Implications for nuclear structure from precision neutron skin measurements

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Weak Elastic Scattering with Nuclei
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(*) Work in progress

(†) Not cover in the talk but give the slides for discussions

The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different predictions for many-body observables** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals (EDFs):

Based on effective interactions solved at the HF level, **EDFs** are **successful** in the description of **ground and excited state** properties such as m, $\langle r^2\rangle^{1/2}$ **or GR** along the **nuclear chart**

Main types of EDFs:

Relativistic mean-field models (RMF), based on Lagrangians where effective mesons carry the interaction:

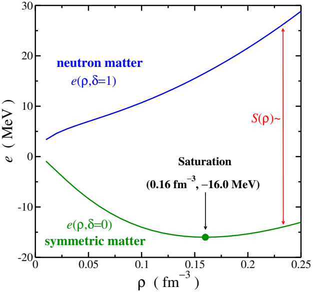
$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma && +\bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ &-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} && -\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\boldsymbol{\tau}\Psi A^{(\rho)\mu}\end{aligned}$$

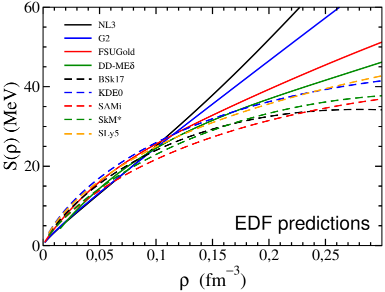
Non-relativistic mean-field models (NRMF), based on Hamiltonians where ef f. interactions are proposed and tested:

$$V_{\rm Nucl}^{\rm eff} = V_{\rm attractive}^{\rm long-range} + V_{\rm repulsive}^{\rm short-range} + V_{\rm SO} + ...$$

-EDFs are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction** in the vacuum
-EDFs derived from a **Mean-Field** $\rightarrow$ we expect bulk properties more accurate as heavier is the nucleus

The Nuclear Equation of State: Infinite System



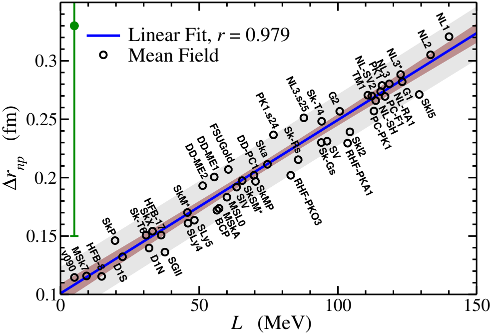


$$e(\rho, \beta) = e(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \quad \text{where } \beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

-**Isovector properties** not well determined in current **EDFs**
-**Parity violating program** (JLab/Mainz), **full E1 response** in stable and exotic nuclei (RCNP/GSI) and measurements on exotic nuclei in **Rare Ion Beam Facilites** worldwide → **better characterization of isovector properties around saturation density**

But how we can better constraint the isovector channel from observables? (Example)

Neutron skin thickness → is one of the most paradigmatic example of an **isovector sensitive observable**.

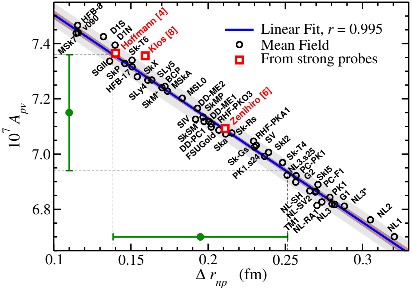


$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \sim \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

$$\text{where } J \equiv S(\rho_0) \text{ and } L \equiv 3\rho_0 \frac{\partial S}{\partial \rho}\Big|_{\rho_0} = 3\rho_0 p_0^{neut}$$

The impact of a neutron skin measurement on other nuclear observables

The neutron skin and the parity violating asymmetry in ^{208}Pb



Physical Review Letters **106**, *252501 (2011)*

(Calculation at a fixed q equal to PREx)

- $\rightarrow$ Electrons interact by exchanging a γ **(couples to** p**)** or a Z_0 **boson (couples to** n**)**
- $\rightarrow$ Ultra-relativistic electrons, **depending on their helicity** ($\pm$), will interact with the nucleus **seeing a slightly different potential**: **Coulomb** $\pm$ **Weak**
- $\rightarrow$ $$A_{pv} \equiv \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$
- $\rightarrow$ Input for the calculation are the ρ_p and ρ_n **(main uncertainty)** and nucleon form factors for the e-m and the weak neutral current.
- $\rightarrow$ In PWBA for small momentum transfer:
$$A_{pv} \approx \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 \langle r_p^2 \rangle^{1/2}}{3F_p(q)} \Delta r_{np}\right)$$

The largest the size of the neutron distribution in nuclei (Δr_{np}), the smaller the parity violating asymmetry.
[Exp. from ew probes: 0.302 ± 0.175 **fm (***Physical Review C* **85**, *032501 (2012)***)].**

Dipole polarizability: definition

From a macroscopic perspective

The electric **polarizability** measures the tendency of the nuclear **charge distribution to be distorted**

$$\alpha_D \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

From a microscopic perspective

The electric **polarizability** is proportional to the **inverse energy weighted sum rule (IEWSR)** of the electric dipole response in nuclei

$$\alpha_D = \frac{8\pi}{9} e^2 \sum \frac{B(E1)}{E}$$

or

$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_{\text{ph. abs.}}(E)}{E^2} dE$$

