How does nuclear structure constrain the equation of state of nuclear matter?

XRM and N. Paar, Prog. Part. Nucl. Phys. 101 (2018) 96-176

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Laser spectroscopy as a tool for nuclear theories
CEA, Orme des Merisiers Campus. October 7th-11th 2019.

Table of contents:

(*) Work in progress
(†) Not covered in the talk [may be some slides at the end :-)]

Nuclear Energy Density Functionals (EDFs):

Based on effective interactions solved at the HF level, **EDFs** are **successful** in the description of **ground and excited state** properties such as m, $\langle r^2 \rangle^{1/2}$ **or GR** along the **nuclear chart**

Main types of EDFs:

Relativistic mean-field models (RMF), based on Lagrangians where effective mesons carry the interaction:

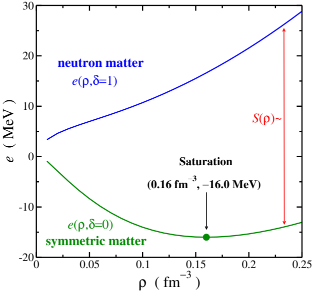
$$\begin{aligned} \mathcal{L}_{int} &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma && +\bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ & -\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} && -\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\boldsymbol{\tau}\Psi A^{(\rho)\mu} \end{aligned}$$

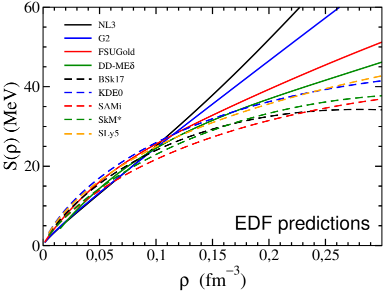
Non-relativistic mean-field models (NRMF), based on Hamiltonians where ef f. interactions are proposed and tested:

$$V_{\text{Nucl}}^{\text{eff}} = V_{\text{attractive}}^{\text{long-range}} + V_{\text{repulsive}}^{\text{short-range}} + V_{\text{SO}} + \ldots$$

-EDFs are **phenomenological** → **not directly connected to any NN** (or NNN) **interaction** in the vacuum
-EDFs derived from a **Mean-Field** → we expect bulk properties more accurate as heavier is the nucleus

The Nuclear Equation of State: Infinite System





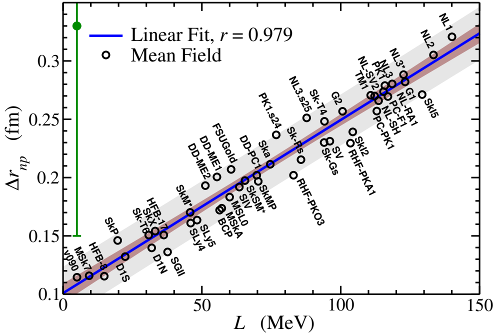
$$e(\rho,\beta) = e(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \quad \text{where} \;\; \beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

$$J \equiv S(\rho_0); \quad L = 3\rho_0 \left.\frac{\partial S(\rho)}{\partial \rho}\right|_{\rho=\rho_0}$$

-**Isovector properties** not well determined in current **EDFs**

But how we can better constraint the isovector channel from observables? (Example)

Neutron skin thickness → is one of the most paradigmatic example of an **isovector sensitive observable**.



Physical Review Letters **106**, 252501 (2011)

$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \sim \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L \;; \qquad J \equiv S(\rho_0) \text{ and } L \equiv 3\rho_0 p_0^{neut}$$

The impact of a neutron skin measurement on other nuclear observables

Dipole polarizability: definition

From a macroscopic perspective

The electric **polarizability** measures the tendency of the nuclear **charge distribution to be distorted**

$$\alpha_D \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

From a microscopic perspective

The electric **polarizability** is proportional to the **inverse energy weighted sum rule (IEWSR)** of the electric dipole response in nuclei

$$\alpha_D = \frac{8\pi}{9} e^2 \sum \frac{B(E1)}{E}$$

or

$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_{\text{ph. abs.}}(E)}{E^2} dE$$

Dipole polarizability: macroscopic approach

The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

Adopting the Droplet Model ($m_{-1} \propto \alpha_D$):

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

Bulk - First derived by Migdal

Surface correction - first derived by J. Meyer, P. Quentin, and B. Jennings, Nucl. Phys. A 385, 269 (1982)

within the same model, connection with the neutron skin thickness:

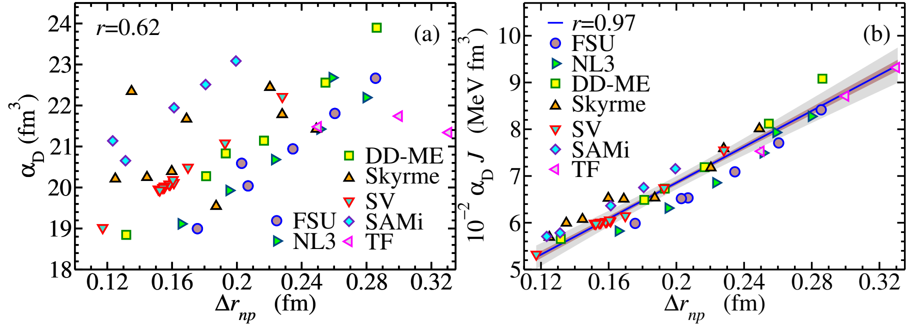
$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\rm surface}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

(*) $\Delta r_{np}^{\rm surface}$ depends on the difference between the neutron and proton difussivities (surface fall-off). This is well constraint in fit to masses and radii → all EDFs agree within a very small dispersion in closed shell such as ^{208}Pb.

Is this correlation appearing also in EDFs?

