The neutron skin thickness in atomic nuclei

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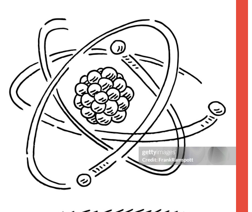
Neutron skin thickness in atomic nuclei

What is it?


Credit: FrankRamspott

The **mean square radius** of the **neutron** (n) or **proton** (p) **density distribution ($\rho_q(\vec{r})$ with $q = n, p$)** is defined as:

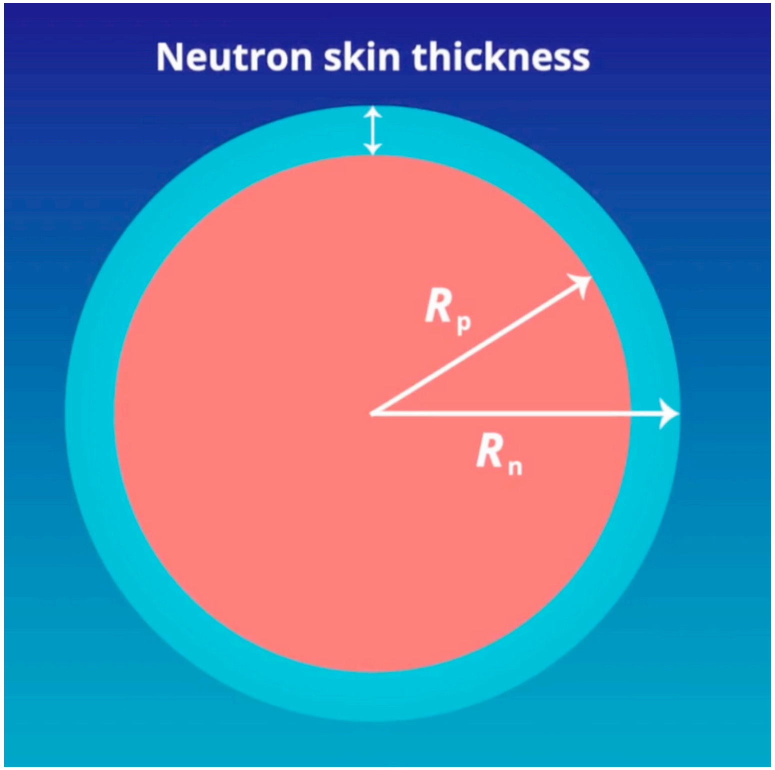
$$\langle r_q^2 \rangle = \int d\vec{r} r^2 \rho_q(\vec{r})$$

<u>**The neutron skin thickness**</u> is defined as the difference between the neutron and proton radii

$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

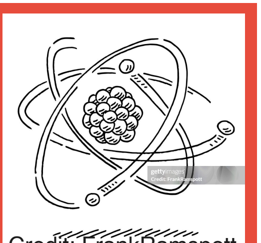
For **atomic nuclei** with $N > Z$ we expect a **neutron skin**, **larger** as **larger the** average repulsion (**pressure**) felt by the neutrons

In **first approximation** (within a Local Density Approximation) **the pressure** that a **neutron uniform system feels** at the densities typical for the atomic nucleus



Neutron skin thickness in atomic nuclei

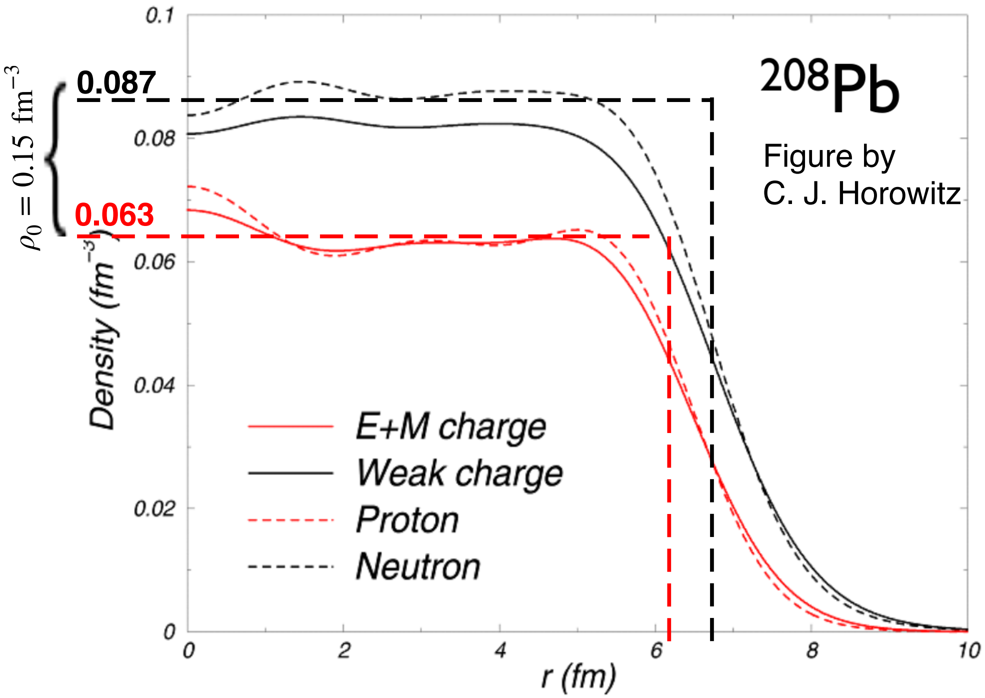
An important physical insight!



Neutron skin thickness is a **proxy to** the pressure (**nuclear force**) of an **ideal uniform system** composed only by neutrons and protons (L)

Droplet Model: $\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$

State-of-the-art microscopic models:



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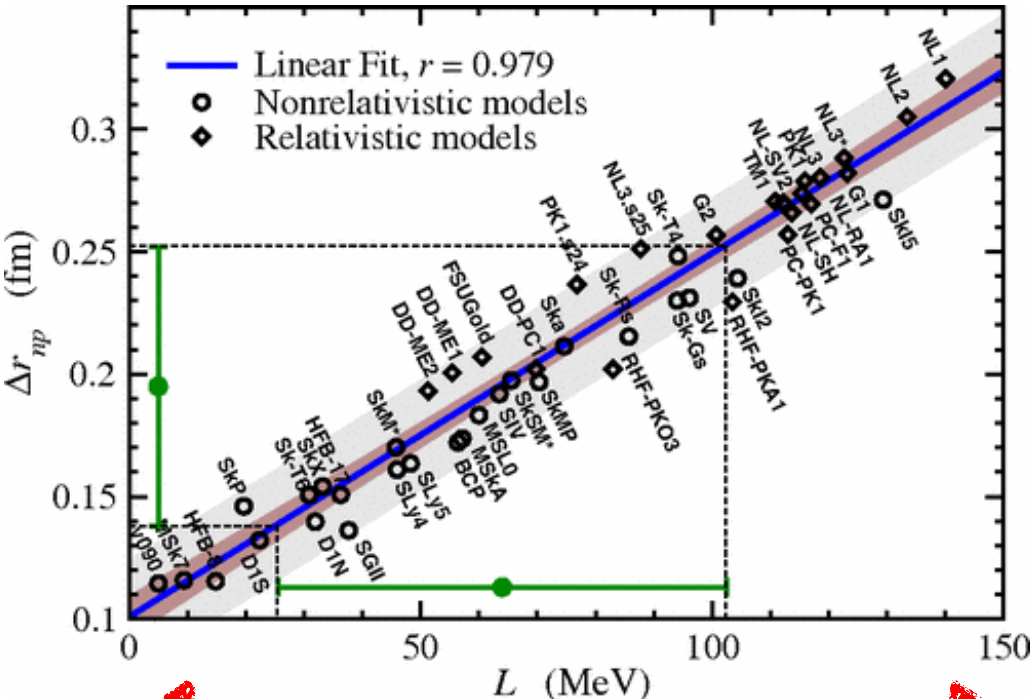


Model predictions are very spread

Neutron skin thickness in atomic nuclei

Why is it important?

Constraints the nuclear equation of state (EoS)

The neutron skin is strongly linked to the density dependence of the energy per particle in a uniform system

[WE WILL DISCUSS THIS IN WHAT FOLLOWS]

Connects nuclei to neutron stars

The same physics governing neutron skins also determines properties of neutron stars related to their size and deformability (quadruple polarizability) as well as how exotic are nuclei present in their crusts.

Tests nuclear structure models

Precise measurements discriminate between competing theoretical models, improving our understanding of nuclear forces.

Impacts reactions and stability of exotic nuclei

Neutron distribution affects reaction cross sections, decay properties, and predictions for very neutron-rich nuclei (relevant for rare-isotope physics).

Informs r-process nucleosynthesis

Better constraints on neutron-rich matter improve models of heavy element formation in astrophysical environments.

Provides electroweak physics tests

Techniques like parity-violating electron scattering probe neutron distributions with minimal strong-interaction uncertainties, offering clean tests of the Standard Model in nuclei.

Neutron skin thickness in atomic nuclei

How it can be calculated (for any nucleus)?

DENSITY FUNCTIONAL THEORY (DFT)

Hohenberg-Kohn theorems

P.Hohenberg, W. Kohn, Phys. Rev. 136, B864 (1964)

→ Assuming a system of **interacting fermions** in a confining **external potential**, there exist a **universal** functional **F[ρ]** of the fermion density **ρ**:

$$E[\rho] = \langle \Psi | T + V + V_{\mathrm{ext}} | \Psi \rangle = F[\rho] + \int V_{\mathrm{ext}}(r)\rho(r) d\vec{r}$$

→ and it can be shown that

$$\min_{\Psi} \langle \Psi | T + V + V_{\mathrm{ext}} | \Psi \rangle = \min_{\rho} E[\rho]$$

so **E[ρ]** has a **minimum** for the **exact ground-state density** where it assumes the **exact energy** as a value.

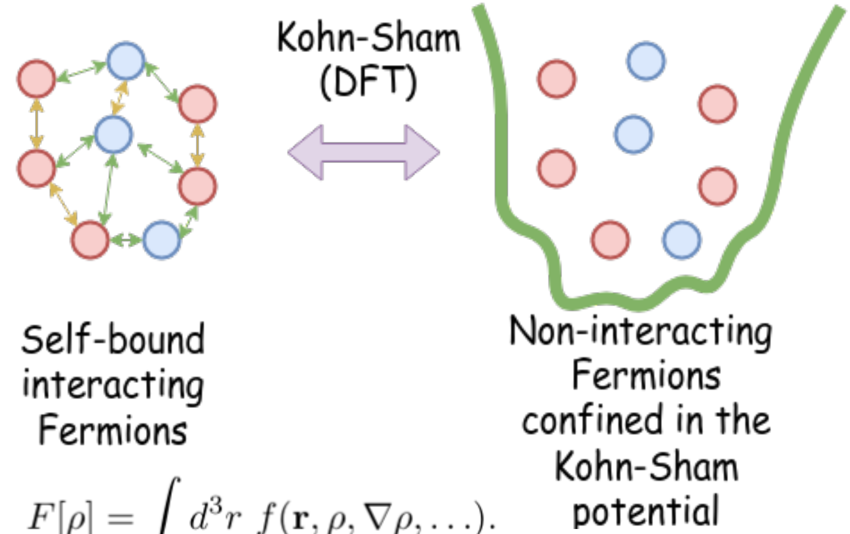
Nuclei are **self-bound** systems $\Rightarrow V_{\mathrm{ext}} = 0$

Kohn-Sham (KS) realization
$F[\rho] \rightarrow T_{\text{non-int.}}[\rho]+V_{KS}[\rho]$
In nuclei no need of external confining potential

For **any interacting system**, there exists a **local single-particle potential V_{ks}(r)**, such that the **exact ground-state** density of the **interacting system** equals the **ground-state density** of the **auxiliary non-interacting system**:

$$\rho_{\text{exact}}(\vec{r}) = \rho_{\text{KS}}(\vec{r}) = \sum_{i=1}^{A} |\phi(\vec{r})|^2$$

where ϕ are single-particle orbitals and the total wave-function correspond to a Slater determinant. The **E[ρ] is unique**



$$F[\rho] = \int d^3r \; f(\mathbf{r}, \rho, \nabla\rho, \ldots).$$

?

$$E[\rho] = T[\rho] + \int V_{\text{KS}}(\vec{r})\rho(\vec{r})d\vec{r}$$

where **T[ρ]** is the **kinetic energy of the non-interacting system** and for which the variational equation

$$0 = \frac{\delta E[\rho]}{\delta\rho} = \frac{\delta T[\rho]}{\delta\rho} + V_{\text{KS}}$$

yields to the **exact ground state density and energy**

Nuclear DFT (in practice)

Exactification of Hartree-Fock

Starting form a Hamiltonian $\mathscr{H}$ or **Lagrangian** $\mathscr{L}$ based on zero-range or finite range effective interactions, one **derives an EDF** by taking the **Hartree-Fock** or **Hartree** expectation value

$$\mathscr{H} = \mathscr{T} + \mathscr{V}$$

$$E[\rho, \vec{\nabla}\rho, \tau, \vec{J}, \ldots] \equiv \langle HF | \mathscr{H} | HF \rangle$$

Example: Skyrme effective interaction [zero-range]

$$\begin{aligned} V(\boldsymbol{r}_1, \boldsymbol{r}_2) = & \; t_0 (1 + x_0 P_\sigma) \delta(\boldsymbol{r}) && \text{central term} \\ & + \frac{1}{2} t_1 (1 + x_1 P_\sigma) \left[\boldsymbol{P}'^2 \delta(\boldsymbol{r}) + \delta(\boldsymbol{r}) \boldsymbol{P}^2 \right] && \\ & + t_2 (1 + x_2 P_\sigma) \boldsymbol{P}' \cdot \delta(\boldsymbol{r}) \boldsymbol{P} && \text{non-local terms} \\ & + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \left[\rho(\boldsymbol{R}) \right]^\sigma \delta(\boldsymbol{r}) && \text{density-dependent term} \\ & + i W_0 \boldsymbol{\sigma} \cdot \left[\boldsymbol{P}' \times \delta(\boldsymbol{r}) \boldsymbol{P} \right] && \text{spin–orbit term .} \end{aligned}$$

[t_i, x_i and W_0 parameters to be adjusted to B, R_{ch}, …]

With this type of **model** one can calculate the properties of **finite nuclei** and of a **uniform ideal system**

Nuclear Equation of State (EoS)

What is it? How is defined?

