Isospin asymmetry in stable and exotic nuclei

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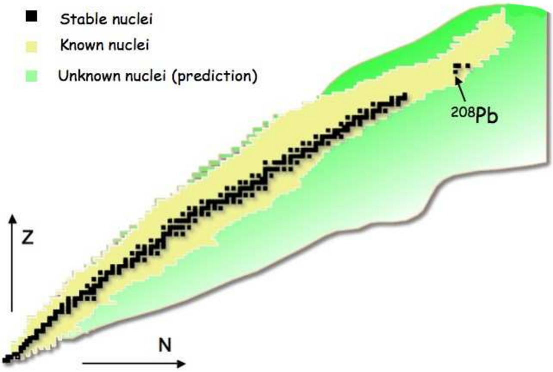
Motivation: Nuclear Chart

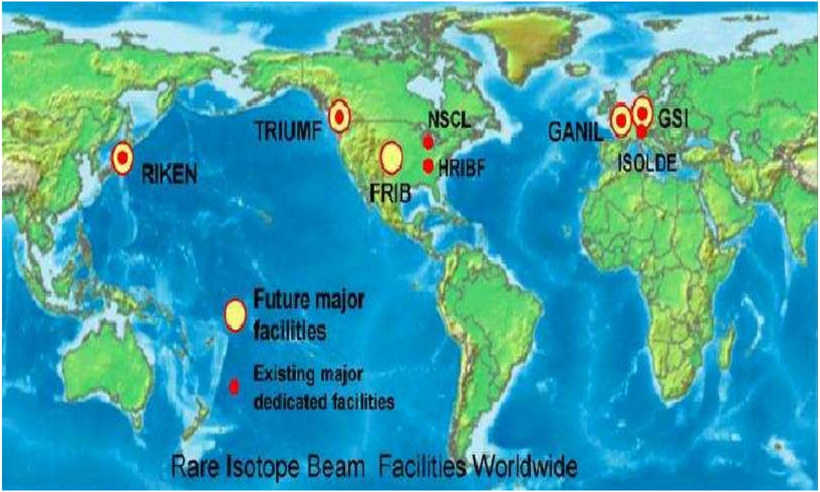


Relative Neutron excess $I \equiv (N - Z)/(N + Z)$

stable nuclei $I \approx 0 - 0.25$

exotic nuclei $I \gtrsim 0.25$

Motivation: Rare Ion Beam Facilities



Thesis works

- Nuclear symmetry energy probed by neutron skin thickness of nuclei. *Phys. Rev. Lett.* **102** (2009) 122502.
- Neutron skin thickness in droplet model with surface width dependence: indications of softness of the nuclear symmetry energy. *Phys. Rev.* C **80** (2009) 024316.
- Impact of the symmetry energy on the outer crust of non-accreting neutron stars. *Phys. Rev.* C **78** (2008) 025807.
- Relativistic Mean Field interaction with density dependent meson-nucleon vertices: DD-MEδ.
- Analysis of bulk and surface contributions in the neutron skin of nuclei. Accepted in *Phys. Rev.* C.
- Influence of the symmetry energy on the giant monopole resonance of neutron-rich nuclei. Accepted in *J. Phys.* G.
- Theoretical study of elastic electron scattering off stable and exotic nuclei *Phys. Rev.* C **78** (2008) 044332.

The Symmetry Energy
and
the Neutron Skin Thickness of nuclei

Definitions

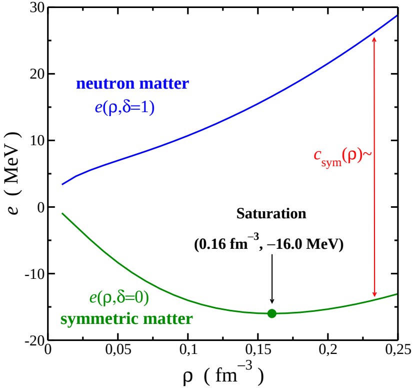
Energy per particle: $e(\rho, \delta)$

The energy per particle (EoS) in asymmetric nuclear matter (infinite system) of total density $\rho = \rho_n + \rho_p$ and asymmetry $\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$ can be written as,