Dipole polarizability: macroscopic approach

The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

Adopting the Droplet Model ($m_{-1} \propto \alpha_D$):

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

Bulk - First derived by Migdal

Surface correction - first derived by J. Meyer, P. Quentin, and B. Jennings, Nucl. Phys. A 385, 269 (1982)

within the same model, connection with the neutron skin thickness:

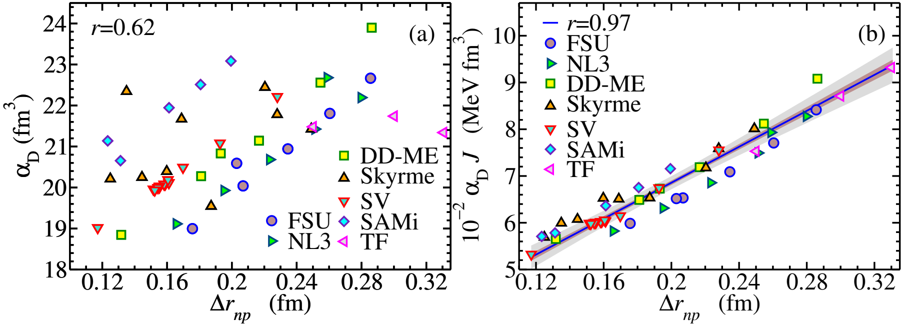
$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\mathrm{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

(*) $\Delta r_{np}^{\mathrm{surface}}$ depends on the difference between the neutron and proton difussivities (surface fall-off). This is well constraint in fit to masses and radii → all EDFs agree within a very small dispersion in closed shell such as ^{208}Pb.

Is this correlation appearing also in EDFs?

Isovector Giant Dipole Resonance in ^{208}Pb:

Dipole polarizability: microscopic results HF+RPA

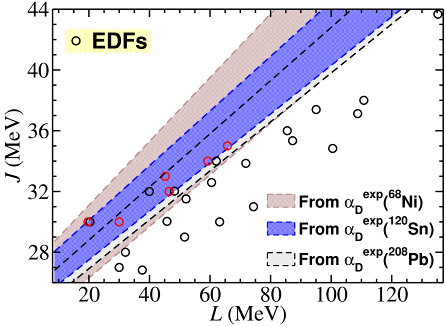


X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

$\alpha_D J$ is linearly correlated with Δr_{np} and no α_D alone within EDFs

Constraints of this analysis on the $J-L$ plane

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1+\frac{5}{3}\frac{L}{J}\frac{\rho_0-\rho_A}{3\rho_0}\right] \text{ where } S(\rho_A)\equiv a_{sym}(A)$$



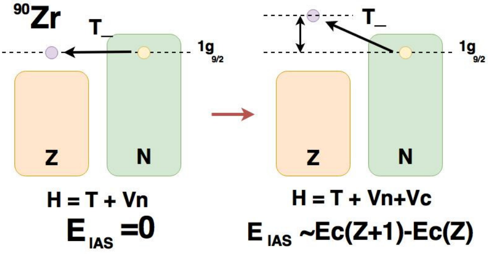
$J=(24.9\pm2.0)+(0.19\pm0.02)L$ for ^{68}Ni
$J=(25.4\pm1.1)+(0.17\pm0.01)L$ for ^{120}Sn
$J=(24.5\pm0.8)+(0.168\pm0.007)L$ for ^{208}Pb

For $S(<\rho>\to\rho_0)\approx J-L\frac{(\rho_0-<\rho>)}{3\rho_0}$

X. Roca-Maza et al. Phys. Rev. C **92**, *064304 (2015)*

Subset of models that reproduce simultaneously measured polarizabilities predict $J=30-35$ **MeV**, $L=20-66$ **MeV; and Δr_{np} in ^{68}Ni, ^{120}Sn, and ^{208}Pb are in the ranges: 0.15-0.19 fm, 0.12-0.16 fm, and 0.13-0.19 fm**

The isobaric analog state energy: E_{IAS}



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$
- **Displacement energy or** E_{IAS}

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

$E_{IAS} \neq 0$ **only due to Isospin Symmtry Breaking terms** $\mathcal{H}$
E_{IAS}^{exp} **usually accuratelly measured !**

Coulomb direct contribution: very simple model

• Assuming indepentent particle model and good isospin for $|0\rangle$ ($\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z$)

$$E_{IAS} \approx E_{IAS}^{C,direct} = \frac{1}{N-Z}\int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{direct}(\vec{r})d\vec{r}$$

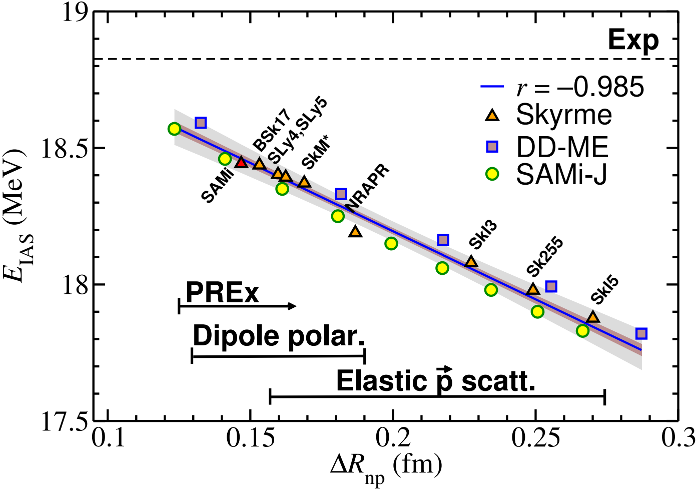
where $U_C^{direct}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{ch}(\vec{r}_1)d\vec{r}_1$

• Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

E_{IAS} in Energy Density Functionals (No Corr.)