Isovector Giant Dipole Resonance in ^{208}Pb:

Dipole polarizability: microscopic results HF+RPA

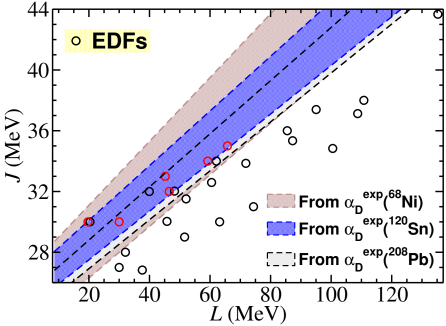


X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

$\alpha_D J$ is linearly correlated with Δr_{np} and no α_D alone within EDFs

Constraints of this analysis on the $J - L$ plane

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{3}\frac{L}{J}\frac{\rho_0 - \rho_A}{3\rho_0}\right] \text{ where } S(\rho_A) \equiv a_{sym}(A)$$



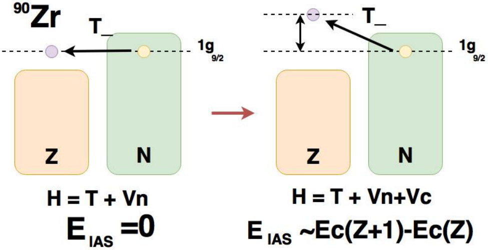
$J = (24.9 \pm 2.0) + (0.19 \pm 0.02)L$ for ^{68}Ni
$J = (25.4 \pm 1.1) + (0.17 \pm 0.01)L$ for ^{120}Sn
$J = (24.5 \pm 0.8) + (0.168 \pm 0.007)L$ for ^{208}Pb

For $S(<\rho> \to \rho_0) \approx J - L\frac{(\rho_0 - <\rho>)}{3\rho_0}$

X. Roca-Maza et al. Phys. Rev. C **92**, *064304 (2015)*

Subset of models that reproduce simultaneously measured polarizabilities predict $J = 30 - 35$ **MeV**, $L = 20 - 66$ **MeV; and** Δr_{np} **in** ^{68}**Ni,** ^{120}**Sn, and** ^{208}**Pb are in the ranges: 0.15-0.19 fm, 0.12-0.16 fm, and 0.13-0.19 fm**

The isobaric analog state energy: E_{IAS}



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$
- **Displacement energy or E_{IAS}**

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

> $E_{IAS} \neq 0$ **only due to Isospin Symmtry Breaking terms** $\mathcal{H}$
> E_{IAS}^{exp} **usually accuratelly measured !**

Coulomb direct contribution: very simple model

- Assuming indepentent particle model and good isospin for $|0\rangle$ ($\langle 0|T_+ T_-|0\rangle = 2T_0 = N - Z$)

$$E_{IAS} \approx E_{IAS}^{C,\mathrm{direct}} = \frac{1}{N-Z}\int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{\mathrm{direct}}(\vec{r}) d\vec{r}$$

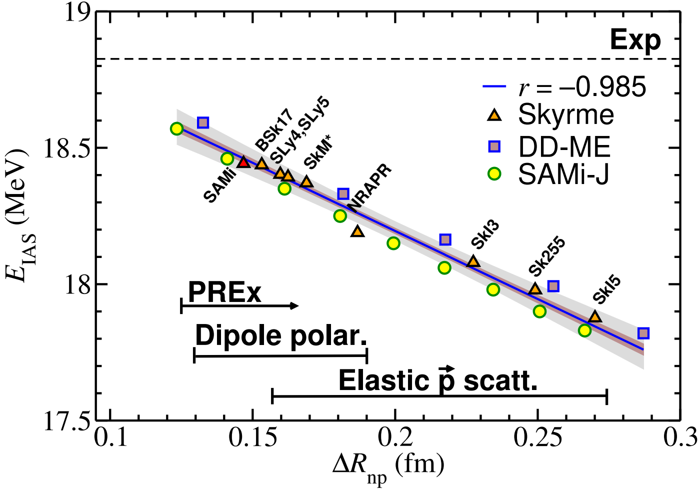
where $U_C^{\mathrm{direct}}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{ch}(\vec{r}_1) d\vec{r}_1$

- Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,\mathrm{direct}} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

E_{IAS} in Energy Density Functionals (No Corr.)



Phys. Rev. Lett. 120, 202501 (2018)

Nuclear models (EDFs) where the nuclear part is isospin symmetric and U_{ch} is calculated from the ρ_p

Corrections: within self-consistent HF+RPA

Within the **HF+RPA** one can **estimate** the E_{IAS} accounting (in an effective way) for **short-range correlations and effects of the continuum** (if a large sp base is adopted).

- **Coulomb exchange** exact (usually Slater approx.):

$$U_C^{x,\text{exact}} \varphi_i(\vec{r}) = -\frac{e^2}{2} \int d^3 r' \, \frac{\varphi_j^*(\vec{r}\,')\varphi_j(\vec{r})}{|\vec{r} - \vec{r}\,'|} \varphi_i(\vec{r}\,')$$

- The **electromagnetic spin-orbit** correction to the nucleon single-particle energy (non-relativistic),

$$\varepsilon_i^{\text{emso}} = \frac{\hbar^2 c^2}{2 m_i^2 c^4} \langle \vec{l}_i \cdot \vec{s}_i \rangle x_i \int \frac{1}{r} \frac{dU_C}{dr} |R_i(r)|^2$$

where x_i: $g_p - 1$ for Z and g_n for N; $g_n = -3.82608545(90)$ and $g_p = 5.585694702(17)$, $R_i \to R_{nl}$ radial wf.