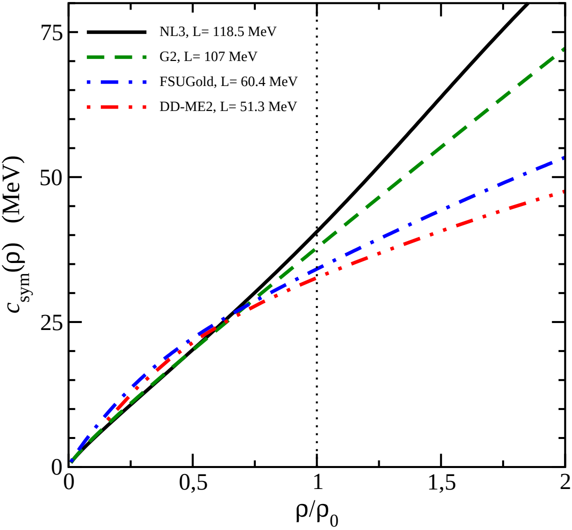
$$e(\rho, \delta) = e(\rho, \delta = 0) + c_{\text{sym}}(\rho)\delta^2 + \ldots$$

where,

$$c_{\text{sym}}(\rho) \equiv \frac{1}{2} \frac{\partial^2 e(\rho, \delta)}{\partial \delta^2} |_{\delta=0}$$



Symmetry energy in Nuclear Models



Symmetry energy around the saturation density

Bulk parameters

The symmetry energy is usually characterized in the literature by the parameters of a Taylor expansion around the saturation density ρ_0,

$$
\begin{aligned}
c_{\rm sym}(\rho) &\approx c_{\rm sym}(\rho_0) + \frac{\partial c_{\rm sym}(\rho)}{\partial \rho}\Big|_{\rho_0} (\rho - \rho_0) + \frac{1}{2}\frac{\partial^2 c_{\rm sym}(\rho)}{\partial \rho^2}\Big|_{\rho_0} (\rho - \rho_0)^2 \\
&\approx J - L\epsilon + \frac{1}{2} K_{\rm sym}\epsilon^2
\end{aligned}
$$

where $\epsilon \equiv \dfrac{\rho_0 - \rho}{3\rho_0}$

With any nuclear interaction one can calculate J, L and $K_{\rm sym}$

The symmetry energy of a finite nucleus

The *semi-empirical mass formula* for the binding energy

It is based on the fact that the baryon density and the binding energy per nucleon are, approximately, the same for all nuclei. In its simplest form,

$$B(N,Z) = a_{\mathrm{v}} A - a_{\mathrm{s}} A^{2/3} - a_{\mathrm{c}} \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

The symmetry energy of a finite nucleus in the Droplet Model

$$a_{\mathrm{sym}}(A) = \frac{J}{1 + \frac{9}{4}\frac{J}{Q} A^{-1/3}}$$

The neutron skin thickness of a nucleus

Definition

$$\Delta R_{np} = \langle {r_n}^2 \rangle^{1/2} - \langle {r_p}^2 \rangle^{1/2}$$

