

# Generalized Heisenberg Uncertainty Principle applied to isospin symmetry breaking operators in atomic nuclei

Author: Àlex Roger Garcia\*

Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisor: Xavier Roca Maza†

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**Abstract:** Based on Sandro Stringari’s article *Heisenberg Uncertainty Inequality and Breaking of Isospin Symmetry in Atomic Nuclei* [1], the generalized Heisenberg’s uncertainty inequality is used to find a lower bound to the breaking of isospin symmetry. To do this, we employ the lowering isospin operator and the isovector giant monopole resonance operator (IVGMR) instead of the original position and linear momentum operators to set up the new uncertainty principle. As an original development of our thesis, we modify the IVGMR operator in order to eliminate the contribution of the isobaric analogue state (IAS). We find, within the Hartree-Fock plus Random Phase Approximation method applied to a sound nuclear effective interaction (Skyrme-type), that the IAS isolates the main contribution to the generalized Heisenberg’s uncertainty principle presented here.

**Keywords:** Heisenberg’s uncertainty principle, isospin symmetry, Hartree-Fock, Random Phase Approximation

**SDGs:** Quality education

## I. INTRODUCTION

Some important magnitudes in the study of the nucleus are the total isospin  $\mathbf{T}$  and its third component  $T_z \equiv \sum_{i=1}^A t_{z,i}$ , which is determined by adding the individual isospin of the nucleus’ protons ( $t_z = -1/2$ ) and neutrons ( $t_z = 1/2$ ). While  $T_z$  commutes with the nuclear Hamiltonian  $\mathcal{H}$ , reflecting the fact that in a closed system, electric charge  $Q = \sum_{i=1}^A [1/2 - t_{z,i}]$  should be conserved,  $\mathbf{T}^2 = T_x^2 + T_y^2 + T_z^2$  does not commute with  $\mathcal{H}$ . This implies that isospin impurities will be present in nuclei. Those are known to be mainly caused by the Coulomb force with a small contribution coming from the isospin symmetry breaking of the strong residual interaction [2].

In 1927, Heisenberg first expressed his uncertainty principle in its most famous form, relating the precision with which we can know a particle’s position and momentum at the same time. In this work we aim to use Heisenberg’s uncertainty principle to determine a lower bound for the breaking of isospin symmetry in atomic nuclei, to better understand how isospin mixing appears under certain experimental observations. In order to do this we closely follow Ref. [1] which uses the generalized Heisenberg uncertainty relation[6]

$$\langle \{A^\dagger, A\} \rangle \langle \{B^\dagger, B\} \rangle \geq |\langle [A^\dagger, B] \rangle|^2. \quad (1)$$

Within the present work, the expectation value brackets  $\langle \rangle$  will indicate a ground state expectation value unless otherwise indicated.

Specifically, in the Sec. II, we employ the same operators as in Ref. [1], that is, the lowering operator in isospin space  $T_-$  –defined below– and the IVGMR operator and derive the corresponding uncertainty principle. In Sec. III, we modify the IVGMR operator extracting the effect of the IAS from the nuclear response associated to such an operator. This is to understand the impact of the IAS on the uncertainty principle. The relations derived theoretically will be tested numerically by means of a Fortran code [3] available in the group that is based on the Hartree-Fock plus Random Phase Approximation applied to a Skyrme effective interaction [5]. In the last Sec. IV we summarize our conclusions.

## II. UNCERTAINTY PRINCIPLE I

In this section, we closely follow Ref. [1] and select the same operators  $A$  and  $B$ . Hence, we define  $A = T_- \equiv \sum_j t_{-,j}$ ;  $B = M_- \equiv \sum_j r_j^2 t_{-,j}$  such that  $t_{+,j}$  ( $t_{-,j}$ ) is the operator that adds (subtracts) 1 from the third component of the isospin of the  $j$ -nucleon. Then, using the convention for the  $z$  component of the isospin for a neutron where the expectation value is  $t_z = +1/2$  and for a proton where the expectation value is  $t_z = -1/2$ , we see that  $t_+$  and  $t_-$  transforms a proton into a neutron and a neutron into a proton respectively. These operators can be written in the usual form:  $t_- = t_x - it_y$  and  $t_+ = t_x + it_y$  which implies that  $t_\pm^\dagger = t_\mp$ . Therefore,

$$\begin{aligned} \langle \{A^\dagger, A\} \rangle &= \langle \{T_+, T_-\} \rangle = \langle T_+ T_- + T_- T_+ \rangle \\ &= 2\langle (T_x^2 + T_y^2) \rangle = 2\langle (\mathbf{T}^2 - T_z^2) \rangle. \end{aligned} \quad (2)$$