Phys. Rev. Lett. 120, 202501 (2018)

Nuclear models (EDFs) where the nuclear part is isospin symmetric and U_{ch} is calculated from the ρ_p

Corrections: within self-consistent HF+RPA

Within the **HF+RPA** one can **estimate** the E_{IAS} accounting (in an effective way) for **short-range correlations and effects of the continuum** (if a large sp base is adopted).

- **Coulomb exchange** exact (usually Slater approx.):

$$U_C^{x,\text{exact}}\varphi_i(\vec{r}) = -\frac{e^2}{2}\int d^3r' \, \frac{\varphi_j^*(\vec{r}')\varphi_j(\vec{r})}{|\vec{r}-\vec{r}'|}\varphi_i(\vec{r}')$$

- The **electromagnetic spin-orbit** correction to the nucleon single-particle energy (non-relativistic),

$$\varepsilon_i^{\text{emso}} = \frac{\hbar^2c^2}{2m_i^2c^4}\langle\vec{l}_i\cdot\vec{s}_i\rangle x_i \int \frac{1}{r}\frac{dU_C}{dr}|R_i(r)|^2$$

where x_i: $g_p - 1$ for Z and g_n for N; $g_n = -3.82608545(90)$ and $g_p = 5.585694702(17)$, $R_i \to R_{nl}$ radial wf.

Corrections:

- **Finite size** effects (assuming spherical symmetry):

$$\rho_{ch}(q) = \left(1 - \frac{q^2}{8m^2}\right)\left[G_{E,p}(q^2)\rho_p(q) + G_{E,n}(q^2)\rho_n(q)\right] - \frac{\pi q^2}{2m^2}\sum_{l,t}\left[2G_{M,t}(q^2) - G_{E,t}(q^2)\right]\langle\vec{l}\cdot\vec{s}\rangle\int_0^\infty dx\frac{j_1(qx)}{qx}|R_{nl}(x)x^2|^2$$

- **Vacuum polarization:** lowest order correction in the fine-structure constant to the Coulomb potential $\frac{eZ}{r}$:

$$V_{vp}(\vec{r}) = -\frac{2}{3}\frac{\alpha e^2}{\pi}\int d\vec{r}'\,\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathcal{K}_1\left(\frac{2}{\lambda\!\!\!^{-}_e}|\vec{r}-\vec{r}'|\right)$$

where e is the fundamental electric charge, α the fine-structure constrant, $\lambda\!\!\!^{-}_e$ the reduced Compton electron wavelength and

$$\mathcal{K}_1(x) \equiv \int_1^\infty dt e^{-xt}\left(\frac{1}{t^2}+\frac{1}{2t^4}\right)\sqrt{t^2-1}$$

Corrections:

- **Isospin symmetry breaking** (Skyrme-like): **two parts** (contact interaction)

charge symmetry breaking +

$$V_{CSB} = V_{nn} - V_{pp}$$

$V_{CSB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{4}[\tau_z(1) + \tau_z(2)]\, s_0 (1 + y_0 P_\sigma)$

τ_z Pauli in isospin space; P_σ are the usual projector operators in spin space.

charge independence breaking*

$$V_{CIB} = \frac{1}{2}(V_{nn} + V_{pp}) - V_{pn}$$

$V_{CIB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{2}\tau_z(1)\tau_z(2) u_0 (1 + z_0 P_\sigma)$

* general operator form $\tau_z(1)\tau_z(2) - \frac{1}{3}\vec{\tau}(1)\cdot\vec{\tau}(2)$. Our prescription $\tau_z(1)\tau_z(2)$ not change structure of HF+RPA.

- Opposite to the other corrections, **ISB contributions depends on new parameters that need to be determined!**

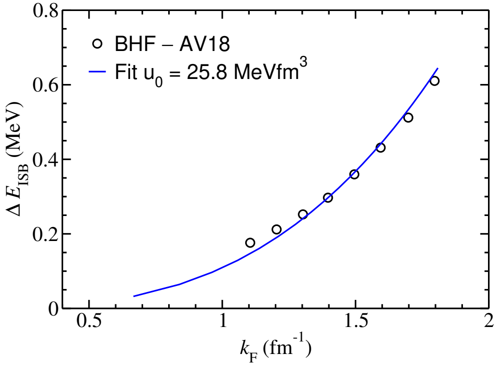
Isospin symmetry breaking in the medium:

• **keeping** things **simple**: **CSB and CIB** interaction just **delta function** depending on s_0 **and** u_0. **Different possibilities:**