Neutron skin thickness in the Droplet Model

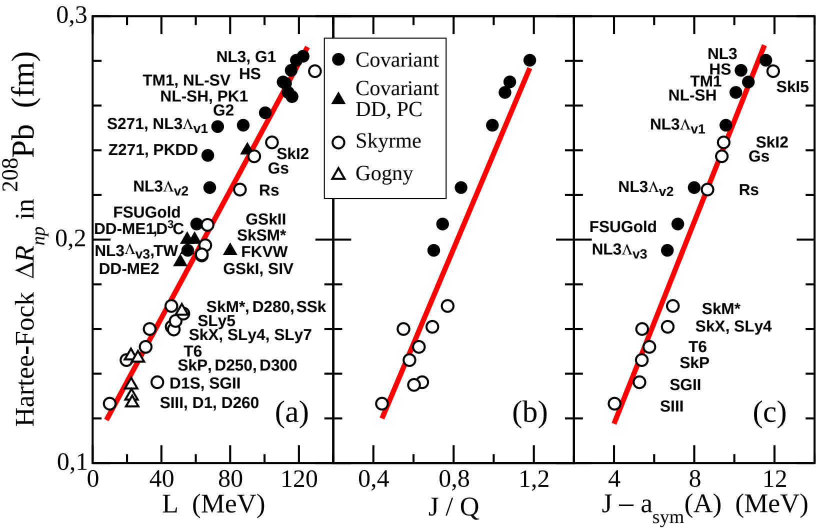
$$\Delta R_{np} = \sqrt{\frac{3}{5}} \left(t - \frac{e^2 Z}{70J} + \frac{2}{5R} \left({b_n}^2 - {b_p}^2 \right) \right)$$

$$t = \frac{3}{2} r_0 \frac{J}{Q} \frac{I - \frac{c_1 Z}{12J} A^{-1/3}}{1 + \frac{9J}{4Q} A^{-1/3}}$$

$$= \frac{2r_0}{3J} \left(J - a_{\mathrm{sym}}(A) \right) A^{1/3} \left(I - \frac{c_1 Z}{12J} A^{-1/3} \right)$$

It is usually assumed that $b_n \approx b_p \approx 1$ fm

ΔR_{np} in ^{208}Pb



$c_{\rm sym}(\rho)$ versus $a_{\rm sym}(A)$

Universal relation in mean-field models

$$c_{\rm sym}(0.1\,{\rm fm}^{-3}) \approx a_{\rm sym}(A = 208)$$

		$A =$	208	$A =$	116	$A =$	40
Model	J	$a_{\rm sym}$	ρ	$a_{\rm sym}$	ρ	$a_{\rm sym}$	ρ
NL3	37.4	25.8	0.103	24.2	0.096	21.1	0.083
NL-SH	36.1	26.0	0.105	24.6	0.099	21.3	0.086
FSUGold	32.6	25.4	0.099	24.2	0.092	21.9	0.078
TF	32.6	24.2	0.094	22.9	0.086	20.3	0.071
SLy4	32.0	25.3	0.100	24.2	0.093	22.0	0.079
SkX	31.1	25.7	0.103	24.8	0.096	22.8	0.084
SkM*	30.0	23.2	0.101	22.0	0.094	19.9	0.079
SIII	28.2	24.1	0.093	23.4	0.088	21.8	0.078
SGII	26.8	21.6	0.104	20.7	0.098	18.9	0.084

$$c_{\rm sym}(\rho_A) = a_{\rm sym}(A)$$

$$\rho_A \simeq \rho_0 - \rho_0/(1 + 0.282A^{1/3}) \text{ for } 40 \lesssim A \lesssim 238$$

The neutron skin thickness and $c_{\rm sym}(\rho)$

summetry properties of the EoS and the ΔR_{np}

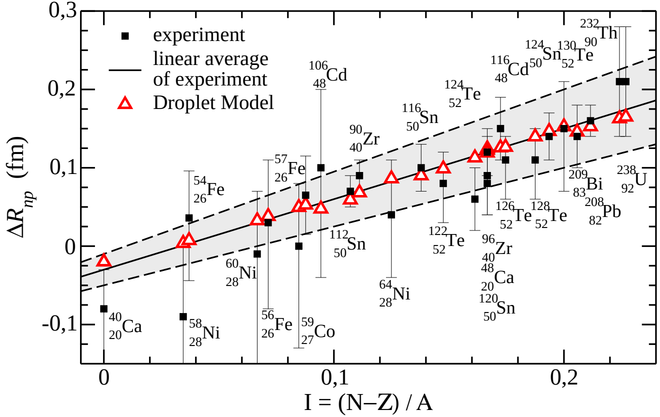
- $c_{\rm sym}(\rho_A) = a_{\rm sym}(A)$
- $c_{\rm sym}(\rho) = J - L\epsilon + \frac{1}{2} K_{\rm sym}\epsilon^2$

ΔR_{np} and the EoS parameters

$$t = \frac{3r_0}{2J} L \left(1 - \frac{K_{\rm sym}}{2L}\epsilon\right) \epsilon A^{1/3} \left(I - \frac{c_1 Z}{12J} A^{-1/3}\right)$$

J is well determined by the experiment ($\approx$ 31.6 MeV) as compared with L and $K_{\rm sym}$.