Evaluating now the anti-commutator of the operator  $B \equiv M_- \equiv \sum_j r_j^2 t_{-,j}$  with  $B^\dagger \equiv M_+ = \sum_j r_j^2 t_{+,j}$  and using

\*Electronic address: arogerga27@alumnos.ub.edu

†Electronic address: xavier.roca.maza@fqa.ub.edu

the completeness relation  $\sum_n |n\rangle\langle n| = \mathbb{I}$  one can write

$$\begin{aligned} \langle \{B^\dagger, B\} \rangle &= \langle M_+ M_- \rangle + \langle M_- M_+ \rangle \\ &= \sigma_-^M + \sigma_+^M \end{aligned} \quad (3)$$

where  $\sigma_+ = \sum_n |\langle n | M_+ | 0 \rangle|^2$  is the so called non-energy weighted sum rule of the IVGMR in the  $T_+$  channel and  $\sigma_- = \sum_n |\langle n | M_- | 0 \rangle|^2$  that in the  $T_-$  channel. These two quantities are experimentally accessible in charge exchange reactions where the excitation of nuclear monopole states is possible (see, e.g. [7, 8] and references therein).

Using  $[t_{+,j}, t_{-,k}] = 2\delta_{jk}t_{z,j}$ , the property of the commutator  $[A + B, C] = [A, C] + [B, C]$  and taking into account the fact that  $r^2$  commutes with  $t_{\pm}$ , as it acts on a vector space that is independent with that of the isospin, we find that

$$\begin{aligned} \langle [A^\dagger, B] \rangle &= \langle [\sum_j t_{+,j}, \sum_k r_k^2 t_{-,k}] \rangle \\ &= 2 \sum_j r_j^2 t_{z,j} \\ &= N \langle r_n^2 \rangle - Z \langle r_p^2 \rangle \end{aligned} \quad (4)$$

where we have used the standard definition for the second moment of the neutron and proton density distributions. That is,

$$\langle r_q^2 \rangle \equiv \frac{\int d\mathbf{r} r^2 \rho_q(\mathbf{r})}{\int d\mathbf{r} \rho_q(\mathbf{r})} \quad (5)$$

where  $q$  stands for neutrons ( $n$ ) and protons ( $p$ ).

Substituting the previous results in Eq. (1), we recover the Uncertainty Principle found in Ref. [1]

$$\langle \mathbf{T}^2 - T_z^2 \rangle \geq \frac{(N \langle r_n^2 \rangle - Z \langle r_p^2 \rangle)^2}{2(\sigma_-^M + \sigma_+^M)} \quad (6)$$

If isospin symmetry breaking was an exact symmetry in atomic nuclei: 1) the l.h.s. of this equation would be zero in  $N = Z$  nuclei while it would be equal to  $\langle T_z \rangle = (N - Z)/2$  for  $N \neq Z$ ; 2) the numerator in the r.h.s. would be again zero for  $N = Z$ , positive for neutron-rich nuclei and negative for proton rich nuclei; and 3) the denominator in the r.h.s. would be equal to  $4\sigma_z$  as we shall detail in what follows.

