Spin-isospin resonances with the new functionals: SAMi, SAMi-T and SAMi-ISB

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  [X. Roca-Maza, G. Colò, H. Sagawa, Phys.Rev. C86 (2012) 031306(R)]
- **Improvement of SAMi: SAMi-T (including tensor terms)**

  [Shihang Shen, G. Colò, X. Roca-Maza, arXiv:1810.09691 (2018)]
- **Improvement of SAMi: SAMi-ISB (including Isospin Symmetry Breaking terms)**

  [X. Roca-Maza, G. Colò, and H. Sagawa, Phys. Rev. Lett. 120 (2018) 202501]

Spin and Isospin excitations in Nuclei

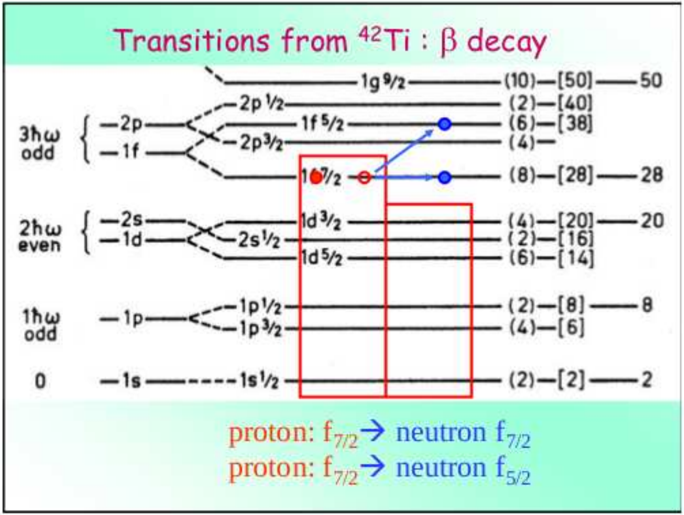
- **Nucleons** are fermions charac. by their spin and isospin
- **Nucleons** with spin (isospin) may **change their state** in **phase**: spin-scalar S=0 modes (isospin-scalar T=0 modes); or **out of phase**: spin-vector S=1 modes (isospin-vector T=1 modes)
- They can be **excited by strong probes** (charge-exhange reactions) and they can **decay via the weak interaction** (axial-vector current couples to the spin and induces $\beta-$decay processes)

One of the most important nuclear excitation modes is the

- **Gamow Teller Resonance** which is a pure **spin-isospin mode** (i.e., from a theoretical picture, it is excited by an operator $\hat{O} \sim \sigma\tau$)

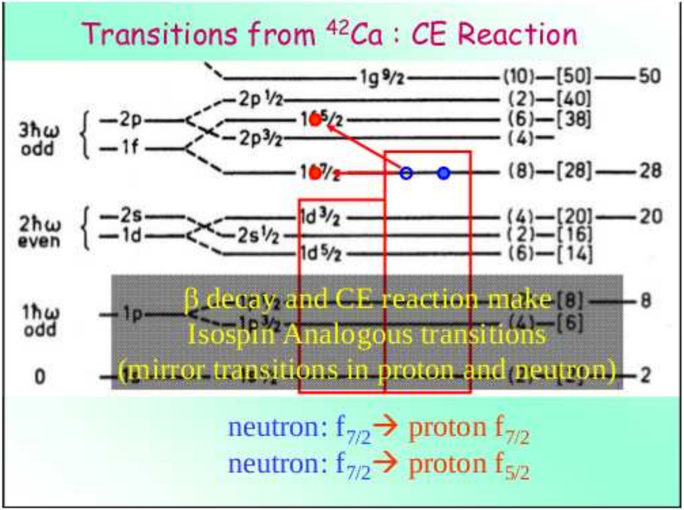
Spin-isospin modes of excitation (such as the **GTR**) give **direct information** on the spin-isospin channel of the **effective interaction**

Example: β−decay transition



Courtesy of Y. Fujita; taken from his lectures http://www.mi.infn.it/~colo/lectures/lectures.html

Example: Gamow Teller transition



Courtesy of Y. Fujita; taken from his lectures **http://www.mi.infn.it/~colo/lectures/lectures.html**

Motivation:

- allowed **GT** transitions mainly detemine **β-decay half-lives**
- **GT** transitions determine **weak interaction rates** essential role in the **core-collapse dynamics** of massive stars leading to supernova explosion
- In neutron-rich environment, **neutrino-induced nucleosynthesis** may take place via **GT** processes
- **GT** matrix elements are necessary for the study of **double-β-decay**
- may be useful in the **calibration of detectors** used to measure neutrinos that reach the Earth

Some comments on the nuclear many-body problem:

- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different predictions** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** **(but still not perfect)** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals:

Main types of successful EDFs derived from the mean-field approximation

- **Relativistic H o HF models,** based on Lagrangians where effective (heavy) mesons carry the interaction.

$$\begin{aligned}\mathcal{L}_{int} \quad &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma \quad +\bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ &-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} \quad -\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\tau\Psi A^{(\rho)\mu} \\ &-e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}\end{aligned}$$

- **Non-relativistic HF models,** based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\text{eff}}_{\text{Nucl}} = V^{\text{long-range}}_{\text{attractive}} + V^{\text{short-range}}_{\text{repulsive}} + V_{\text{SO}}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction**

Drawbacks on current EDFs ???

On the one side,

- **we expect** that the **H(F)+RPA** method based on nuclear effective interactions of the **Skyrme, Gogny or Relativistic** (can be understood as an **approximate realization of an EDF**) ⇒ **reasonable description of g.s. energy and density of the system** (<one-body operators>)

On the other side,

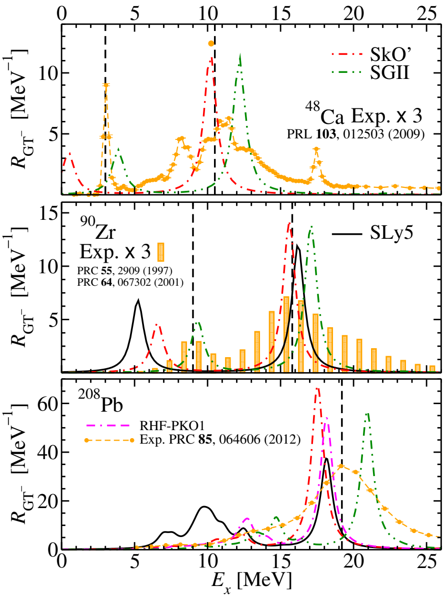
- there are still many **open problems** ... we will concentrate here on how to

improve the spin-isospin properties of our EDF

Motivation: Gamow Teller Resonance

The E_x is not properly described in H(F)+RPA

- **SGII**[a]: earliest attempt to give a quantitative description of the GTR
- **SkO'**[b]: accurate in ground state finite nuclear properties and improves the GTR
- **PKO1**[c]: relativistic HF, reasonable GTR still not perfect
- Relativistic H[d]: residual interaction modified *ad-hoc*



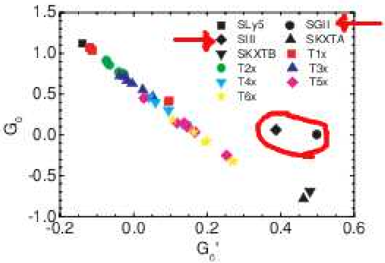
[a] PLB **106**, 379 (1981), [b] PRC **60**, 014316 (1999), [c] PRL **101**, 122502 (2008), [d] PRC 69, 054303

Motivation: which gs properties are important for describing the E_x^{GTR}?