Experiment: Antiprotonic atoms



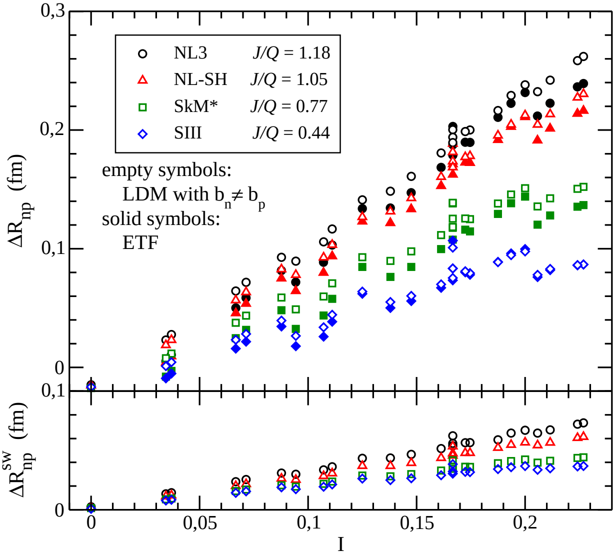
Empirical indications at $\rho < \rho_0$: $c_{\rm sym} = J\left(\frac{\rho}{\rho_0}\right)^{\gamma}$

We find $L = 75 \pm 25$ MeV

Surface width contribution to the neutron skin thickness

- Our prediction for L points towards a relatively soft symmetry energy
- Mean Field models predict different surface widths for the proton and neutron density profiles and, therefore, a surface contribution to the neutron skin thickness of nuclei may be important.

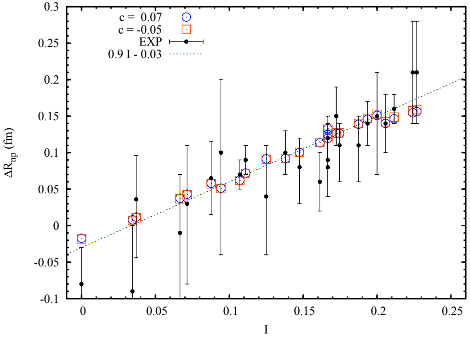
Droplet model with surface width correction



DM inspired ansatz

$$\Delta R_{np} = \sqrt{\frac{3}{5}}\left(t - \frac{e^2 Z}{70J}\right) + \left(0.3\frac{J}{Q} + p\right) I$$

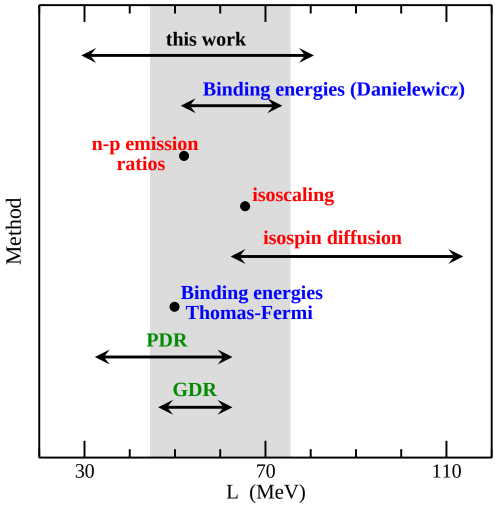
Density content of the symmetry energy



With $\rho_0 = 0.16$ fm^{-3}, $28 < J < 35$ MeV (suggested by MF models) and $-0.05 < p < 0.07$ fm we found