<u>Definition:</u> the **energy** (E) per **nucleon** (A=N+Z), $e \equiv E/A$, of an **uniform system of neutrons** (N) **and protons** (Z) as a function of the **neutron** (ρ_n = N/V) and **proton** (ρ_p = Z/V) **densities …**

→ **… at <u>zero temperature</u>:** room temperature 10^2K→10^{-8} MeV while "cold" neutron stars are at about 10^{10}K→1 MeV. **Separation energy** in stable nuclei (equivalent to ionization energy in atoms) is of **several MeV**.

→ … **<u>unpolarized</u>:** energy favors **couples** of neutrons and protons **occupying the same state but with opposite spins** (equivalent to electrons in atoms)

→ … **<u>isospin symmetric</u>:** neutron-neutron, proton-proton and neutron-proton **nuclear interaction** are very **similar** among them. **Masses** of neutrons and protons are almost **degenerate**. Hence neutrons and protons can be thought as **two states** of the **same particle** with different isobaric spin or **isospin** (in analogy with spin): the **nucleon.**

→ … and **<u>no Coulomb</u>: idealized uniform system** (focus on strong interaction, much stronger than Coulomb at nuclear scale $\sim 10^{15}$ fm). Real systems are finite and electrically neutral so no divergences in adding Coulomb.

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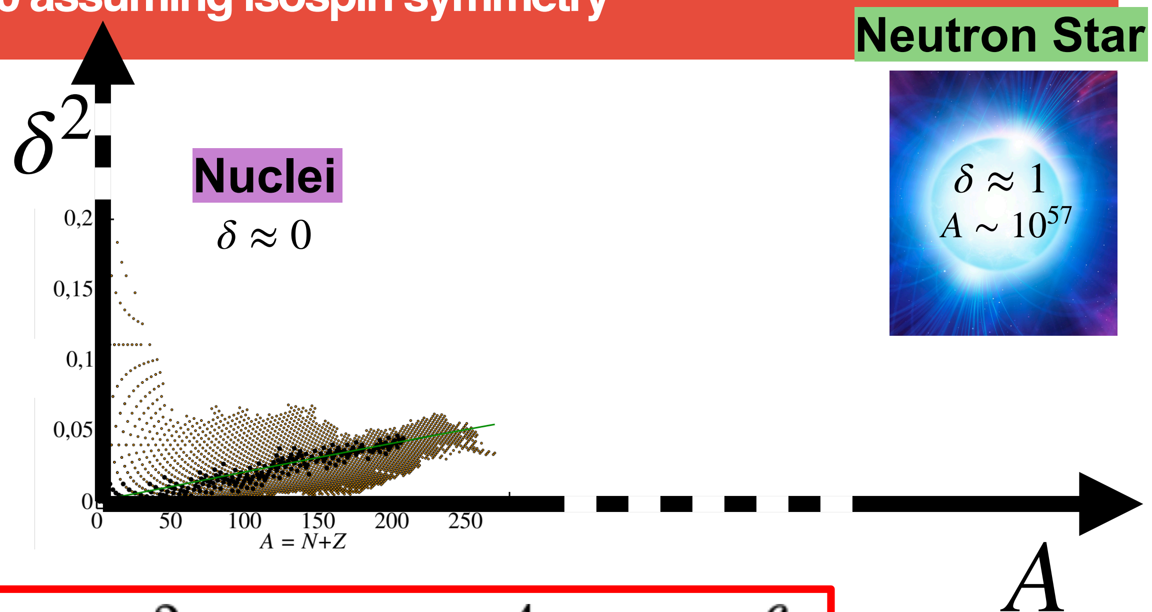
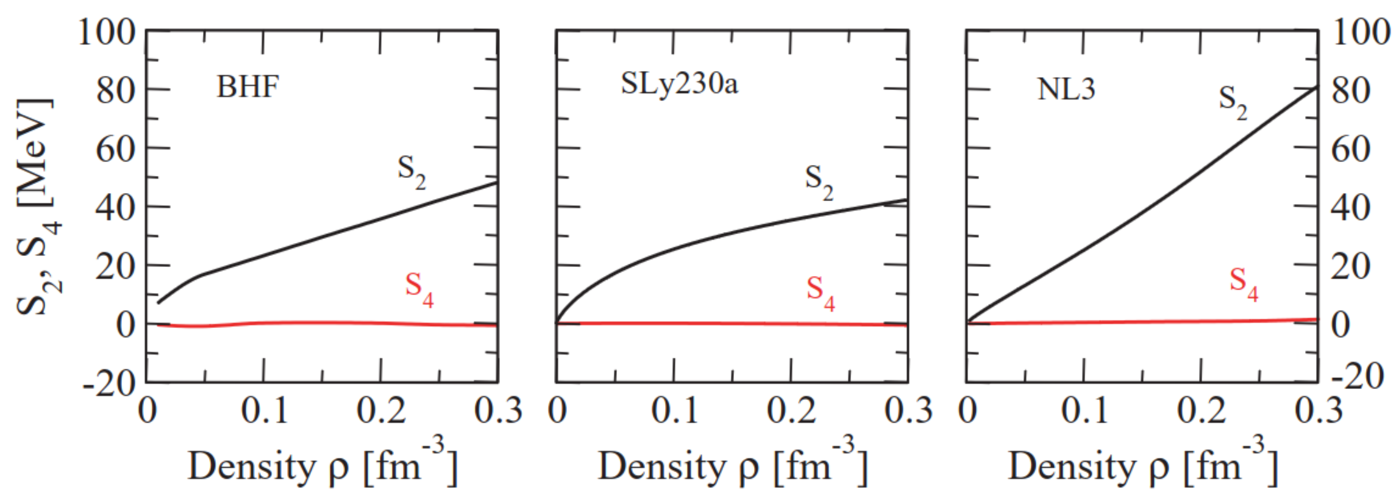
Nuclear Equation of State (EoS)

Unpolarized, uniform nuclear matter at T=0 assuming isospin symmetry

It is convenient to write the energy per nucleon (***e***) as a **function** of the **total density** $\rho=\rho_n+\rho_p$ and the **relative difference** $\delta=(\rho_n-\rho_p)/\rho$ for $\delta \to 0$:



$$e(\rho,\delta) = e(\rho,0) + S_2(\rho)\delta^2 + S_4(\rho)\delta^4 + \mathcal{O}[\delta^6]$$



Isaac Vidaña, Constança Providência, Artur Polls, and Arnau Rios
Phys. Rev. C **80**, 045806 – Published 23 October 2009

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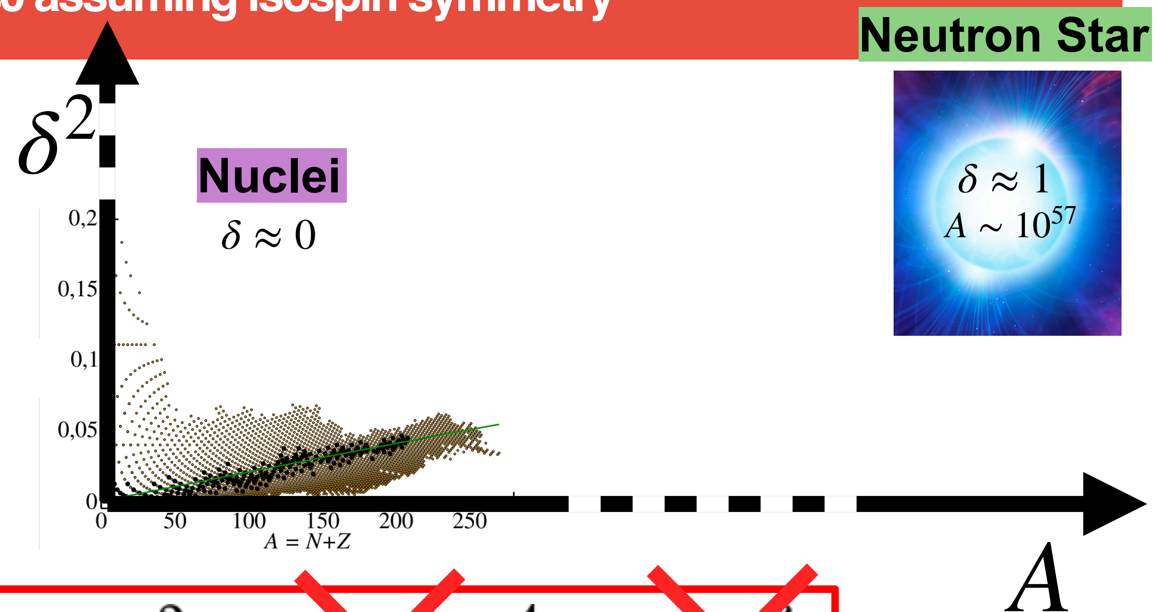

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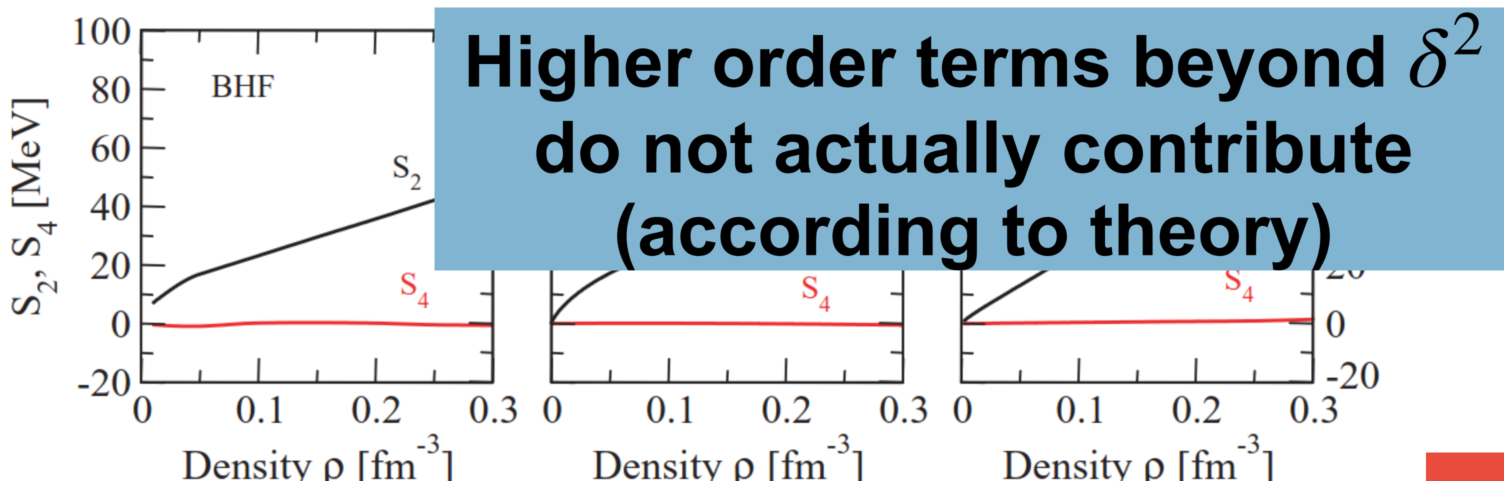

Higher order terms beyond δ^2 do not actually contribute (according to theory)

Isaac Vidaña, Constança Providência, Artur Polls, and Arnau Rios
Phys. Rev. C **80**, 045806 – Published 23 October 2009

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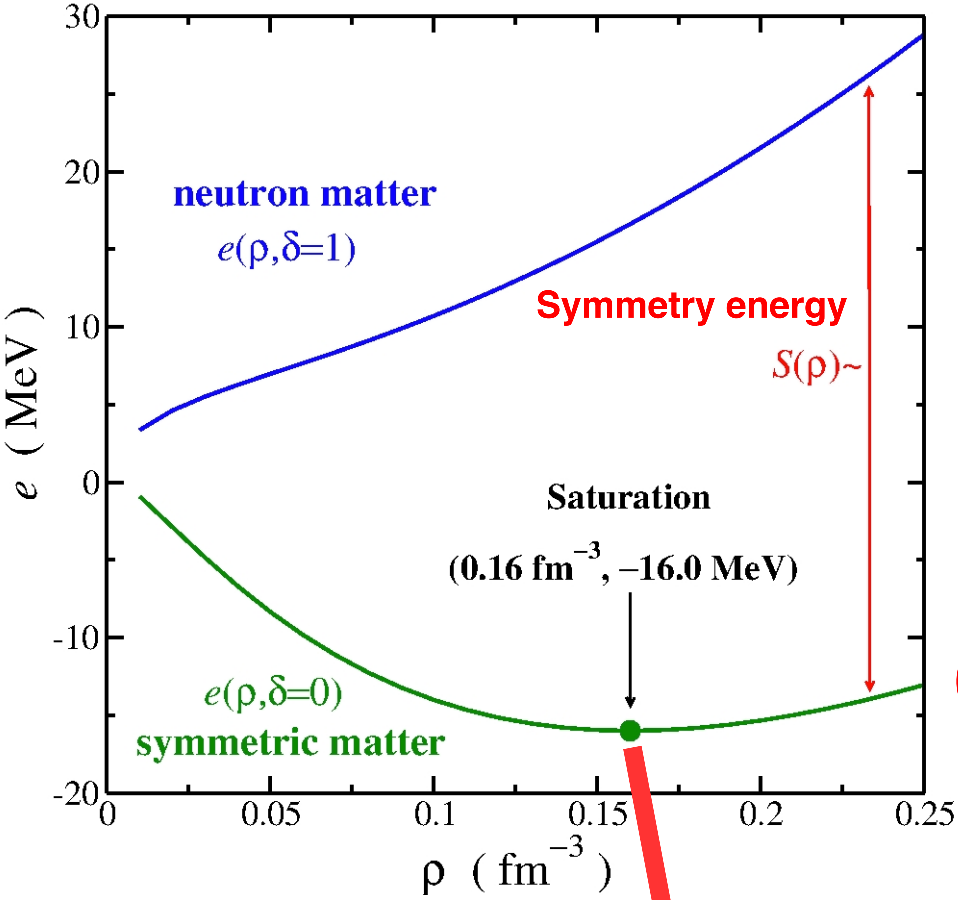
$$e(\rho,\delta) = e(\rho,0) + S_2(\rho)\delta^2$$



It is customary to **Taylor expand** e(ρ,0) and S(ρ) around **nuclear saturation density** ρ_0 ~ 0.16 fm^{-3}

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\rm sym}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