The study[a] of the GTR and the spin-isospin Landau-Migdal parameter G_0' using several Skyrme sets,

- concluded that G_0' **is not the only important quantity** in determining the **excitation energy of the GTR**
- **spin-orbit splittings** also influences the GTR

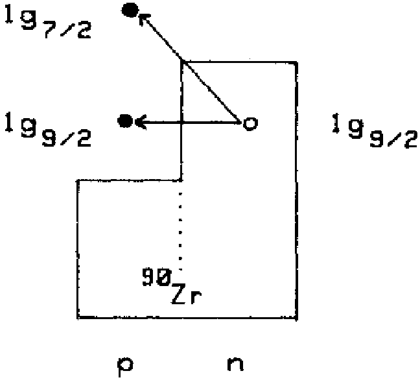
- Empirical indications[b] suggest that $G_0' > G_0 > 0$
- **Not a very common feature** within available Skyrme forces[c]



[a] M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C **65**, 054322 (2002); [b] T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005); T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999),[c] Li-Gang Cao, G. Colo, and H. Sagawa, Phys. Rev. C **81**, 044302 (2010)

Why spin-orbit splittings are important?

$E_x^1 \approx$
$\epsilon_{\pi 1 g_{7/2}} - \epsilon_{\nu 1 g_{9/2}} + \epsilon_{ph}^1$
$E_x^2 \approx$
$\epsilon_{\pi 1 g_{9/2}} - \epsilon_{\nu 1 g_{9/2}} + \epsilon_{ph}^2$
$\Delta E_x \approx \Delta\epsilon_{\pi 1 g} + \Delta\epsilon_{ph}$



Schematic picture of single-particle transitions involved in the Gamow Teller Resonance of ^{90}Zr. Transitions excited by $\sigma\tau_-$ operator.

F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992)

Skyrme Model

Hamiltonian[a]

Includes **central tensor terms (J^2 terms)** due to the coupling of tensor and spin and gradients terms and **two spin-orbit parameters** (same as SkO and some SkI forces)

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}} + \mathcal{H}_{\text{SO}} + \mathcal{H}_{\text{sg}} + \mathcal{H}_{\text{Coul}}$$

$$\begin{aligned}
\mathcal{K} &= \hbar^2\tau/2m \\
\mathcal{H}_0 &= (1/4)t_0[(2+x_0)\rho^2 - (2x_0+1)(\rho_n^2+\rho_p^2)] \\
\mathcal{H}_3 &= (1/24)t_3\rho^\alpha[(2+x_3)\rho^2 - (2x_3+1)(\rho_n^2+\rho_p^2)] \\
\mathcal{H}_{\text{eff}} &= (1/8)[t_1(2+x_1)+t_2(2+x_2)]\tau\rho \\
&\quad + (1/8)[t_2(2x_2+1) - t_1(2x_1+1)](\tau_n\rho_n + \tau_p\rho_p) \\
\mathcal{H}_{\text{fin}} &= (1/32)[3t_1(2+x_1) - t_2(2+x_2)](\nabla\rho)^2 \\
&\quad - (1/32)[3t_1(2x_1+1) + t_2(2x_2+1)][(\nabla\rho_n)^2 + (\nabla\rho_p)^2] \\
\mathcal{H}_{\text{SO}} &= (1/2)W_0\mathbf{J}\cdot\nabla\rho + (1/2)W_0'(\mathbf{J}\cdot_{\mathbf{n}}\nabla\rho_n + \mathbf{J_p}\cdot\nabla\rho_p) \\
\mathcal{H}_{\text{sg}} &= -(1/16)(t_1x_1+t_2x_2)\mathbf{J}^2 + (1/16)(t_1-t_2)(\mathbf{J_n}^2 + \mathbf{J_p}^2)
\end{aligned}$$

[a] E. Chabanat et al., Nucl. Phys. A **635**, 231 (1998); E. Chabanat *et al.*, ibid. **643**, 441 (1998)

Fitting Protocol

χ^2 definition: $$\chi^2 = \frac{1}{N_{data}} \sum_i^{N_{data}} \frac{(\mathcal{O}_i^{theo.} - \mathcal{O}_i^{data})^2}{(\Delta\mathcal{O}_i^{data})^2}$$

Landau-Migdal parameters in infinite nuclear matter G_0 and G_0' fixed to **0.15** and **0.35**, respectively, at ρ_0.

Table: Data and *pseudo*-data $\mathcal{O}_i$, adopted errors for the fit $\Delta\mathcal{O}_i$ and selected finite nuclei and EoS.

$\mathcal{O}_i$	$\Delta\mathcal{O}_i$	
B	1.00 MeV	^{40,48}Ca, ^{90}Zr, ^{132}Sn and ^{208}Pb
r_c	0.01 fm	^{40,48}Ca, ^{90}Zr and ^{208}Pb
ΔE_{SO}	$0.04\times\mathcal{O}_i$	$\pi 1g$ in ^{90}Zr and $\pi 2f$ in ^{208}Pb
$e_n(\rho)$	$0.20\times\mathcal{O}_i$	R. B. Wiringa *et al.*, PRC **38**, 1010 (1988)

Skyrme Aizu Milano interaction: SAMi

Parameter set and nuclear matter properties:

Table: SAMi parameter set and saturation properties with the estimated standard deviations inside parenthesis