$30 < L < 80$ MeV

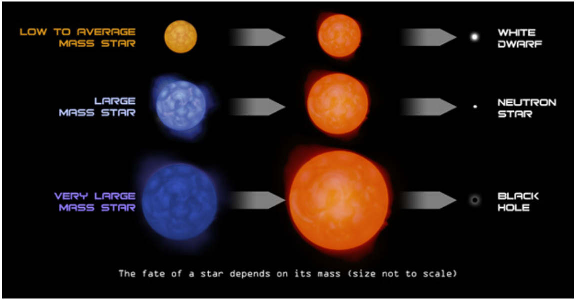
Comparison with other *L* predictions



The compatible range for *L* range from 45 to 75 MeV

The Symmetry Energy and the Outer Crust of a Neutron Star

Introduction



R (Km)	$\bar{\rho}$ (gr/cm^3)	v_{scape}/c	g/g_{Earth} (surface)	P (dyn/cm^2)
10	$10^{14} - 10^{15}$	0.5	10^{11}	$0 - 10^{35}$

*Orientative properties of a typical neutron star of mass $M = M_{\mathrm{Sun}}$.

Formalism

Total energy per nucleon

$$e(A,Z,\rho=\rho_n+\rho_p) = e_N(A,Z) + e_{lat}(A,Z,\rho) + e_{el}(\rho)$$

The different contributions

$$e_N(A,Z) = \frac{M(A,Z)}{A}$$

$$e_{lat}(A,Z,\rho) = -C_{lat}\frac{Z^2}{A^{4/3}p_F}$$

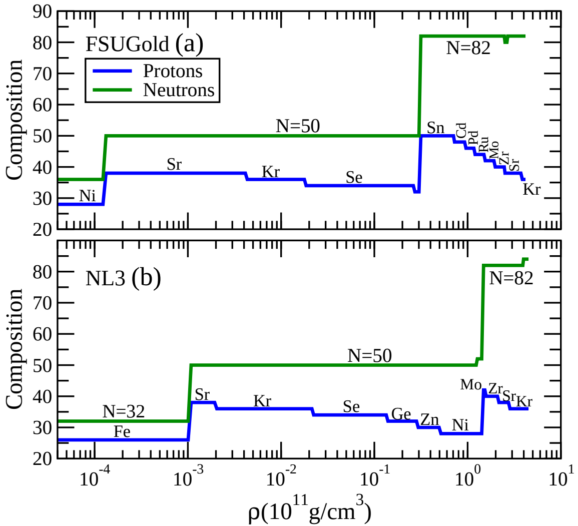
where $C_{lat} = 0.00341$ and

$$p_F = (3\pi^2\rho)^{1/3} = p_{F_{el}}(A/Z)^{1/3}\ (N_{el}=Z)$$

$$e_{el}(\rho) = \frac{m_{el}^4}{8\pi^2\rho}\left(x_F y_F(x_F^2+y_F^2) - \ln(x_F+y_F)\right)$$

where $x_F \equiv p_{F_{el}}$ and $y_F \equiv \frac{\epsilon_{F_{el}}}{m_{el}} = \sqrt{1+x_F^2}$

Results: Composition of the outer crust



ΔR_{np} and the outer crust

$$\Delta R_{np}^{\mathrm{NL3}}(^{208}\mathrm{Pb}) = 0.28 \text{ fm and } \Delta R_{np}^{\mathrm{FSUGold}}(^{208}\mathrm{Pb}) = 0.20 \text{ fm}$$

The larger the neutron skin of ^{208}Pb, the more exotic the composition of the outer crust

Relativistic Mean Field interaction with Density Dependent Meson-Nucleon Vertices: DD-MEδ

Relativistic Mean Field Models

Standard Relativistic Mean Field Models

- Interaction: σ, ω and ρ mesons (and the γ)
- Usually fitted to finite nuclei properties (binding energies, charge radii, etc.) and to some properties of the EoS at saturation (E/A, ρ_0, K_0, etc.)
- The coupling constants do not depend on the density.

DDMEδ

- Interaction: σ, ω, ρ and δ mesons (and the γ)
- Fitted to a microscopic EoS obtained from *ab initio* calculations based on a bare nucleon-nucleon potential and, in addition, to finite nuclei properties (binding energies and charge radii).
- The coupling constants depend on the density.

