Dipole Polarizability and the neutron skin thickness

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INTRODUCTION

The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different predictions for many-body observables** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals (EDFs):

Based on effective interactions solved at the HF level, **EDFs** are **successful** in the description of **ground and excited state** properties such as m, $\langle r^2 \rangle^{1/2}$ **or GR** along the **nuclear chart**

Main types of EDFs:

Relativistic mean-field models (RMF), based on Lagrangians where effective mesons carry the interaction:

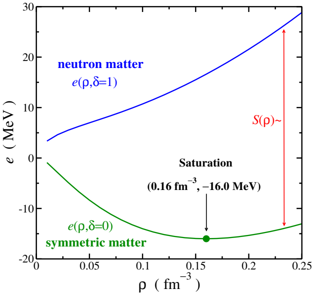
$$\begin{aligned}\mathcal{L}_{int} \quad &= \bar{\Psi}\Gamma_{\sigma}(\bar{\Psi},\Psi)\Psi\Phi_{\sigma} \quad + \bar{\Psi}\Gamma_{\delta}(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_{\delta} \\ &- \bar{\Psi}\Gamma_{\omega}(\bar{\Psi},\Psi)\gamma_{\mu}\Psi A^{(\omega)\mu} \quad - \bar{\Psi}\Gamma_{\rho}(\bar{\Psi},\Psi)\gamma_{\mu}\boldsymbol{\tau}\Psi A^{(\rho)\mu}\end{aligned}$$

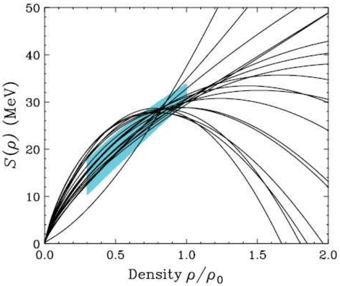
Non-relativistic mean-field models (NRMF), based on Hamiltonians where ef f. interactions are proposed and tested:

$$V_{\rm Nucl}^{\rm eff} = V_{\rm attractive}^{\rm long-range} + V_{\rm repulsive}^{\rm short-range} + V_{\rm SO} + ...$$

-EDFs are **phenomenological** → **not directly connected to any NN** (or NNN) **interaction** in the vacuum
-EDFs derived from a **Mean-Field** → we expect bulk properties more accurate as heavier is the nucleus

The Nuclear Equation of State: Infinite System



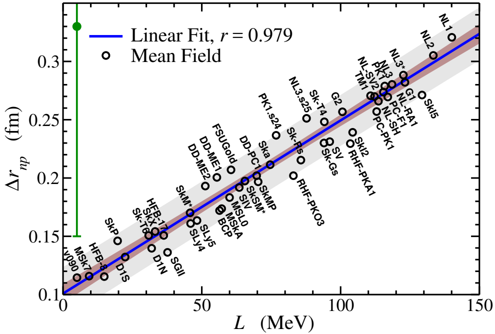


$$e(\rho,\beta) = e(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \quad \text{where } \beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

-**Isovector properties** not well determined in current **EDFs**
-**Parity violating program** (JLab/Mainz), **full E1 response** in stable and exotic nuclei (RCNP/GSI) and measurements on exotic nuclei in **Rare Ion Beam Facilites** worldwide → **better characterization of isovector properties around saturation density**

But how we can better constraint the isovector channel from observables? (Example)

Neutron skin thickness → is one of the most paradigmatic example of an **isovector sensitive observable**.



$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \sim \frac{1}{12}\frac{IR}{J} L$$

$$\text{where } J \equiv S(\rho_0) \text{ and } L \equiv 3\rho_0 \frac{\partial S}{\partial \rho}\Big|_{\rho_0} = 3\rho_0 p_0^{neut}$$

THE DIPOLE POLARIZABILITY

Dipole polarizability: definition

From a macroscopic perspective

The electric **polarizability** measures the tendency of the nuclear **charge distribution to be distorted**

$$\alpha_D \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

From a microscopic perspective

The electric **polarizability** is proportional to the **inverse energy weighted sum rule (IEWSR)** of the electric dipole response in nuclei

$$\alpha_D = \frac{8\pi}{9} e^2 \sum \frac{B(E1)}{E}$$

or

$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_{\text{ph. abs.}}(E)}{E^2} dE$$

In more detail (from theory) ...