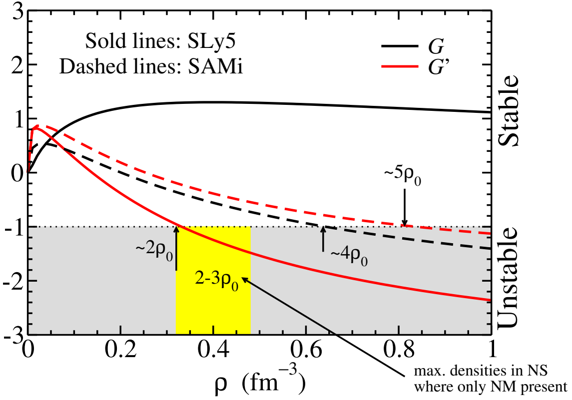
	value(σ)			value(σ)	
t_0	−1877.75(75)	MeV fm^3	ρ_∞	0.159(1)	fm^{-3}
t_1	475.6(1.4)	MeV fm^5	e_∞	−15.93(9)	MeV
t_2	−85.2(1.0)	MeV fm^5	m^*_{IS}	0.6752(3)	
t_3	10219.6(7.6)	MeV fm$^{3+3\alpha}$	m^*_{IV}	0.664(13)	
x_0	0.320(16)		J	28(1)	MeV
x_1	−0.532(70)		L	44(7)	MeV
x_2	−0.014(15)		K_∞	245(1)	MeV
x_3	0.688(30)		G_0	0.15	(fixed)
W_0	137(11)		G'_0	0.35	(fixed)
W'_0	42(22)				
α	0.25614(37)				

SAMi: spin and spin-isospin instabilities in NM

Qualitative test

Imposing that spin and isospin dof at the Fermi surface are stable under generalized deformations [S.-O. Bäckman *et al.*, Nucl. Phys. A **321**, 10 (1979)]

$$1 + G_0 > 0 \qquad 1 + G'_0 > 0$$



Gamow Teller Resonance in ^{48}Ca, ^{90}Zr and ^{208}Pb

Operator:
$\sum_{i=1}^{A} \boldsymbol{\sigma}(i)\tau_{\pm}(i)$

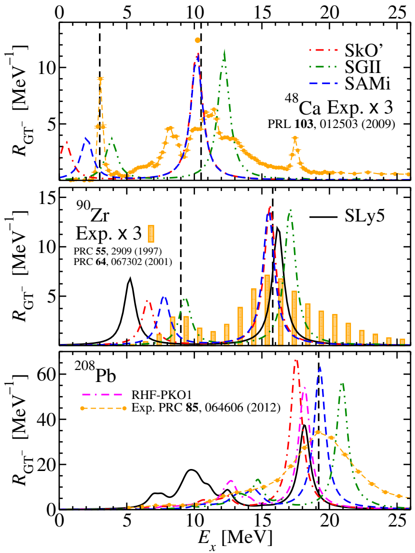


Figure: Gamow Teller strength distributions in ^{48}Ca (upper panel), ^{90}Zr (middle panel) and ^{208}Pb (lower panel) as measured in the experiment [T. Wakasa *et al.*, Phys. Rev. C **55**, 2909 (1997), K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009), A. Krasznaborkay *et al.*, Phys. Rev. C **64**, 067302 (2001), H. Akimune *et al.*, Phys. Rev. C **52**, 604 (1995) and T. Wakasa et al., Phys. Rev. C **85**, 064606 (2012)] and predicted by SLy5, SkO', SGII and SAMi forces.

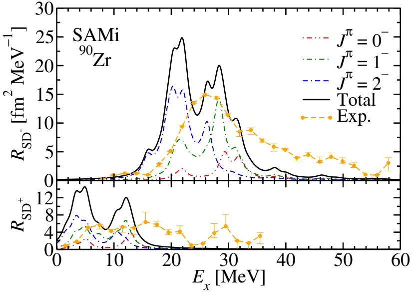
Spin Dipole Resonances in ^{90}Zr and ^{208}Pb

Operator:

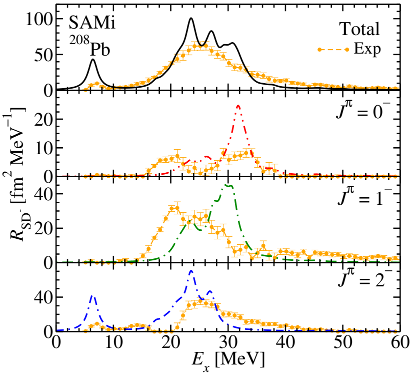
$$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_i^L \left[Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i) \right]_{JM}$$

Sum Rule:

$$\int [R_{SD^-}(E) - R_{SD^+}(E)] dE = \frac{9}{4\pi} \left(N\langle r_n^2 \rangle - Z\langle r_p^2 \rangle \right)$$



K. Yako *et al.*, Phys. Rev. C **74**, 051303(R) (2006)



T. Wakasa *et al.*, Phys. Rev. C **85**, 064606 (2012)

Spin-Dipole Resonance (SDR)

Total SDR reasonably well described but different channels in the SDR **NOT well** described.

Tensor terms produce a softening of 1^- states and a hardening of 0^- and 2^- states. Diagonal matrix elements:

$$V^{\lambda}_{T,AS} = (a_\lambda \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} T + b_\lambda \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} U)\langle \vec{\tau}_1 \cdot \vec{\tau}_2 \rangle \text{ for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix}$$

[C. L. Bai, *et al.*, Phys. Rev. Lett. 105, 072501 (2010)]

- This **general trend** would correct in the **proper** direction the **1^- and 2^- channels as predicted by SAMi** while the 0^- might be worsten (**Experimentally**, note that the **largest** contribution in **^{208}Pb** comes from the **1^- channel**).

Tensor terms

- New radioactive ion beam facilities → **evolution of the spin-orbit splittings with neutron excess** have been highlighted

  [J. P. Schiffer, *et al.*, Phys. Rev. Lett. 92, 162501 (2004)]

- Importance of an **effective neutron-proton tensor force to explain this evolution** was suggested

  [T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe, and Y. Akaishi, Phys. Rev. Lett. 95 , 232502 (2005)]

$$\vec{U}_q^{\mathrm{spin-orbit}} = \frac{1}{2}\left[W_0 \vec{\nabla}\rho + W_0' \vec{\nabla}\rho_q\right] + \left[\alpha \vec{J}_q + \beta \vec{J}_{1-q}\right]$$

where $\alpha = \alpha_C + \alpha_T$ and $\beta = \beta_C + \beta_T$ **(Non Rel. SO mean-field)**

- A number of experimental **data could be reproduced, to a certain extent better by including tensor terms in EDFs: Gogny** [T.Otsuka, T. Matsuo, and D. Abe, Phys. Rev. Lett.97, 162501 (2006)], **Skyrme** [T. Lesinski, M. Bender, K. Bennaceur, T. Duguet, and J. Meyer,Phys. Rev. C 76, 014312 (2007)], and **relativistic** frameworks [W. Long, H. Sagawa, J. Meng, and N. Van Giai, EPL 82, 12001 (2008)]

Tensor terms

• Difficult to **single out observables** that can uniquely determine the effects of the **tensor force**: **single-particle data senistive to beyond mean-field effects** (such as **PVC** [G. Colò, *et al.*, PRC 50, 1496 (1994) or A. V. Afanasjev and E. Litvinova, PRC 92, 044317 (2015)])

• This explain the **difficulties in determining tensor terms in EDFs** (c.f. [T. Lesinski *et al.*, Phys. Rev. C 76, 014312 (2007)])

• **Fit different channels of the SDR** (or other resonances such as M1) via HF+RPA calculations is **expensive computationally**.