K → how **compressible** is matter @ ρ_0

J → **penalty energy** for systems with $\rho_n \neq \rho_p$ @ $\rho_n + \rho_p = \rho_0$

L → **pressure** for systems with $\rho_n \neq \rho_p$ @ ρ_0

$$P = -\left.\frac{\partial E}{\partial V}\right|_{A=\text{ const.}} = \frac{1}{3}\rho\delta^2 L(\rho)$$

$$P(\rho=\rho_0,\delta=0) = 0 \text{ MeV fm}^{-3}$$

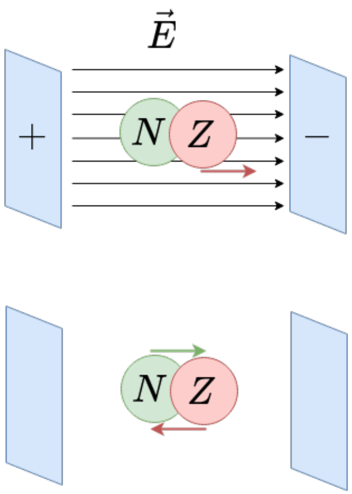
Electric Dipole Polarizability: introduction

The **electric dipole polarizability** measures the **tendency** of the nuclear **charge distribution** to be **distorted**

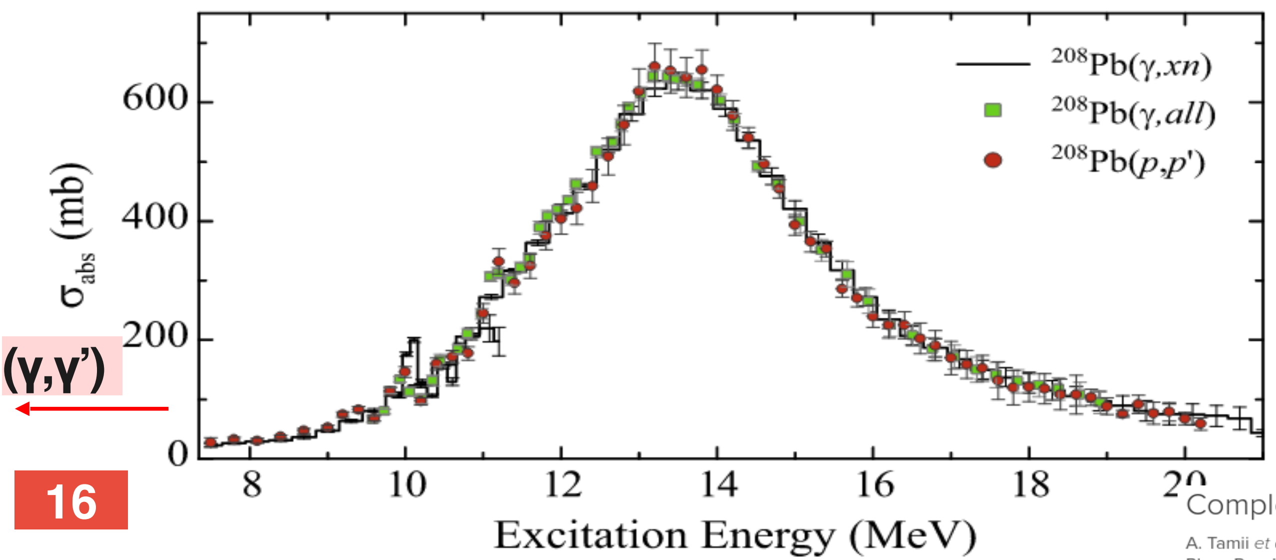
$$\alpha = \frac{\text{electric dipole moment}}{\text{external electric field applied}}$$

Microscopically, it relates with the **photo-absorption cross-section**

$$\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int \frac{\sigma_{\text{abs}}}{\omega^2}\, d\omega,$$



Measured using polarized proton scattering at very forward angles (dominated by E1 and M1 well separated)



$$\alpha_D(^{208}\text{Pb}) = 19.6 \pm 0.6\ \text{fm}^3$$

$$\Delta r_{np} = 0.156^{+0.025}_{-0.021}\ \text{fm}$$

Complete Electric Dipole Response and the Neutron Skin in ^{208}Pb

A. Tamii *et al.*

Phys. Rev. Lett. **107**, 062502 – Published 3 August 2011

Electric Dipole Polarizability: theory

Theoretically, the total **photo-absorption cross section**, can be written as

$$\sigma_{\gamma-\text{abs}} = 4\pi^2\alpha \sum_{\nu}(E_\nu - E_0)|\langle\nu|F_{\text{dipole}}|0\rangle|^2$$

Dipole operator subtract CM motion

$$\frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\hat{r}_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\hat{r}_i)$$

And, thus,

$$\alpha_D = 2\sum_{\nu\neq 0}\frac{|\langle\nu|F_{\text{dipole}}|0\rangle|^2}{E_\nu - E_0} \equiv 2m_{-1}$$

Considering the G.S. perturbed by an **external field** λF (with λ → 0):
$\langle\mathscr{H}\rangle = \langle\mathscr{H}_0 + \lambda F_{\text{dipole}}\rangle$; The variation in the expectation energy can be written as:

$$\delta\langle\mathcal{H}\rangle = \lambda^2\sum_{\nu\neq 0}\frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^3) = \lambda^2 m_{-1} + \mathcal{O}(\lambda^3)$$

$$m_{-1} = \frac{1}{2}\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\lambda^2}\bigg|_{\lambda=0}$$

Dielectric Theorem

Electric Dipole Polarizability: simple model & correlations

Applying the **dielectric theorem** to the **Droplet Model** Hamiltonian (first Migdal and latter on Meyer et al. NPA385, 269) one can find

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1+\frac{5}{2}\frac{\Delta r_{np}+\sqrt{\frac{3}{5}\frac{e^2 Z}{70J}}-\Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$

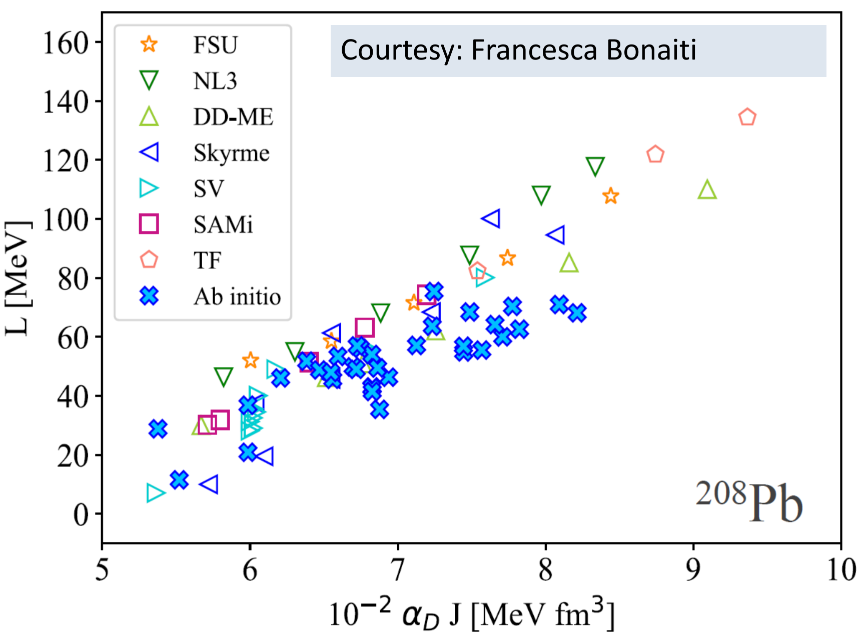
J = e(PNM)−e(SNM) → Symmetry energy at ρ_0

$\Delta r_{np} \equiv \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$ → Neutron skin thickness

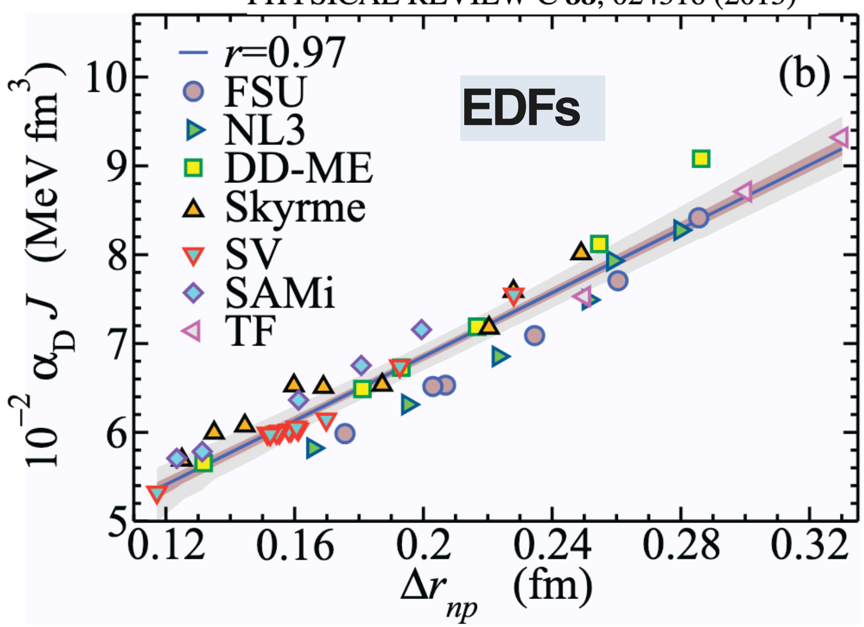
Using microscopic calculations (Energy Density Functionals & Ab initio)

DFT data from: Roca-Maza, Phys. Rev. C **88**, 024316 (2013)

Ab initio data from: Hu, Nat. Phys. **18**, 1196-1200 (2022)

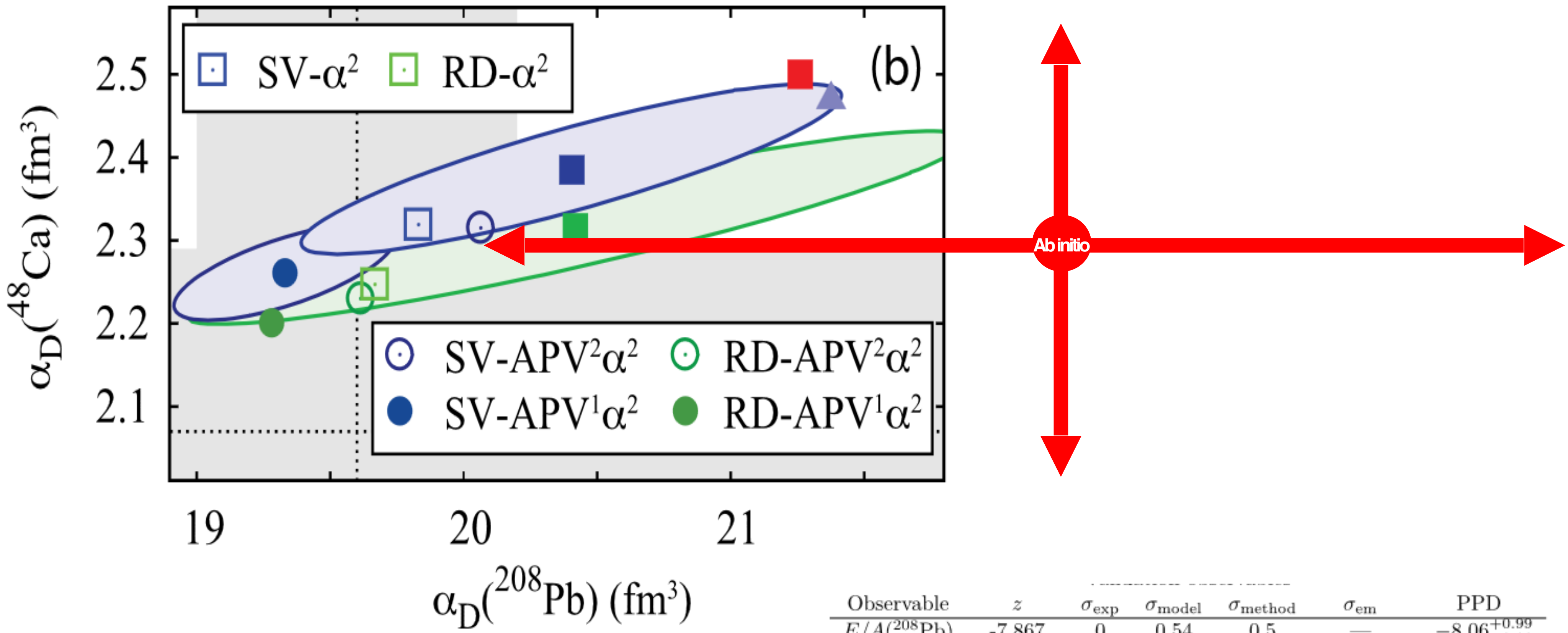


PHYSICAL REVIEW C **88**, 024316 (2013)



$$\alpha_D J = (301 \pm 32) + (1922 \pm 73)\Delta r_{np}$$

Electric Dipole Polarizability in ^{48}Ca and ^{208}Pb



Observable	z	$\sigma_{\rm exp}$	$\sigma_{\rm model}$	$\sigma_{\rm method}$	$\sigma_{\rm em}$	PPD
$E/A(^{208}\mathrm{Pb})$	-7.867	0	0.54	0.5	—	$-8.06^{+0.99}_{-0.88}$
$R_\mathrm{p}(^{208}\mathrm{Pb})$	5.45	0	0.17	0.05	—	$5.43^{+0.21}_{-0.23}$
$\alpha_D(^{48}\mathrm{Ca})$	2.07	0.22	0.06	0.1	—	$2.30^{+0.31}_{-0.26}$
$\alpha_D(^{208}\mathrm{Pb})$	20.1	0.6	0.59	0.8	—	$22.6^{+2.1}_{-1.8}$

Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

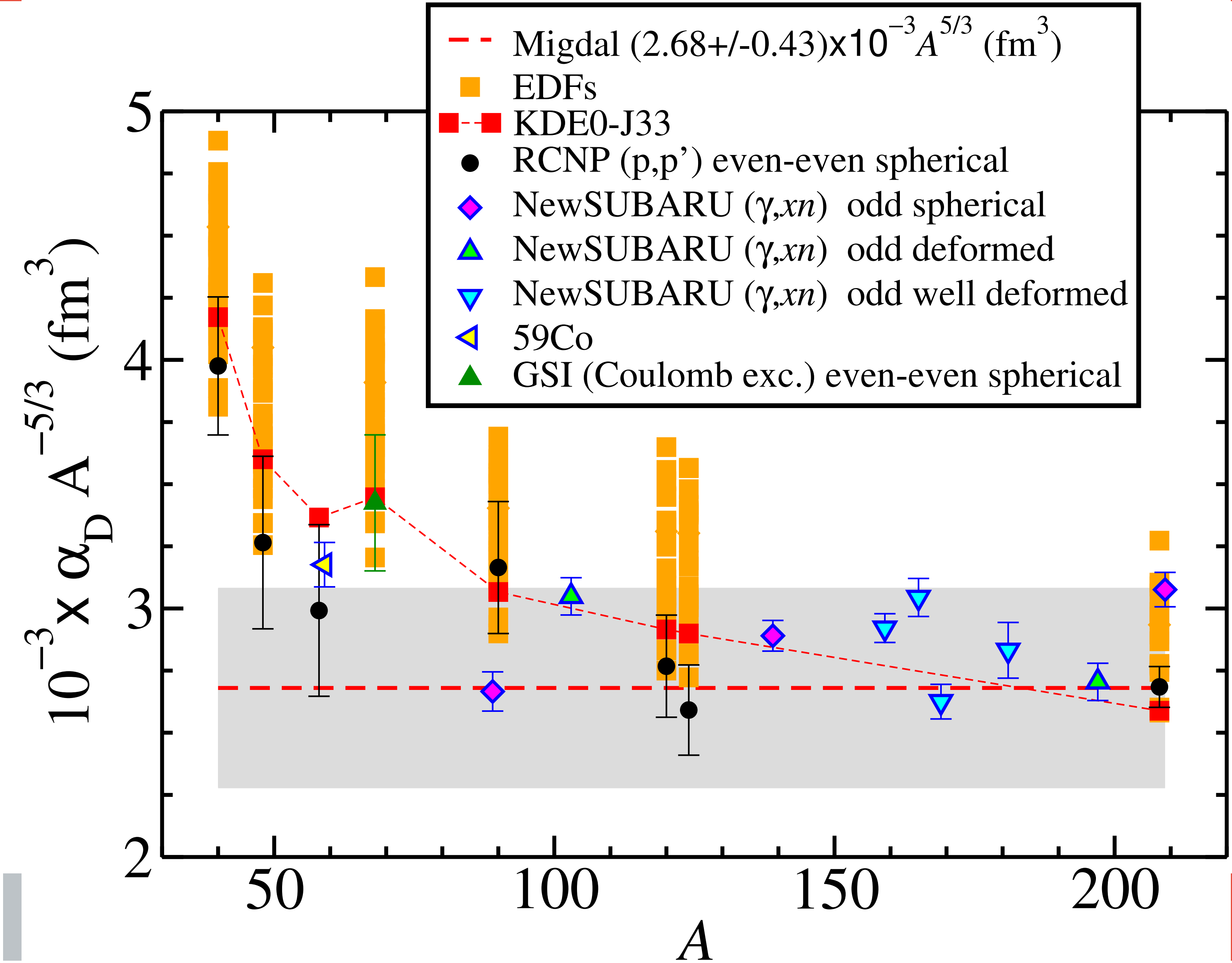
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. **129**, 232501 – Published 2 December 2022

Ab initio predictions link the neutron skin of ^{208}Pb to nuclear forces

Baishan Hu, Weiguang Jiang, Takayuki Miyagi, Zhonghao Sun, Andreas Ekström, Christian Forssén, Gaute Hagen, Jason D. Holt, Thomas Papenbrock, S. Ragnar Stroberg & Ian Vernon

Nature Physics **18**, 1196–1200 (2022) | Cite this article

Electric Dipole Polarizability: How nuclear models perform for other nuclei?



Parity Violating Asymmetry: introduction

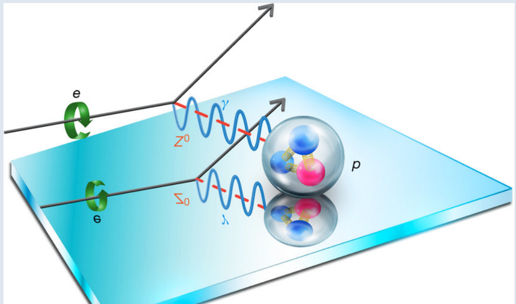
Elastic electron scattering by nuclei:

→ **Parity conserving**

Exchange of **γ** that essentially couples to protons $\mathbf{Q_p^c=1; Q_n^c=0}$

→ **Parity violating**

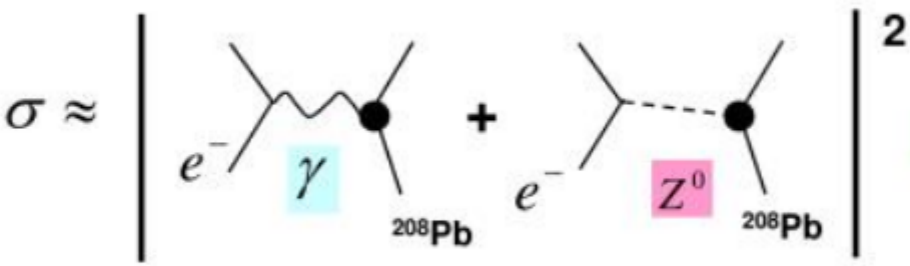
Exchange $\mathbf{Z_0}$ that essentially couples to neutrons $\mathbf{Q_p^W=0.07; Q_n^W=-0.99}$



Nature **557**, 207–211 (2018)

Ultra-relativistic electrons (m_e→0) with spin **aligned** (+) or **anti-aligned** (-) to the beam line, in approaching the nucleus:

$$\left[\vec{\alpha}\cdot\vec{p}+V_{\mathrm{Coulomb}}(\vec{r})\pm V_{\mathrm{Weak}}(\vec{r})\right]\Psi_{\pm}=E\Psi_{\pm}$$



One can get **advantage** form this **interference pattern** between **electromag.** and **weak** interact.

$$A_{PV}=\frac{\frac{d\sigma_+}{d\Omega}-\frac{d\sigma_-}{d\Omega}}{\frac{d\sigma_+}{d\Omega}+\frac{d\sigma_-}{d\Omega}}\approx\frac{\mathrm{Weak}}{\mathrm{Coulomb}}$$

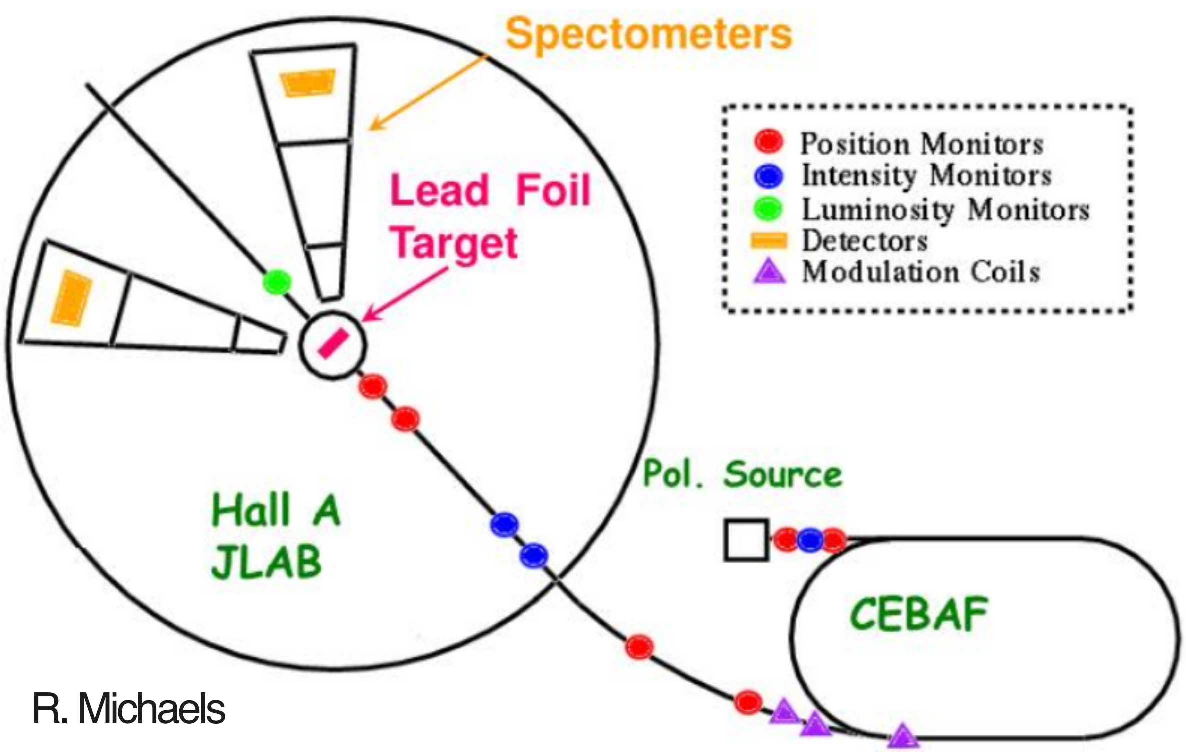
Parity Violating Asymmetry: experiment

Lead and Calcium

Radius Experiments

PREx & CREx @ JLab

$$A_{PV} \sim \frac{G_F}{4\sqrt{2}\pi\alpha} Q^2 \sim 10^{-4} (Q[\mathrm{GeV}])^2$$



R. Michaels

→**Spectrometers ~ 5^0**

→**e^- beam E ~ GeV**

$$Q^2_{\mathrm{PREx}} = 0.00616\ \mathrm{GeV}^2$$

$$Q^2_{\mathrm{CREx}} = 0.0297\ \mathrm{GeV}^2$$

→ **Demanding experiment:**

For **10^6 electrons** or more only **one** of them **interact weakly** with the nucleus

New experiment on ^{208}Pb @ Mainz !!! (MREX)

Accurate Determination of the Neutron Skin Thickness of ^{208}Pb through Parity-Violation in Electron Scattering
D. Adhikari *et al.* (PREX Collaboration)
Phys. Rev. Lett. **126**, 172502 – Published 27 April 2021

Precision Determination of the Neutral Weak Form Factor of ^{48}Ca
D. Adhikari *et al.* (CREX Collaboration)
Phys. Rev. Lett. **129**, 042501 – Published 20 July 2022

$$\Delta r_{np} = 0.283 \pm 0.071\ \mathrm{fm}$$

Parity Violating Asymmetry: theory

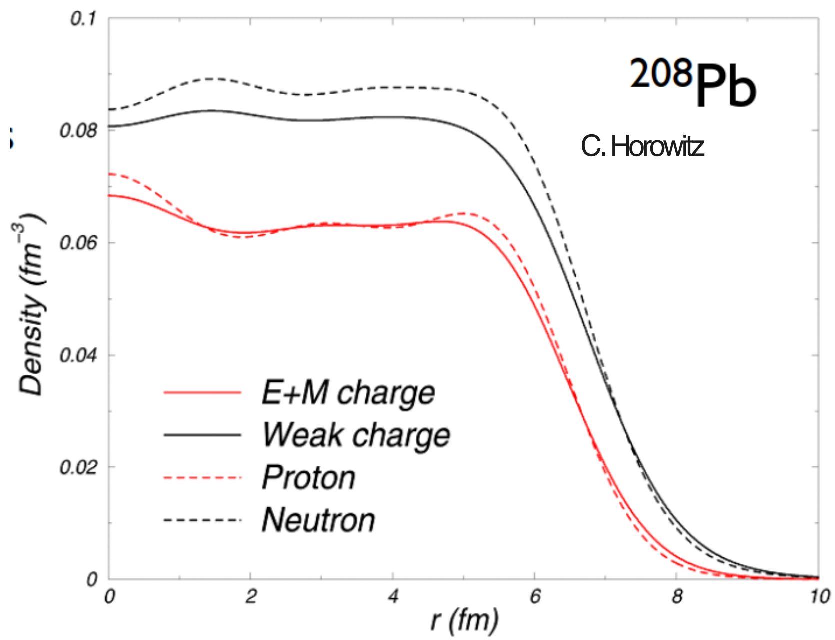
To solve the **Dirac equation**

$$\left[\vec{\alpha}\cdot\vec{p} + V_{\text{Coulomb}}(\vec{r}) \pm V_{\text{Weak}}(\vec{r})\right]\Psi_{\pm} = E\Psi_{\pm}$$

The main inputs are the **electromagnetic ρ_{ch}** and **weak charge ρ_{W}** distributions

$$V_{\text{Coulomb}}(\vec{r}) = Z\alpha \int \frac{\rho_{\text{ch}}(\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{r}' \qquad V_{\text{Weak}}(\vec{r}) = \frac{G_F}{2\sqrt{2}}\rho_{\text{W}}(\vec{r})$$

We need to know the **neutron and proton distributions** in nuclei and the **electromagnetic and weak** charge **form factors** of the **neutron** and the **proton**



Parity Violating Asimmetry: simple model & correlations

The **parity violating asymmetry** within the **PWBA:**

$$A_{PV}^{\rm PWBA} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[\underbrace{4\sin^2\theta_W}_{\approx 1} + \frac{F_n(Q)-F_p(Q)}{F_p(Q)}\right]$$