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

  $$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \; (\Delta L = 1 \rightarrow Dipole)$$

- is proportional to the **static polarizability** for small oscillations $\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**

> The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

STATISTIC UNCERTAINTIES AND CORRELATIONS IN EDFs

Example on the dipole polarizability

Covariance analysis: χ^2 test

- Observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$ (e.g. of an EDF)

$$\chi^2(\mathbf{p}) = \frac{1}{m - n_p - 1} \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

- Assuming that the χ^2 can be approximated by an hyper-parabola around the minimum $\mathbf{p}_0$,

$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j})$$

where $\mathcal{M} \equiv \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2$ (curvature m.) and $\mathcal{E} \equiv \mathcal{M}^{-1}$ (error m.).

- errors between predicted observables $\mathcal{A}$

$$\Delta \mathcal{A} = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A}$$

- correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}}$$

where, $C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$

Example: Fitting protocol of SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

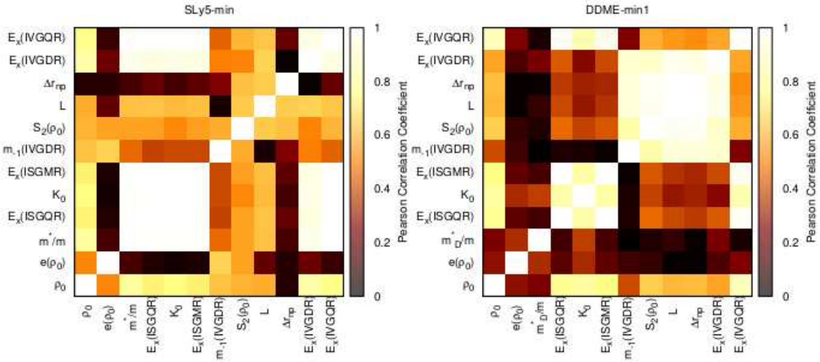
SLy5-min:

- **Binding energies** of ^{40,48}Ca, ^{56}Ni, ^{130,132}Sn and ^{208}Pb with a fixed adopted error of 2 MeV
- the **charge radius** of ^{40,48}Ca, ^{56}Ni and ^{208}Pb with a fixed adopted error of 0.02 fm
- the **neutron matter** Equation of State calculated by Wiringa *et al.* (1988) for densities between 0.07 and 0.40 fm^{-3} with an adopted error of 10%
- the **saturation energy** ($e(\rho_0) = -16.0 \pm 0.2$ MeV) and **density** ($\rho_0 = 0.160 \pm 0.005$ fm^{-3}) of **symmetric nuclear matter**.

DD-ME-min1:

- **binding energies, charge radii, diffraction radii and surface thicknesses** of 17 even-even spherical nuclei, ^{16}O, ^{40,48}Ca, ^{56,58}Ni, ^{88}Sr, ^{90}Zr, ^{100,112,120,124,132}Sn, ^{136}Xe, ^{144}Sm and ^{202,208,214}Pb. The assumed errors of these observables are 0.2%, 0.5%, 0.5%, and 1.5%, respectively.

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

The **neutron skin** is **correlated** with **L** in both models but **NOT** with α_D. **(I will come back on that latter)**

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

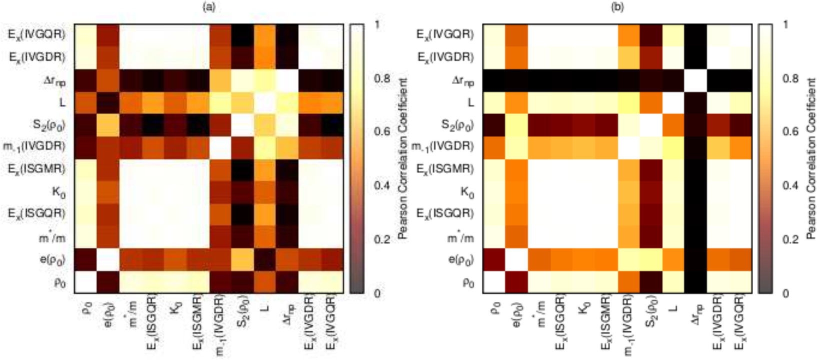
	SLy5-min			DDME-min1			
A	A_0		$\sigma(A_0)$	A_0		$\sigma(A_0)$	units
SNM							
ρ_0	0.162	±	0.002	0.150	±	0.001	fm^{-3}
$e(\rho_0)$	−16.02	±	0.06	−16.18	±	0.03	MeV
m^*/m	0.698	±	0.070	0.573	±	0.008	
J	32.60	±	0.71	33.0	±	1.7	MeV
K_0	230.5	±	9.0	261	±	23	MeV
L	47.5	±	4.5	55	±	16	MeV
^{208}Pb							
E_x^{ISGMR}	14.00	±	0.36	13.87	±	0.49	MeV
E_x^{ISGQR}	12.58	±	0.62	12.01	±	1.76	MeV
Δr_{np}	0.1655	±	0.0069	0.20	±	0.03	fm
E_x^{IVGDR}	13.9	±	1.8	14.64	±	0.38	MeV
m_{-1}^{IVGDR}	4.85	±	0.11	5.18	±	0.28	MeV^{-1} fm^2
E_x^{IVGQR}	21.6	±	2.6	25.19	±	2.05	MeV

