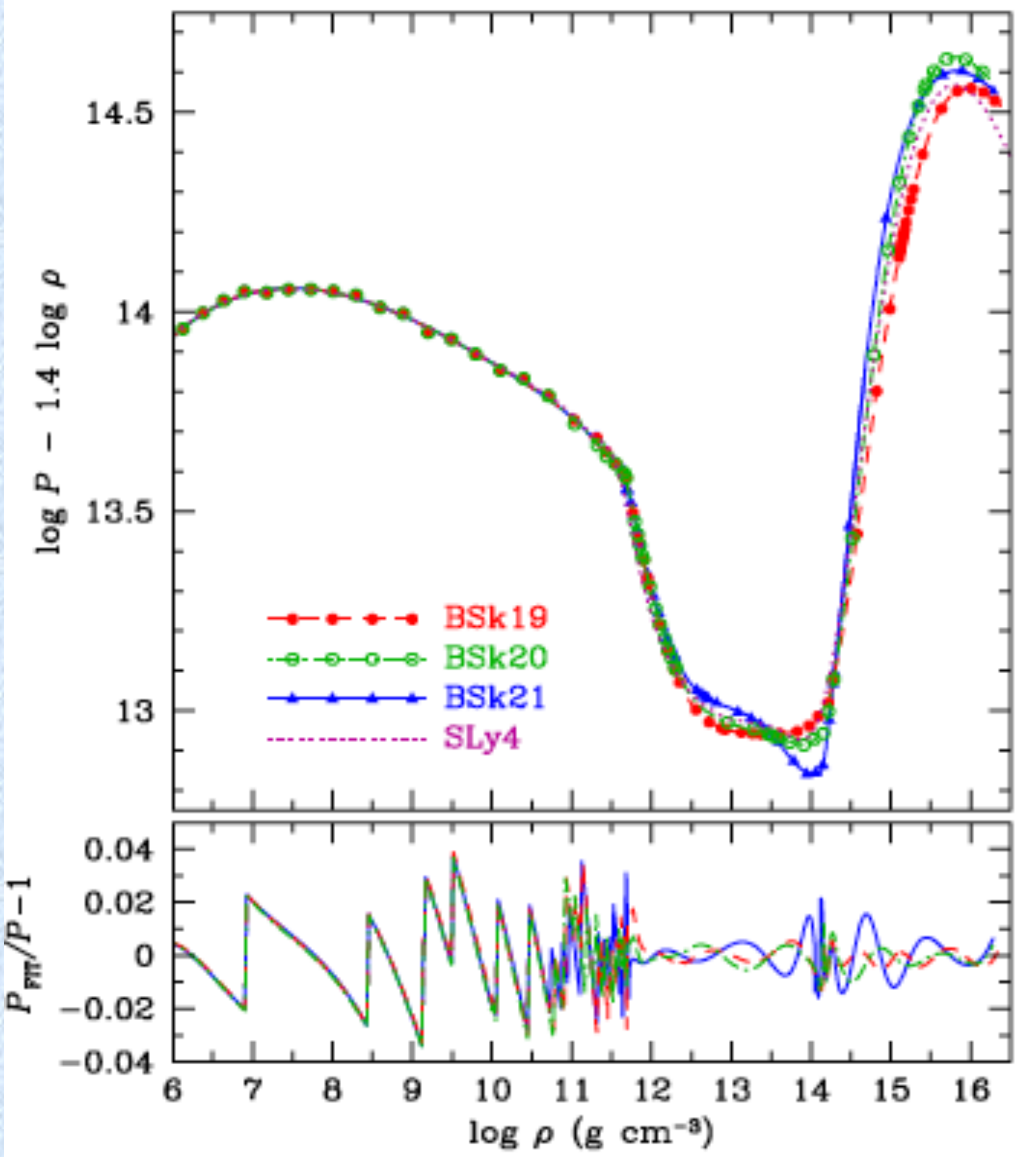


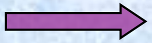
# EoS of NS: adiabatic index



Realistic EoSs hardly parametrised by polytropes!

# EoS of NS: analytical representation

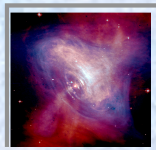




Analytical fits are very accurate !

- ✓ BSk19, BSk20, BSk21 already available
- ✓ BSk22-25 to come soon!

Potekhin, Fantina, Chamel *et al.*, A&A 560, A48 (2013)
Pearson *et al.*, in preparation



# Computing the NS structure

- Nuclear models: **BSk 19-20-21** & **BSk 22-23-24-25-26**

- Build NS :
  - ✧ **non-rotating NS** → solve Tolman-Oppenheimer-Volkoff (TOV) equations:

$$\frac{dP}{dr} = -\frac{G\rho\mathcal{M}}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{\mathcal{M}c^2}\right)\left(1-\frac{2G\mathcal{M}}{rc^2}\right)^{-1}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \rho$$