Lagrangian

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{int}$$

$$\begin{aligned}
\mathcal{L}_N &= \bar{\Psi}\left(i\gamma_\mu\partial^\mu - M\right)\Psi \\
\mathcal{L}_M &= \frac{1}{2}\sum_{i=\sigma,\delta}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i - m_i^2\Phi_i^2\right) \\
&\quad -\frac{1}{2}\sum_{j=\omega,\rho,\gamma}\left(\frac{1}{2}F_{\mu\nu}^{(j)}F^{(j)\mu\nu} - m_j^2A_\mu^{(j)}A^{(j)\mu}\right) \\
\mathcal{L}_{int} &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma + \bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\
&\quad -\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} - \bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\boldsymbol{\tau}\Psi A^{(\rho)\mu} \\
&\quad -e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}
\end{aligned}$$

Density dependence of the meson-nucleon vertices

σ and ω

$$\Gamma_i(\rho) = g_i(\rho_0) f_i(x = \frac{\rho}{\rho_0})$$

$$f(x) = a\frac{1+b(x+d)^2}{1+c(x+d)^2}$$

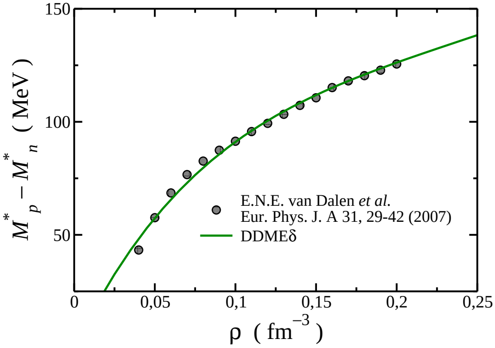
(Same density dependence for σ and ω)

ρ and δ

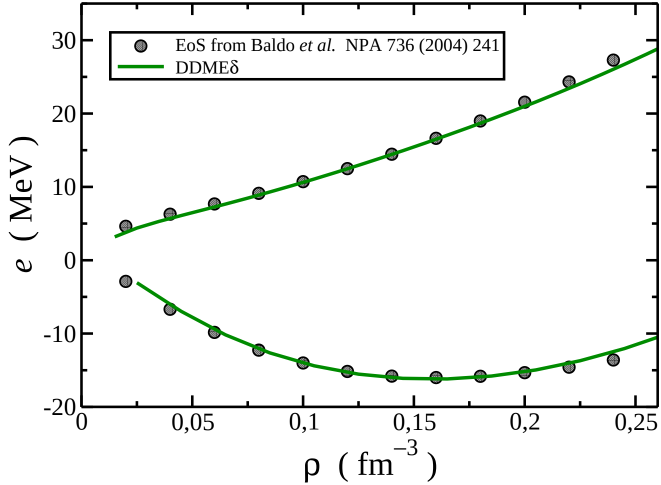
$$\Gamma_\rho(\rho) = g_\rho(\rho_0) e^{-a_\rho(x-1)}$$

$$\Gamma_\delta(\rho) = g_\rho(\rho_0) \times$$

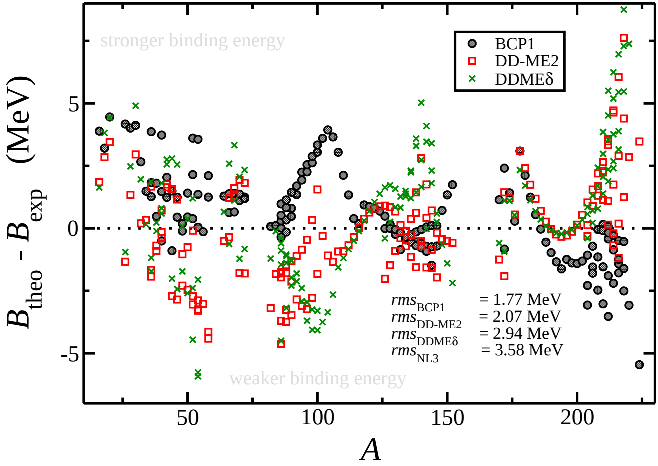
$$\times \left[e^{-a_\delta(x-1)} + b_\delta(x-1)\right]$$