#### A. Uncertainty Principle assuming isospin symmetry in $\sigma_-^M + \sigma_+^M$

Considering  $\mathcal{H}$  is symmetrical with respect to rotations in isospin space, so that  $\langle M_x^2 \rangle = \langle M_y^2 \rangle = \langle M_z^2 \rangle \equiv \sigma_z^M$ , and using

$$\begin{aligned} t_{+,i} t_{-,j} &= t_{x,i} t_{x,j} - i t_{x,i} t_{y,j} + i t_{y,i} t_{x,j} + t_{y,i} t_{y,j}, \\ t_{-,j} t_{+,i} &= t_{x,j} t_{x,i} + i t_{x,j} t_{y,i} - i t_{y,j} t_{x,i} + t_{y,j} t_{y,i} \end{aligned} \quad (7)$$

we find

$$\begin{aligned} \sigma_-^M + \sigma_+^M &= \langle 0 | \sum_{i,j} r_i^2 r_j^2 (t_{+,i} t_{j,-} + t_{-,i} t_{+,j}) | 0 \rangle \\ &= 2(\langle M_x^2 \rangle + \langle M_y^2 \rangle) = 4\sigma_z^M \end{aligned} \quad (8)$$

Then, for  $N = Z$  we have  $\langle T_z \rangle = 0$ , so in that case Eq. (6) becomes

$$\langle \mathbf{T}^2 \rangle \geq \frac{N(\langle r_n^2 \rangle - \langle r_p^2 \rangle)^2}{8\sigma_z^M} \quad (9)$$

which would reduce to an equality since  $\langle \mathbf{T}^2 \rangle = 0$  and  $\langle r_n^2 \rangle = \langle r_p^2 \rangle$ . Hence the last expression is most useful for  $N = Z$  nuclei in the case isospin symmetry breaking is assumed to be negligible only on  $\sigma_\mu^M$  and not on the other operators since, then, the relation will not be trivial. In such a case, the relation is useful to understand the non-charge exchange isovector giant monopole resonance (IVGMR), as it is associated with the operator  $\sum_i r_i^2 t_{z,i}$ . In a macroscopic picture, Giant Resonances are collective oscillations of the nucleons forming the atomic nucleus, being the IVGMR a mode of oscillation in which protons and neutrons vibrate out of phase keeping a spherical shape of the atomic nucleus, such that when the protons are expanding, neutrons are compressing [10]. Note that it is possible to use perturbation theory up to second order to transform inequality (9) into an equality, as presented in the Appendix A.

#### B. Uncertainty Principle assuming isospin symmetry breaking

If isospin symmetry is broken  $\mathbf{T}^2$  does not commute with  $\mathcal{H}$ . In general, however, we can expand the ground state wave function  $|0\rangle$  in a bases of good isospin  $|T, T_z\rangle$  as

$$|0\rangle = \sum_i c_i |T + i, T\rangle \quad (10)$$

where for simplicity of notation we use  $T_z \equiv T$  and it has been seen that the dominant terms are  $c_0$  and  $c_1$  [9]. Neglecting all other terms and defining  $c_1 = \epsilon$ , so that  $|0\rangle = \sqrt{1 - \epsilon^2} |T, T\rangle + \epsilon |T + 1, T\rangle$ , one may write,

$$\langle \mathbf{T}^2 - T_z^2 \rangle = 2(T + 1)\epsilon^2 + T \quad (11)$$

and Eq. (6) can be written as

$$2(T + 1)\epsilon^2 + T \geq \frac{(N \langle r_n^2 \rangle - Z \langle r_p^2 \rangle)^2}{2(\sigma_-^M + \sigma_+^M)} \quad (12)$$

Then for  $N = Z$ ,  $T = 0$

$$2\epsilon^2 \geq \frac{N^2(\langle r_n^2 \rangle - \langle r_p^2 \rangle)^2}{2(\sigma_-^M + \sigma_+^M)} \quad (13)$$

### C. Isobaric analogue state

An isobaric analog state (IAS) is a nuclear state with the same total isospin, spin-parity, and structure as a state in another isobar, differing only in the value of  $T_z$ . The operator that excites such a state is  $T_{\pm} = \sum_{i=1}^A t_{\pm,i}$ . That is, it differs only by a factor  $r^2$  with respect to  $M_{\pm}$  defined previously. Therefore, both operators excite the IAS and only  $M_{\pm}$  excite the so called IVGMR at higher energies (cf. Sec. III A).