➡ EoS $P(\rho)$ to close the system

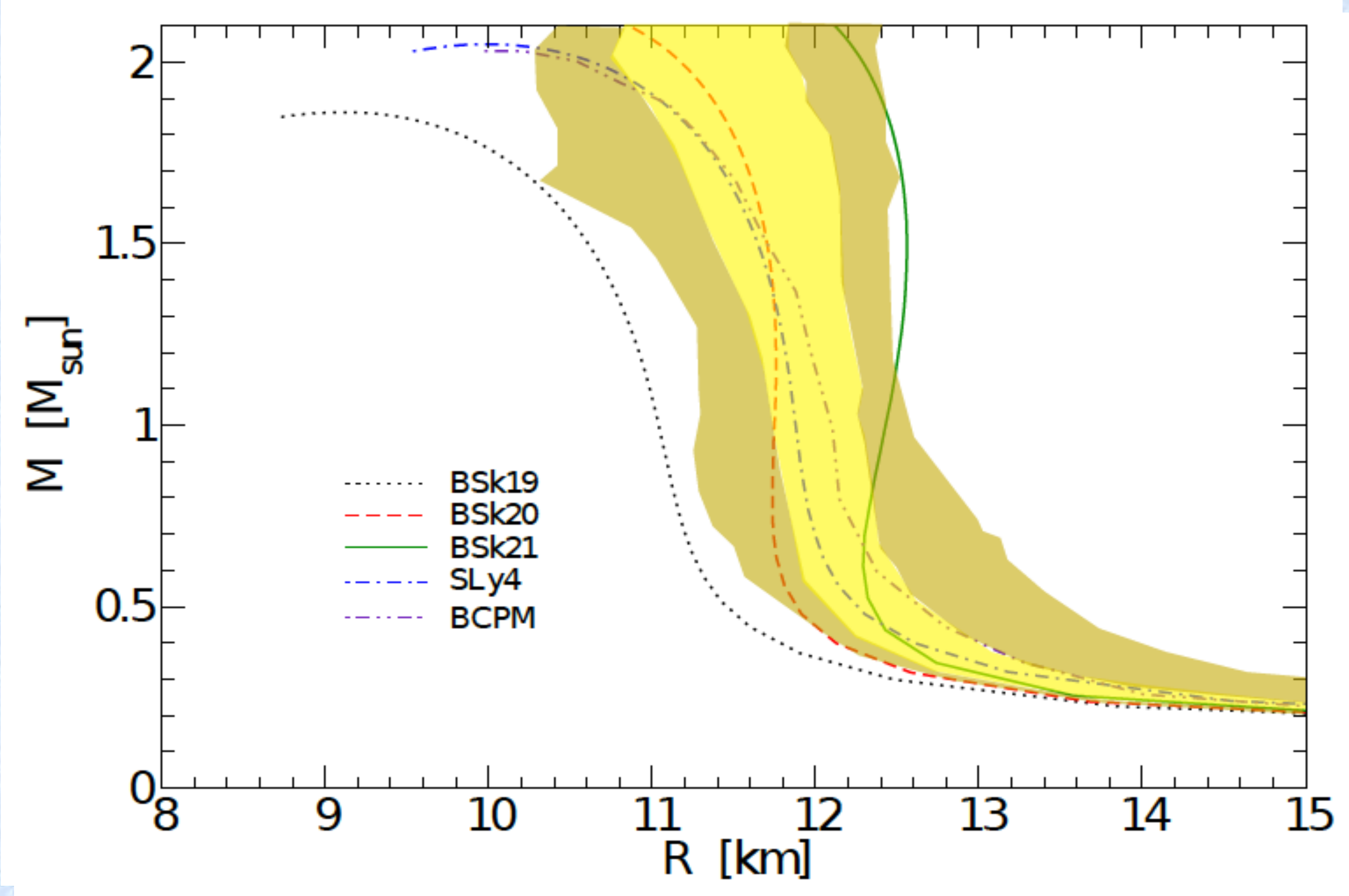
  - ✧ **rigidly rotating** NSs

→ stationary axi-symmetric configurations in GR.

**LORENE** library (http://www.lorene.obspm.fr)

Refs on LORENE: Gourgoulhon, arXiv: 1003.5015 (lectures given at 2010 CompStar school)
Gourgoulhon *et al.*, A&A 349, 851 (1999)
Granclément & Novak, Liv. Rev. Relativ. 12, 1 (2009)

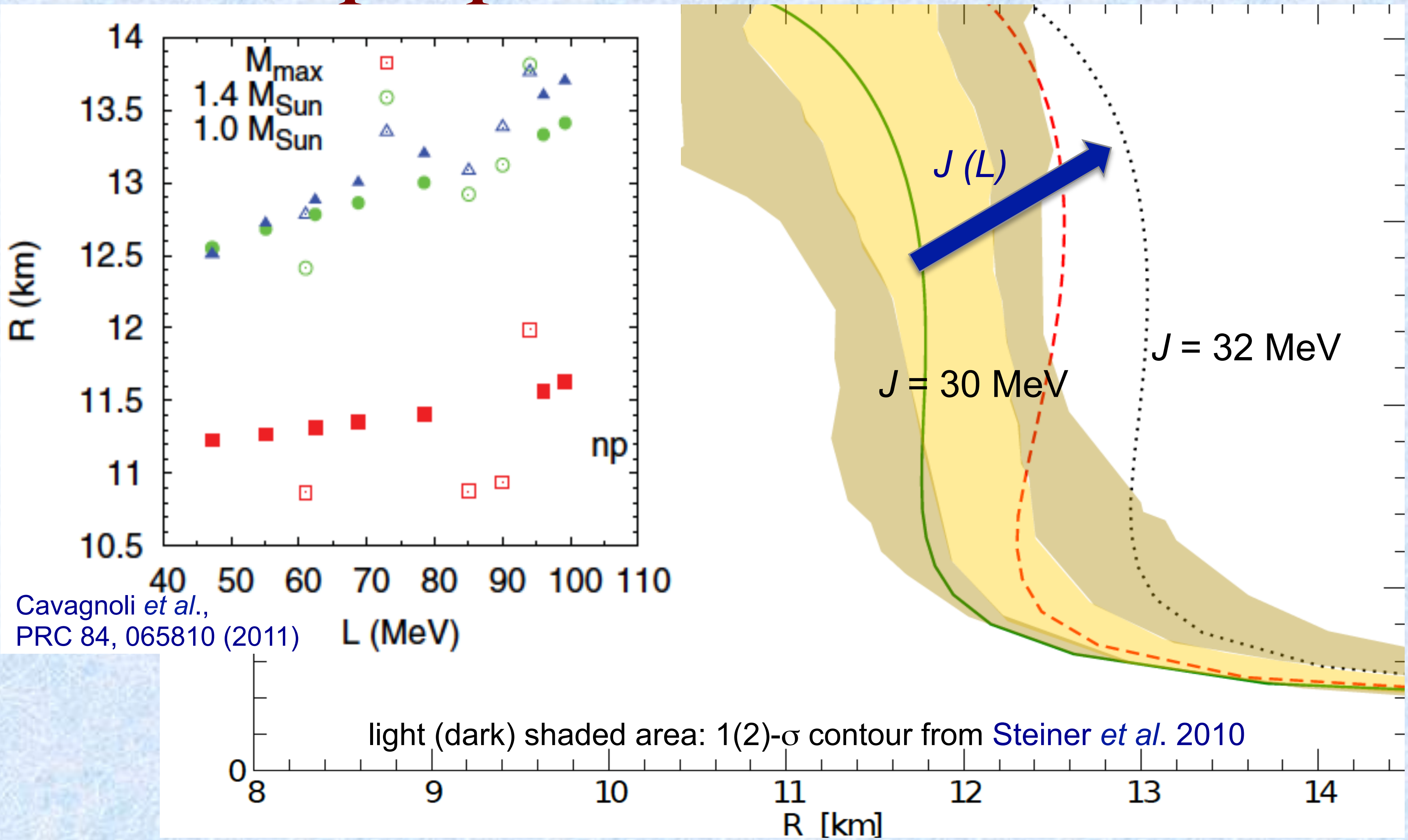
# NS properties: *M-R relation*



dark (light) shaded area: 1(2)-σ contour from Steiner *et al*. 2010

Fantina *et al*., Astron. Astrophys. 559, A128 (2013)