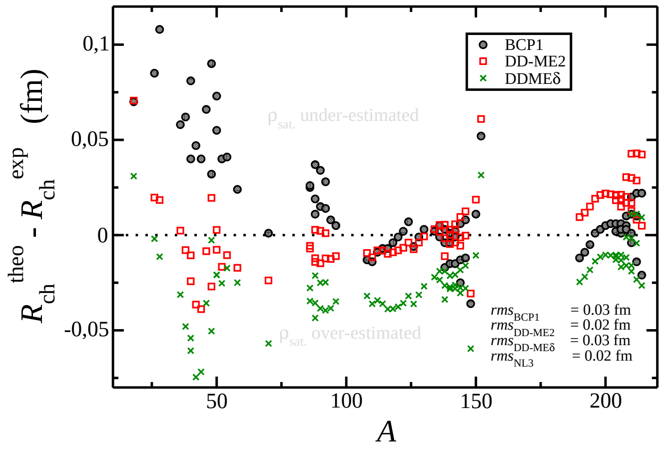
Results: EoS


EoS from Baldo *et al.* NPA 736 (2004) 241
DDMEδ
30
20
10
0
-10
-20
0
0,05
0,1
0,15
0,2
0,25
e (MeV)
ρ (fm^{-3})

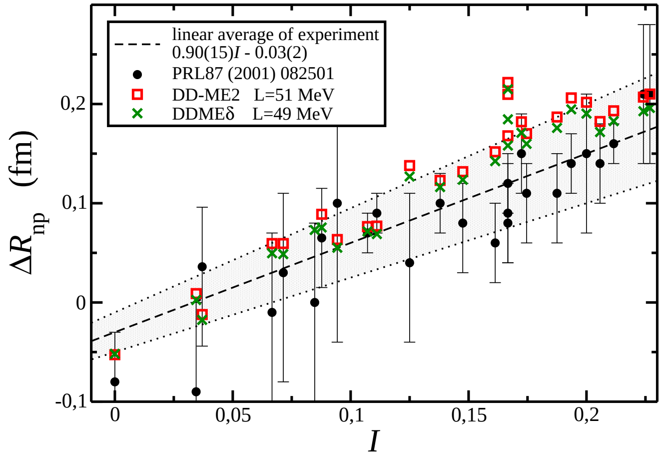
Results: Binding energies



Results: Charge Radii



Results: Neutron Skins



Energy density functional

Achievements of DDMEδ

- Reproduce the EoS of *ab initio* calculations up to $2\rho_0$.
- Reproduce the binding energies and charge radii of spherical nuclei with the same level of accuracy than other RMF models.

But this is a first step...

Perspectives

- Reduce the free parameters to 4 (g_σ, g_ω, g_ρ and m_σ)
- Achieve the same level of accuracy than standard RMF models in describing finite nuclei properties.
- Reproduce the EoS of *ab initio* calculations at higher densities.

Conclusions

Conclusions

- $c_{\mathrm{sym}}(\rho_A) = a_{\mathrm{sym}}(A)$
- $L = 75 \pm 25$ MeV (without surface contributions to ΔR_{np})
- $L = 55 \pm 25$ MeV (with surface contributions to ΔR_{np})
- Compatible range for predictions coming form different observables point towards a soft symetry energy $L = 60 \pm 15$ MeV
- Mean Field models and the analyzed experiment on antiprotonic atoms indicate the importance of the of the surface correction to the neutron skin thickness of nuclei.
- The stiffer the symmetry energy the more exotic the composition of the outer crust and the larger the neutron skin of medium and heavy elements.
- With the RMF model DDMEδ it is possible to reproduce the microscopic EoS obtained from *ab initio* calculations based on a bare nucleon-nucleon potential and to describe at the same time basic properties of finite nuclei.