→ **Fitting** to (two) experimentally known **IAS energies**

→ **Derive from theory**

→ **our option:** u_0 **to reproduce BHF** (symmetric nuclear matter) and s_0 **to reproduce** E_{IAS} **in** ^{208}**Pb**



Physics Letters B 445, 259 (1999)

Re-fit of SAMi: SAMi-ISB

- All these **corrections** are relatively **small** but **modify binding energies, neutron and proton distributions, etc.** ⇒ a **re-fit of the interaction is needed**.
- Use **SAMi fitting protocol** (special care for spin-isospin resonances) including all corrections and **find SAMi-ISB**

Table: Saturation properties

	SAMi	SAMi-ISB	
ρ_∞	0.159	0.1613(6)	fm^{-3}
e_∞	−15.93	−16.03(2)	MeV
m^*_{IS}	0.6752	0.730(19)	
m^*_{IV}	0.664	0.667(120)	
J	28	30.8(4)	MeV
L	44	50(4)	MeV
K_∞	245	235(4)	MeV

SAMi-ISB finite nuclei properties

El.	N	B [MeV]	B^{exp} [MeV]	r_c [fm]	r_c^{exp} [fm]	ΔR_{np} [fm]
Ca	28	417.67	415.99	3.49	3.47	0.214
Zr	50	783.60	783.89	4.26	4.27	0.097
Sn	82	1102.75	1102.85	4.73	–	0.217
Pb	126	1635.78	1636.43	5.50	5.50	0.151

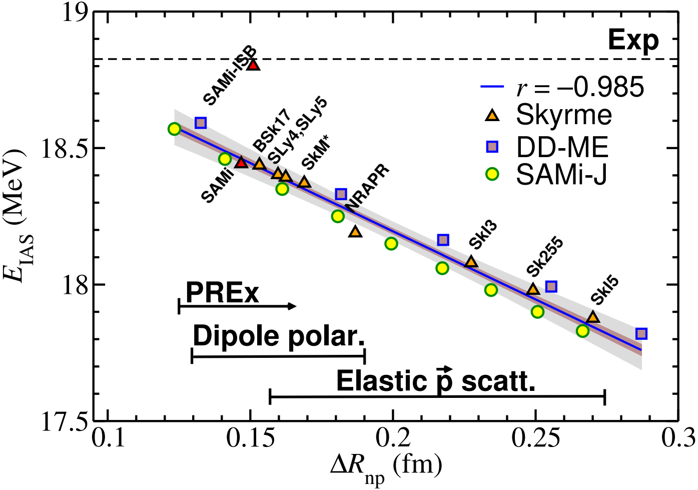
Corrections on E_{IAS} for ^{208}Pb one by one

	E_{IAS} [MeV]	Correction [keV]
No corrections[a]	18.31	
Exact Coulomb exchange	18.41	+100
n/p mass difference	18.44	+30
Electromagnetic spin-orbit	18.45	+10
Finite size effects	18.40	-50
Vacuum polarization (V_{ch})	18.53	+130
Isospin symmetry breaking	**18.80(5)**	+270

[a] From Skyrme Hamiltonian where the nuclear part is isospin symmetric and V_{ch} is calculated from the ρ_p

$E_{IAS}^{exp} = 18.83 \pm 0.01$ MeV. *Nuclear Data Sheets 108, 1583 (2007).*

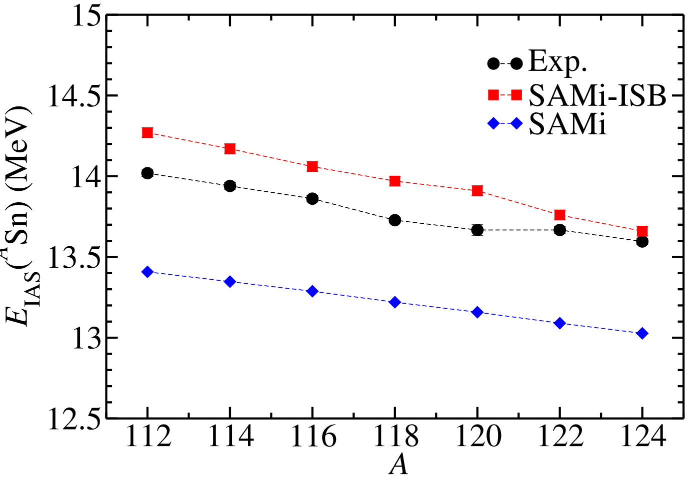
E_{IAS} with SAMi-ISB



Phys. Rev. Lett. 120, 202501 (2018)

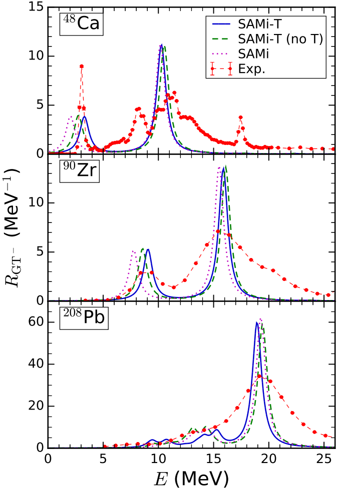
Measurement of $\Delta r_{np} \rightarrow$ determine ISB in the nuclear medium