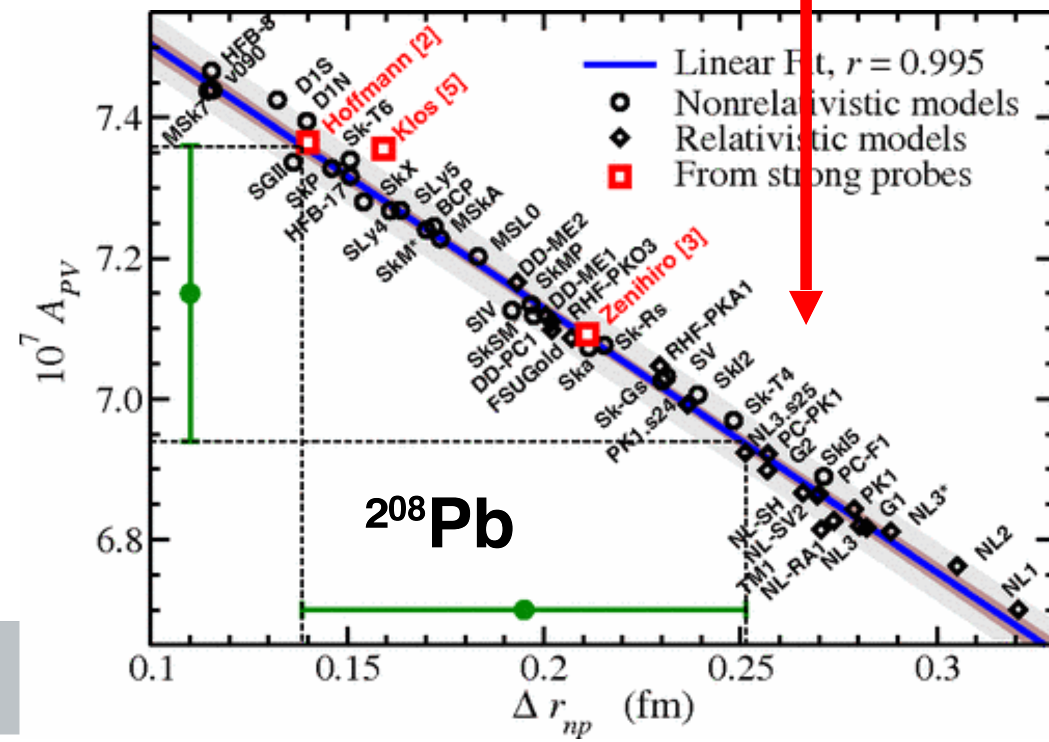
$$\xrightarrow{Q\to 0} \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[1 - \frac{Q^2\langle r_p^2\rangle}{3}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right]$$

30% due to Coulomb distortions (PREX) does not blur **the main physics**

The **parity violating asymmetry** within the **DWBA:**

○ ◇ → Energy Density Functionals

□ → Expeirmental ρ_n form hadronic probes + ρ_p from e$^-$ scattering



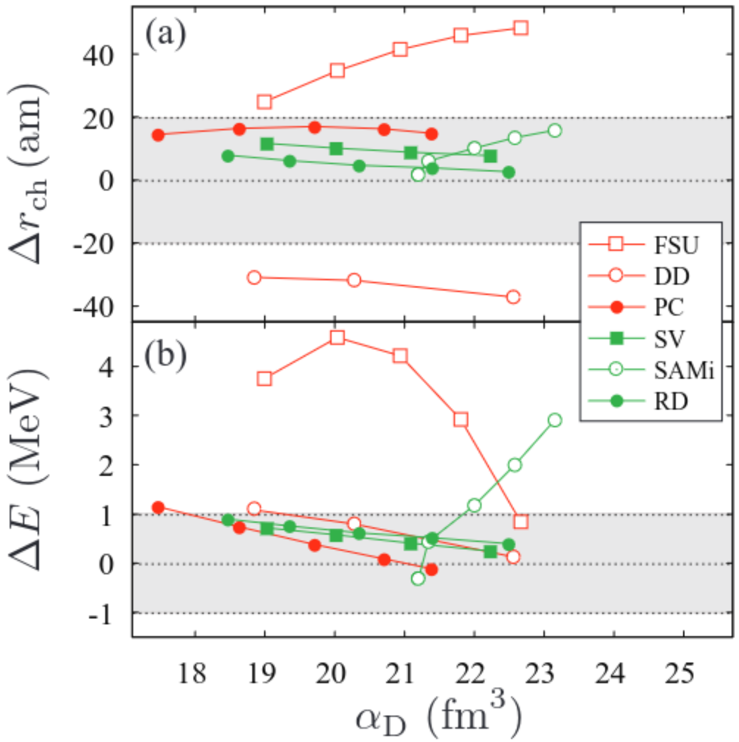
Neutron Skin of ^{208}Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment

X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda
Phys. Rev. Lett. **106**, 252501 – Published 21 June 2011

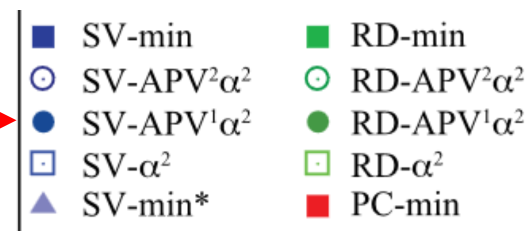
Parity Violating Asymmetry: measurements vs. theory

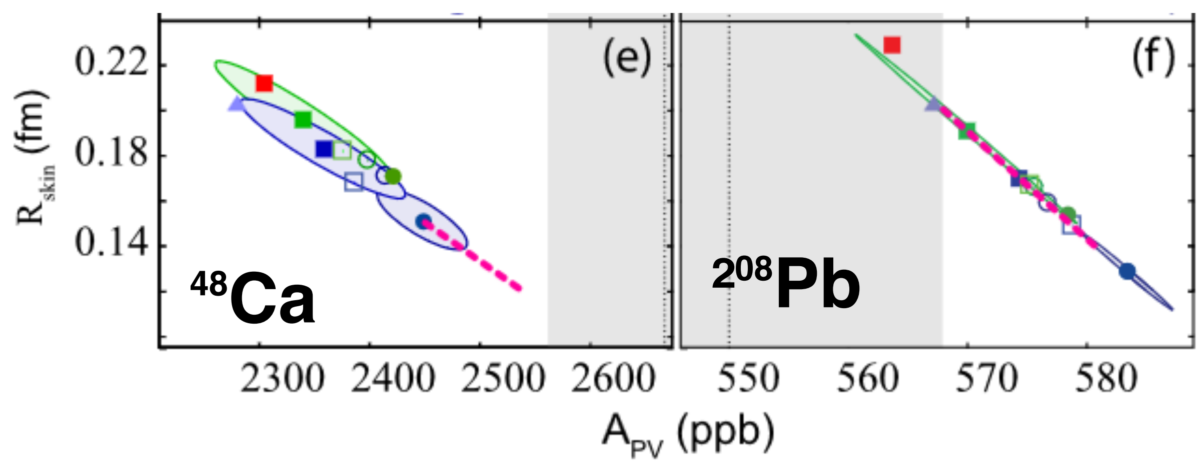
Selecting those **models** (we consider) **well calibrated** to **masses** and **radii**

Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca → **No simultaneous description**



Fitting protocol also including **A$_{PV}$** and **α$_D$**





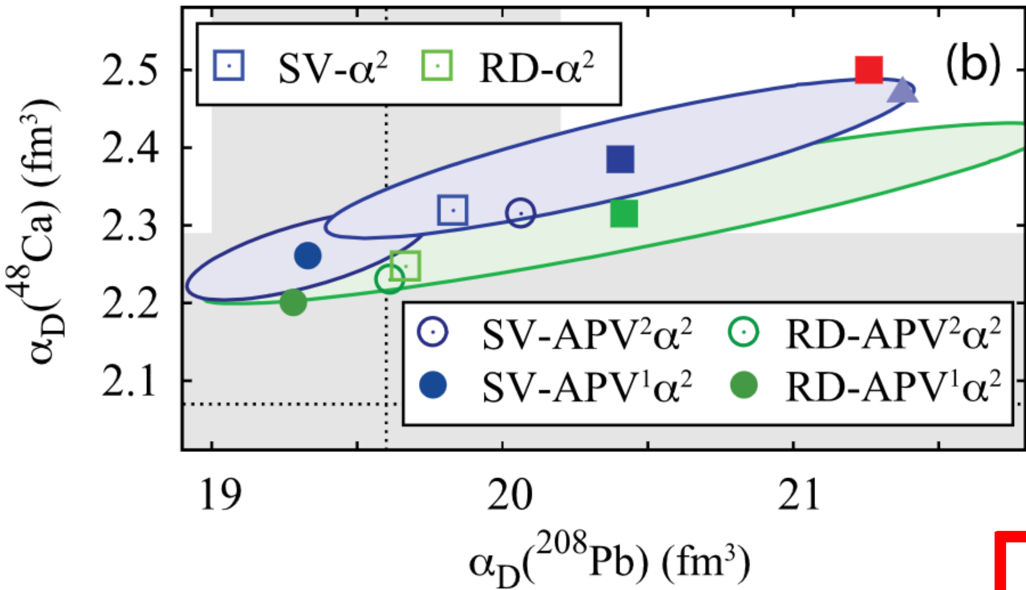
Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. **129**, 232501 – Published 2 December 2022

Magenta dashed lines from extrapolated Δr_{np} given in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Summary: model performance

A_{PV} (sensitive to Δr_{np}) and α_D (sensitive to J and Δr_{np}) in ^{48}Ca and ^{208}Pb



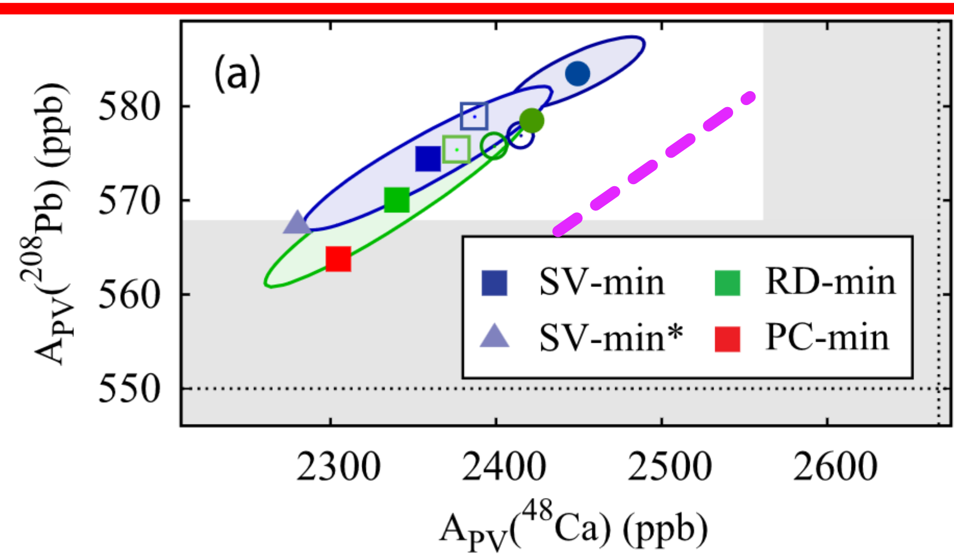
Simultaneous description of **dipole polarizabilities** → point to a **good understanding** of **symmetry energy (J)** and **neutron skins (Δr_{np})**

Ab-initio (B. Hu) Nature Physics (2022)

$\alpha_D(^{48}\mathrm{Ca})$ $2.30^{+0.31}_{-0.26}$

$\alpha_D(^{208}\mathrm{Pb})$ $22.6^{+2.1}_{-1.8}$

No simultaneous description of **parity violating asymmetries** (ground state observable) → point to a **deficient understanding** of **neutron skins**



Magenta dashed lines from extrapolated Δr_{np} given in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

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$A_{\rm PV}$ theoretical corrections?

In all analysis: Coulomb potential has been considered at tree level while $Q_{\rm Weak}$ has been corrected at one-loop level (interference of the γ and Z_0 exchange):

$$[-i\vec{\alpha}\cdot\vec{\nabla}+V_{\rm Coul}\pm V_{\rm Weak}]\Psi_\pm = E\Psi_\pm$$

First order QED includes: vacuum polarization (VP), self-energy and e^- vertex correction (V-SE) [Jakubassa-Amundsen (2024) JPG: Nucl. Part. Phys. 51 035105]

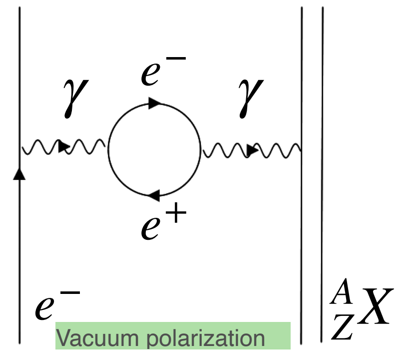
$$[-i\vec{\alpha}\cdot\vec{\nabla}+V_{\rm Coul}\pm V_{\rm Weak}+V_{\rm VP}+V_{\rm V-SE}]\Psi_\pm = E\Psi_\pm$$

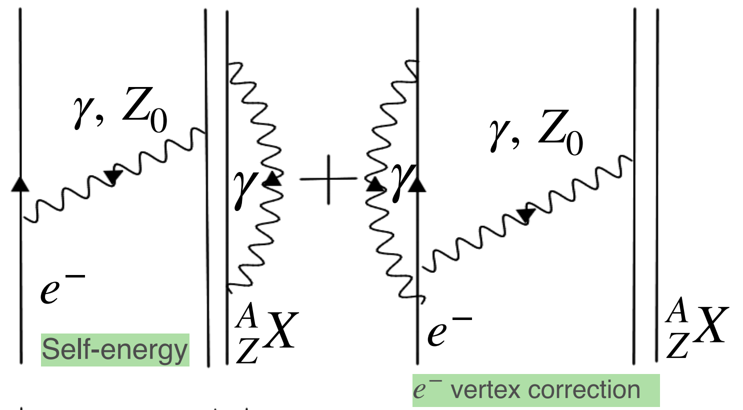
Dispersion corrections: interference of the γ and Z_0 exchange have been shown to be relevant for estimating the $Q_{\rm weak}$.