Statistical uncertainties depend on the fitting protocol, that is on the **data (or pseudo-data) and associated errors used for the fits: Let us see an example...**

Covariance analysis: modifying the χ^2

→ **SLy5-a:** χ^2 as in SLy5-min except for the neutron EoS (relaxed the required accuracy = increasing associated error).

→ **SLy5-b:** χ^2 as in SLy5-min except the neutron EoS (not employed) and used instead a tight constraint on the Δr_{np} in ^{208}Pb



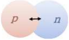
J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

- When a **constraint** on a property is **relaxed**, **correlations** of other observables with such a property should become **larger** → **SLy5-a: α_D is now better correlated with Δr_{np}**
- When a **constraint** on a property is **enhanced** —artificially or by an accurate experimental measurement— **correlations** of other observables with such a property should become **small** → **SLy5-b: Δr_{np} is not correlated with any other observable**

SISTEMATIC UNCERTAINTIES AND CORRELATIONS IN EDFs

Example on the dipole polarizability

Dipole polarizability: macroscopic approach



The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

Adopting the Droplet Model ($m_{-1} \propto \alpha_D$):

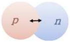
$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

within the same model, connection with the neutron skin thickness:

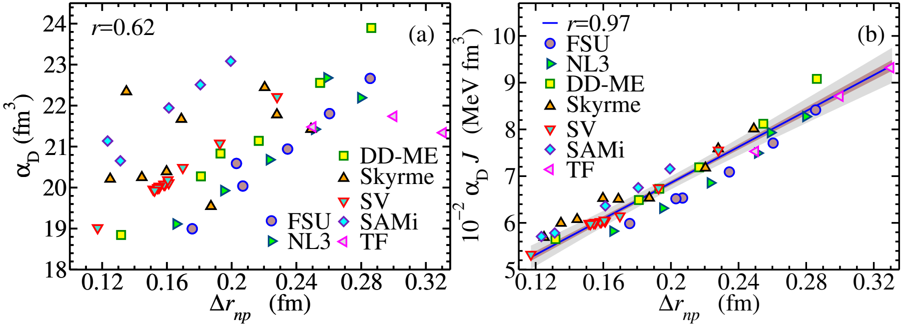
$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Is this correlation appearing also in EDFs?

Isovector Giant Dipole Resonance in ^{208}Pb:

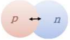


Dipole polarizability: microscopic results HF+RPA



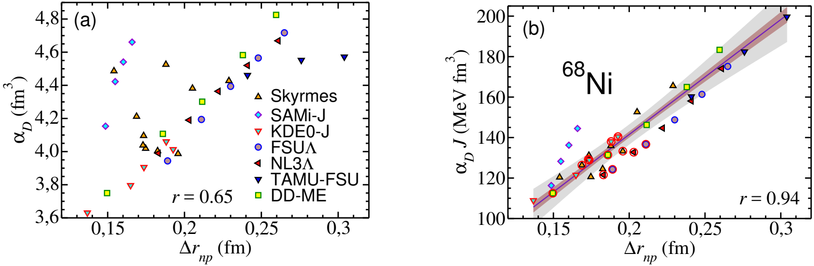
X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

$\alpha_D J$ is linearly correlated with Δr_{np} and no α_D alone within EDFs

Isovector Giant Dipole Resonance in ^{68}Ni: 

What about other nuclei?