Corrections:

- **Finite size** effects (assuming spherical symmetry):

$$\rho_{ch}(q) = \left(1 - \frac{q^2}{8m^2}\right)\left[G_{E,p}(q^2)\rho_p(q) + G_{E,n}(q^2)\rho_n(q)\right]$$
$$- \frac{\pi q^2}{2m^2}\sum_{l,t}\left[2G_{M,t}(q^2) - G_{E,t}(q^2)\right]\langle \vec{l}\cdot\vec{s}\rangle \int_0^\infty dx \frac{j_1(qx)}{qx}|R_{nl}(x)x^2|^2$$

- **Vacuum polarization:** lowest order correction in the fine-structure constant to the Coulomb potential $\frac{eZ}{r}$:

$$V_{vp}(\vec{r}) = \frac{2}{3}\frac{\alpha e^2}{\pi}\int d\vec{r}' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathcal{K}_1\left(\frac{2}{\lambda\!\!\!^{-}_e}|\vec{r}-\vec{r}'|\right)$$

where e is the fundamental electric charge, α the fine-structure constrant, $\lambda\!\!\!^{-}_e$ the reduced Compton electron wavelength and

$$\mathcal{K}_1(x) \equiv \int_1^\infty dt e^{-xt}\left(\frac{1}{t^2} + \frac{1}{2t^4}\right)\sqrt{t^2-1}$$

Corrections:

- **Isospin symmetry breaking** (Skyrme-like): **two parts** (contact interaction)

charge symmetry breaking +

$$V_{CSB} = V_{nn} - V_{pp}$$

$$V_{CSB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{4}\left[\tau_z(1) + \tau_z(2)\right] s_0 (1 + y_0 P_\sigma)$$

τ_z Pauli in isospin space; P_σ are the usual projector operators in spin space.

charge independence breaking*

$$V_{CIB} = \frac{1}{2}(V_{nn} + V_{pp}) - V_{pn}$$

$$V_{CIB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{2}\tau_z(1)\tau_z(2) u_0 (1 + z_0 P_\sigma)$$

* general operator form $\tau_z(1)\tau_z(2) - \frac{1}{3}\vec{\tau}(1)\cdot\vec{\tau}(2)$. Our prescription $\tau_z(1)\tau_z(2)$ not change structure of HF+RPA.

- Opposite to the other corrections, **ISB contributions depends on new parameters that need to be determined!**

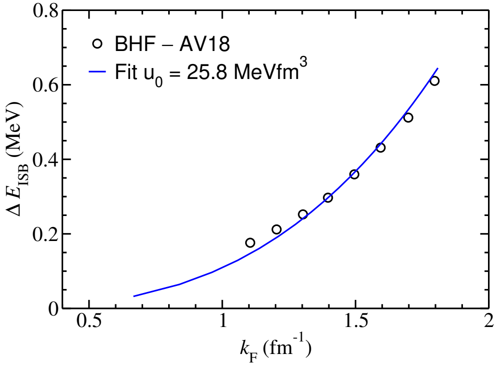
Isospin symmetry breaking in the medium:

- **keeping** things **simple**: **CSB and CIB** interaction just **delta function** depending on s_0 **and** u_0. **Different possibilities:**

→ **Fitting** to (two) experimentally known **IAS energies**

→ **Derive from theory**

→ **our option:** u_0 **to reproduce BHF** (symmetric nuclear matter) and s_0 **to reproduce** E_{IAS} **in** ^{208}**Pb**



Physics Letters B 445, 259 (1999)

Re-fit of SAMi: SAMi-ISB

- All these **corrections** are relatively **small** but **modify binding energies, neutron and proton distributions, etc.** $\Rightarrow$ a **re-fit of the interaction is needed**.
- Use **SAMi fitting protocol** (special care for spin-isospin resonances) including all corrections and **find SAMi-ISB**

Table: Saturation properties

	SAMi	SAMi-ISB	
ρ_∞	0.159	0.1613(6)	fm^{-3}
e_∞	−15.93	−16.03(2)	MeV
m^*_{IS}	0.6752	0.730(19)	
m^*_{IV}	0.664	0.667(120)	
J	28	30.8(4)	MeV
L	44	50(4)	MeV
K_∞	245	235(4)	MeV

SAMi-ISB finite nuclei properties

El.	N	B [MeV]	B^{exp} [MeV]	r_c [fm]	r_c^{exp} [fm]	ΔR_{np} [fm]
Ca	28	417.67	415.99	3.49	3.47	0.214
Zr	50	783.60	783.89	4.26	4.27	0.097
Sn	82	1102.75	1102.85	4.73	–	0.217
Pb	126	1635.78	1636.43	5.50	5.50	0.151

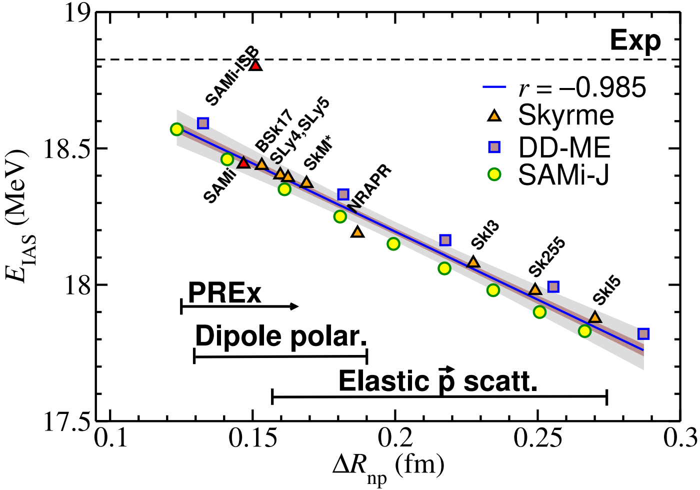
Corrections on E_{IAS} for ^{208}Pb one by one

	E_{IAS} [MeV]	Correction [keV]
No corrections[a]	18.31	
Exact Coulomb exchange	18.41	+100
n/p mass difference	18.44	+30
Electromagnetic spin-orbit	18.45	+10
Finite size effects	18.40	-50
Vacuum polarization (V_{ch})	18.53	+130
Isospin symmetry breaking	**18.80(5)**	+270

[a] From Skyrme Hamiltonian where the nuclear part is isospin symmetric and V_{ch} is calculated from the ρ_p