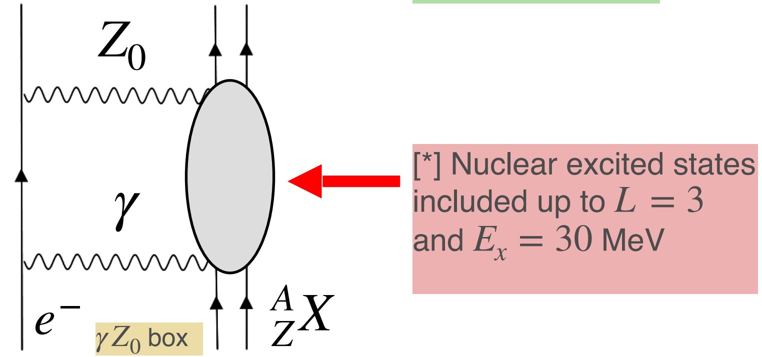
On A_{PV} @ PREx kin., corrections **small & cancel**

- Box diagram -0.1 %
- QED 0.09%

[Jakubassa-Amundsen and XRM, arXiv:2507.15380]
[XRM and Jakubassa-Amundsen PRL 134, 192501 (2025)]



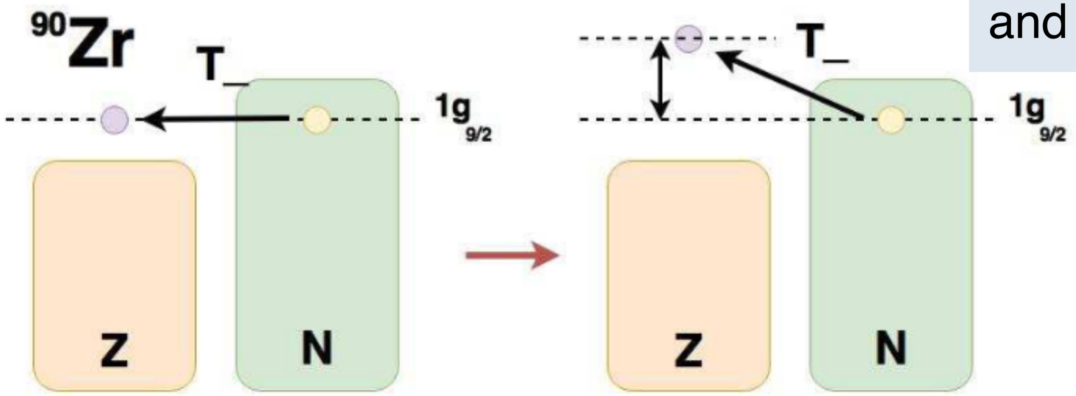




Isobaric Analog State:
Example + theory

$$F = T_{\pm} = \sum_{i}^{A} t_{\pm}(i)$$

Charge-exchange experiments



H = T + Vn

E_{IAS} =0

H = T + Vn+Vc

E_{IAS} ~Ec(Z+1)-Ec(Z)

Isospin algebra analogous to spin algebra s→ t and τ → σ (Pauli matrices with t=τ/2)

$$t_{-}|n\rangle = \frac{1}{2}|p\rangle$$

$$t_{+}|p\rangle = -\frac{1}{2}|n\rangle$$

$$T_{+}^{\dagger} = T_{-} \qquad T_{-}^{\dagger} = T_{+}$$

$$[T_z, T_{\pm}] = \pm T_{\pm}$$

$$[T_{+}, T_{-}] = 2T_z$$

→ non-energy weighted sum rule:

$$m_0^{-} - m_0^{+} = \langle 0|T_{+}T_{-}|0\rangle - \langle 0|T_{-}T_{+}|0\rangle$$
$$= \langle 0|[T_{+}, T_{-}]|0\rangle = \langle 0|2T_z|0\rangle$$
$$= N - Z$$

Note: If not isospin-mixing it would be zero!!

→ energy weighted sum rule:

$$m_1 = \sum_{\nu}(E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \langle 0|T_{+}[\mathcal{H}, T_{-}]|0\rangle$$

$[\mathscr{H}, T\_] \neq 0$ only if $\mathscr{H}$ contains terms that **breaks isospin invariance**

Isobaric Analog State:
Theory

$$F = T_\pm = \sum_i^A t_\pm(i)$$

→ Hence, the **centroid energy** m_1/m_0:

$$E_{\text{IAS}} = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle} = \frac{1}{N-Z}\langle 0|T_+[\mathcal{H}, T_-]|0\rangle$$

→ Assuming a simple model: **indepenent particle** model with only **Coulomb breaking isospin symmetry** (neglect exchange effects)

$$E_{\text{IAS}}^{\text{C,direct}} = \frac{1}{N-Z}\int \left[\rho_n(\vec{r}) - \rho_p(\vec{r})\right] U_{\text{C}}^{\text{direct}}(\vec{r})d\vec{r}\,,$$

$$U_{\text{C}}^{\text{direct}}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{\text{ch}}(\vec{r}_1)d\vec{r}_1$$

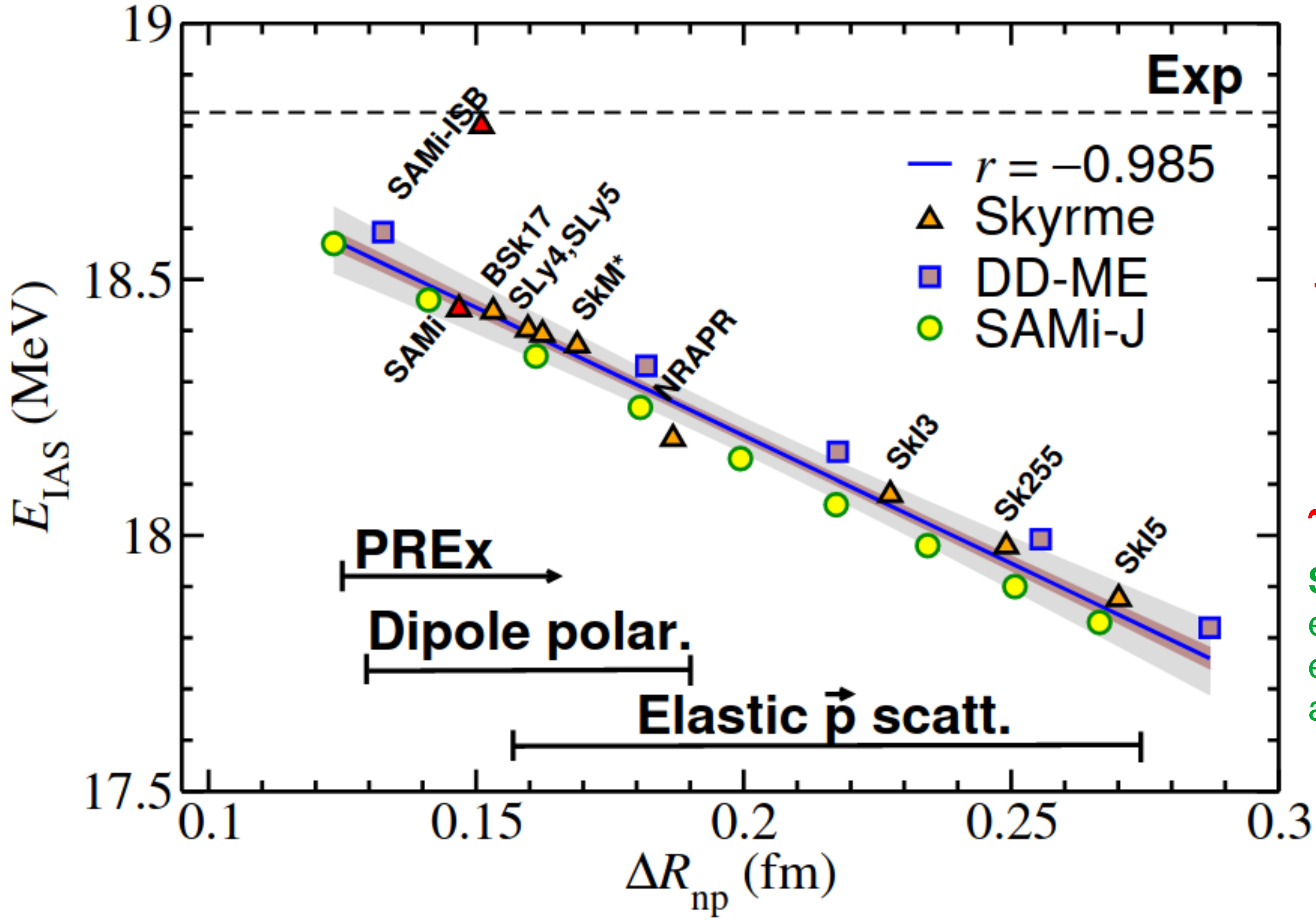
→ Assuming sharp sphere to describe ρ_n and ρ_p and $\rho_{ch} = \rho_p$

$$U_{\text{C}}^{\text{direct}}(\vec{r}) = \begin{cases} \frac{Ze^2}{2R_p}\left(3 - \frac{r^2}{R_p^2}\right) & \text{for } r < R_p \\ \frac{Ze^2}{r} & \text{for } r > R_p \end{cases}$$

$$\begin{aligned} E_{\text{IAS}} &\approx E_{\text{IAS}}^{\text{C,direct}} \\ &\approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \frac{1}{2}\frac{N}{N-Z}\frac{R_n - R_p}{R_p}\right) \\ &\approx \frac{6}{5}\frac{Ze^2}{r_0A^{1/3}}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta R_{\text{np}}}{r_0A^{1/3}}\right) \end{aligned}$$

Isobaric Analog State:

Theory+experiment



Exp errors in IAS ~ tens of keV

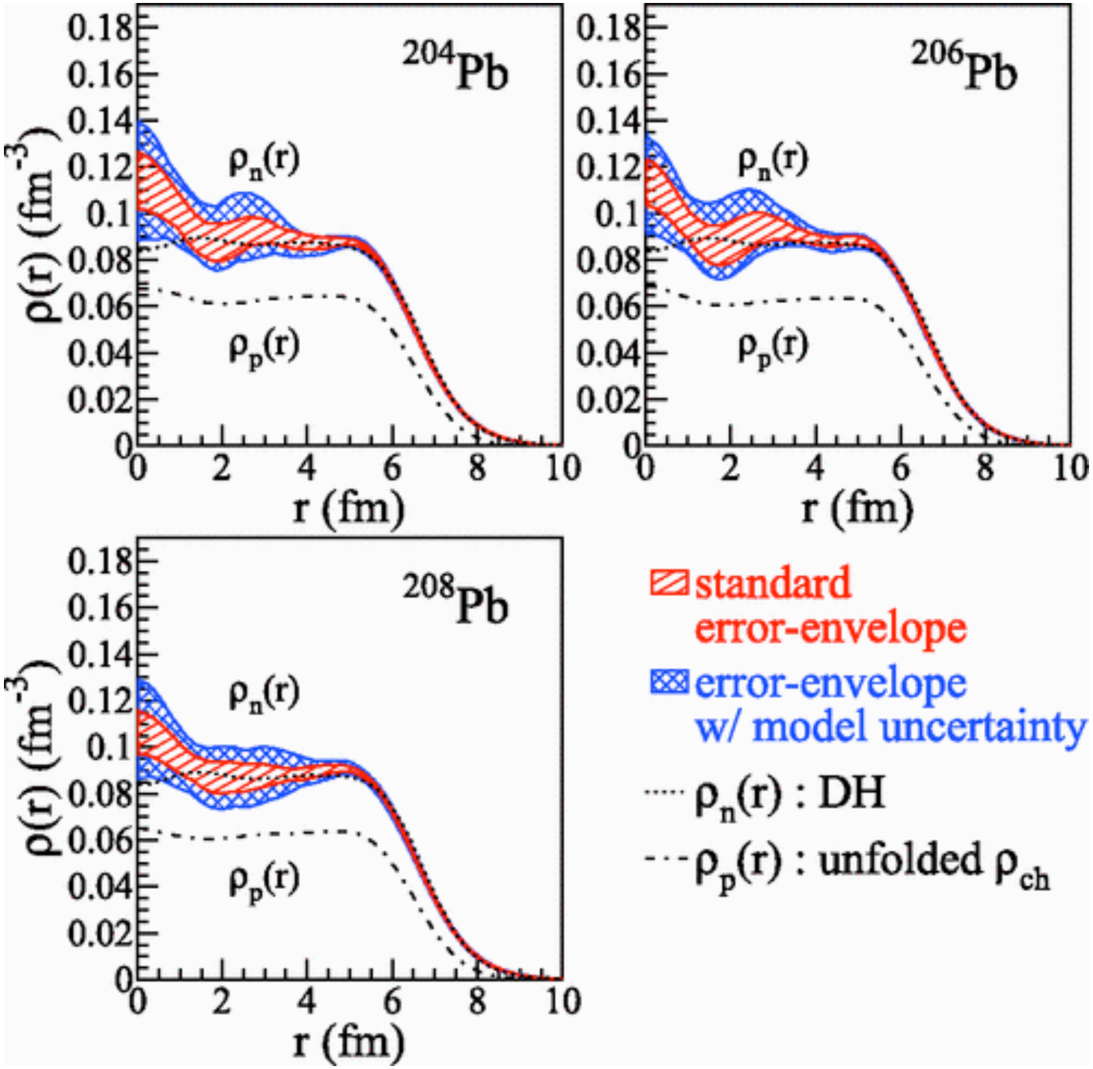
(or smaller)

Exp width IAS

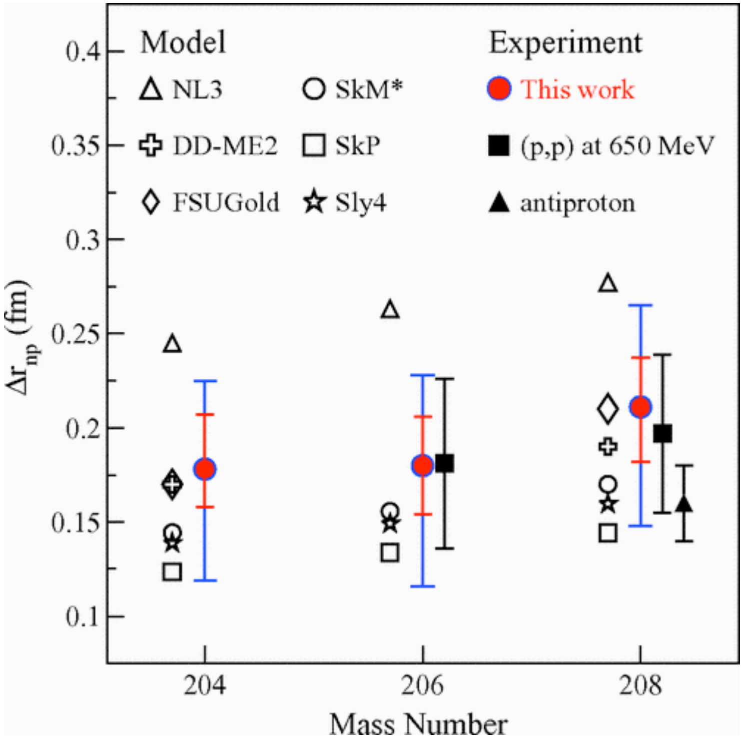
~ hundreds of keV

SAMi-ISB: includes ISB effects due to Coulomb exchange, QED corrections and nuclear ISB

RCNP - proton elastic scattering



Polarized **proton** elastic **scattering** @ 295MeV sensitive to the overall **strong size** of the **nucleus**



$$\Delta r_{\rm np} = 0.211^{+0.054}_{-0.063}\ \text{fm}$$

Neutron density distributions of ^{204,206,208}Pb deduced via proton elastic scattering at $E_p = 295$ MeV