• *ab initio* calculations may provide reliable **connection between few-body and many-body** physics.

• **Brueckner-Hartree-Fock (BHF)** theory do not contain effects like **PVC**, and **can be used directly as a benchmark** for EDFs.

Neutron drops from *ab initio*

- The **neutron drop is an ideal system** composed of a finite number of neutrons confined in an external field: can give an interesting **insight on shell structure and finite size properties**.
- Attempts to build EDFs driven by *ab initio* calculations in neutron drops [S. Gandolfi, J. Carlson, and S. C. Pieper, PRL 106, 012501 (2011) or J. Bonnard, M. Grasso, and D. Lacroix, PRC 98, 034319 (2018)]
- Essential **neutron-proton component of the tensor** to study nuclei ⇒ RBHF calculations also in **neutron-proton drops** [Shihang Shen, G. Colò, X. Roca-Maza, arXiv:1810.09691 (2018)].

• We use **neutron and neutron-proton drops** (RBHF with Bonn A) as benchmark to **fix the tensor terms**.
• **Refit full parameter space including tensor terms**.
• Requires minimal **modification of SAMi fitting protol**: neutron matter EoS changed by drops **(relative change in SO splittings and total energy in neutron drops)**.

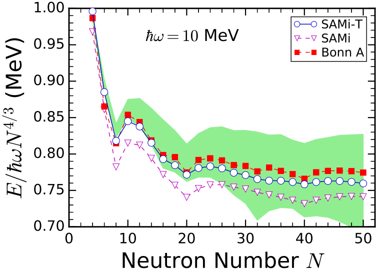
SAMi-T: parameter set and saturation properties

	Value	Error		Value
t_0	-2199.38 MeV fm^3	372.	ρ_0	0.164(1) fm^{-3}
t_1	533.036 MeV fm^5	20.7	e_0	$-16.15(3)$ MeV
t_2	-88.1692 MeV fm^5	12.6	m^*_{IS}/m	0.634(19)
t_3	11293.5 MeV fm$^{3+3\gamma}$	2014.	m^*_{IV}/m	0.625(122)
x_0	0.514710	0.178	J	29.7(6) MeV
x_1	-0.531674	0.593	L	46(12) MeV
x_2	-0.026340	0.117	K_0	244(5) MeV
x_3	0.944603	0.481	G_0	0.08 (fixed)
γ	0.179550	0.047	G'_0	0.29 (fixed)
W_0	130.026 MeV fm^5	8.2		
W'_0	101.893 MeV fm^5	18.6		
α	73.0 MeV fm^5	0.8	nn and pp	
β	101.8 MeV fm^5	1.2	np	

- strengths of the **SO terms in SAMi-T are larger** when compared with SAMi.
- **tensor terms known to reduce the SO splittings** of spin unsaturated systems such as ^{90}Zr or ^{208}Pb that have been fitted in both functionals.
- **SAMi-T needs larger SO strength to reproduce the same data.**

SAMi-T: neutron and neutro-proton drops

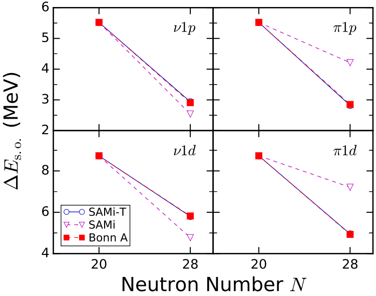
Neutron drops: total energy



The shaded area represents the values spanned by quantum Monte-Carlo calculations using AV8'+UIX and AV8'+IL7

S. Gandolfi, J. Carlson, and S. C. Pieper, PRL. 106, 012501 (2011) and P. Maris, J. P. Vary, S. Gandolfi, J. Carlson, and S. C. Pieper, PRC 87, 054318 (2013)

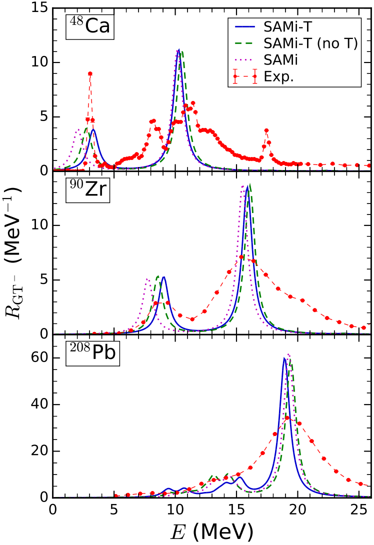
Neutron-proton drops: spin-orbit splittings

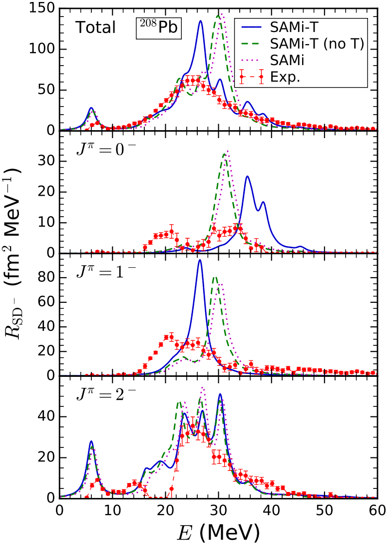


Trends in RBHF: **constrain trends tensor term** (neutron and neutron-proton drops have been included)

Experimental SO splittings: **constrain SO term** (mainly absolute value, no trends)