# NS properties: *M-R relation*



Cavagnoli *et al.*,
PRC 84, 065810 (2011)

*J (L)*

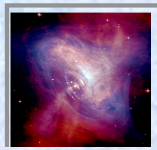
*J* = 30 MeV

*J* = 32 MeV

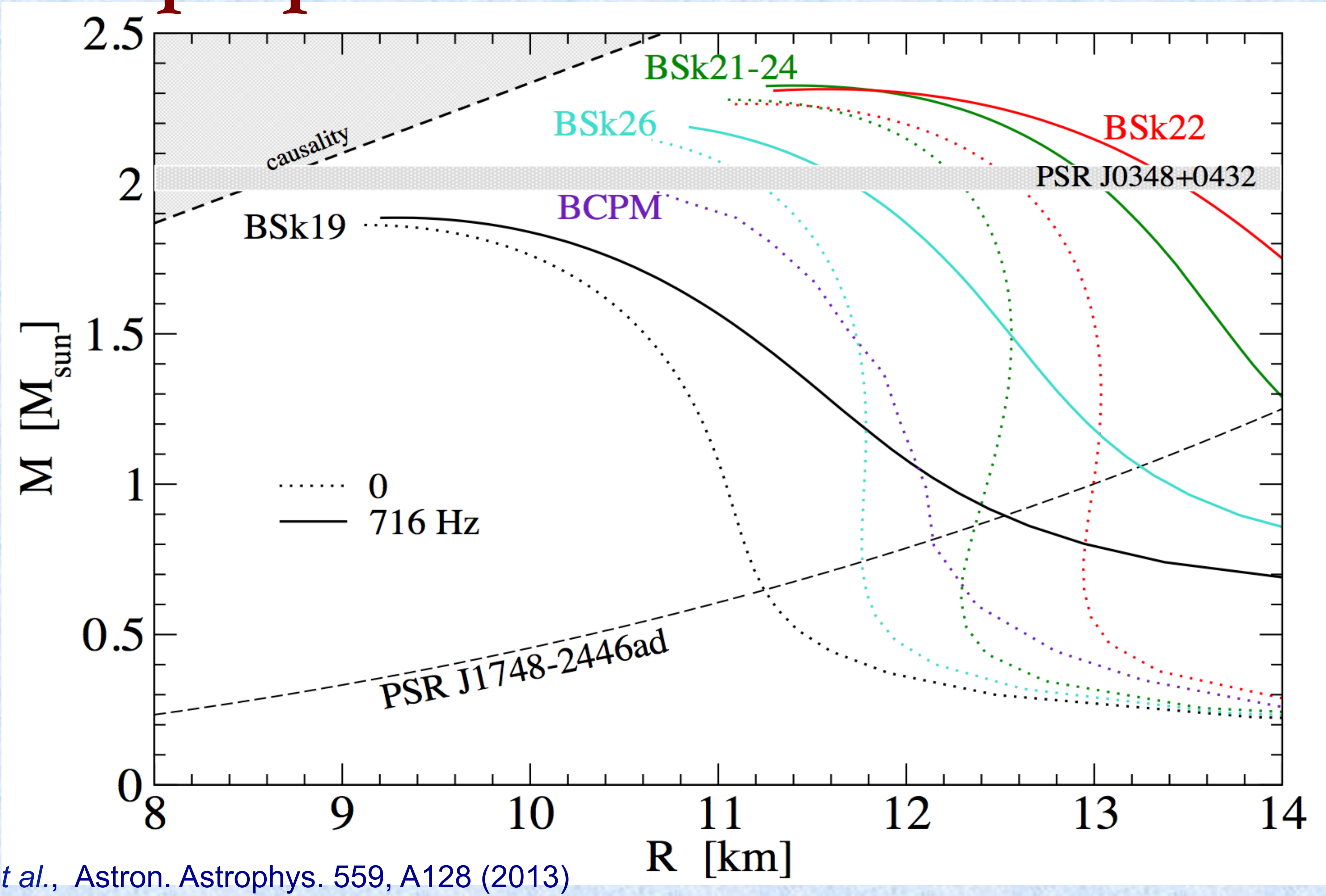
light (dark) shaded area: 1(2)-σ contour from Steiner *et al*. 2010

➡ BSk24, 26 compatible with astrophysical “observations"

Pearson, Chamel, Fantina, Goriely, Eur. Phys. J. A 50, 43 (2014)
Fantina *et al*., AIP Conf. 1645, 92 (2015)

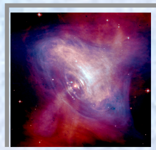


# NS properties: *M-R relation with rotation*



Fantina *et al.*, Astron. Astrophys. 559, A128 (2013)

→ nucleonic EoS BSk19 too soft...