The isobaric analog state can be defined in terms of the ground state  $|0\rangle$  of the parent nucleus ( $N > Z$ ) as

$$|IAS\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle^{1/2}} \quad (14)$$

It is notable that the excitation energy  $E_{IAS}$  will be different than zero only if the nuclear Hamiltonian breaks isospin symmetry  $[\mathcal{H}, T_-] \neq 0$ . Assuming negligible isospin mixing in the nuclear wave function of  $N > Z$  nuclei, i.e.  $T_+|0\rangle \approx 0$  then using  $E_{IAS} = \langle IAS|\mathcal{H}|IAS\rangle - \langle 0|\mathcal{H}|0\rangle$  we arrive at

$$E_{IAS} \approx \frac{\langle 0|[T_+, [\mathcal{H}, T_-]]|0\rangle}{N - Z} \quad (15)$$

The contribution of the IAS to  $\sigma_-^M$  can be evaluated within the same approximation, as

$$\begin{aligned} \sigma_-^M(IAS) &\equiv |\langle IAS|M_-|0\rangle|^2 \\ &= \frac{\langle 0|M_+T_-|0\rangle\langle 0|T_+M_-|0\rangle}{\langle 0|T_+T_-|0\rangle} \\ &\approx \frac{\langle 0|[M_+, T_-]|0\rangle\langle 0|[T_+, M_-]|0\rangle}{\langle 0|[T_+, T_-]|0\rangle} \\ &= \frac{(N\langle r_n^2\rangle - Z\langle r_p^2\rangle)^2}{N - Z} \end{aligned} \quad (16)$$

Within the same approximation, it can be easily seen that  $\sigma_+^M(IAS) \approx 0$ . Comparing these results with,

$$\begin{aligned} \sigma_-^M - \sigma_+^M &= \langle 0|[M_+, M_-]|0\rangle \\ &= N\langle r_n^4\rangle - Z\langle r_p^4\rangle, \end{aligned} \quad (17)$$

one realizes that the IAS contribution  $[\sigma_-^M(IAS)]$  to  $\sigma_-^M + \sigma_+^M$  has to be of the same order of magnitude as  $\sigma_-^M + \sigma_+^M$  themselves. This lead us to analyze in the following section the effect of the IAS to Eq. (6) by removing its effect in the operator  $B \equiv M_-$ .

Note that we can rewrite Eq. (6) within these same assumptions as

$$\begin{aligned} \langle T^2 - T_z^2 \rangle &\geq \frac{N - Z}{2} \frac{\sigma_-(IAS)}{\sigma_-^M + \sigma_+^M} \\ \sigma_-^M + \sigma_+^M &\geq \sigma_-(IAS) \end{aligned} \quad (18)$$

This result, although trivial, shows the coherency of our approximations.

### III. EFFECT OF THE ISOBARIC ANALOG STATE

Based on the discussion in the previous section, we now choose  $A = T_-$  and  $\tilde{B} = \sum_i (r_i^2 - \langle r_e^2 \rangle) t_{-,i}$  where the new operator  $\tilde{B}$  subtract the effect of the IAS in  $\sigma_-^M$  and  $\sigma_+^M$  (cf. Sec. III A). The quantity  $\langle r_e^2 \rangle$  corresponds to the mean square radius of the density distribution  $\rho_e \equiv \sum_i \psi_i$  with  $i$  running for single-particle states belonging to the neutron excess, for nuclei with  $N > Z$ ,

$$\langle r_e^2 \rangle = \frac{\int dr r^2 \rho_e(\mathbf{r})}{N - Z}. \quad (19)$$

Remembering the Generalized Heisenberg's Uncertainty Principle given in Eq. (1), we now proceed to derive a new version using operator  $A$  and  $\tilde{B}$  and defining  $\tilde{B}^\dagger \tilde{B} = \tilde{\sigma}_-^M$  and  $\tilde{B} \tilde{B}^\dagger = \tilde{\sigma}_+^M$  in the same way previously done. For the matrix element in the r.h.s. of Eq. (1), we find

$$\begin{aligned} \langle 0|[A^\dagger, B]|0\rangle &= N(\langle r_n^2 \rangle - \langle r_e^2 \rangle) \\ &\quad - Z(\langle r_p^2 \rangle - \langle r_e^2 \rangle) \end{aligned} \quad (20)$$