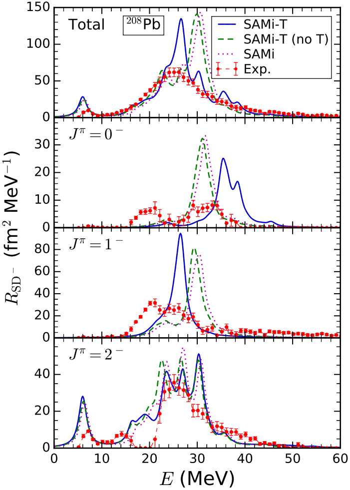
Prediction: E_{IAS} in the Sn isotopic chain



Phys. Rev. Lett. 120, 202501 (2018)

SAMi-T: Skyrme functional with tensor terms from ab initio calculations (Gamow-Teller and Spin Dipole)





Despcription of low and high energy GT peaks

Improved 1^- channel gives largest contribution

Shihang Shen et al. arXiv:1810.09691

SDR and the neutron skin thickness Δr_{np}:

$$\int [R_{SD^-}(E) - R_{SD^+}(E)] dE = \frac{9}{4\pi}(N\langle r_n^2\rangle - Z\langle r_p^2\rangle)$$
$$\approx (N-Z)\langle r_p^2\rangle \left(1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)$$

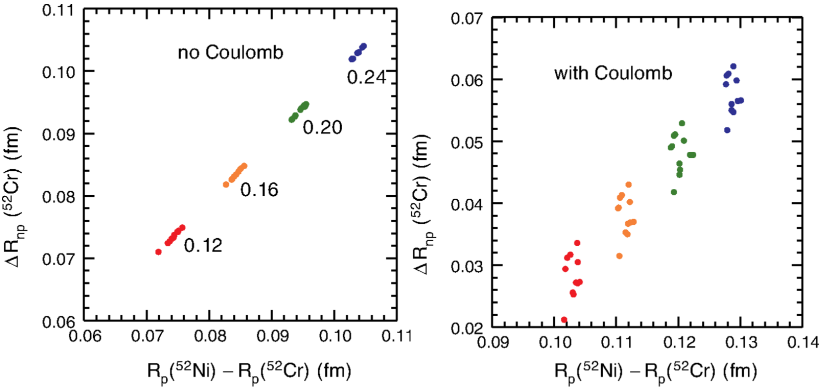
• Experimental NEWSR in ^{208}**Pb** is 1004^{+24}_{-23} **fm^2**; SAMi is **1224 fm^2**; and **SAMi-T 1260± 10 fm^2** **(some strength is missing in the experimental measurement ?** $\Delta r_{np} \approx 0.05$ **fm**).

• Experimental NEWSR in ^{90}**Zr** is 148 ± 12 **fm^2**; SAMi is **150 fm^2**; and SAMi-T 147 ± 1 **fm^2** $\Rightarrow$ **neutron skin should be properly determined by SAMi and SAMi-T**

Differences in the proton radii of mirror nuclei

If isospin symmetry conserved (ISC) in nuclei

- $r_n(N,Z) = r_p(Z,N)$
- $\Delta r_{np}(N,Z) \equiv r_n(N,Z) - r_p(N,Z) = r_p(Z,N) - r_p(N,Z)$



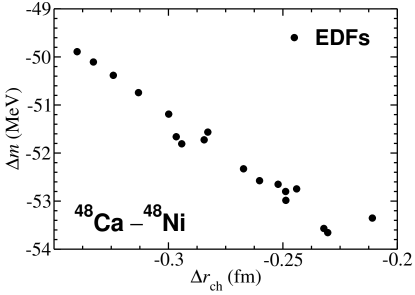
B. A. Brown, Phys. Rev. Lett. 119 122502 (2017)

$\Delta r_{ch}(N,Z)$ is better correlated than $\Delta r_{np}(N,Z)$ with L. (Using EDFs fitted by the same group). ←no explanation given.

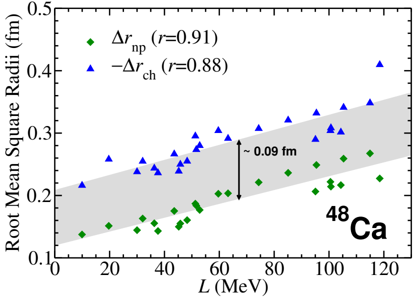
Differences in the charge radii of mirror nuclei

$$\Delta m \equiv m(N,Z) - m(Z,N)$$
$$\approx \frac{3}{5}\frac{Z(Z-1)e^2}{R_{ch}^{(N,Z)}} - \frac{3}{5}\frac{N(N-1)e^2}{R_{ch}^{(Z,N)}}$$
$$\approx -E_C \frac{(N-Z)(N+Z-1)}{Z(Z-1)}\left(1 + \frac{\Delta R_{ch}}{R_{ch}^{(N,Z)}}\right)$$

$$\Delta r_{np}(N,Z) + \Delta r_{ch}(N,Z) \approx$$
$$-\frac{e^2 N}{35J}\left(1 - \frac{Z}{4N}\right) + \sqrt{\frac{3}{5}}\frac{5}{2}\frac{b_n^2(N,Z) - b_p^2(Z,N)}{R}$$
$$\approx -0.04\,\mathrm{fm}$$



Work in progress



Work in progress

Correlation of $\Delta r_{np}(N,Z)$ or $\Delta r_{ch}(N,Z)$ with L seems to be the same in EDFs $\Rightarrow$ from $\Delta r_{np}(N,Z)$ derive R_{ch} and $M(N,Z)$ of very exotic proton rich nuclei

Co-workers:

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H. Sagawa (Aizu U. and RIKEN, Japan)

*Passed away summer 2018.