$E_{IAS}^{exp} = 18.83 \pm 0.01$ MeV. *Nuclear Data Sheets 108, 1583 (2007).*

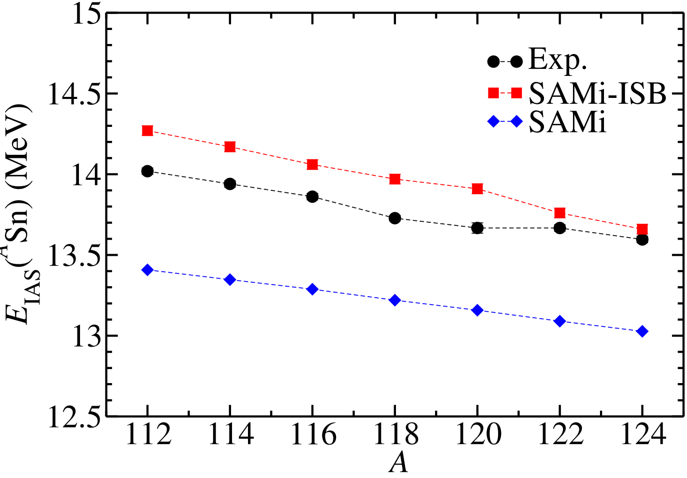
E_{IAS} with SAMi-ISB



Phys. Rev. Lett. 120, 202501 (2018)

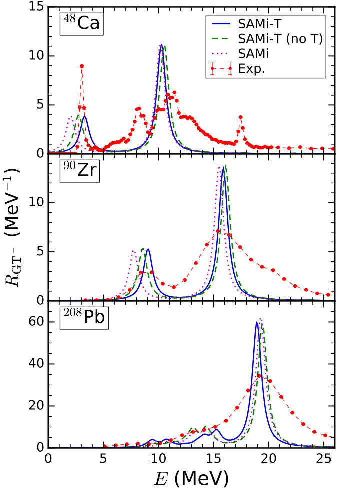
Measurement of Δr_{np} → determine ISB in the nuclear medium

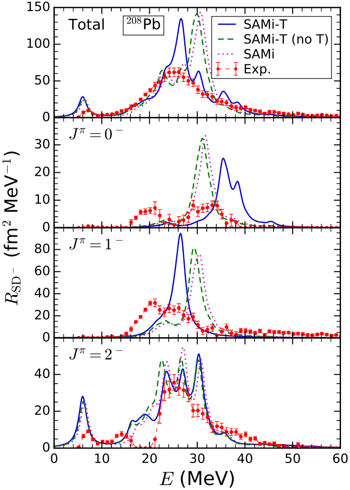
Prediction: E_{IAS} in the Sn isotopic chain



Phys. Rev. Lett. 120, 202501 (2018)

SAMi-T: Skyrme functional with tensor terms (Gamow-Teller and Spin Dipole)





Despcription of low and high energy GT peaks

Improved 1^- channel gives largest contribution

Shihang Shen et al. Phys. Rev. C 99, 034322 (2019)

SDR and the neutron skin thickness Δr_{np}:

$$\int [R_{SD^-}(E) - R_{SD^+}(E)]dE = \frac{9}{4\pi}(N\langle r_n^2\rangle - Z\langle r_p^2\rangle)$$
$$\approx (N-Z)\langle r_p^2\rangle \left(1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)$$

• Experimental NEWSR in ^{208}**Pb** is 1004^{+24}_{-23} **fm^2**; SAMi is **1224 fm^2**; and **SAMi-T 1260± 10 fm^2** **(some strength is missing in the experimental measurement ?** $\Delta r_{np} \approx 0.05$ **fm**).

• Experimental NEWSR in ^{90}**Zr** is 148 ± 12 **fm^2**; SAMi is **150 fm^2**; and SAMi-T 147 ± 1 **fm^2** $\Rightarrow$ **neutron skin should be properly determined by SAMi and SAMi-T**

Conclusions

The electromagnetic response of the nucleus is related with the neutron skin via the Electric Dipole polarizability (also other multipolarities show the same characteristics)

Isobaric Analog State energy relates the electromagnetic charge radius, ISB effects and the neutron skin thickness of nuclei

Spin dipole resonance NEWSR relates with the neutron and proton average sizes

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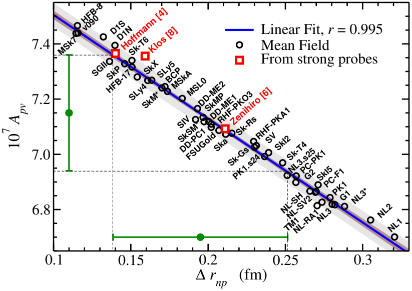
*Passed away summer 2018.

Thank you for your attention!

EXTRA EXAMPLES

The neutron skin and the parity violating asymmetry in ^{208}Pb



Physical Review Letters **106**, *252501 (2011)*

(Calculation at a fixed q equal to PREx)

→ Electrons interact by exchanging a γ **(couples to** p**)** or a Z_0 **boson (couples to** n**)**

→ Ultra-relativistic electrons, **depending on their helicity (±)**, will interact with the nucleus **seeing a slightly different potential**: **Coulomb** ± **Weak**

→ $$A_{pv} \equiv \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Input for the calculation are the ρ_p and ρ_n **(main uncertainty)** and nucleon form factors for the e-m and the weak neutral current.

→ In PWBA for small momentum transfer:

$$A_{pv} \approx \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 \langle r_p^2\rangle^{1/2}}{3F_p(q)} \Delta r_{np}\right)$$

The largest the size of the neutron distribution in nuclei (Δr_{np}), the smaller the parity violating asymmetry.