Which leads to

$$\langle T^2 - T_z^2 \rangle \geq \frac{[N(\langle r_n^2 \rangle - \langle r_e^2 \rangle) - Z(\langle r_p^2 \rangle - \langle r_e^2 \rangle)]^2}{2(\tilde{\sigma}_+^M + \tilde{\sigma}_-^M)} \quad (21)$$

#### A. Numerical Results

We now test these results by using a Fortran program to calculate the different quantities appearing in Eqs. (6) and (21) within the Hartree-Fock plus Random Phase Approximation method (HF+RPA) applied to a Skyrme type nuclear effective interaction [3]. This is an iterative variational method that uses a Slater determinant as a wave function. As an example we will study the tin isotopic chain, from  $^{104}\text{Sn}$  to  $^{132}\text{Sn}$ . Specifically, in Table I the numerical values for  $\sigma_{\pm}^M$ ,  $\tilde{\sigma}_{\pm}^M$  and  $\epsilon^2$  are shown as a function of the mass number  $A$ . First of all, the table shows that the isospin mixing of the nuclear wave function is at the percent level or below and decreases as the neutron excess increases. This trend is expected, as proton and neutron wave functions exhibit the largest overlap in  $N = Z$  nuclei.

In Table II the root of the mean square radii for the neutrons  $\langle r_n^2 \rangle^{1/2}$ , protons  $\langle r_p^2 \rangle^{1/2}$  and the excess neutrons  $\langle r_e^2 \rangle^{1/2}$  are shown for different isotopes of Sn in relation to the mass number of the nucleus.

A	$\frac{\sigma_{\pm}^M}{4\pi}$	$\frac{\sigma_{\pm}^M}{4\pi}$	$\frac{\tilde{\sigma}_{\pm}^M}{4\pi}$	$\frac{\tilde{\sigma}_{\pm}^M}{4\pi}$	$\epsilon^2$ (%)
132	226.42	2665.2	182.14	764.63	0.43167
128	231.78	2303.1	185.65	704.91	0.41127
124	237.95	1958.8	189.77	647.95	0.39577
120	245.03	1632.9	194.66	593.98	0.39313
116	265.73	1279.3	210.45	500.90	0.43611
112	292.75	947.13	231.06	406.50	0.59374
110	304.91	801.38	240.04	368.89	0.70041
106	298.62	600.76	228.97	363.26	1.2382
104	310.43	455.91	237.72	324.29	1.5136

Table I: Numerical values predicted within the HF+RPA using the Skyrme interaction SAMi [5] for  $\sigma_{\pm}^M$  (fm<sup>4</sup>),  $\tilde{\sigma}_{\pm}^M$  (fm<sup>4</sup>) and  $\epsilon^2$  are shown as a function of the mass number A.

A	$\sqrt{\langle r_n^2 \rangle}$	$\sqrt{\langle r_p^2 \rangle}$	$\sqrt{\langle r_e^2 \rangle}$
132	4.8891	4.6783	5.3675
128	4.8331	4.6503	5.3177
124	4.7751	4.6230	5.2629
120	4.7153	4.5963	5.2007
116	4.6471	4.5655	5.1315
112	4.5841	4.5421	5.0738
110	4.5550	4.5316	5.0517
106	4.4737	4.4759	5.1366
104	4.4346	4.4622	5.1113

Table II: Numerical values predicted within the HF+RPA using the Skyrme interaction SAMi [5] for the mean square radii  $\langle r_n^2 \rangle$ ,  $\langle r_p^2 \rangle$  and  $\langle r_e^2 \rangle$  in fm as a function of the mass number A.

Now, remembering that  $\langle \mathbf{T}^2 - T_z^2 \rangle = 2(T+1)\epsilon^2 + T$  and that  $T = \frac{|N-Z|}{2}$  we can test equations (6) and (21), obtaining the results shown at table III. In this table the l.h.s of equations (6) and (21) is compared to their r.h.s to test the validity of our results. Note that we have also represented  $\langle T_z \rangle$  in terms of A, which will become useful to study the isospin impurity component of  $\langle \mathbf{T}^2 - T_z^2 \rangle$ .