Thank you for your attention!

EXTRA EXAMPLES

Isovector Giant Resonances (some considerations)

→ In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**

→ **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"

→ The **excitation energy (E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

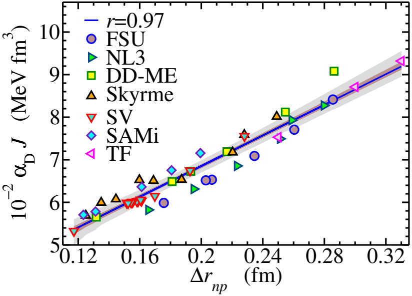
$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \rightarrow E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{\mathbf{S(\rho)}}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

→ The **dipole polarizability ($\alpha \sim \int \frac{\sigma_{\gamma-abs}}{\text{Energy}^2} \sim$ IEWSR)** measures the tendency of the nuclear charge distribution to be distorted, that is, from a **macroscopic** point of view

$$\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

Dipole polarizability and the neutron skin in ^{208}Pb



Physical Review C **85** *041302 (2012);* **88** *024316 (2013);* **92**, *064304 (2015)*

Macroscopic model:

→ Using the **dielectric theorem:** m_{-1} moment can be computed from the expectation value of the Hamiltonian in the constrained (D dipole operator) ground state $\mathcal{H}' = \mathcal{H} + \lambda D$

→ Assuming the **Droplet Model** (heavy nucleus):

$$\alpha_D \approx \alpha_D^{bulk}\left[1 + \frac{1}{5}\frac{L}{J}\right] \text{ where}$$

$$\alpha_D^{bulk} \equiv \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J} \text{ (Migdal first derived)}$$

→ $L \approx \dfrac{\alpha_D^{exp} - \alpha_D^{bulk}}{\alpha_D^{bulk}} 5J$

By using the Droplet Model one can also find:

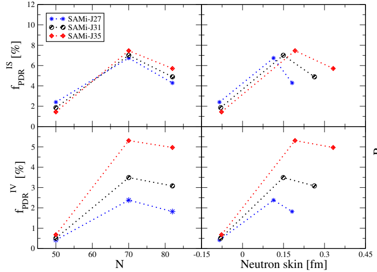
$$\alpha_D J \approx \frac{\pi e^2}{54} A\langle r^2\rangle \left[1 + \frac{5}{2}\frac{\Delta r_{np} - \Delta r_{np}^{coul} - \Delta r_{np}^{surf}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

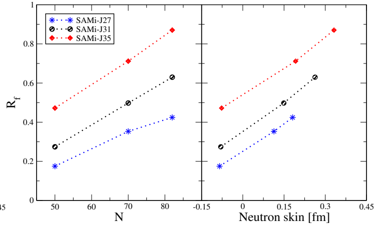
For a fixed value of the symmetry energy at saturation, the larger the neutron skin in ^{208}Pb, the larger the dipole polarizability.

Pygmy Dipole Resonance

(Pygmy: low-energy excited state appearing in the dipole response of $N \neq Z$ nuclei)

- Sn isotopes, SAMi-J interactions
- Outermost neutrons contributing to the IS and IV pygmy state (f_{PDR} fraction of the EWSR and $R_f = f_{PDR}^{IV}/f_{PDR}^{IS}$)

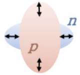




WARNING: we lack of a clear understanding of the physical reason for this correlation. Models used fitted by the same group → possible bias.

S. Burrello et al. arXiv:1807.10118

IV-IS GQRs and the neutron skin in ^{208}Pb



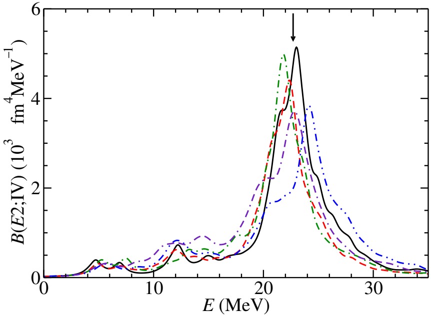
Within the Quantum Harmonic Oscillator approach

$$E_x^{IV} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m}\frac{V_{sym}\langle r^2\rangle}{(\hbar\omega_0)^2\langle r^4\rangle}}$$

and EDF calculations, one can deduce

$$V_{sym} \approx 8(S(\rho_A) - S^{kin}(\rho_0))$$

$$S^{kin}(\rho_0) \approx \varepsilon_{F_0}/3 \text{ (Non-Rel)}$$

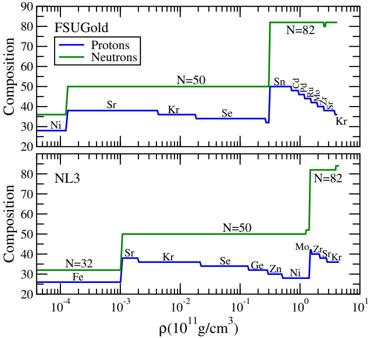


$$S(\rho_A) \approx J - L\frac{\rho_0 - \rho_A}{3\rho_0} \approx \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$

The larger the neutron skin in ^{208}Pb, the smallest the difference between the IS and IV excitation energies in GQRs.