[Exp. from ew probes: 0.302 ± 0.175 **fm (***Physical Review C* **85***, 032501 (2012)***)].**

Isovector Giant Resonances (some considerations)

→ In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**

→ **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"

→ The **excitation energy (E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

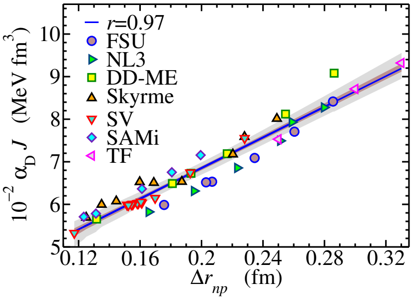
$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \to E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{\mathbf{S(\rho)}}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

→ The **dipole polarizability ($\alpha \sim \int \frac{\sigma_{\gamma-abs}}{\text{Energy}^2} \sim$ IEWSR)** measures the tendency of the nuclear charge distribution to be distorted, that is, from a **macroscopic** point of view

$$\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

Dipole polarizability and the neutron skin in ^{208}Pb



Physical Review C **85** *041302 (2012);* **88** *024316 (2013);* **92**, *064304 (2015)*

Macroscopic model:

→ Using the **dielectric theorem:** m_{-1} moment can be computed from the expectation value of the Hamiltonian in the constrained (D dipole operator) ground state $\mathcal{H}' = \mathcal{H} + \lambda D$

→ Assuming the **Droplet Model** (heavy nucleus):

$$\alpha_D \approx \alpha_D^{\rm bulk}\left[1 + \frac{1}{5}\frac{L}{J}\right] \text{ where}$$

$$\alpha_D^{\rm bulk} \equiv \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J} \text{ (Migdal first derived)}$$

→ $$L \approx \frac{\alpha_D^{\rm exp} - \alpha_D^{\rm bulk}}{\alpha_D^{\rm bulk}} 5J$$

By using the Droplet Model one can also find:

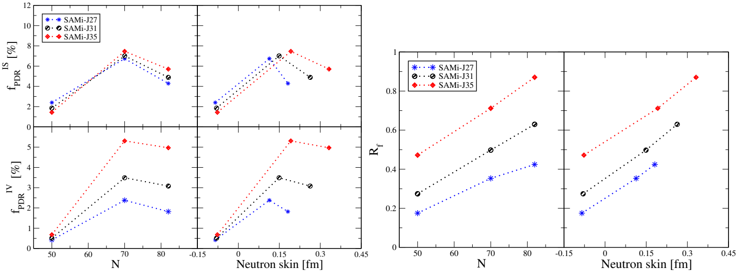
$$\alpha_D J \approx \frac{\pi e^2}{54} A\langle r^2\rangle\left[1 + \frac{5}{2}\frac{\Delta r_{np} - \Delta r_{np}^{\rm coul} - \Delta r_{np}^{\rm surf}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

For a fixed value of the symmetry energy at saturation, the larger the neutron skin in ^{208}Pb, the larger the dipole polarizability.

Pygmy Dipole Resonance

(Pygmy: low-energy excited state appearing in the dipole response of $N \neq Z$ nuclei)

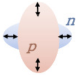
- Sn isotopes, SAMi-J interactions
- Outermost neutrons contributing to the IS and IV pygmy state (f_{PDR} fraction of the EWSR and $R_f = f^{IV}_{PDR}/f^{IS}_{PDR}$)



WARNING: we lack of a clear understanding of the physical reason for this correlation. Models used fitted by the same group → possible bias.

S. Burrello et al. Phys. Rev. C 99, 054314 (2019)

IV-IS GQRs and the neutron skin in ^{208}Pb



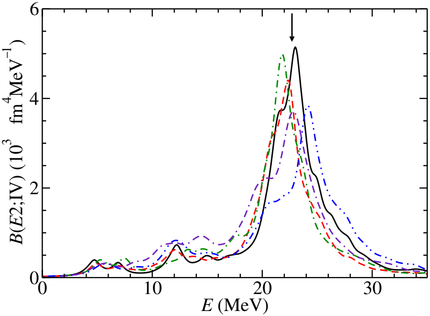
Within the Quantum Harmonic Oscillator approach

$$E_x^{IV} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m}\frac{V_{sym}\langle r^2\rangle}{(\hbar\omega_0)^2\langle r^4\rangle}}$$

and EDF calculations, one can deduce

$$V_{sym} \approx 8(S(\rho_A) - S^{kin}(\rho_0))$$

$$S^{kin}(\rho_0) \approx \varepsilon_{F_0}/3 \text{ (Non-Rel)}$$



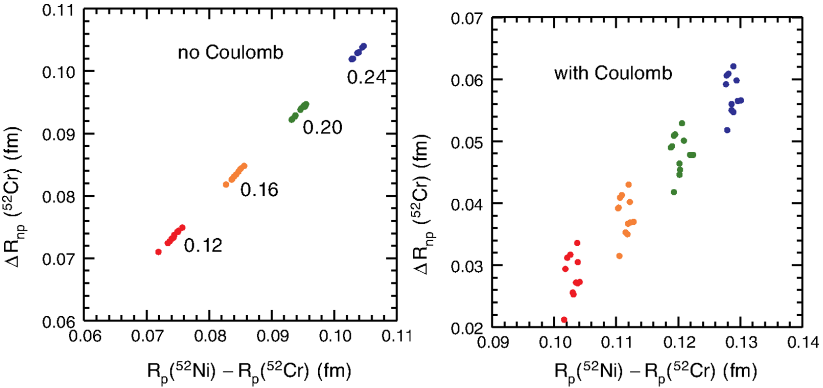
$$S(\rho_A) \approx J - L\frac{\rho_0 - \rho_A}{3\rho_0} \approx \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$

The larger the neutron skin in ^{208}Pb, the smallest the difference between the IS and IV excitation energies in GQRs.