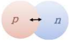
Dipole polarizability: microscopic results HF+RPA



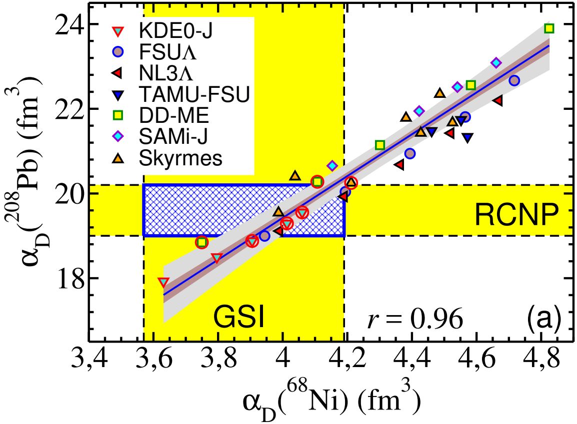
X. Roca-Maza et al. Phys. Rev. C **92**, *064304 (2015)*

Experimental dipole polarizability $\alpha_D = 3.40 \pm 0.23$ fm^3 D. M. Rossi *et al.*, PRL 111, 242503 (GSI). $\alpha_D = 3.88 \pm 0.31$ fm^3 "full" response D. M. Rossi, T. Aumann, and K. Boretzky.

^{208}Pb vs ^{68}Ni:



Dipole polarizability: microscopic results HF+RPA

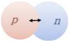


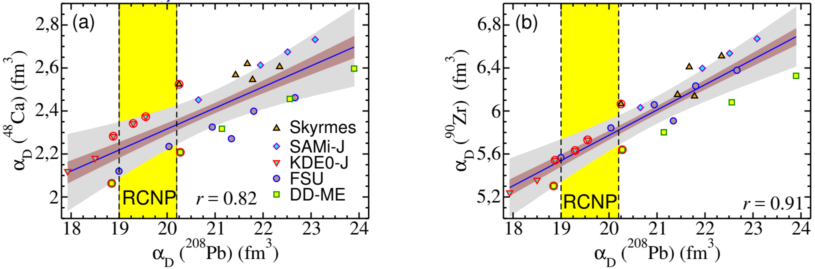
X. Roca-Maza et al. Phys. Rev. C **92**, *064304 (2015)*

Just as an indication DM would predict:

$$\alpha_{\rm D}(A=208)/\alpha_{\rm D}(A=68) \sim (208/68)^{5/3}$$

Can we use this correlation to predict the polarizability in other nuclei?





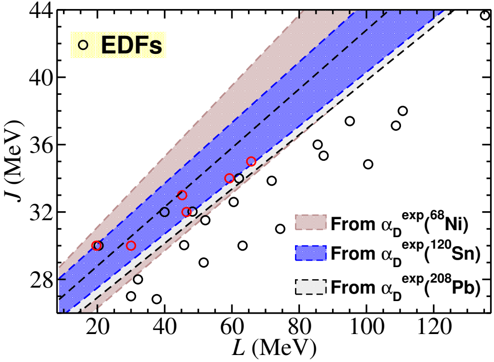
X. Roca-Maza et al. Phys. Rev. C ***92****, 064304 (2015)*

Nucleus	Δr_{np} (fm)	α_D (fm^3)
^{48}Ca	0.15−0.18 (0.16 ± 0.01)	2.06−2.52 (2.30 ± 0.14)
^{90}Zr	0.058−0.077 (0.067 ± 0.008)	5.30−6.06 (5.65 ± 0.23)

Table: Estimates for the neutron skin thickness and electric dipole polarizability of ^{48}Ca and ^{90}Zr from models that predict α_{exp} in ^{68}Ni, ^{120}Sn and ^{208}Pb.

Constraints of this analysis on the J − L plane

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{3}\frac{L}{J}\frac{\rho_0 - \rho_A}{3\rho_0}\right] \text{ where } S(\rho_A) \equiv a_{sym}(A)$$



X. Roca-Maza et al. Phys. Rev. C **92**, *064304 (2015)*

$$J = (24.9 \pm 2.0) + (0.19 \pm 0.02)L \text{ for } {}^{68}\text{Ni}$$
$$J = (25.4 \pm 1.1) + (0.17 \pm 0.01)L \text{ for } {}^{120}\text{Sn}$$
$$J = (24.5 \pm 0.8) + (0.168 \pm 0.007)L \text{ for } {}^{208}\text{Pb}$$