The neutron skin in ^{208}Pb and the structure and composition of a neutron star outer crust

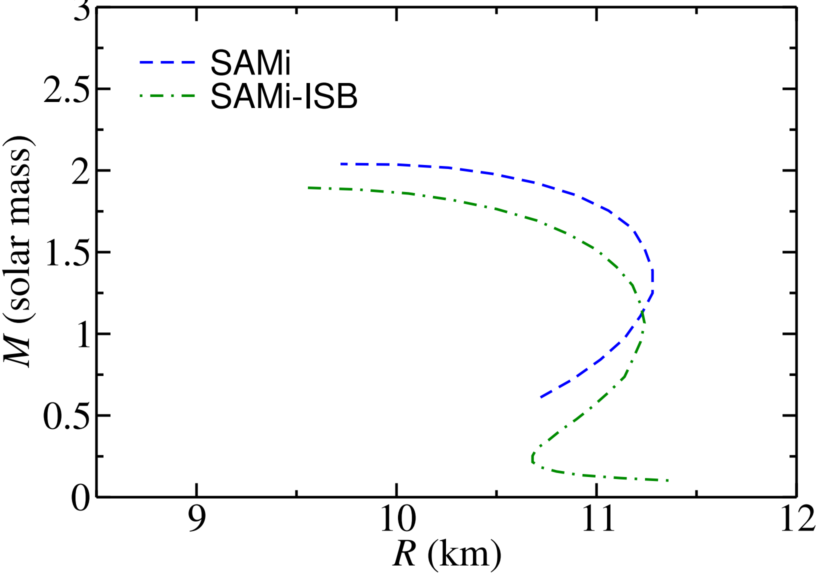
- → span **7** orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g/cm to the **neutron drip** $\sim 10^{11}$ g/cm)
- → it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- → $T \sim 10^6$ K ~ 0.1 keV → one can treat **nuclei and electrons at $T = 0$ K**
- → At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}**Fe** nuclei.
- → As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$ and therefore $Z \downarrow$ with constant (approx) number of N
- → As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a dif ferent $N-$*plateau*
  $\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{Fe}}{8a_{sym}}$ where $(A_0, Z_0) = {}^{56}\text{Fe}_{26}$
- → The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$)**.
  $[M(N, Z) + m_n < M(N+1, Z)]$



Physical Review C **78**, *025807 (2008)*

The larger the neutron skin of ^{208}Pb ($L \uparrow$), the more exotic the composition of the outer crust.

Mass Radius ISB



EXTRA MATERIAL

In more detail (from theory) ...

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \ (\Delta L = 1 \to Dipole)$$

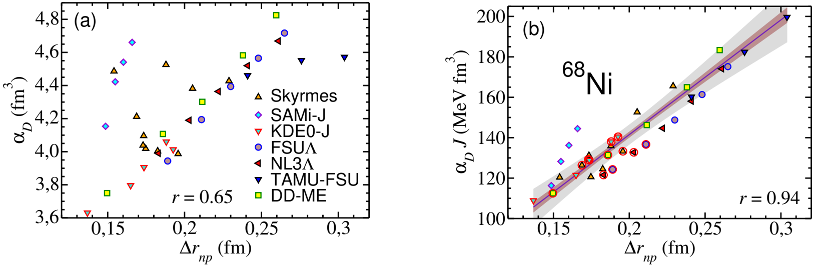
- is proportional to the **static polarizability** for small oscillations $\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**

The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

Isovector Giant Dipole Resonance in ^{68}Ni: 

What about other nuclei?

Dipole polarizability: microscopic results HF+RPA



X. Roca-Maza et al. Phys. Rev. C **92**, *064304 (2015)*

Experimental dipole polarizability $\alpha_D = 3.40 \pm 0.23$ fm^3 D. M. Rossi *et al.*, PRL 111, 242503 (GSI). $\alpha_D = 3.88 \pm 0.31$ fm^3 “full” response D. M. Rossi, T. Aumann, and K. Boretzky.

STATISTIC UNCERTAINTIES AND CORRELATIONS IN EDFs

Example on the dipole polarizability

Covariance analysis: χ^2 test

- Observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$ (e.g. of an EDF)
$$\chi^2(\mathbf{p}) = \frac{1}{m - n_p - 1} \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta\mathcal{O}_i^{\text{ref.}}} \right)^2$$
- Assuming that the χ^2 can be approximated by an hyper-parabola around the minimum $\mathbf{p}_0$,
$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j})$$
where $\mathcal{M} \equiv \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2$ (curvature m.) and $\mathcal{E} \equiv \mathcal{M}^{-1}$ (error m.).
- errors between predicted observables $\mathcal{A}$
$$\Delta\mathcal{A} = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A}$$
- correlations between predicted observables,
$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}}$$
where, $C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$

Example: Fitting protocol of SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