Differences in the proton radii of mirror nuclei

If isospin symmetry conserved (ISC) in nuclei

- $r_n(N, Z) = r_p(Z, N)$
- $\Delta r_{np}(N, Z) \equiv r_n(N, Z) - r_p(N, Z) = r_p(Z, N) - r_p(N, Z)$



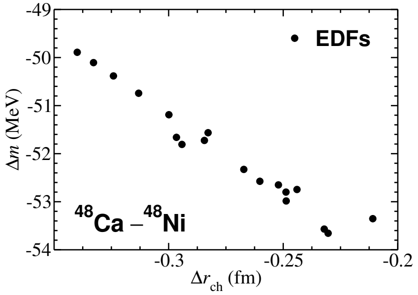
B. A. Brown, Phys. Rev. Lett. 119 122502 (2017)

$\Delta r_{ch}(N, Z)$ is better correlated than $\Delta r_{np}(N, Z)$ with L. (Using some set of EDFs).

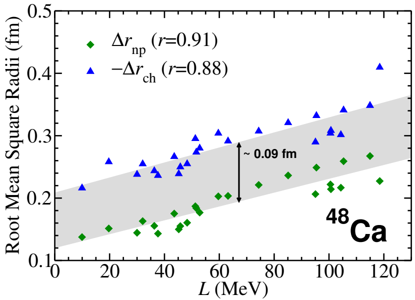
Differences in the charge radii of mirror nuclei

$$\begin{aligned}\Delta m &\equiv m(N,Z) - m(Z,N)\\ &\approx \frac{3}{5}\frac{Z(Z-1)e^2}{R_{ch}^{(N,Z)}} - \frac{3}{5}\frac{N(N-1)e^2}{R_{ch}^{(Z,N)}}\\ &\approx -E_C\frac{(N-Z)(N+Z-1)}{Z(Z-1)}\left(1+\frac{\Delta R_{ch}}{R_{ch}^{(N,Z)}}\right)\end{aligned}$$

$$\begin{aligned}&\Delta r_{np}(N,Z) + \Delta r_{ch}(N,Z) \approx\\ &-\frac{e^2 N}{35J}\left(1-\frac{Z}{4N}\right) + \sqrt{\frac{3}{5}}\frac{5}{2}\frac{b_n^2(N,Z) - b_p^2(Z,N)}{R}\\ &\approx -0.04\,\mathrm{fm}\end{aligned}$$



Work in progress

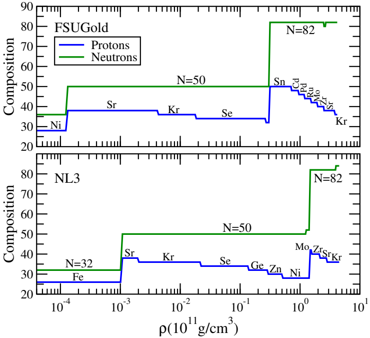


Work in progress

Correlation of $\Delta r_{np}(N,Z)$ or $\Delta r_{ch}(N,Z)$ with L seems to be the same for other set of EDFs

The neutron skin in ^{208}Pb and the structure and composition of a neutron star outer crust

- → span **7** orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g/cm to the **neutron drip** $\sim 10^{11}$ g/cm)
- → it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- → $T \sim 10^6$ K ~ 0.1 keV → one can treat **nuclei and electrons at $T = 0$ K**
- → At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}**Fe** nuclei.
- → As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$ and therefore $Z \downarrow$ with constant (approx) number of N
- → As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a dif ferent N*−plateau*
  $\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{Fe}}{8 a_{sym}}$ where $(A_0, Z_0) = {}^{56}\text{Fe}_{26}$
- → The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$)**.
  $[M(N, Z) + m_n < M(N+1, Z)]$



Physical Review C **78**, *025807 (2008)*

The larger the neutron skin of ^{208}Pb **(L ↑)**, the more exotic the composition of the outer crust.

EXTRA MATERIAL

In more detail (from theory) ...

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

  $$F_{JM} = \sum_{i}^{A} r^{J} Y_{JM}(\hat{r})\tau_z(i) \; (\Delta L = 1 \rightarrow Dipole)$$

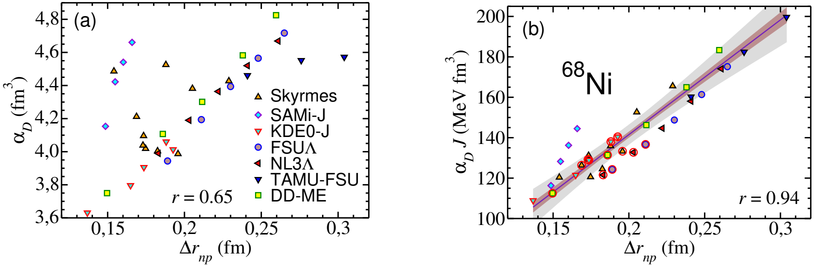
- is proportional to the **static polarizability** for small oscillations $\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**

The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

Isovector Giant Dipole Resonance in ^{68}Ni:

What about other nuclei?

Dipole polarizability: microscopic results HF+RPA



X. Roca-Maza et al. Phys. Rev. C **92**, *064304 (2015)*

Experimental dipole polarizability $\alpha_D = 3.40 \pm 0.23$ fm^3 D. M. Rossi *et al.*, PRL 111, 242503 (GSI). $\alpha_D = 3.88 \pm 0.31$ fm^3 "full" response D. M. Rossi, T. Aumann, and K. Boretzky.

Spin-Dipole Resonance (SDR)

Total SDR reasonably well described but different channels in the SDR **NOT well** described.

Tensor terms produce a softening of 1^- states and a hardening of 0^- and 2^- states. Diagonal matrix elements:

$$V_{T,AS}^{\lambda} = (a_\lambda \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} T + b_\lambda \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} U) \langle \vec{\tau}_1 \cdot \vec{\tau}_2 \rangle \text{ for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix}$$

[C. L. Bai, *et al.*, Phys. Rev. Lett. 105, 072501 (2010)]

- This **general trend** would correct in the **proper** direction the **1^- and 2^- channels as predicted by SAMi** while the 0^- might be worsten (**Experimentally**, note that the **largest** contribution in **^{208}Pb** comes from the **1^- channel**).

