A hand is pointing to a specific element on a large, colorful periodic table of elements. The table is divided into blocks of different colors: yellow, red, blue, and green. The background is slightly blurred, focusing attention on the hand and the table.

The nuclear symmetry energy and the breaking of the isospin symmetry: how do they reconcile with each other?

Xavier Roca-Maza

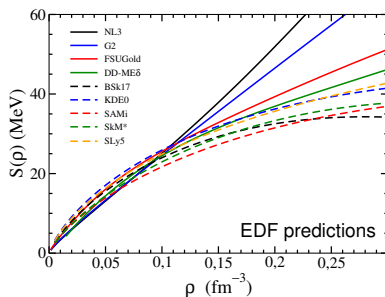
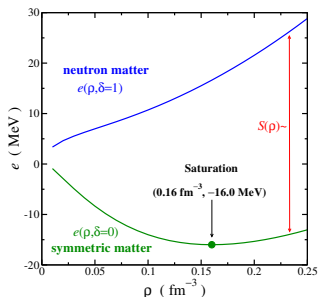
**Università degli Studi di Milano and INFN, Milano section
in collaboration with Gianluca Colò and Hiroyuki Sagawa**

**NSRT18 - Nuclear Structure and Related Topics
Burgas, Bulgaria, June 3rd - 9th, 2018**

Table of contents:

- **Brief introduction**
 - Nuclear Equation of State: the symmetry energy
 - The neutron skin thickness
- **The isobaric analog state**
 - Displacement energy
 - Corrections to isospin symmetric \mathcal{H}_{eff}
 - Refitted interaction
 - Results
- **Conclusions**

The Nuclear Equation of State: Infinite System



- Expansion for small asymmetries $\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$:

$$e(\rho, \delta) = e(\rho, \delta = 0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]$$

- Expansion on the density around saturation $x \equiv \frac{\rho - \rho_0}{3\rho}$:

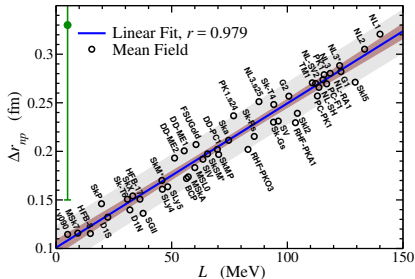
$$e(\rho, \delta) = \left[e(\rho_0, \delta = 0) + \frac{1}{2}K_0x^2 \right] + \left[J + Lx + \frac{1}{2}K_{\text{sym}}x^2 \right] \delta^2 + \mathcal{O}[\delta^2, \rho^3]$$

Uncertainties on $S(\rho)$ around saturation (mainly due to L) **impact** on many nuclear physics and astrophysics **observables**.

Example: L and the neutron skin in ^{208}Pb

$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

$$\text{Macroscopic model: } \Delta r_{np} \sim \frac{1}{12} \frac{(N-Z)R}{A} \frac{R}{J} L \quad (L \propto p_0^{\text{neut}})$$

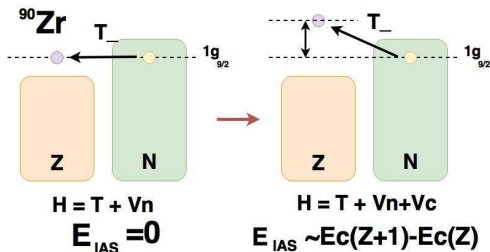


Physical Review Letters **106**, 252501 (2011)

The faster the symmetry energy increases with density (L), the largest the size of the neutron skin in (heavy) nuclei.

[Exp. from strongly interacting probes: $\sim 0.15 - 0.22$ fm (*Physical Review C* **86** 015803 (2012))].

The isobaric analog state energy: E_{IAS}



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$

- **Displacement energy or E_{IAS}**

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

$E_{IAS} \neq 0$ only due to Isospin Symmetry Breaking terms \mathcal{H}
 E_{IAS}^{exp} usually accurately measured !

Contributions

$[\mathcal{H}, T_-] \neq 0$? essentially **Coulomb potential** but not only

Table: Estimate of the different effects on E_{IAS} in ^{208}Pb .