A	$\langle \mathbf{T}^2 - T_z^2 \rangle$	$\langle T_z \rangle$	r.h.s Eq. 6	r.h.s Eq. 21
132	16.15	16	10.31	0.13
128	14.12	14	8.31	0.12
124	12.10	12	6.93	0.10
120	10.09	10	5.30	0.084
116	8.08	8	3.78	0.082
112	6.08	6	2.36	0.088
110	5.08	5	1.71	0.091
106	3.10	3	0.628	0.10
104	2.09	2	0.229	0.10

Table III: Comparison of  $\langle \mathbf{T}^2 - T_z^2 \rangle$  and the right hand side of the expressions obtained with Heisenberg's inequality, using the numerical values shown in tables I and II.

As we can see, the results obtained using  $\tilde{B} = \sum_i (r_i^2 - \langle r_e^2 \rangle) t_{-,i}$  when compared to those using  $B = \sum_i r_i^2 t_{-,i}$

imply that the IAS is the main contributor to the r.h.s of the Uncertainty Principles derived here. Since our main aim is to isolate isospin symmetry breaking in nuclei in a Heisenberg-like Uncertainty principle and the l.h.s of Eqs. (6) and (21) contain non isospin symmetry breaking contributions [cf. Eq. (11)], if we evaluate just the isospin symmetry breaking part, i.e. estimated here as  $2(T+1)\epsilon^2$  or columns two minus three in Table III, we see that the result of this subtraction and the last column coincides in good approximation. Hence the r.h.s. of Eq. (21) would be the best experimental proxy to measuring isospin symmetry breaking in nuclei, according to our work.

To better see the transition peaks caused by the operators  $\tilde{B}$  and  $\tilde{B}^\dagger$  and compare them to the peaks caused by  $B \equiv M_-$  and  $B^\dagger \equiv M_+$  we have represented them graphically, obtaining Figs. 1 and 2.

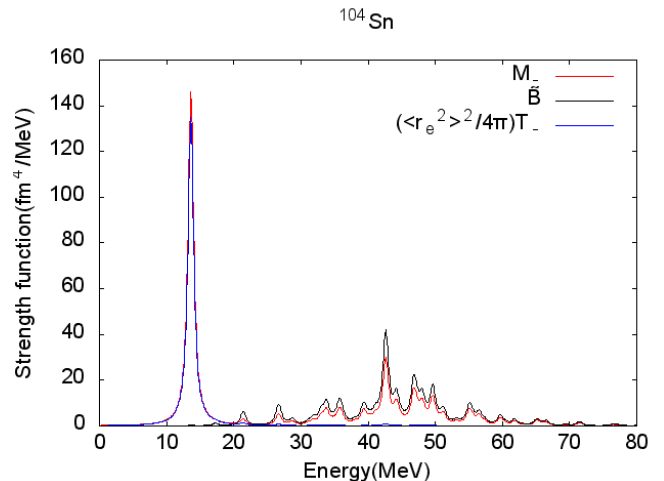


Figure 1: Reduced transition probabilities of the operators  $M_-$  and  $\tilde{B}$  exiting the nucleus to a certain energy, and of  $T_-$ , which excites the IAS, multiplied by a constant for dimensional considerations.

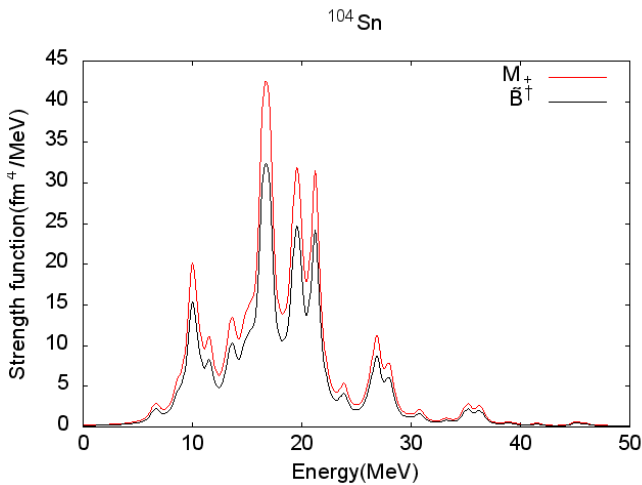


Figure 2: Reduced transition probabilities of the operators  $M_+$  and  $\tilde{B}^\dagger$  exiting the nucleus to a certain energy.