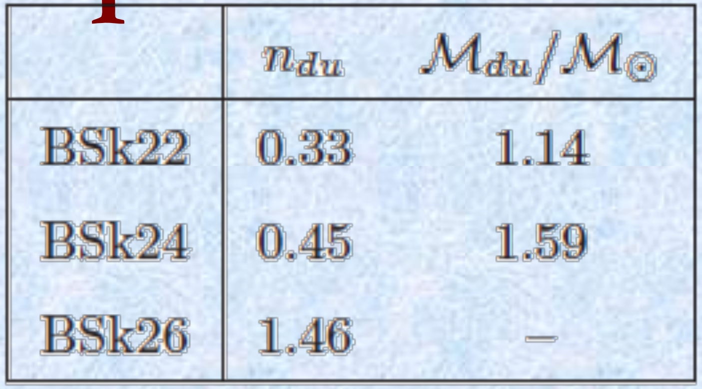


# $Y_p$ and Direct URCA process

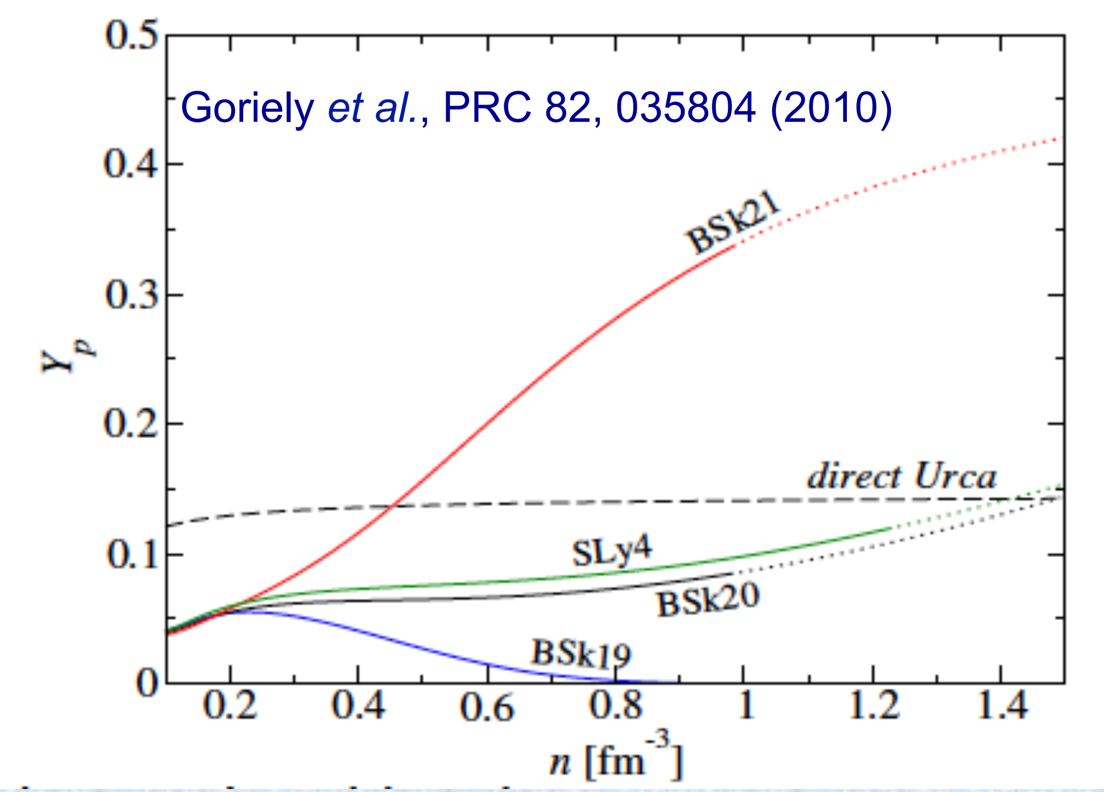
$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$p + e^- \rightarrow n + \nu_e$$

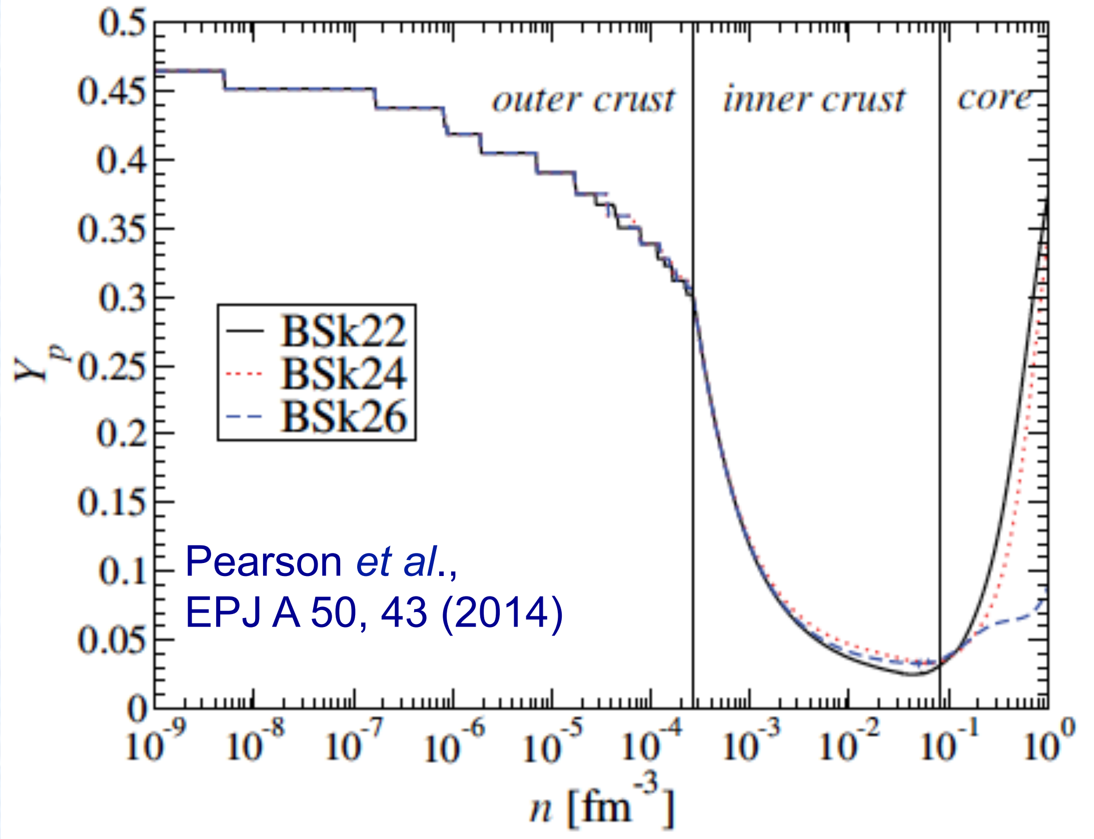
direct URCA possible if $Y_p \approx$ 11-15%



| | $n_{du}$ | $\mathcal{M}_{du}/\mathcal{M}_\odot$ |
|---|---|---|
| BSk22 | 0.33 | 1.14 |
| BSk24 | 0.45 | 1.59 |
| BSk26 | 1.46 | – |