SAMi-T: Gamow-Teller and Spin Dipole Resonances





Despcription of low and high energy GT peaks

Improved 1^- channel gives largest contribution

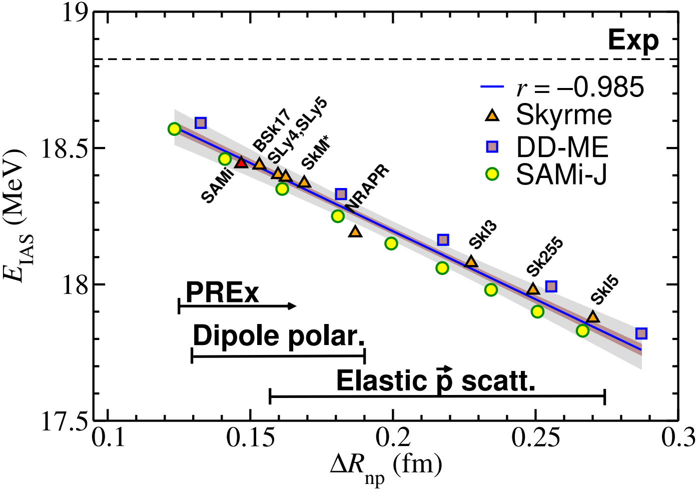
SDR and the neutron skin thickness Δr_{np}:

$$\int [R_{SD^-}(E) - R_{SD^+}(E)] dE = \frac{9}{4\pi}(N\langle r_n^2\rangle - Z\langle r_p^2\rangle)$$
$$\approx (N-Z)\langle r_p^2\rangle \left(1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)$$

- Experimental NEWSR in ^{208}**Pb** is 1004^{+24}_{-23} **fm^2**; SAMi is **1224 fm^2**; and **SAMi-T 1260± 10 fm^2** (**some strength is missing in the experimental measurement ?** $\Delta r_{np} \approx 0.05$ **fm**).
- Experimental NEWSR in ^{90}**Zr** is 148 ± 12 **fm^2**; SAMi is **150 fm^2**; and SAMi-T 147 ± 1 **fm^2** ⇒ **neutron skin should be properly determined by SAMi and SAMi-T**

Not only spin-isospin channel but also isospin channel alone seems to be reasonably well determined by the neutron drops (we also checked other observables)

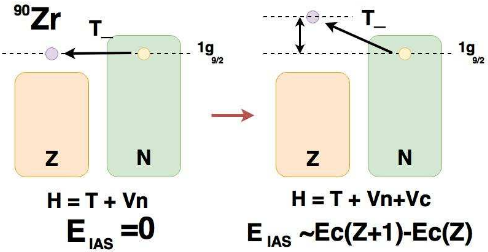
In contrast with E_{IAS}? Isobaric Aanalog Resonance - pure isospin resonance **($\mathcal{O} = \sum_{i=1}^{A} \tau_{-}(i)$)**



X. Roca-Maza, G. Colò, and H. Sagawa, Phys. Rev. Lett. 120, 202501 (2018)

Standard nuclear EDFs where the nuclear part of the Hamiltionian is isospin symmetric and only the Coulomb

The isobaric analog state energy: E_{IAS}



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+ T_-|0\rangle}$
- **Displacement energy or** E_{IAS}

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+ T_-|0\rangle}$$

$E_{IAS} \neq 0$ **only due to Isospin Symmtry Breaking terms** $\mathcal{H}$
E_{IAS}^{exp} **usually accuratelly measured !**

Contributions

$[\mathcal{H}, T_-] \neq 0$? essentially **Coulomb potential** but not only

Table: Estimate of the different effects on E_{IAS} in ^{208}Pb.

	E_{IAS} Correction
Coumb direct	$\sim$ 20 MeV
Coulomb exchange	$\sim$ -300 keV
n-p mass difference	$\sim$ tens keV
Electromagnetic spin-orbit	$\sim$ - tens keV
Finite size effects	$\sim$ - 100 keV
Short range correlations	$\sim$ 100 keV
sospin impurity	$\sim$ -100 keV
Isospin symmetry breaking	$\sim$ - 250 keV
	$\sim$ 19 MeV

Physical Review Letters **23**, 484 (1969).

$E_{IAS}^{exp} = 18.83 \pm 0.01$ MeV. *Nuclear Data Sheets 108, 1583 (2007).*

Coulomb direct contribution: very simple model

• Assuming indepentent particle model and good isospin for $|0\rangle$ ($\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z$)

$$E_{IAS} \approx E_{IAS}^{C,direct} = \frac{1}{N-Z}\int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{direct}(\vec{r})d\vec{r}$$

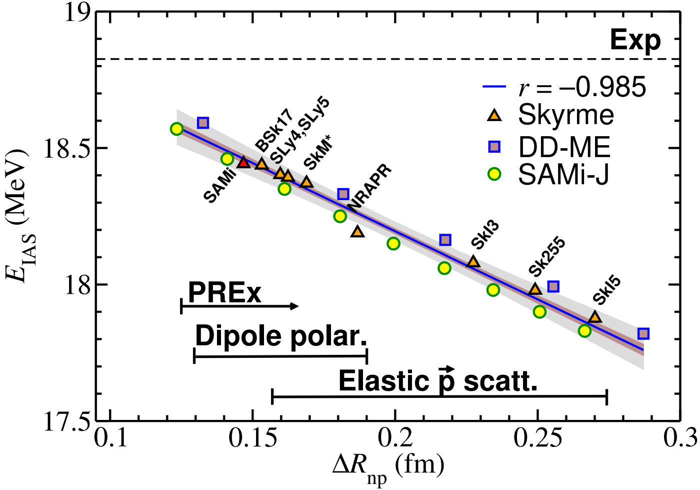
where $U_C^{direct}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{ch}(\vec{r}_1)d\vec{r}_1$

• Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

As already shown...



X. Roca-Maza, G. Colò, and H. Sagawa, Phys. Rev. Lett. 120, 202501 (2018)

Standard nuclear EDFs where the nuclear part of the Hamiltionian is isospin symmetric and only the Coulomb term breaks isospin symmetry

Corrections: within self-consistent HF+RPA

Within the **HF+RPA** one can **estimate** the E_{IAS} accounting (in an effective way) for **short-range correlations and effects of the continuum** (if a large sp base is adopted).

- **Coulomb exchange** exact (usually Slater approx.):

$$U_C^{x,\text{exact}}\varphi_i(\vec{r}) = -\frac{e^2}{2}\int d^3r' \, \frac{\varphi_j^*(\vec{r}')\varphi_j(\vec{r})}{|\vec{r}-\vec{r}'|}\varphi_i(\vec{r}')$$

- The **electromagnetic spin-orbit** correction to the nucleon single-particle energy (non-relativistic),

$$\varepsilon_i^{\text{emso}} = \frac{\hbar^2 c^2}{2m_i^2 c^4}\langle \vec{l}_i \cdot \vec{s}_i\rangle x_i \int \frac{1}{r}\frac{dU_C}{dr}|R_i(r)|^2$$

where x_i: $g_p - 1$ for Z and g_n for N; $g_n = -3.82608545(90)$ and $g_p = 5.585694702(17)$, $R_i \to R_{nl}$ radial wf.