J. Zenihiro[1,*], H. Sakaguchi[1,†], T. Murakami[1], M. Yosoi[1,†], Y. Yasuda[1,†], S. Terashima[1,‡], Y. Iwao[1], H. Takeda[2], M. Itoh[3,§] *et al.*

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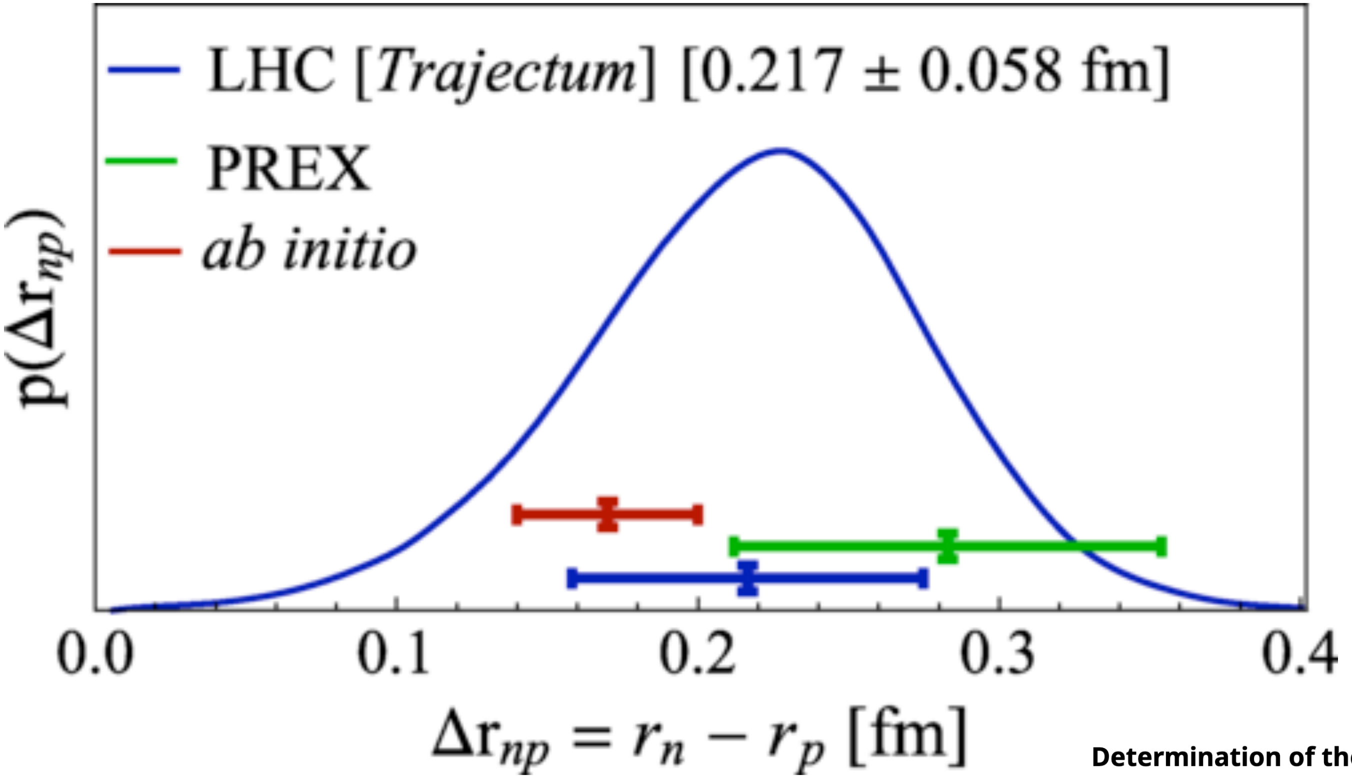
Phys. Rev. C **82**, 044611 – **Published 22 October, 2010**

LHC - Relativistic Heavy Ion Collisions

Particle distributions and **collective flow** in 208Pb + 208Pb **RHIC @ LHC** are **sensitive** to the overall strong **size** of the **nuclei involved.**



$$\Delta r_{\rm np} = 0.217 \pm 0.058 \text{ fm}$$

Determination of the Neutron Skin of ^{208}Pb from Ultrarelativistic Nuclear Collisions

Giuliano Giacalone[1], Govert Nijs[2], and Wilke van der Schee[3,4]

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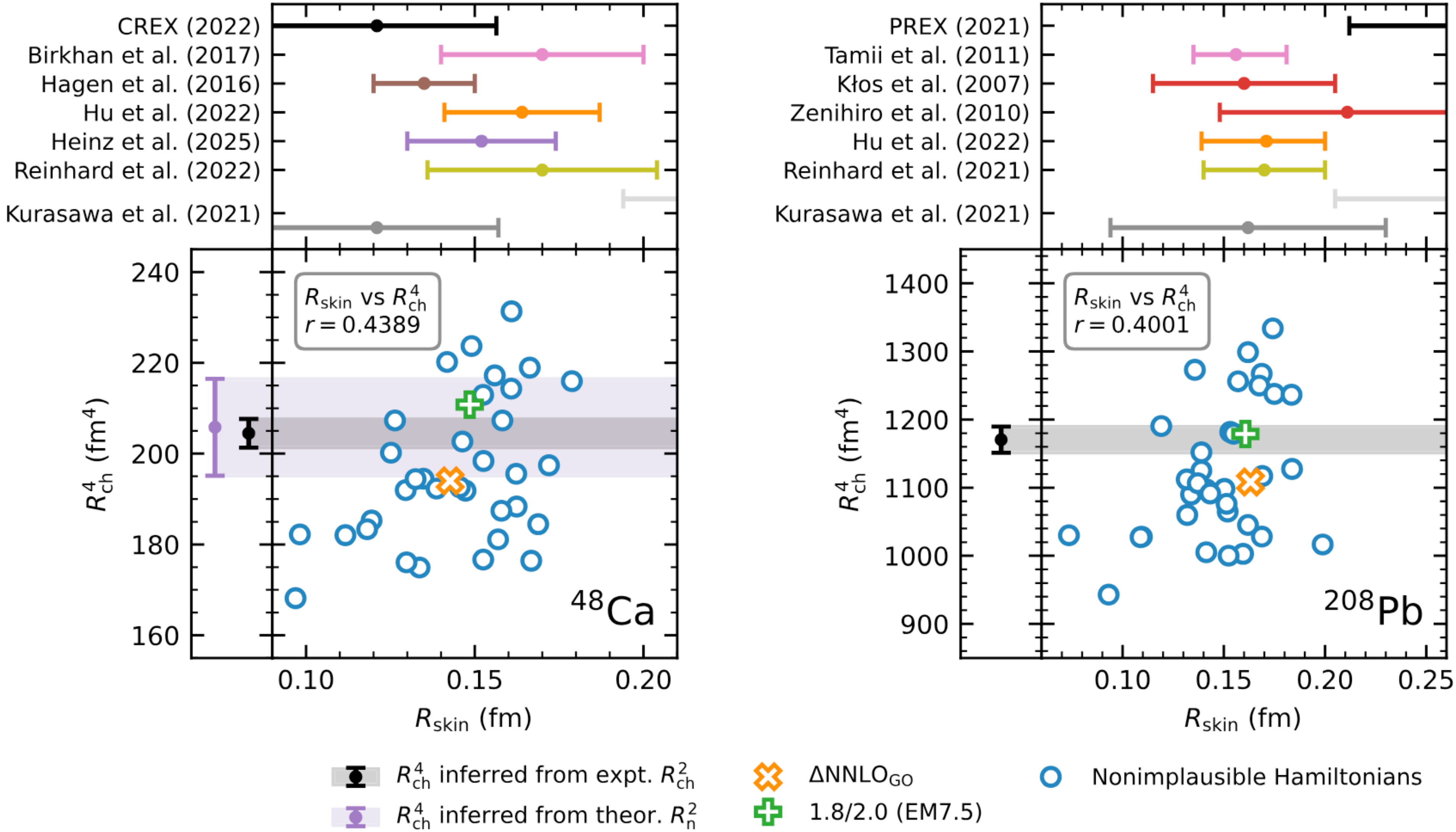
Phys. Rev. Lett. **131**, 202302 – **Published 15 November, 2023**

Summary on Δr_{np} in ^{48}Ca and ^{208}Pb



Ab initio computations of the fourth-order charge density moments of ^{48}Ca and ^{208}Pb

T. Miyagi [a], M. Heinz [b c], A. Schwenk [d e f]

Summary

Different ways to learn about the **neutron distribution** in atomic nuclei using **different observables** provide **different answers**

However, within **1.5–2 σ all looks fine**, both from phenomenologic **Energy Density Functionals** and from **Ab Initio**

From Theory:

→ An effort to better understand the **parity violating asymmetry** and the **beam normal spin asymmetry**[*] in ^{208}**Pb** is needed [PREX & CREX, PRL 128, 142501 (2022)].

→ An effort to better understand the **systematics** on the **dipole polarizability** would be of interest [see e.g. Sn chain in Bassauer et al. PLB 810, 135804 (2020)].

From Experiment (low-energy)**:**

→ An effort to improve the accuracy in the **parity violating asymmetry in** ^{208}**Pb** (and measure **other Q values**) **is needed**. Measuring other nuclei would also be desirable.

→ **Systematic** measurements of the **dipole polarizability** along **neutron rich** isotopic chains (e.g. N>74 Sn isotopes) could help testing models and improve our understanding of this observable.

[*] Not discussed here, in principle different physics rule this observable

Collaborators

→ Gianluca **Colò** (University of Milan)

→ Pietro **Klausner** (University of Milan)

→ Tomoya **Naito** (University of Tokyo)

→ Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)

→ Jorge **Piekarewicz** (Florida State University)

→ Nils **Paar** & Dario **Vretenar** (University of Zagreb)

→ Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)

→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)

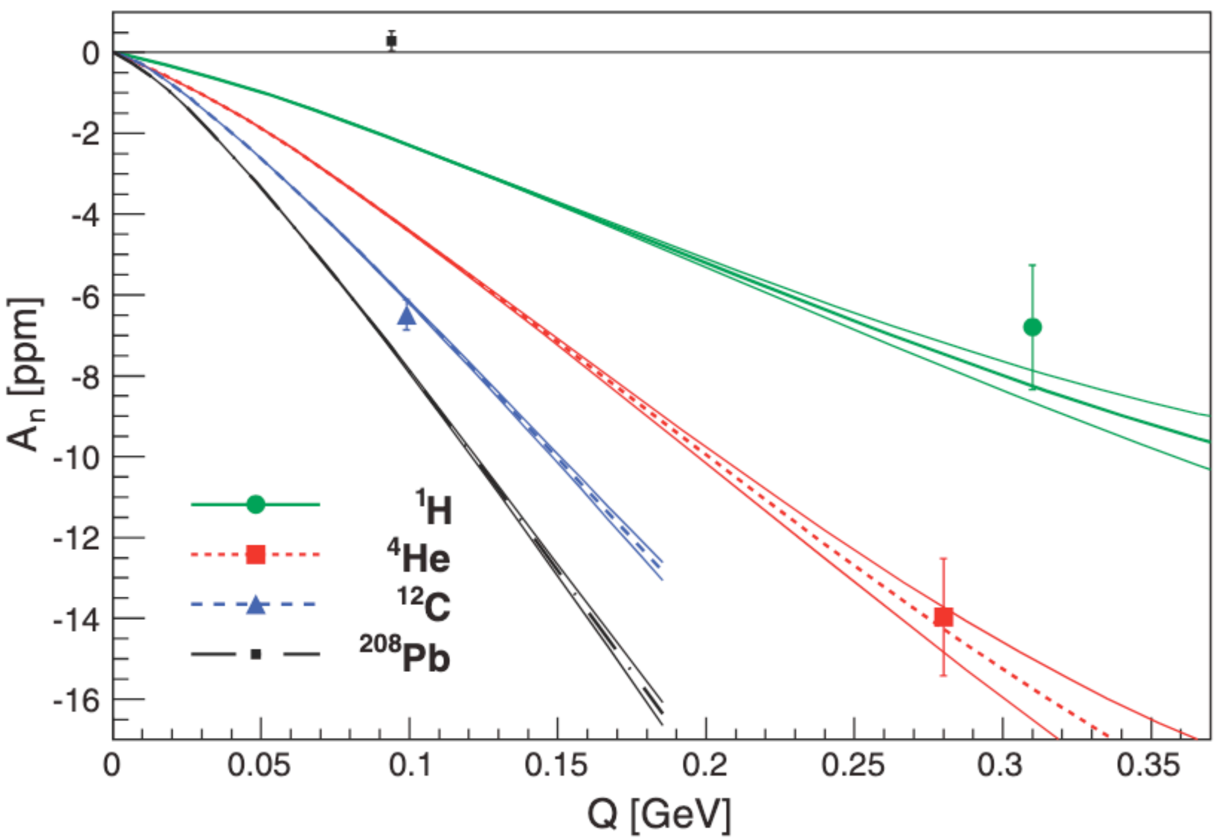
→ Witold **Nazarewicz** (FRIB and Michigan State University)

→ Doris H. **Jakubassa-Amunsden** (Ludwig-Maximilians-Universität München)

Beam normal spin asymmetry (A_n)

(aka Alayzing power or Sherman function)

Is the asymmetry in the scattering cross section when the incident electron beam is polarized perpendicular (longitudinal → A_{pv}) to the scattering plane



⇒ Important to understand A_{pv} since it represents a potentially large systematic correction to measured asymmetries.

In Born approximation (one-photon exchange), A_n = 0 because time-reversal symmetry forbids such an asymmetry.

Arises from interference between one-photon exchange and the absorptive part of two-photon exchange amplitudes.