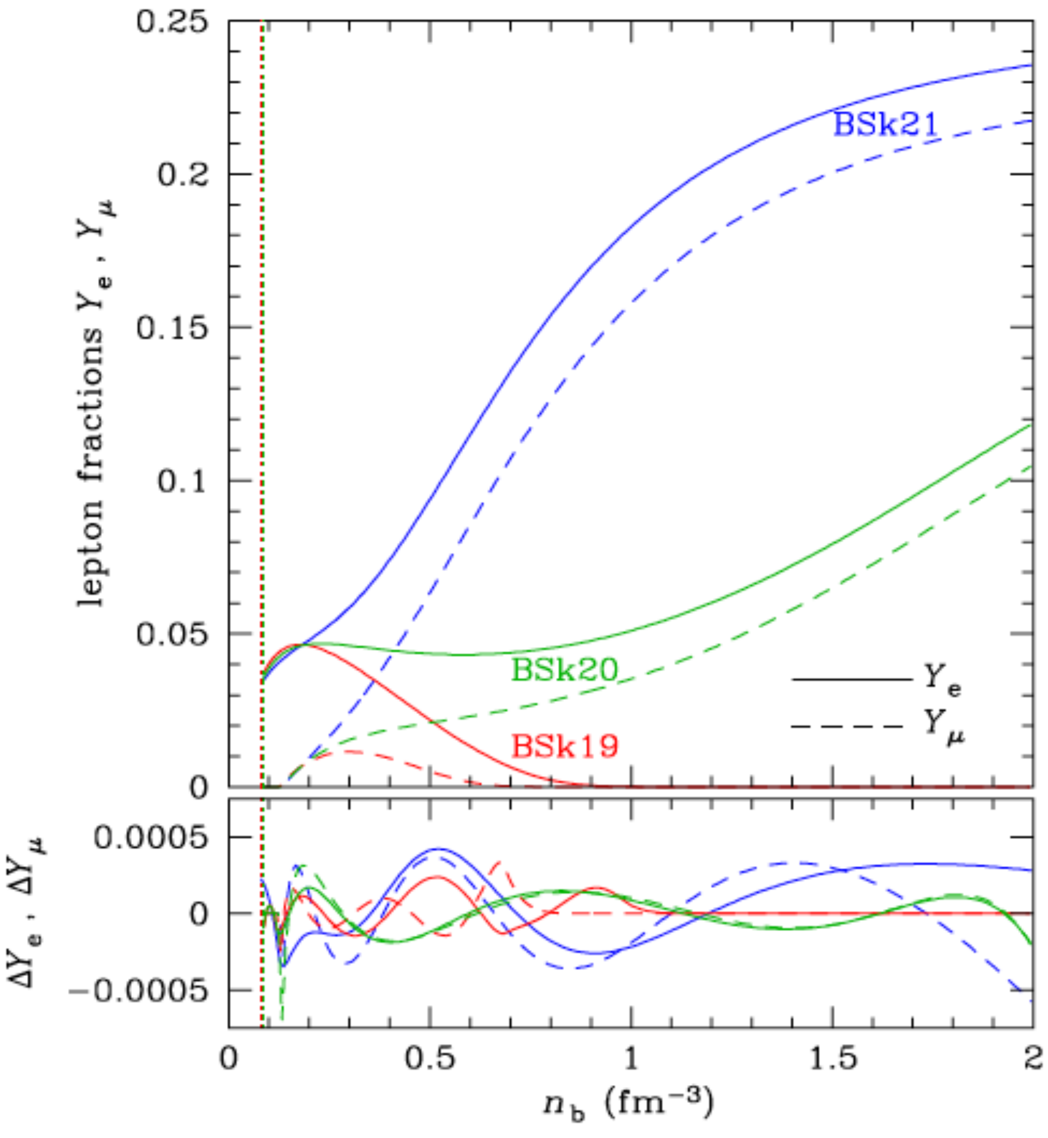
Goriely *et al.*, PRC 82, 035804 (2010)



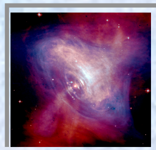
Pearson *et al.*,
EPJ A 50, 43 (2014)

⇨ BSk21 & BSk24 compatible with existence of direct URCA process for $n$ > 0.45 1/fm$^3$ (or $M$ > 1.59 $M_{sun}$)