We see that the operator  $\tilde{B}_-$  has removed the IAS peak and that the IAS has a small effect on the  $T_+$  channel of the IVGMR ( $B^\dagger$ ), shown in Fig. 2. The IAS corresponds to the largest and first peak of the  $B \equiv M_-$  spectrum. We expected that the results using  $\tilde{B}^\dagger$  would not change much as compared to those using  $B^\dagger \equiv M_+$  since  $|IAS\rangle = (1/\sqrt{N-Z})T_-|0\rangle$  and thus  $M_+$  doesn't access the IAS in  $N > Z$  nuclei.

We also test Eq. (9) obtained for  $N = Z$  nuclei as shown in table IV, which represents  $\sigma_z^M$ ,  $\sqrt{\langle r_n^2 \rangle}$  and  $\sqrt{\langle r_p^2 \rangle}$  in terms of the nucleus used and compares the r.h.s of Eq. (9) with  $\langle T^2 - T_z^2 \rangle = 2\epsilon^2$ , finding that  $2\epsilon^2 >$  r.h.s Eq. (9) in all cases, as expected.

Nucl.	$2\epsilon^2$ [%]	$\sigma_z^M$ [fm <sup>4</sup> ]	$\sqrt{\langle r_n^2 \rangle}$ [fm]	$\sqrt{\langle r_p^2 \rangle}$ [fm]	r.h.s Eq. (9) [ $\times 10^{-5}$ ]
<sup>16</sup> O	0.13951	295.54	2.625	2.6484	5.1522
<sup>40</sup> Ca	1.26298	1351.4	3.3425	3.3896	18.599
<sup>56</sup> Ni	2.3862	2278.5	3.6603	3.7153	25, 253
<sup>100</sup> Sn	8.748	6302.8	4.3523	4.4352	52.624

Table IV: Comparison of  $2\epsilon^2$  and the r.h.s of Eq. (9), using numerical values predicted within the HF+RPA using the Skyrme interaction SAMi [5] for  $\langle r_n^2 \rangle$ ,  $\langle r_p^2 \rangle$  and  $\sigma_z^M$ .

#### IV. CONCLUSION

In conclusion, we have followed Stringari's work [1] to study a lower bound to the breaking of isospin symmetry derived with Heisenberg's generalized uncertainty principle. In the case of  $N = Z$  nuclei, the l.h.s of the inequality directly correlates with the amount of isospin impurities in the nucleus  $\langle T^2 - T_z^2 \rangle \approx 2\epsilon^2$  while for  $N \neq Z$  it amounts to  $\langle T^2 - T_z^2 \rangle \approx 2(T+1)\epsilon^2 + T$ . Regarding the r.h.s of the Uncertainty Principles derived here, we have found that depending on the operator used,  $B$  or  $\tilde{B}$ , one isolates in a very different way effects related to the isospin symmetry breaking in the nuclear medium. We show that using  $\tilde{B}$  we eliminate the contribution of the IAS to the creation of isospin impurities and find that it generates a much smaller lower bound to the breaking of isospin symmetry, as compared to the operator  $B$ . We conclude that the information on isospin symmetry breaking must then be encoded in the IAS.

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## Principi d'Incertesa Generalitzat de Heisenberg aplicat a operadors de trencament de simetria d'isospín a nuclis atòmics.

Author: Àlex Roger Garcia\*

*Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.*

Advisor: Xavier Roca Maza†

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**Resum:** Utilitzant l'article *Heisenberg Uncertainty Inequality and Breaking of Isospin Symmetry in Atomic Nuclei* [1] de Sandro Stringari, s'ha fet servir la desigualtat de Heisenberg generalitzada per trobar un límit inferior al trencament de simetria d'isospí. Per fer això, fem servir l'operador de reducció d'isospí i l'operador associat amb els 'isovector giant monopole' (IVGMR) en lloc dels operadors de posició i de moment lineal per establir un nou principi d'incertesa. Com a desenvolupament original per a la nostra tesi, modifiquem l'operador del IVGMR per eliminar la contribució de l'estat anàleg isobàric (IAS). Trobem que, aplicant el mètode de Hartree-Fock a una interacció nuclear efectiva (tipus Skyrme), el IAS aïlla la contribució principal al principi d'incertesa de Heisenberg generalitzat presentat aquí. **Paraules clau:** Principi d'incertesa de Heisenberg, isospí, simetria, mètode de Hartree-Fock.