	E_{IAS} Correction
Coumb direct	~ 20 MeV
Coulomb exchange	~ -300 keV
n-p mass difference	\sim tens keV
Electromagnetic spin-orbit	$\sim -$ tens keV
Finite size effects	$\sim -$ 100 keV
Short range correlations	\sim 100 keV
sospin impurity	$\sim -$ 100 keV
Isospin symmetry breaking	$\sim -$ 250 keV
	~ 19 MeV

Physical Review Letters **23**, 484 (1969).

$E_{IAS}^{\text{exp}} = 18.83 \pm 0.01$ MeV. *Nuclear Data Sheets 108*, 1583 (2007).

Coulomb direct contribution: very simple model

- Assuming independent particle model and good isospin for $|0\rangle$
($\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z$)

$$E_{IAS} \approx E_{IAS}^{C,direct} = \frac{1}{N - Z} \int [\rho_n(\vec{r}) - \rho_p(\vec{r})] U_C^{direct}(\vec{r}) d\vec{r}$$

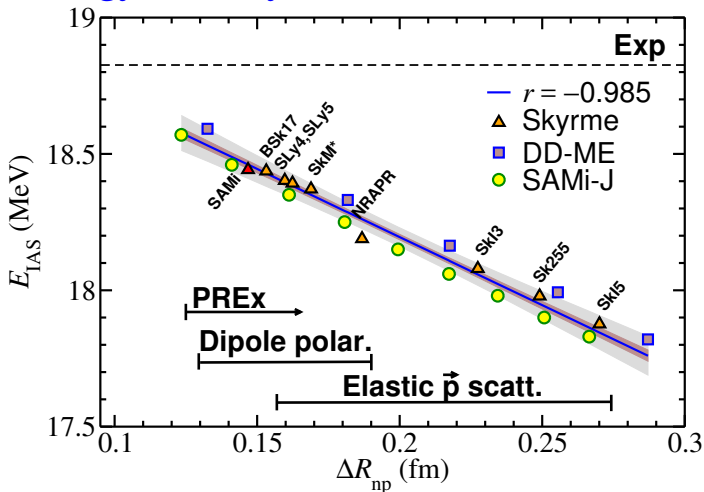
where $U_C^{direct}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|} \rho_{ch}(\vec{r}_1) d\vec{r}_1$

- Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5} \frac{Ze^2}{R_p} \left(1 - \sqrt{\frac{5}{12}} \frac{N}{N - Z} \frac{\Delta r_{np}}{R_p} \right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

E_{IAS} in Energy Density Functionals (No Corr.)



Phys. Rev. Lett. 120, 202501 (2018)

Nuclear models (EDFs) where the nuclear part is isospin symmetric and U_{ch} is calculated from the ρ_p

Corrections: within self-consistent HF+RPA

Within the **HF+RPA** one can **estimate** the E_{IAS} accounting (in an effective way) for **short-range correlations and effects of the continuum** (if a large sp base is adopted).

- **Coulomb exchange** exact (usually Slater approx.):

$$U_C^{x,\text{exact}} \varphi_i(\vec{r}) = -\frac{e^2}{2} \int d^3r' \frac{\varphi_j^*(\vec{r}') \varphi_j(\vec{r})}{|\vec{r} - \vec{r}'|} \varphi_i(\vec{r}')$$

- The **electromagnetic spin-orbit** correction to the nucleon single-particle energy (non-relativistic),

$$\varepsilon_i^{\text{emso}} = \frac{\hbar^2 c^2}{2m_i^2 c^4} \langle \vec{l}_i \cdot \vec{s}_i \rangle x_i \int \frac{1}{r} \frac{dU_C}{dr} |R_i(r)|^2$$

where x_i : $g_p - 1$ for Z and g_n for N ; $g_n = -3.82608545(90)$ and $g_p = 5.585694702(17)$, $R_i \rightarrow R_{nl}$ radial wf.