# $Y_e, Y_\mu$ in NS matter (core)



Potekhin, Fantina, Chamel *et al.*, A&A 560, A48 (2013)
fit by A.Y. Potekhin

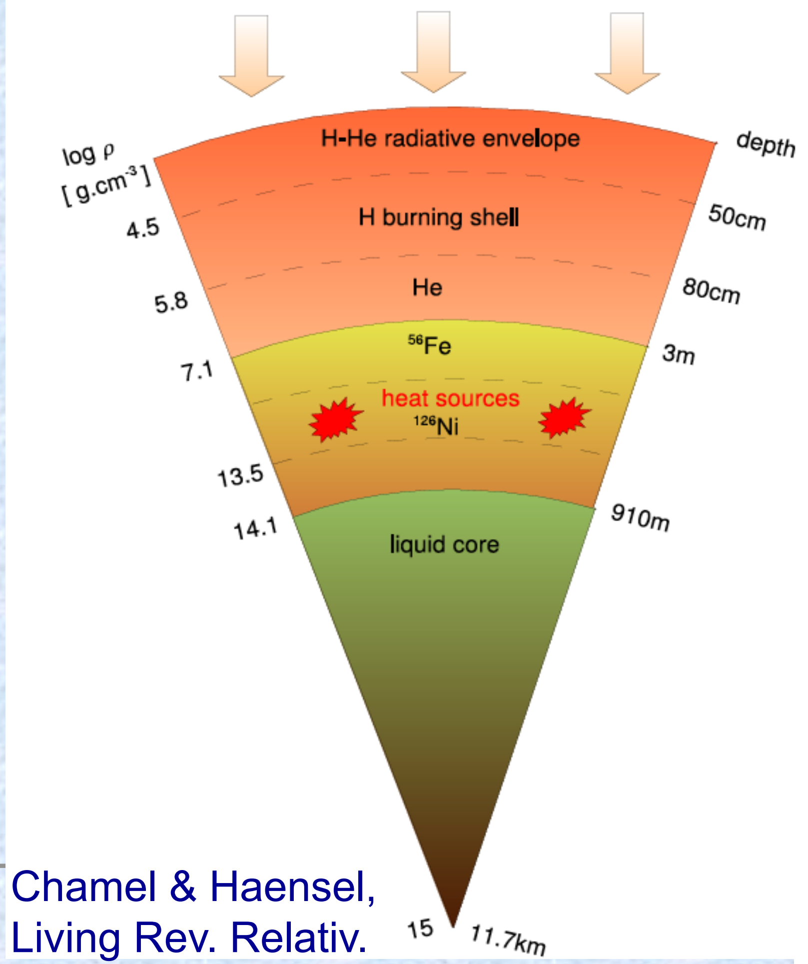


# Outline

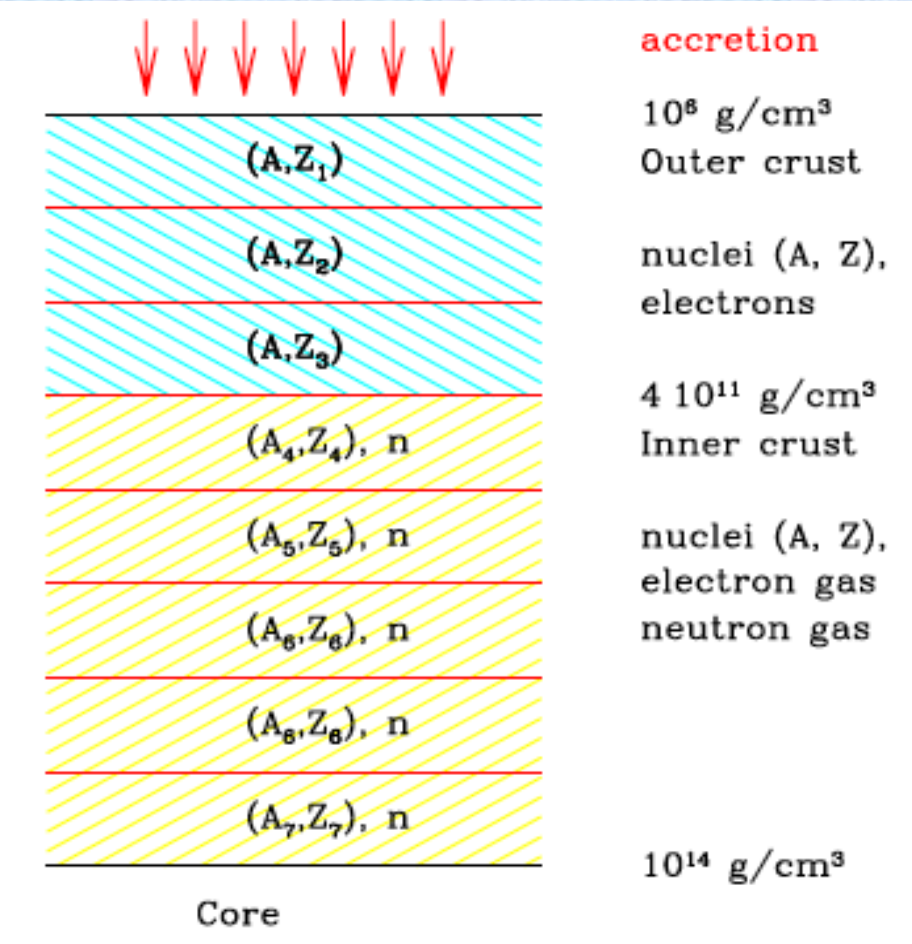
- Astrophysical framework and motivations
- Effective nuclear models
  - Nuclear functionals and the Brussels-Montreal BSk model
- Equations of state (EoSs) of dense matter
  - *Catalysed* NS and astrophysical constraints
  - ***Accreted*** crust (work in progress)
- Conclusions & Outlook

# Accreted NS: model, heat sources

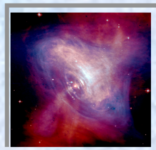
- Thernodynamics:
  - ✧ bbc lattice, ground state = minimum of *g* at constant *A*
  - ✧ use of the *same* BSk models as for catalysed matter
    - → more microscopic model, proton shell corrections included



Chamel & Haensel, Living Rev. Relativ. 11, 10 (2008)



Zdunik, talk at MODE, May 2014



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- Heat sources:
  - ✧ outer crust: with increasing *P*, if $\mu_b(A,Z-1) < \mu_b(A,Z)$ → EC can occur:

$$(A,Z) + e^- \to (A,Z-1) + \nu_e$$

(in quasi-equilibrium)

$$(A,Z-1) + e^- \to (A,Z-2) + \nu_e + Q_c$$

(energy release)

  - ✧ inner crust: n emission also possible:

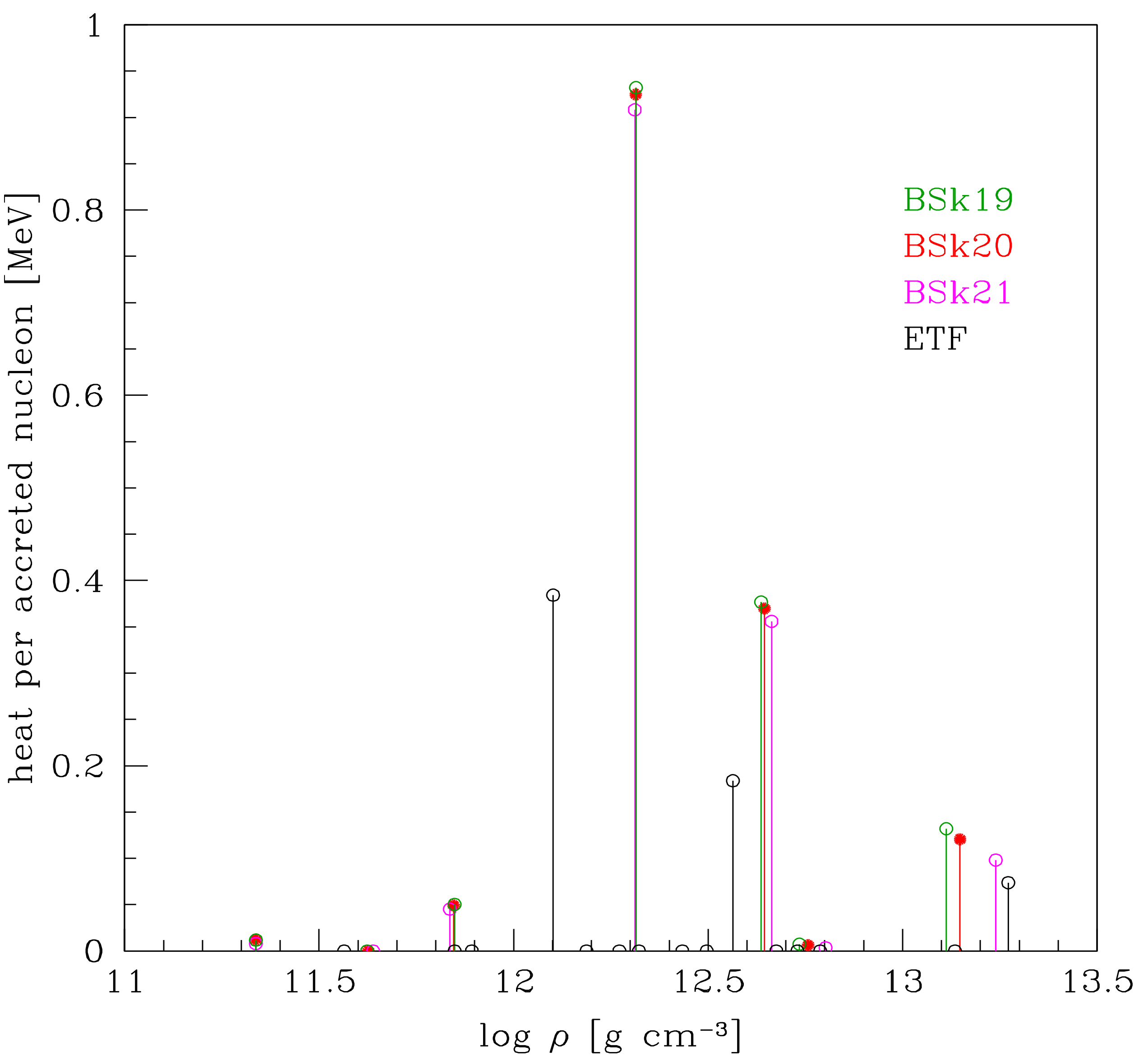
$$(A,Z) + e^- \to (A,Z-1) + \nu_e$$

$$(A,Z-1) + e^- \to (A-k,Z-2) + kn + \nu_e + Q_c$$

  - ✧ pycnonuclear reactions: $(A,Z) + (A,Z) \to (2A,2Z) + Q_p$

N.B.: uncertainties in pycno reactions → it occurs when $Z=Z_{min}$

# Accreted NS: deep heat sources



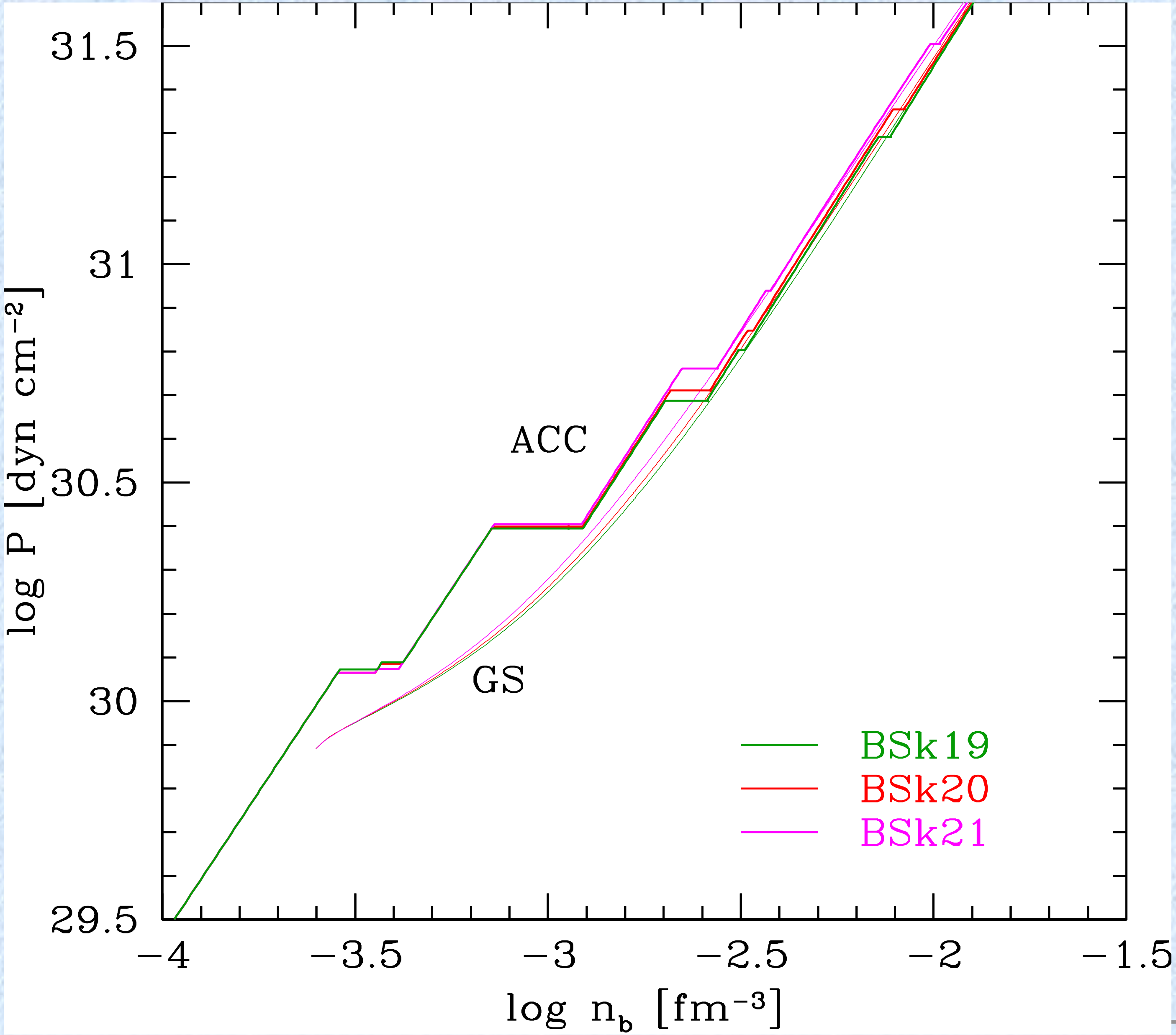
PRELIMINARY! Figures by J. L. Zdunik

Heat sources in the inner crust
Initial composition: $^{56}$Fe ahes

Main energy sources located at 300-500 below NS surface

→ we can compute heat released and position of the sources
→ impact of shell effects

# Accreted NS: EoS



EoS for catalysed (GS) and accreted (ACC) crust.