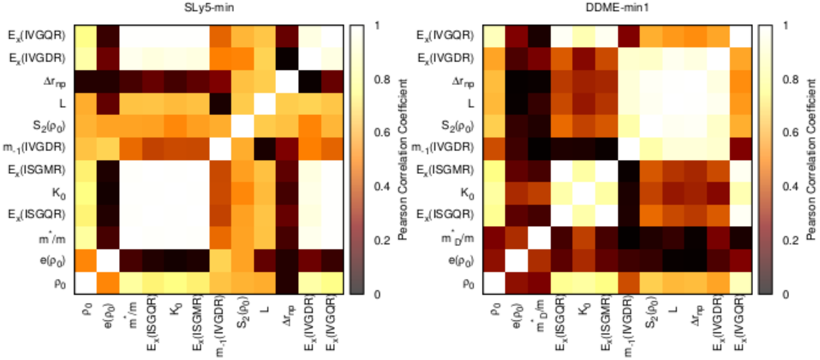
SLy5-min:

- **Binding energies** of ^{40,48}Ca, ^{56}Ni, ^{130,132}Sn and ^{208}Pb with a fixed adopted error of 2 MeV
- the **charge radius** of ^{40,48}Ca, ^{56}Ni and ^{208}Pb with a fixed adopted error of 0.02 fm
- the **neutron matter** Equation of State calculated by Wiringa *et al.* (1988) for densities between 0.07 and 0.40 fm^{-3} with an adopted error of 10%
- the **saturation energy** ($e(\rho_0) = -16.0 \pm 0.2$ MeV) and **density** ($\rho_0 = 0.160 \pm 0.005$ fm^{-3}) of **symmetric nuclear matter**.

DD-ME-min1:

- **binding energies, charge radii, diffraction radii and surface thicknesses** of 17 even-even spherical nuclei, ^{16}O, ^{40,48}Ca, ^{56,58}Ni, ^{88}Sr, ^{90}Zr, ^{100,112,120,124,132}Sn, ^{136}Xe, ^{144}Sm and ^{202,208,214}Pb. The assumed errors of these observables are 0.2%, 0.5%, 0.5%, and 1.5%, respectively.

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

The **neutron skin** is **correlated** with **L** in both models but **NOT** with α_D. **(I will come back on that latter)**

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

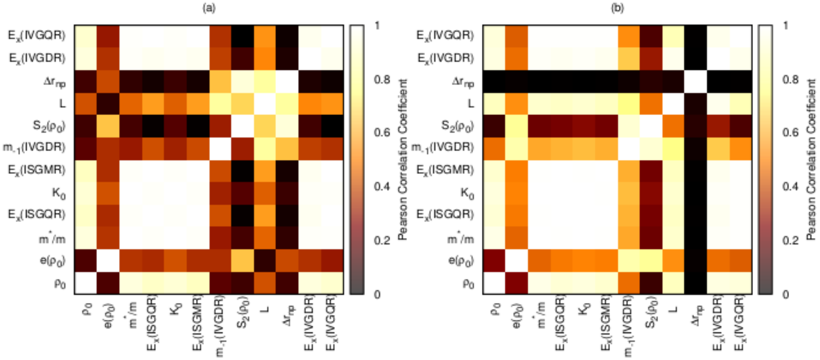
A	SLy5-min A_0		$\sigma(A_0)$	DDME-min1 A_0		$\sigma(A_0)$	units
SNM							
ρ_0	0.162	±	0.002	0.150	±	0.001	fm^{-3}
$e(\rho_0)$	−16.02	±	0.06	−16.18	±	0.03	MeV
m^*/m	0.698	±	0.070	0.573	±	0.008	
J	32.60	±	0.71	33.0	±	1.7	MeV
K_0	230.5	±	9.0	261	±	23	MeV
L	47.5	±	4.5	55	±	16	MeV
^{208}Pb							
E_x^{ISGMR}	14.00	±	0.36	13.87	±	0.49	MeV
E_x^{ISGQR}	12.58	±	0.62	12.01	±	1.76	MeV
Δr_{np}	0.1655	±	0.0069	0.20	±	0.03	fm
E_x^{IVGDR}	13.9	±	1.8	14.64	±	0.38	MeV
m_{-1}^{IVGDR}	4.85	±	0.11	5.18	±	0.28	$MeV^{-1}\ fm^2$
E_x^{IVGQR}	21.6	±	2.6	25.19	±	2.05	MeV

Statistical uncertainties depend on the fitting protocol, that is on the **data (or pseudo-data) and associated errors used for the fits: Let us see an example...**

Covariance analysis: modifying the χ^2

→ **SLy5-a:** χ^2 as in SLy5-min except for the neutron EoS (relaxed the required accuracy = increasing associated error).

→ **SLy5-b:** χ^2 as in SLy5-min except the neutron EoS (not employed) and used instead a tight constraint on the Δr_{np} in ^{208}Pb



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

- When a **constraint** on a property is **relaxed**, **correlations** of other observables with such a property should become **larger** → **SLy5-a: α_D is now better correlated with Δr_{np}**
- When a **constraint** on a property is **enhanced** —artificially or by an accurate experimental measurement— **correlations** of other observables with such a property should become **small** → **SLy5-b: Δr_{np} is not correlated with any other observable**