New Measurements of the Transverse Beam Asymmetry for Elastic Electron Scattering from Selected Nuclei

S. Abrahamyan[45], A. Acha[10], A. Afanasev[11], Z. Ahmed[33], H. Albataineh[6], K. Aniol[3], D. S. Armstrong[7], W. Armstrong[35], J. Arrington[1] *et al.* (HAPPEX and PREX Collaborations)

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Phys. Rev. Lett. **109**, 192501 – **Published 5 November, 2012**

DOI: https://doi.org/10.1103/PhysRevLett.109.192501

New Measurements of the Beam-Normal Single Spin Asymmetry in Elastic Electron Scattering over a Range of Spin-0 Nuclei

D. Adhikari[1], H. Albataineh[2], D. Androic[3], K. Aniol[4], D. S. Armstrong[5], T. Averett[5], C. Ayerbe Gayoso[5], S. Barcus[6], V. Bellini[7] *et al.* (PREX and CREX Collaborations)

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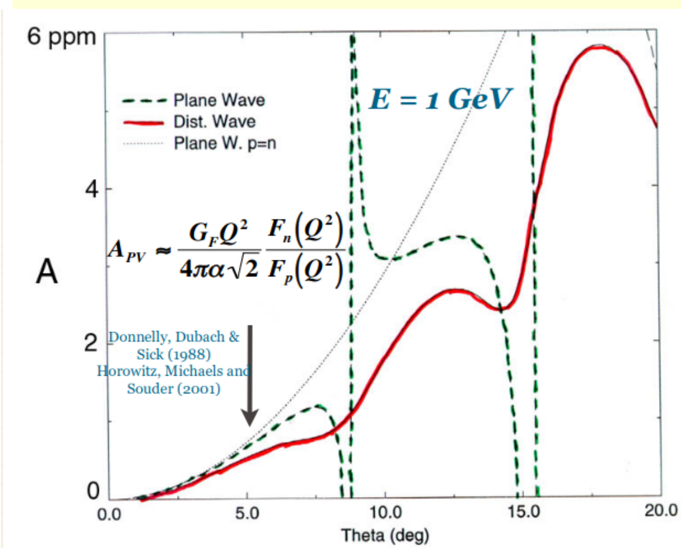
Phys. Rev. Lett. **128**, 142501 – **Published 8 April, 2022**

DOI: https://doi.org/10.1103/PhysRevLett.128.142501

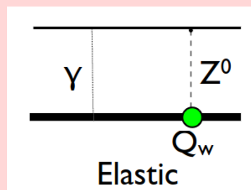
Parity Violating Asimmetry: theory

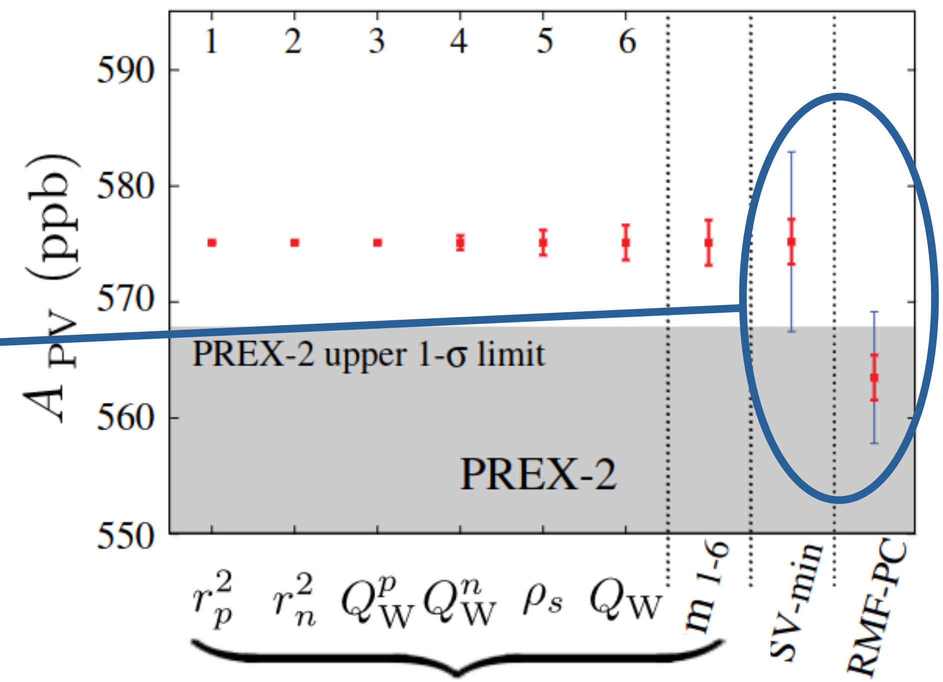
The **main uncertainties** come from **nuclear model errors** on the description of the **neutron distribution (blue error bars)**

→ **Coulomb distortions are important** (order αZ)



Are **γZ_0 box** corrections **important**? not included





Hadronic uncertainties less relevant

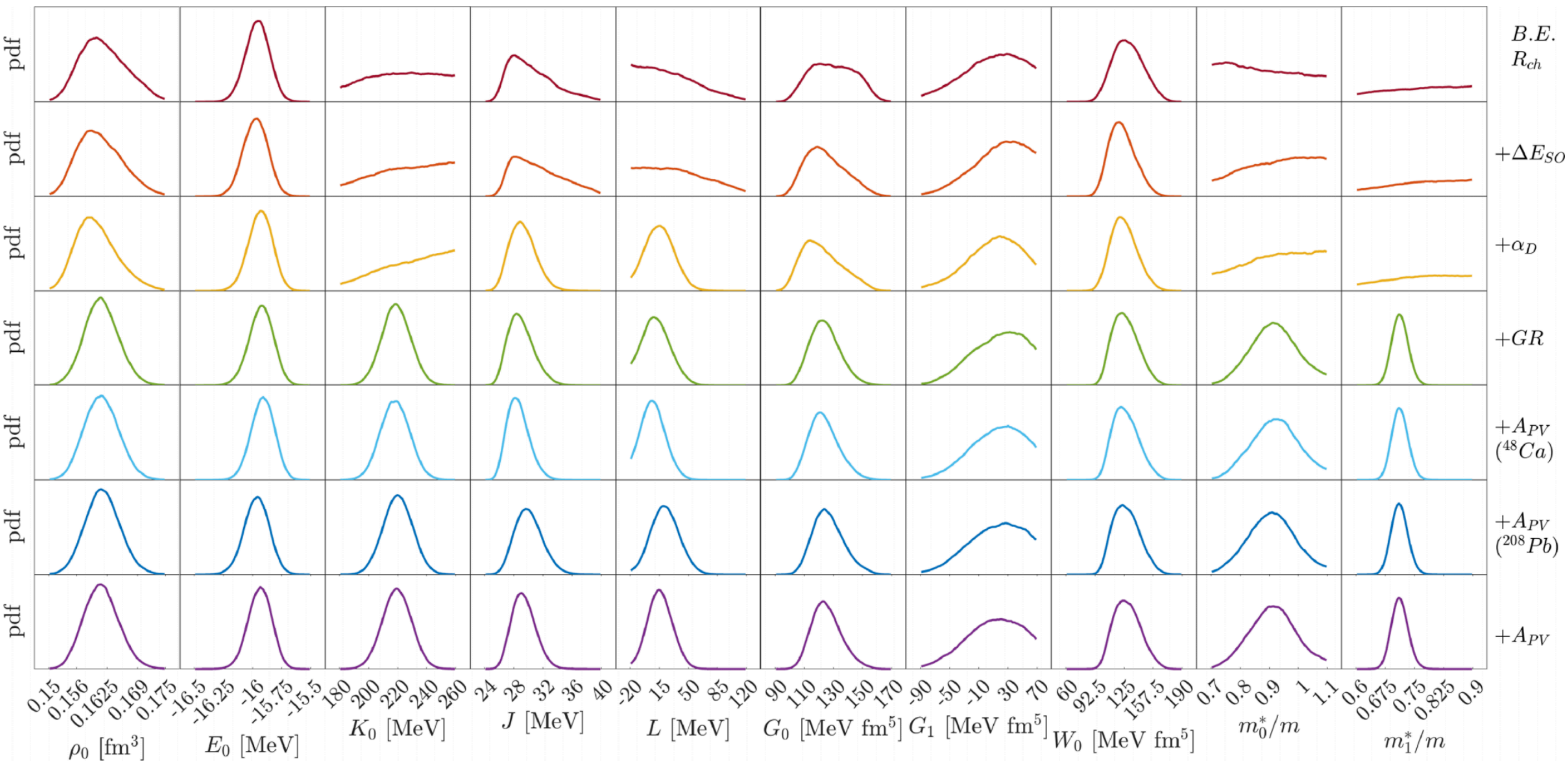
Parameter	Value
$\langle r_p^2 \rangle$ (fm^2)	0.726 ± 0.019
$\langle r_n^2 \rangle$ (fm^2)	-0.1161 ± 0.0022
μ_p	2.792 847
μ_n	-1.9130
$Q_p^{(W)}$	0.0713 ± 0.0001
$Q_n^{(W)}$	-0.9888 ± 0.0011
ρ_s	-0.24 ± 0.70
κ_s	-0.017 ± 0.004
$Q_{126,82}^{(W)}$	-117.9 ± 0.3

Information Content of the Parity-Violating Asymmetry in ^{208}Pb

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. **127**, 232501 – Published 29 November 2021

Bayesian inference Skyrme EDF: can we accommodate α_D and $A_{\rm PV}$ without compromising other observables?

By P. Klausner



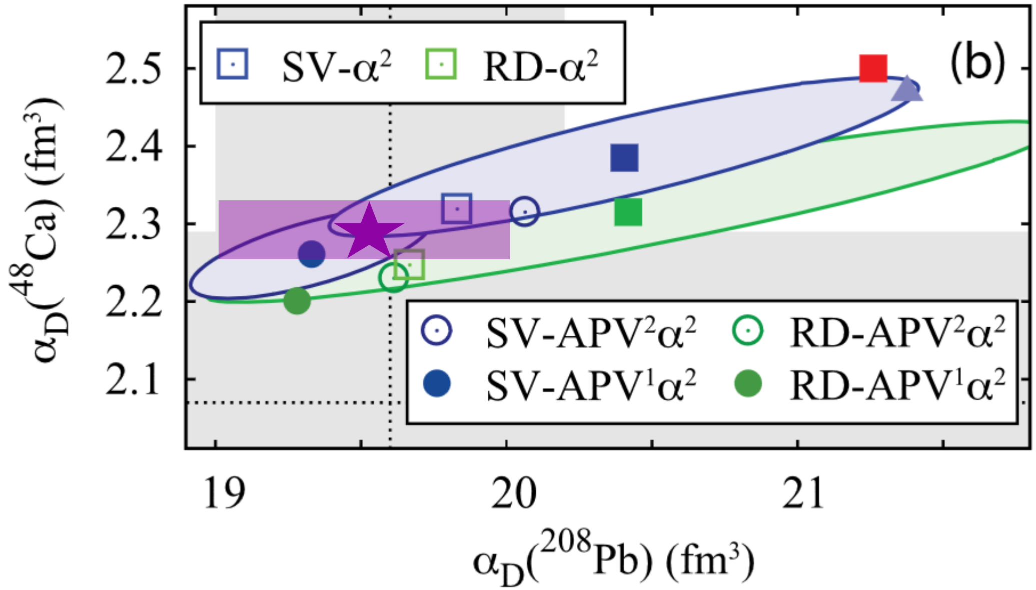
Pietro Klausner, Gianluca Colò, Xavier Roca-Maza, and Enrico Vigezzi
Phys. Rev. C 111 014311 (2025)

Bayesian inference Skyrme EDF: can we accommodate α_D and $A_{\rm PV}$ without compromising other observables?

Pietro Klausner, Gianluca Colò, Xavier Roca-Maza, and Enrico Vigezzi
Phys. Rev. C 111 014311 (2025)

By P. Klausner

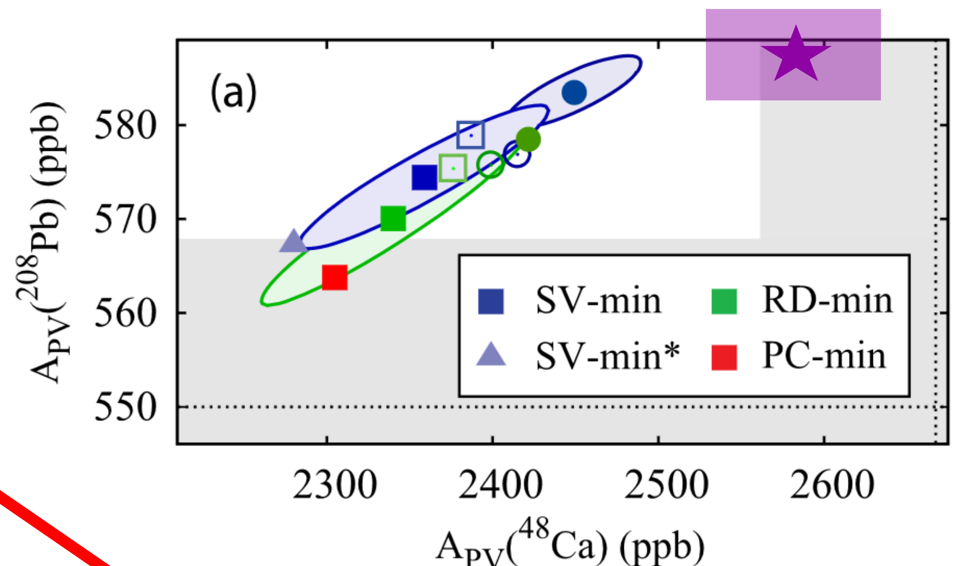
	α_D (fm^3)	$m(1)$ (MeV fm^2)	$A_{\rm PV}$ (ppb)
^{208}Pb	19.5 ± 0.5	958 ± 22	589 ± 5
^{48}Ca	2.30 ± 0.08	–	2591 ± 54



	B.E. (MeV)	$R_{\rm ch}$ (fm)	$\Delta E_{\rm SO}$ (MeV)
^{208}Pb	1636 ± 1.8	5.49 ± 0.03	2.34 ± 0.16
^{48}Ca	417 ± 1.2	3.51 ± 0.02	1.92 ± 0.20
^{40}Ca	342 ± 1.6	3.50 ± 0.02	–
^{56}Ni	482 ± 1.4	–	–
^{68}Ni	590 ± 1.0	–	–
^{100}Sn	826 ± 1.6	–	–
^{132}Sn	1103 ± 1.7	4.71 ± 0.03	–
^{90}Zr	784 ± 1.3	4.27 ± 0.02	–

Isoscalar resonances

	$E^{\rm IS}_{\rm GMR}$ (MeV)	$E^{\rm IS}_{\rm GQR}$ (MeV)
^{208}Pb	13.5 ± 0.3	10.8 ± 0.4
^{90}Zr	17.8 ± 0.4	–



Keeping ground and excited state **properties** within typical **Skyrme-EDF accuracy**