Tensor terms

- New radioactive ion beam facilities $\rightarrow$ **evolution of the spin-orbit splittings with neutron excess** have been highlighted

  [J. P. Schiffer, *et al.*, Phys. Rev. Lett. 92, 162501 (2004)]

- Importance of an **effective neutron-proton tensor force to explain this evolution** was suggested

  [T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe, and Y. Akaishi, Phys. Rev. Lett. 95 , 232502 (2005)]

$$\vec{U}_q^{\text{spin-orbit}} = \frac{1}{2}\left[W_0 \vec{\nabla}\rho + W_0' \vec{\nabla}\rho_q\right] + \left[\alpha \vec{J}_q + \beta \vec{J}_{1-q}\right]$$

where $\alpha = \alpha_C + \alpha_T$ and $\beta = \beta_C + \beta_T$ **(Non Rel. SO mean-field)**

- A number of experimental **data could be reproduced, to a certain extent better by including tensor terms in EDFs: Gogny** [T.Otsuka, T. Matsuo, and D. Abe, Phys. Rev. Lett.97, 162501 (2006)], **Skyrme** [T. Lesinski, M. Bender, K. Bennaceur, T. Duguet, and J. Meyer,Phys. Rev. C 76, 014312 (2007)], and **relativistic** frameworks [W. Long, H. Sagawa, J. Meng, and N. Van Giai, EPL 82, 12001 (2008)]

Tensor terms

• Difficult to **single out observables** that can uniquely determine the effects of the **tensor force**: **single-particle data senistive to beyond mean-field effects** (such as **PVC** [G. Colò, *et al.*, PRC 50, 1496 (1994) or A. V. Afanasjev and E. Litvinova, PRC 92, 044317 (2015)])

• This explain the **difficulties in determining tensor terms in EDFs** (c.f. [T. Lesinski *et al.*, Phys. Rev. C 76, 014312 (2007)])

• **Fit different channels of the SDR** (or other resonances such as M1) via HF+RPA calculations is **expensive computationally**.

• *ab initio* calculations may provide reliable **connection between few-body and many-body** physics.

• **Brueckner-Hartree-Fock (BHF)** theory do not contain effects like **PVC**, and **can be used directly as a benchmark** for EDFs.

Neutron drops from *ab initio*

- The **neutron drop is an ideal system** composed of a finite number of neutrons confined in an external field: can give an interesting **insight on shell structure and finite size properties**.
- Attempts to build EDFs driven by *ab initio* calculations in neutron drops [S. Gandolfi, J. Carlson, and S. C. Pieper, PRL 106, 012501 (2011) or J. Bonnard, M. Grasso, and D. Lacroix, PRC 98, 034319 (2018)]
- Essential **neutron-proton component of the tensor** to study nuclei ⇒ RBHF calculations also in **neutron-proton drops** [Shihang Shen, G. Colò, X. Roca-Maza, arXiv:1810.09691 (2018)].

• We use **neutron and neutron-proton drops** (RBHF with Bonn A) as benchmark to **fix the tensor terms**.
• **Refit full parameter space including tensor terms**.
• Requires minimal **modification of SAMi fitting protol**: neutron matter EoS changed by drops **(relative change in SO splittings and total energy in neutron drops)**.

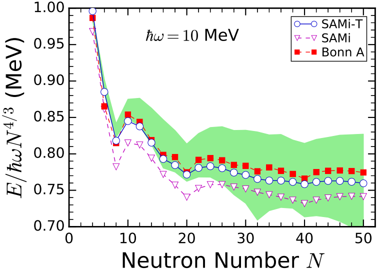
SAMi-T: parameter set and saturation properties

	Value	Error		Value
t_0	-2199.38 MeV fm^3	372.	ρ_0	0.164(1) fm^{-3}
t_1	533.036 MeV fm^5	20.7	e_0	$-16.15(3)$ MeV
t_2	-88.1692 MeV fm^5	12.6	m^*_{IS}/m	0.634(19)
t_3	11293.5 MeV fm$^{3+3\gamma}$	2014.	m^*_{IV}/m	0.625(122)
x_0	0.514710	0.178	J	29.7(6) MeV
x_1	-0.531674	0.593	L	46(12) MeV
x_2	-0.026340	0.117	K_0	244(5) MeV
x_3	0.944603	0.481	G_0	0.08 (fixed)
γ	0.179550	0.047	G'_0	0.29 (fixed)
W_0	130.026 MeV fm^5	8.2		
W'_0	101.893 MeV fm^5	18.6		
α	73.0 MeV fm^5	0.8	nn and pp	
β	101.8 MeV fm^5	1.2	np	

- strengths of the **SO terms in SAMi-T are larger** when compared with SAMi.
- **tensor terms known to reduce the SO splittings** of spin unsaturated systems such as ^{90}Zr or ^{208}Pb that have been fitted in both functionals.
- **SAMi-T needs larger SO strength to reproduce the same data.**

SAMi-T: neutron and neutro-proton drops

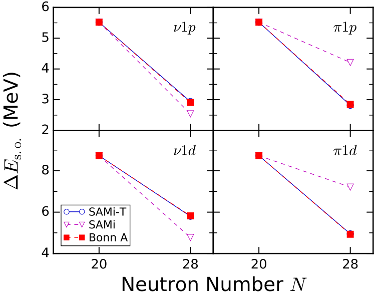
Neutron drops: **total energy**



The shaded area represents the values spanned by quantum Monte-Carlo calculations using AV8'+UIX and AV8'+IL7

S. Gandolfi, J. Carlson, and S. C. Pieper, PRL. 106, 012501 (2011) and P. Maris, J. P. Vary, S. Gandolfi, J. Carlson, and S. C. Pieper, PRC 87, 054318 (2013)

Neutron-proton drops:
spin-orbit splittings



Trends in RBHF: **constrain trends tensor term** (neutron and neutron-proton drops have been included)

Experimental SO splittings: **constrain SO term** (mainly absolute value, no trends)