Corrections:

- **Finite size** effects (assuming spherical symmetry):

$$\begin{aligned}\rho_{ch}(q) &= \left(1-\frac{q^2}{8m^2}\right)\left[G_{E,p}(q^2)\rho_p(q)+G_{E,n}(q^2)\rho_n(q)\right] \\ &- \frac{\pi q^2}{2m^2}\sum_{l,t}\left[2G_{M,t}(q^2)-G_{E,t}(q^2)\right]\langle\vec{l}\cdot\vec{s}\rangle\int_0^\infty dx\frac{j_1(qx)}{qx}|R_{nl}(x)x^2|^2\end{aligned}$$

- **Vacuum polarization:** lowest order correction in the fine-structure constant to the Coulomb potential $\frac{eZ}{r}$:

$$V_{vp}(\vec{r}) = -\frac{2}{3}\frac{\alpha e^2}{\pi}\int d\vec{r}'\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathcal{K}_1\left(\frac{2}{\lambda\!\!\!^{-}_e}|\vec{r}-\vec{r}'|\right)$$

where e is the fundamental electric charge, α the fine-structure constrant, $\lambda\!\!\!^{-}_e$ the reduced Compton electron wavelength and

$$\mathcal{K}_1(x) \equiv \int_1^\infty dt e^{-xt}\left(\frac{1}{t^2}+\frac{1}{2t^4}\right)\sqrt{t^2-1}$$

Corrections:

- **Isospin symmetry breaking** (Skyrme-like): **two parts** (contact interaction)

charge symmetry breaking +

$V_{\mathrm{CSB}} = V_{nn} - V_{pp}$

$V_{\mathrm{CSB}}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{4}\left[\tau_z(1) + \tau_z(2)\right] s_0 (1 + y_0 P_\sigma)$

τ_z Pauli in isospin space; P_σ are the usual projector operators in spin space.

charge independence breaking*

$V_{\mathrm{CIB}} = \frac{1}{2}(V_{nn} + V_{pp}) - V_{pn}$

$V_{\mathrm{CIB}}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{2}\tau_z(1)\tau_z(2) u_0 (1 + z_0 P_\sigma)$

* general operator form $\tau_z(1)\tau_z(2) - \frac{1}{3}\vec{\tau}(1)\cdot\vec{\tau}(2)$.

Our prescription $\tau_z(1)\tau_z(2)$ not change structure of HF+RPA.

- Opposite to the other corrections, **ISB contributions depends on new parameters that need to be determined!**

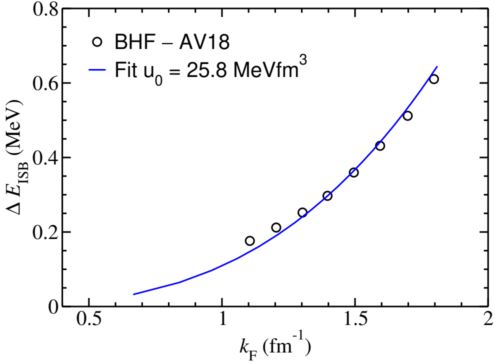
Isospin symmetry breaking in the medium:

- **keeping** things **simple**: **CSB and CIB** interaction just **delta function** depending on s_0 **and** u_0. **Different possibilities:**

→ **Fitting** to (two) experimentally known **IAS energies**

→ **Derive from theory**

→ **our option:** u_0 **to reproduce BHF** (symmetric nuclear matter) and s_0 **to reproduce** E_{IAS} **in** ^{208}**Pb**



Physics Letters B 445, 259 (1999)

Re-fit of SAMi: SAMi-ISB

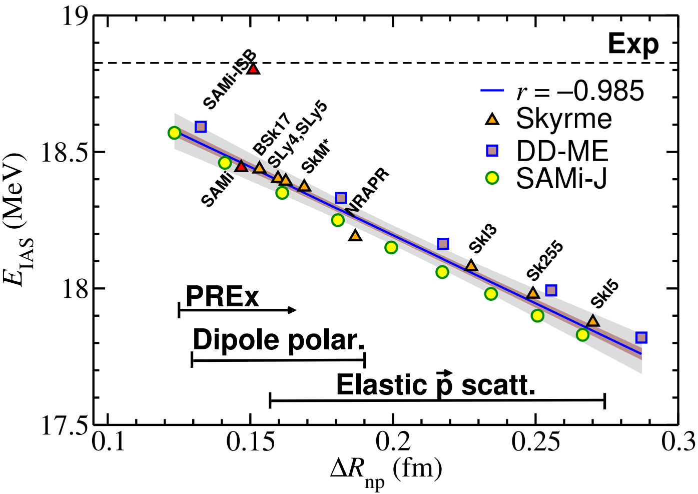
• All these **corrections** are relatively **small** but **modify binding energies, neutron and proton distributions, etc.** $\Rightarrow$ a **re-fit of the interaction is needed**.

• Use **SAMi fitting protocol** (special care for spin-isospin resonances) including all corrections and **find SAMi-ISB**

Table: Saturation properties

	SAMi	SAMi-ISB	
ρ_∞	0.159(1)	0.1613(6)	fm^{-3}
e_∞	−15.93(9)	−16.03(2)	MeV
m^*_{IS}	0.6752(3)	0.730(19)	
m^*_{IV}	0.664(13)	0.667(120)	
J	28(1)	30.8(4)	MeV
L	44(7)	50(4)	MeV
K_∞	245(1)	235(4)	MeV

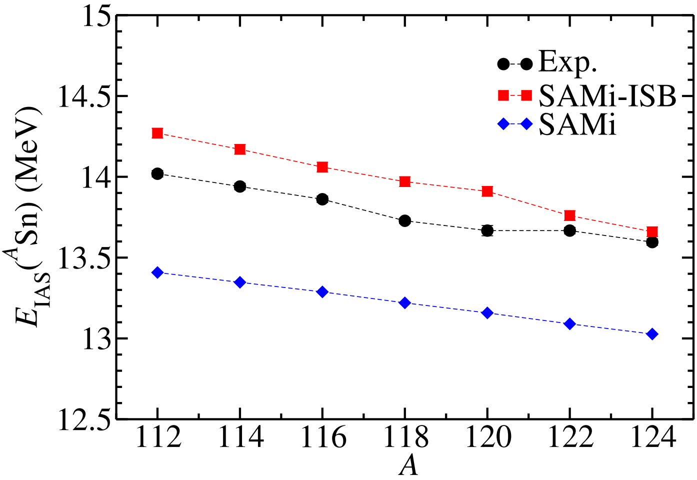
E_{IAS} with SAMi-ISB



Phys. Rev. Lett. 120, 202501 (2018)