For $S(<\rho> \to \rho_0) \approx J - L\dfrac{(\rho_0 - <\rho>)}{3\rho_0}$

CONCLUSIONS

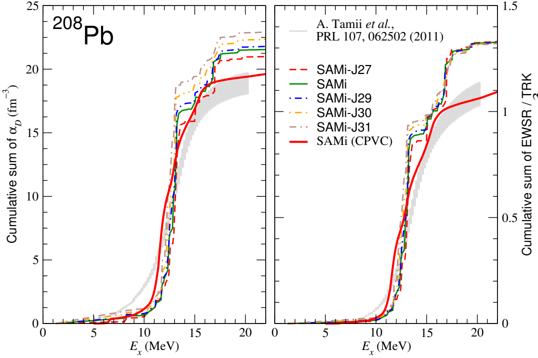
Conclusions:

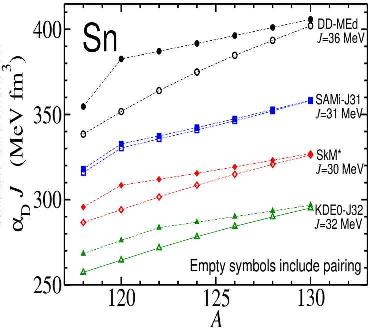
- We have **studied** theoretically **how sensitive is the isovector** channel of the interaction to a **measurement** of the **dipole polarizability** in a heavy nucleus such as ^{208}Pb.
- we have proposed a **physically meaningful correlation** between the polarizability and the properties of the effective interaction: $\alpha_D J$ vs Δr_{np} and not α_D alone.
- Our **results** for ^{208}Pb can be **extended to other nuclei** such as the exotic ^{68}Ni.
- Within our approach, we have **derived three bands in the** $J - L$ **plane** consistent with the recent measurements of the polarizability in ^{68}Ni, ^{120}Sn and ^{208}Pb
- The **slope shown by the derived bands** in the $J - L$ is **not strictly followed by the models** used for the analysis
- Subset of models that reproduce simultaneously the measured polarizabilities are employed to predict $J = 30 - 35$ **MeV,** $L = 20 - 66$ **MeV; and the values for** Δr_{np} **in** ^{68}**Ni,** ^{120}**Sn, and** ^{208}**Pb are in the ranges: 0.15-0.19 fm, 0.12-0.16 fm, and 0.13-0.19 fm**

thank you!