Corrections:

- **Finite size** effects (assuming spherical symmetry):

$$\begin{aligned}\rho_{\text{ch}}(q) &= \left(1 - \frac{q^2}{8m^2}\right) [G_{E,p}(q^2)\rho_p(q) + G_{E,n}(q^2)\rho_n(q)] \\ &- \frac{\pi q^2}{2m^2} \sum_{l,t} [2G_{M,t}(q^2) - G_{E,t}(q^2)] \langle \vec{l} \cdot \vec{s} \rangle \int_0^\infty dx \frac{j_1(qx)}{qx} |\mathcal{R}_{nl}(x)x^2|^2\end{aligned}$$

- **Vacuum polarization:** lowest order correction in the fine-structure constant to the Coulomb potential $\frac{eZ}{r}$:

$$V_{\text{vp}}(\vec{r}) = -\frac{2}{3} \frac{\alpha e^2}{\pi} \int d\vec{r}' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \mathcal{K}_1 \left(\frac{2}{\lambda_e} |\vec{r} - \vec{r}'| \right)$$

where e is the fundamental electric charge, α the fine-structure constant, λ_e the reduced Compton electron wavelength and

$$\mathcal{K}_1(x) \equiv \int_1^\infty dt e^{-xt} \left(\frac{1}{t^2} + \frac{1}{2t^4} \right) \sqrt{t^2 - 1}$$

Corrections:

- **Isospin symmetry breaking** (Skyrme-like): **two parts**
(contact interaction)

charge symmetry breaking +

$$V_{\text{CSB}} = V_{\text{nn}} - V_{\text{pp}}$$

$$V_{\text{CSB}}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{4} [\tau_z(1) + \tau_z(2)] s_0 (1 + y_0 P_\sigma)$$

τ_z Pauli in isospin space; P_σ are the usual projector operators in spin space.

charge independence breaking*

$$V_{\text{CIB}} = \frac{1}{2} (V_{\text{nn}} + V_{\text{pp}}) - V_{\text{pn}}$$

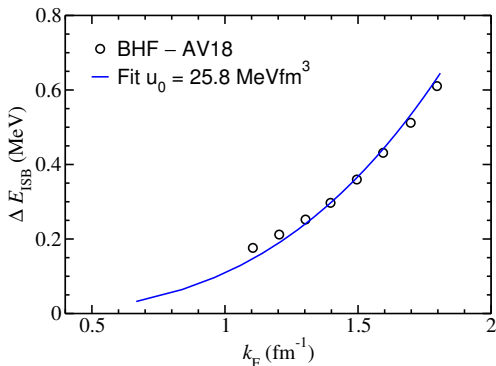
$$V_{\text{CIB}}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{2} \tau_z(1) \tau_z(2) u_0 (1 + z_0 P_\sigma)$$

* general operator form $\tau_z(1) \tau_z(2) - \frac{1}{3} \vec{\tau}(1) \cdot \vec{\tau}(2)$. Our prescription $\tau_z(1) \tau_z(2)$ not change structure of HF+RPA.

- Opposite to the other corrections, **ISB contributions depends on new parameters that need to be determined!**

Isospin symmetry breaking in the medium:

- **keeping** things **simple**: **CSB** and **CIB** interaction just **delta function** depending on s_0 and u_0 . **Different possibilities**:
 - **Fitting** to (two) experimentally known **IAS energies**
 - **Derive from theory**
 - **our option**: u_0 to reproduce **BHF** (symmetric nuclear matter) and s_0 to reproduce E_{IAS} in ^{208}Pb



Re-fit of SAMi: SAMi-ISB

- All these **corrections** are relatively **small** but **modify binding energies, neutron and proton distributions, etc.**
⇒ a **re-fit of the interaction is needed.**
- Use **SAMi fitting protocol** (special care for spin-isospin resonances) including all corrections and **find SAMi-ISB**

Table: Saturation properties

	SAMi	SAMi-ISB	
ρ_∞	0.159(1)	0.1613(6)	fm^{-3}
e_∞	-15.93(9)	-16.03(2)	MeV
m_{IS}^*	0.6752(3)	0.730(19)	
m_{IV}^*	0.664(13)	0.667(120)	
J	28(1)	30.8(4)	MeV
L	44(7)	50(4)	MeV
K_∞	245(1)	235(4)	MeV

SAMi-ISB finite nuclei properties

El.	N	B [MeV]	B ^{exp} [MeV]	r _c [fm]	r _c ^{exp} [fm]	ΔR _{np} [fm]
Ca	28	417.67	415.99	3.49	3.47	0.214
Zr	50	783.60	783.89	4.26	4.27	0.097
Sn	82	1102.75	1102.85	4.73	–	0.217
Pb	126	1635.78	1636.43	5.50	5.50	0.151