Initial composition: $^{56}$Fe ahes

**Accreted crust EoS significantly stiffer than GS one for :**

**$\rho = 5 \times 10^{11} - 6 \times 10^{12}$ g/cm$^3$**

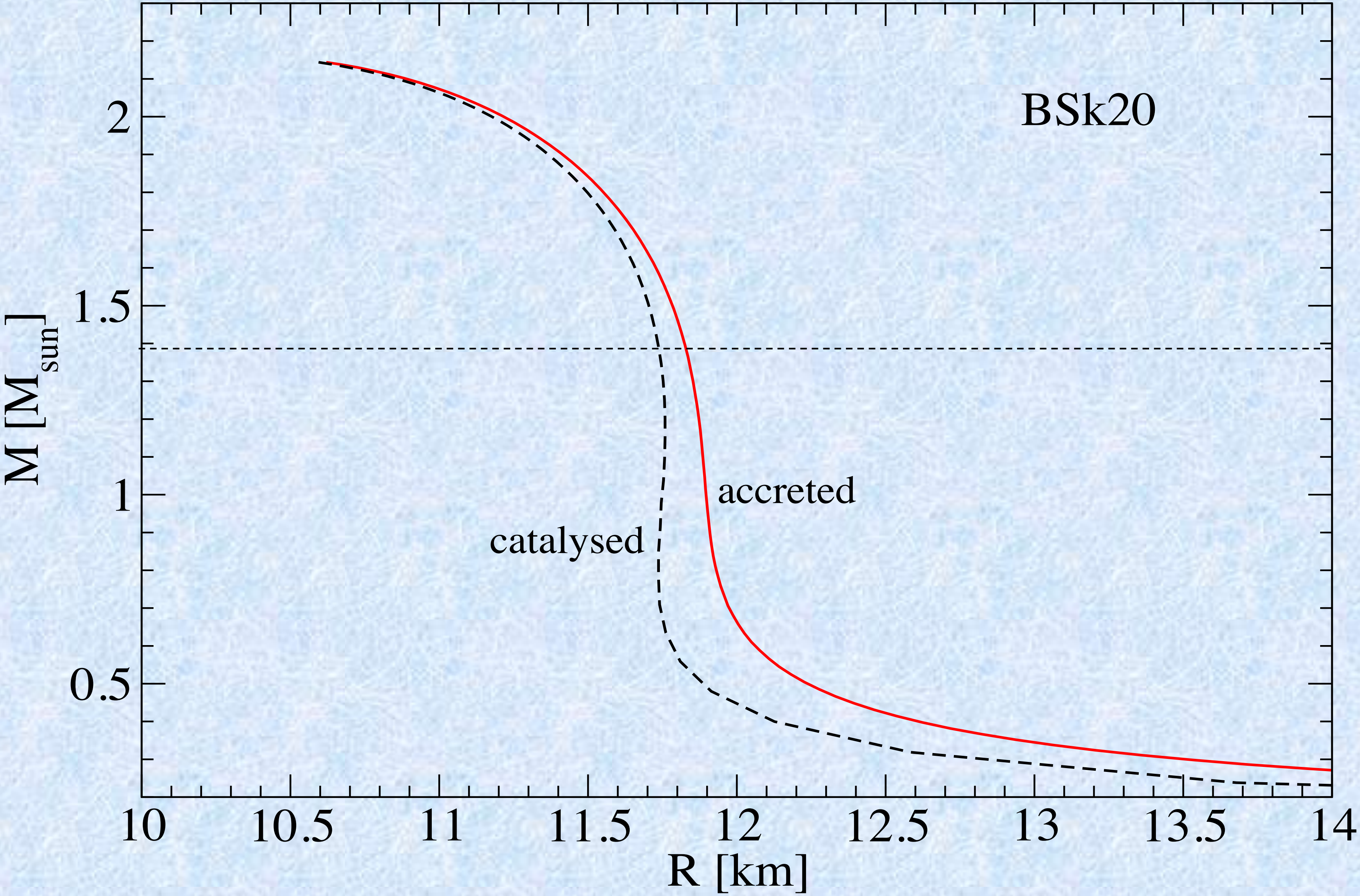
↓

Typically, for 1.4 $M_{sun}$ NS, one expects : $R_{ACC} - R_{GS} \approx 100$ m

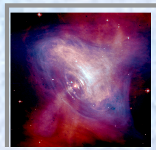
(see e.g. Haensel&Zdunik, A&A, 1990)

PRELIMINARY! Figure by J. L. Zdunik

# Accreted NS: EoS



PRELIMINARY!



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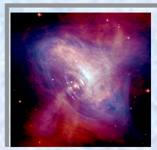

# Conclusions & Outlooks

- Unified EoSs for NS matter → same nuclear model to describe all regions of NS

- Properties of NS (outer) crust very sensitive
  to the details of the nuclear structure far from the valley of stability
  → **combine constraints from nuclear physics & astrophysics**

- EoSs **BSk 19-20-21** at T=0 for catalysed matter available as:
  - ***tables*** : Fantina *et al.*, A&A 559, A128 (2013), doi: 10.1051/0004-6361/201321884
  - ***fit*** : Potekhin *et al.*, A&A 560, A48 (2013) at: http://www.ioffe.ru/astro/NSG/BSk/
    Fit: EoS, density profiles, electrical conductivities → can be used in NS calculations!

- ***Unified EoSs*** for the **BSk22-25** series will appear soon (tables + fits, paper in prep.) !
  Fit (in progress): EoS, density profiles, *chemical potentials*

- Perspectives :
  - Finite T for SN cores
  - Accreting NSs
  - Magnetars

# Grazie