STATISTIC UNCERTAINTIES AND CORRELATIONS IN EDFs

Example on the dipole polarizability

Covariance analysis: χ^2 test

- Observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$ (e.g. of an EDF)
$$\chi^2(\mathbf{p}) = \frac{1}{m - n_p - 1} \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$
- Assuming that the χ^2 can be approximated by an hyper-parabola around the minimum $\mathbf{p}_0$,
$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j})$$
where $\mathcal{M} \equiv \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2$ (curvature m.) and $\mathcal{E} \equiv \mathcal{M}^{-1}$ (error m.).
- errors between predicted observables $\mathcal{A}$
$$\Delta \mathcal{A} = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A}$$
- correlations between predicted observables,
$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}}$$
where, $C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$

Example: Fitting protocol of SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

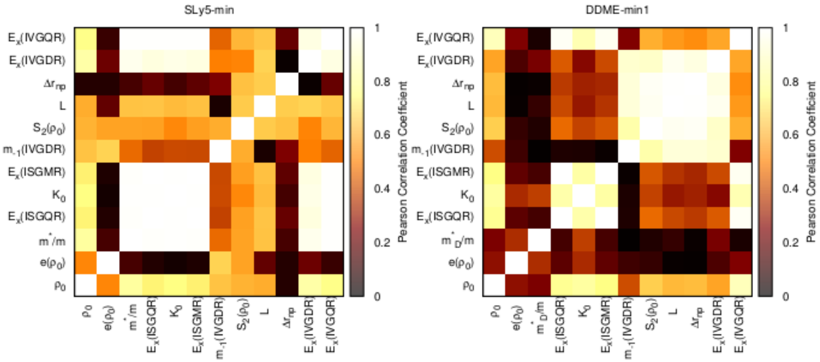
SLy5-min:

- **Binding energies** of ^{40,48}Ca, ^{56}Ni, ^{130,132}Sn and ^{208}Pb with a fixed adopted error of 2 MeV
- the **charge radius** of ^{40,48}Ca, ^{56}Ni and ^{208}Pb with a fixed adopted error of 0.02 fm
- the **neutron matter** Equation of State calculated by Wiringa *et al.* (1988) for densities between 0.07 and 0.40 fm^{-3} with an adopted error of 10%
- the **saturation energy** ($e(\rho_0) = -16.0 \pm 0.2$ MeV) and **density** ($\rho_0 = 0.160 \pm 0.005$ fm^{-3}) of **symmetric nuclear matter**.

DD-ME-min1:

- **binding energies, charge radii, diffraction radii and surface thicknesses** of 17 even-even spherical nuclei, ^{16}O, ^{40,48}Ca, ^{56,58}Ni, ^{88}Sr, ^{90}Zr, ^{100,112,120,124,132}Sn, ^{136}Xe, ^{144}Sm and ^{202,208,214}Pb. The assumed errors of these observables are 0.2%, 0.5%, 0.5%, and 1.5%, respectively.

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

The **neutron skin** is **correlated** with **L** in both models but **NOT** with α_D. **(I will come back on that latter)**

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

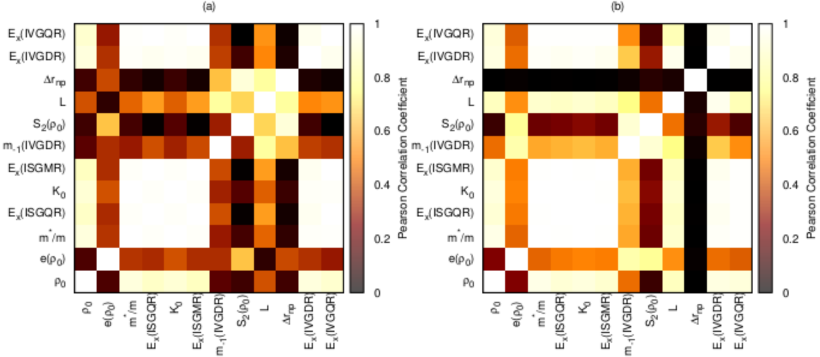
A	SLy5-min A_0		$\sigma(A_0)$	DDME-min1 A_0		$\sigma(A_0)$	units
SNM							
ρ_0	0.162	±	0.002	0.150	±	0.001	fm^{-3}
$e(\rho_0)$	−16.02	±	0.06	−16.18	±	0.03	MeV
m^*/m	0.698	±	0.070	0.573	±	0.008	
J	32.60	±	0.71	33.0	±	1.7	MeV
K_0	230.5	±	9.0	261	±	23	MeV
L	47.5	±	4.5	55	±	16	MeV
^{208}Pb							
E_x^{ISGMR}	14.00	±	0.36	13.87	±	0.49	MeV
E_x^{ISGQR}	12.58	±	0.62	12.01	±	1.76	MeV
Δr_{np}	0.1655	±	0.0069	0.20	±	0.03	fm
E_x^{IVGDR}	13.9	±	1.8	14.64	±	0.38	MeV
m_{-1}^{IVGDR}	4.85	±	0.11	5.18	±	0.28	MeV^{-1} fm^2
E_x^{IVGQR}	21.6	±	2.6	25.19	±	2.05	MeV

Statistical uncertainties depend on the fitting protocol, that is on the **data (or pseudo-data) and associated errors used for the fits: Let us see an example...**

Covariance analysis: modifying the χ^2

→ **SLy5-a:** χ^2 as in SLy5-min except for the neutron EoS (relaxed the required accuracy = increasing associated error).

→ **SLy5-b:** χ^2 as in SLy5-min except the neutron EoS (not employed) and used instead a tight constraint on the Δr_{np} in ^{208}Pb



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

- When a **constraint** on a property is **relaxed**, **correlations** of other observables with such a property should become **larger** → **SLy5-a: α_D is now better correlated with Δr_{np}**
- When a **constraint** on a property is **enhanced** —artificially or by an accurate experimental measurement— **correlations** of other observables with such a property should become **small** → **SLy5-b: Δr_{np} is not correlated with any other observable**