Measurement of Δr_{np} → determine ISB in the nuclear medium

Prediction: E_{IAS} in the Sn isotopic chain



Phys. Rev. Lett. 120, 202501 (2018)

Conclusions:

- we have **successfully determined new Skyrme** functionals which **account** for the most relevant quantities in order to improve the description of **charge-exchange nuclear resonances**:
  - the **hierarchy** and **positive values** of the spin and spin-isospin Landau-Migdal parameters G_0 **and** G_0'
  - the **proton spin-orbit splittings** of different **high angular momenta** single-particle levels
  - and for **SAMi-T** also trends of **spin-orbit splittings** derived from RBHF in **neutron-proton drops**.
- the **GTR** and **SDR** are reasonably well described by **SAMi and further improved by SAMi-T**
- **No EDF** is **compatible** with experimental data and **IAR** data at the same time **unless ISB terms are included** (**SAMi-ISB**)
- **SAMi** functionals do **not deteriorate** the description of other **nuclear observables such as masses, radii, and non-charge exchange resonances**
- **applicability in nuclear physics and astrophysics**

Thank you for your attention!

Extra Material

Landau-Migdal vs Skyrme parameters

- The p-h interaction is derived as the second functional derivative of the total energy with respect to density at the Fermi surface. Matrix elements of the form:

$$\langle \vec{k}_1 \vec{k}_2 | V | \vec{k}_1 \vec{k}_2 \rangle = \sum_{\alpha=1,\tau,\sigma,\tau\cdot\sigma} \frac{\delta \mathcal{H}}{\delta \rho_\alpha \delta \rho_\alpha} =$$
$$N_0^{-1} (F + F' \boldsymbol{\tau_1 \tau_2} + G \boldsymbol{\sigma_1 \sigma_2} + G' \boldsymbol{\tau_1 \tau_2 \sigma_1 \sigma_2})$$

* $N_0 = 2k_F m^* / \hbar^2 \pi^2$ is the density of states per energy at the Fermi surface
* Parameters are functions of $\vec{k}_1$ and $\vec{k}_2$

* $\vec{k}_1$ and $\vec{k}_2$ are taken at the Fermi surface (Landau parameters are only functions of the angle between them and the Fermi momentum $\Rightarrow F = \sum_l F_l P_l(\cos\theta)$).

- One can do the same functional derivative with Skyrme and compare both expressions to find the relation between the Landau-Migdal and the Skyrme parameters (here just two of them):

$$\mathbf{G_0} N_0 = -\frac{1}{4}t_0 + \frac{1}{2}t_0 x_0 - \frac{1}{8}t_1 k_F^2 + \frac{1}{4}t_1 x_1 k_F^2 + \frac{1}{8}t_2 k_F^2 + \frac{1}{4}t_2 x_2 k_F^2 - \frac{1}{24}t_3 \rho^\alpha + \frac{1}{12}t_3 x_3 \rho^\alpha$$
$$\mathbf{G_0'} N_0 = -\frac{1}{4}t_0 - \frac{1}{8}t_1 k_F^2 + \frac{1}{8}t_2 k_F^2 - \frac{1}{24}t_3 \rho^\alpha$$

Empirical constraints on G_0 and G_0'

- **Gamow-Teller Resonance** using RPA based on the Woods-Saxon potential have been studied and the **Landau-Migdal parameters estimated by comparing experiment** with theoretical calculations in Refs. [T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005) and T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999)].
  - Landau-Migdal parameter G_0' **dominates the excitation energy in the GTR**
- In our fit, **we do not use the obtained values as pseudodata** because **our theoretical framework is different and the results are associated to different** m^* (our sp energies are based on HF calculations instead of a Wood-Saxon potential).
- **We use** the empirical result in which a **hierarchy** between spin and spin-isospin parameters is suggested:

$$G_0' > G_0 > 0$$

Equation of State: SAMi vs *ab*–initio calculations

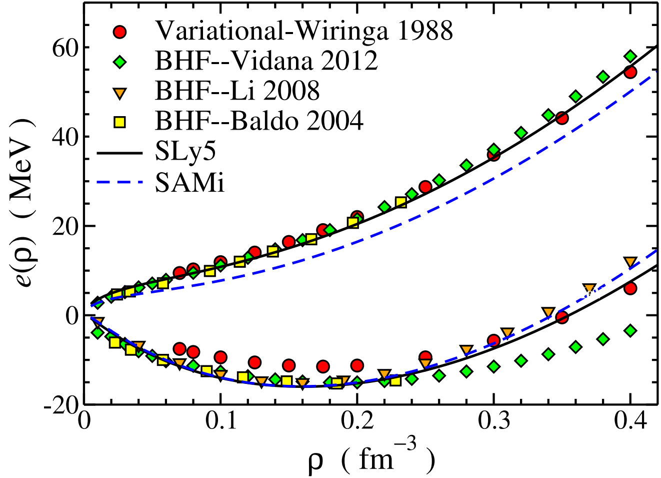


Figure: Neutron and symmetric matter EoS as predicted by the HF SAMi (dashed line) and SLy5 (solid line) interactions and by the benchmark microscopic calculations of R. B. Wiringa *et al.*, PRC **38**, 1010 (1988) (circles). State-of-the-art BHF calculations are shown by diamonds I. Vidaña, private communication, triangles Z. H. Li *et al.*, Phys. Rev. C **77**, 034316 (2008) and squares M. Baldo *et al.*, Nucl. Phys. A **736**, 241 (2004).

Finite Nulcei: spherical double-magic nuclei

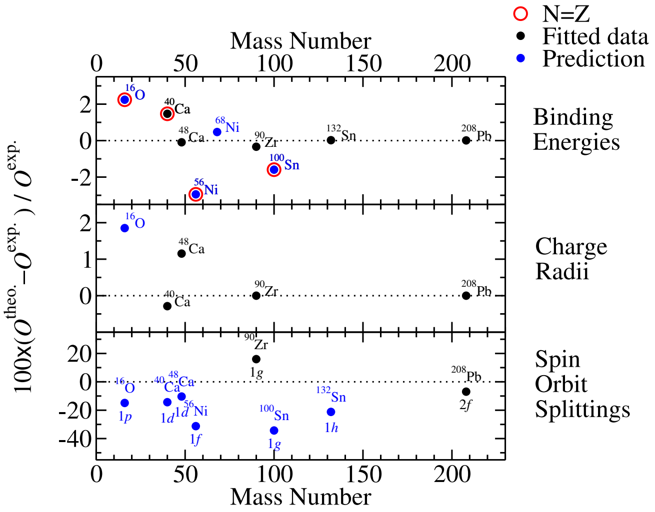


Figure: Finite nuclei properties as predicted by the HF SAMi (black circles) and some predictions (blue circles) for spherical double-magic nuclei. Experimental data taken from Refs. G. Audi *et al.*, NPA **729**, 337 (2003), I. Angeli, ADNDT **87**, 185 (2004), M. Zalewski *et al.*, PRC **77**, 024316 (2008)