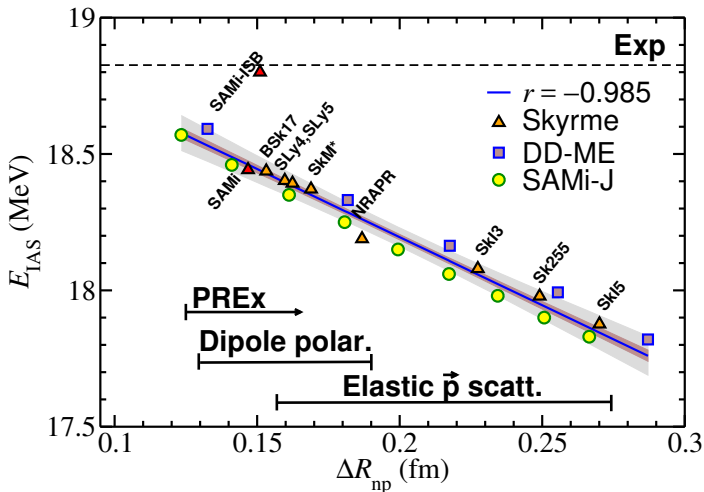
Corrections on E_{IAS} for ²⁰⁸Pb one by one

	E _{IAS} [MeV]	Correction [keV]
No corrections ^a	18.31	
Exact Coulomb exchange	18.41	+100
n/p mass difference	18.44	+30
Electromagnetic spin-orbit	18.45	+10
Finite size effects	18.40	-50
Vacuum polarization (V _{ch})	18.53	+130
Isospin symmetry breaking	18.80(5)	+270

^aFrom Skyrme Hamiltonian where the nuclear part is isospin symmetric and V_{ch} is calculated from the ρ_p

$$E_{IAS}^{\text{exp}} = 18.83 \pm 0.01 \text{ MeV. } \textit{Nuclear Data Sheets 108, 1583 (2007).}$$

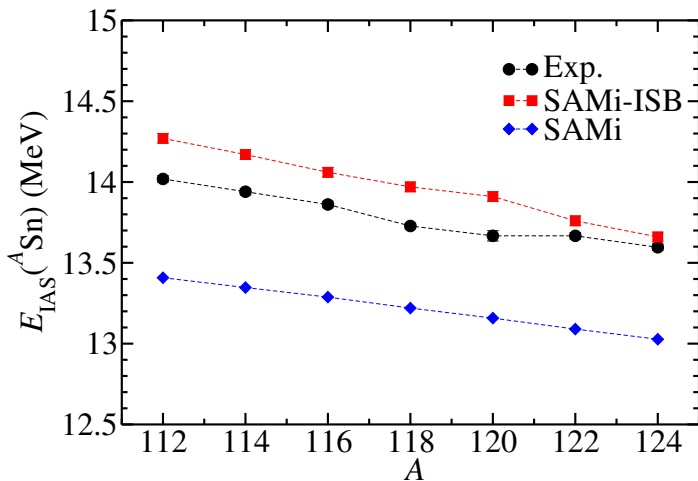
E_{IAS} with SAMi-ISB



Phys. Rev. Lett. 120, 202501 (2018)

Measurement of Δr_{np} \rightarrow determine ISB in the nuclear medium

Prediction: E_{IAS} in the Sn isotopic chain



Phys. Rev. Lett. 120, 202501 (2018)

Conclusions

- The most **promising observables** to constraint the symmetry energy are those that can be **measured** via processes with little or **no direct influence from the strong force** (e.g. electromagnetic or weak probes).
- Alternatively, E_{IAS} has **no** dependence on the **isospin conserving** part (largest) of the **strong interaction**.
- **EDFs** of common use in nuclear physics show a **linear dependence between E_{IAS} and Δr_{np}**
- **EDFs do not properly** describe the experimental E_{IAS}
- **Modification of \mathcal{H}_{eff} requires a refit** of the interaction including **new ISB parameters**.
- **One can reconcile good reproduction of experimental charge radii, binding energies, E_{IAS} ...**

- **A better knowledge of ISB contributions in the medium may lead to an accurate determination of neutron skin thickness via E_{IAS} (or the other way around)**

**Thank you for your
attention!**