**ODSs:** Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible (SDGs)

### Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible		16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures			

## Appendix A: Perturbation theory

We test Eq. (9) using perturbation theory and introducing an isospin breaking perturbation, into a system with isospin symmetry, of the type  $V_{1C} = \alpha \sum_j r_j^2 t_{z,j}$ . The latter represents the largest isospin symmetry breaking term of the Coulomb interaction [9]. If we assume that the operator  $\sum_j r_j^2 t_{z,j} = M_z$  only excites a single state  $T = 1$  with excitation energy  $E_M - E_0$  we should find that  $\langle \mathbf{T}^2 \rangle = 2\alpha^2 \sigma_z^M / (E_M - E_0)^2$  and  $N(\langle r_n^2 \rangle - \langle r_p^2 \rangle) = 2\langle \sum_j r_j^2 t_{z,j} \rangle = 4\alpha \sigma_z^M / (E_M - E_0)$ . If we define the ground state  $|0\rangle$  as  $|T, T\rangle$  and the excited state as  $|T+1, T\rangle$ , then, using perturbation theory up to first order

$$|0'\rangle = |T, T\rangle + \alpha \sum_{k>0} \frac{\langle k|M_z|T, T\rangle}{E_0 - E_k^{(0)}} |k\rangle \quad (\text{A1})$$

As only a single state is excited,  $\sum_{k>0} \frac{\langle k|M_z|T, T\rangle}{E_0 - E_k^{(0)}} |k\rangle = \frac{\langle T+1, T|M_z|T, T\rangle}{E_0 - E_M} |T+1, T\rangle$ , noting that  $\sigma_z^M = |\langle T+1, T|M_z|T, T\rangle|^2$ .

$$\begin{aligned} \langle \mathbf{T}^2 \rangle &= \langle T, T|\mathbf{T}^2|T, T\rangle + \\ &+ \frac{\alpha^2}{(E_0 - E_M)^2} \sigma_z^M \langle T+1, T|\mathbf{T}^2|T+1, T\rangle = \\ &= \frac{2\alpha^2 \sigma_z^M}{(E_0 - E_M)^2} \end{aligned} \quad (\text{A2})$$

As  $\langle T, T|\mathbf{T}^2|T, T\rangle = 0$ ,  $\langle T+1, T|\mathbf{T}^2|T+1, T\rangle = 2$  and the crossed terms  $\langle T, T|\mathbf{T}^2|T+1, T\rangle$  and its adjunct are zero because the two states with different  $T$  are orthogonal.

$$\begin{aligned} \langle M_z \rangle &= \langle T, T|M_z|T, T\rangle + \\ &+ \frac{\alpha}{E_0 - E_M} \langle T+1, T|M_z|T, T\rangle \langle T, T|M_z|T+1, T\rangle + \\ &+ \frac{\alpha}{E_0 - E_M} \langle T, T|M_z|T+1, T\rangle \langle T+1, T|M_z|T, T\rangle + \\ &+ \frac{\alpha^2 \sigma_z^M}{(E_0 - E_M)^2} \langle T+1, T|M_z|T+1, T\rangle \end{aligned} \quad (\text{A3})$$

As the only terms that survive are the crossed terms

$$2\langle M_z \rangle = \frac{4\alpha \sigma_z^M}{E_0 - E_M} = N(\langle r_n^2 \rangle - \langle r_p^2 \rangle) \quad (\text{A4})$$

Now we can see that the inequality (15) is transformed into an equality, as

$$\langle \mathbf{T}^2 \rangle = \frac{N^2(\langle r_n^2 \rangle - \langle r_p^2 \rangle)^2}{8\sigma_z^M} \quad (\text{A5})$$