Giant Monopole and Dipole Resonances in ^{208}Pb

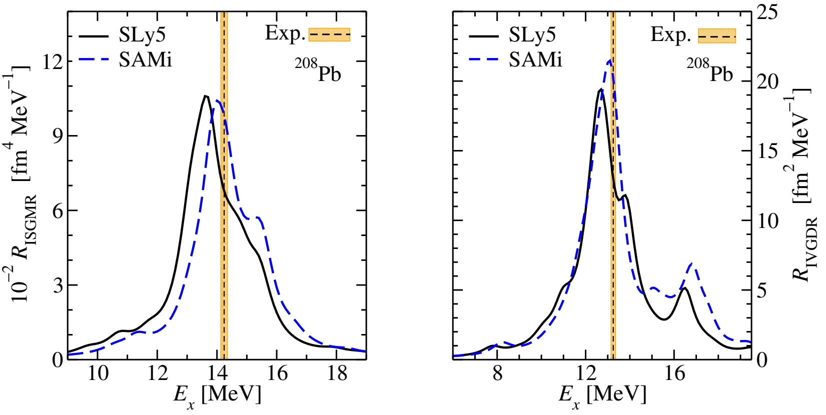


Figure: Strength function at the relevant excitation energies in ^{208}Pb as predicted by SLy5 and the SAMi interaction for GMR and GDR. A Lorentzian smearing parameter equal to 1 MeV is used. Experimental data for the centroid energies are also shown: $E_c(GMR) = 14.24 \pm 0.11$ MeV [D. H. Youngblood, et al., Phys. Rev. Lett. **82**, 691 (1999)] and $E_c(GDR) = 13.25 \pm 0.10$ MeV [N. Ryezayeva et al., Phys. Rev. Lett. **89**, 272502 (2002)].

Motivation:

Gamow Teller Resonance II: quenching of the strength

- **Experimentally**, the **GTR** exhausts **60–70%** of the **Ikeda sum rule**: $\int [R_{GT^-}(E) - R_{GT^+}(E)] dE = 3(N - Z)$
- To **explain** the problem, two possibilities that go beyond RPA correlations have been drawn:
  - the effects of the second-order configuration mixing: **2p-2h correlations**
  - within the quark model, a **n(p)** can become a **p(n)** or a $\Delta^+(\Delta^{++})$ under the action of the GT^- operator and since there is **no Pauli blocking for $\Delta - h$ excitations** $\Rightarrow$ it may **contribute to the GTR**.
- The **experimental analysis of ^{90}Zr** $\Rightarrow$ **quenching** (2/3) has to be **mainly attributed to 2p-2h** coupling and not to $\Delta-$isobar effects much smaller [T. Wakasa *et. al.*, Phys. Rev. C **55**, 2909 (1997)].
- E_x **GTR in nuclei** mainly in the region of several **tens of MeV** and the $\Delta - h$ states are hundreds of MeV above the GT $\Rightarrow$ **hard to excite the Δ** in the nuclear medium.

Covariance analysis: χ^2 test

Observables $\mathcal{O}$ are used to calibrate the parameters $\mathbf{p}$ of a given model. The optimum parametrization $\mathbf{p}_0$ is determined by a least-squares fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the χ^2 is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned} \chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j}) \\ &\equiv \sum_{i,j}^{n} (p_i - p_{0i}) \mathcal{M}_{ij} (p_j - p_{0j}) \end{aligned}$$

where $\mathcal{M}$ is the curvature matrix.

Covariance analysis: χ^2 test

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(\mathbf{p})$),

$$\Delta\mathcal{A} = \sqrt{\sum_{\imath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\imath} \partial_{p_\imath} A} \tag{1}$$

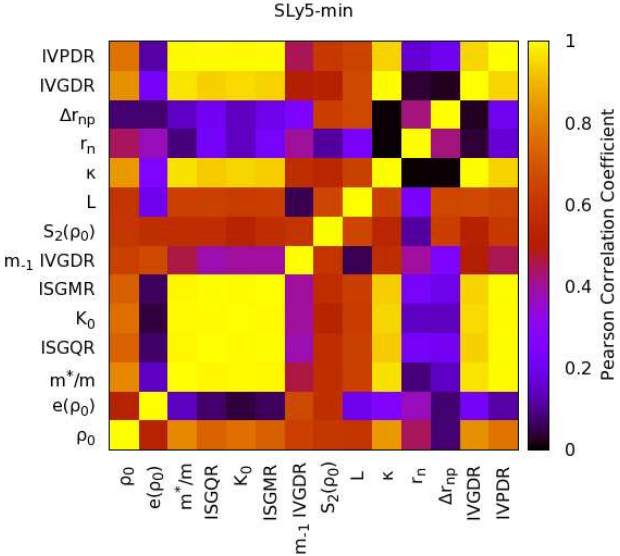
$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}} \tag{2}$$

where,

$$C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{\imath\jmath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\jmath} \partial_{p_\jmath} B$$

Covariance analysis: SLy5-min as an example



Covariance analysis: SLy5-min as an example

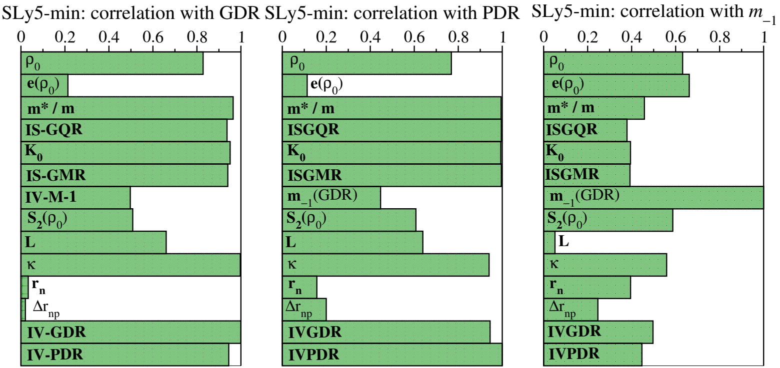


Figure: Pearson product-moment correlation coefficient for the IVGDR (left panel), IVPDR (middle panel) and m_{-1}(IVGDR) (right panel) with all other studied properties as predicted by the covariance analysis of SLy5.