EXTRA MATERIAL

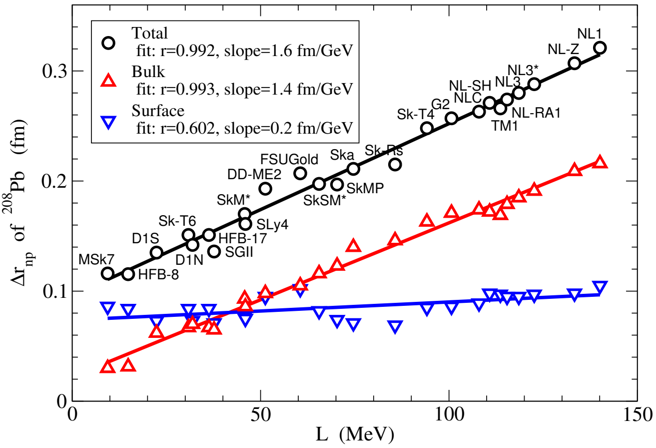
Warnings: RPA versus experiment

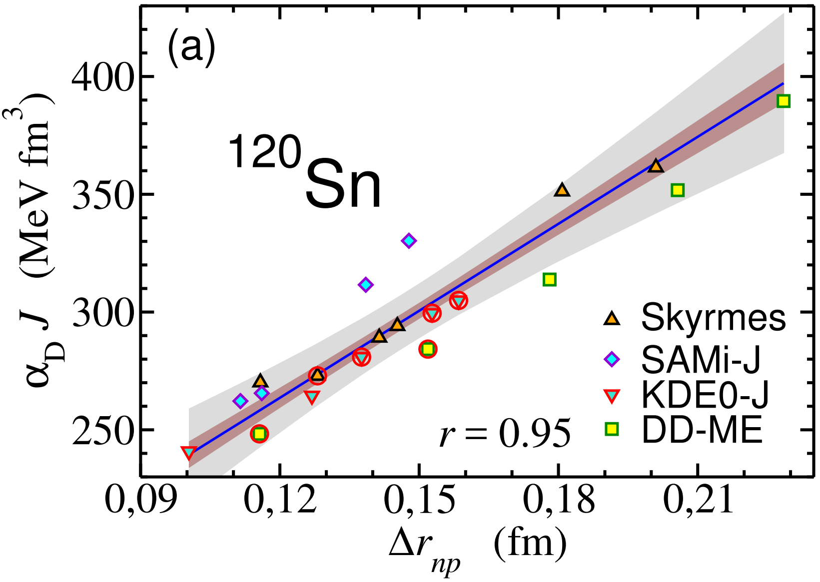




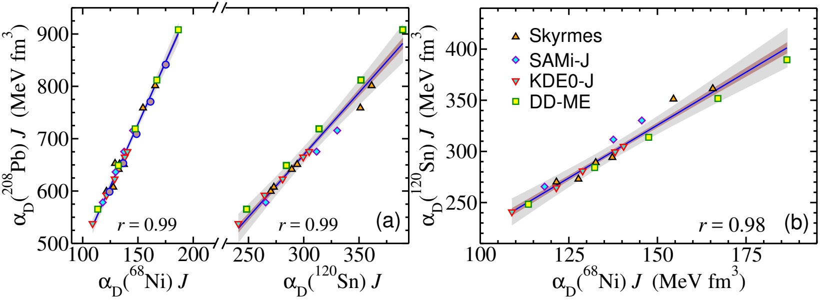
- Important to take into account the **full energy range to compare with RPA** results. (we expect RPA to be quantitative for excitation energy and sum rules but not in details of the response function)
- **RPA do not reproduce the resonance width**, maximum possible error: $\Delta\alpha_D \lesssim -\alpha_D \frac{\Gamma^2}{4E_x^2} < 2\%$ in ^{208}Pb
- Including **pairing correlations for Sn**: Δr_{np} and α_D tend to be smaller by few % (0-8% in ^{120}Sn and studied models).
- **quasi-deuteron contributions** should be substracted from exp.

Δr_{np} in ^{208}Pb:

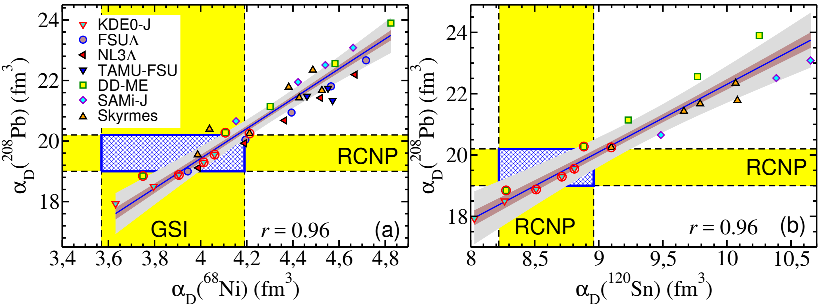




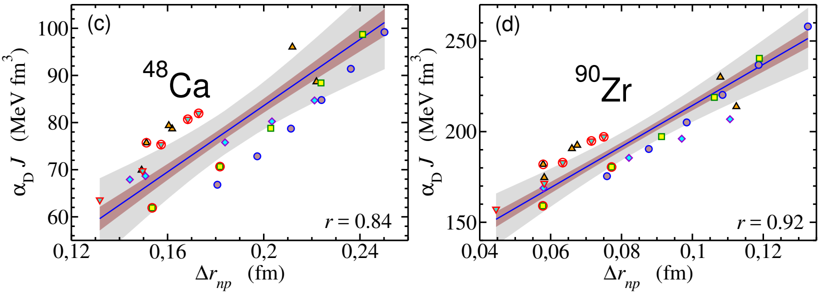
(a)
^{120}Sn
$\alpha_D J$ (MeV fm^3)
400
350
300
250
0,09
0,12
0,15
0,18
0,21
Δr_{np} (fm)
$r = 0.95$
Skyrmes
SAMi-J
KDE0-J
DD-ME



α_D(208Pb) J (MeV fm3)
α_D(68Ni) J
α_D(120Sn) J
r = 0.99
r = 0.99
(a)
α_D(120Sn) J (MeV fm3)
α_D(68Ni) J (MeV fm3)
Skyrmes
SAMi-J
KDE0-J
DD-ME
r = 0.98
(b)



KDE0-J
FSUΛ
NL3Λ
TAMU-FSU
DD-ME
SAMi-J
Skyrmes
RCNP
GSI
r = 0.96
(a)
α_D(208Pb) (fm3)
α_D(68Ni) (fm3)
3,4
3,6
3,8
4
4,2
4,4
4,6
4,8
18
20
22
24
RCNP
RCNP
r = 0.96
(b)
α_D(120Sn) (fm3)
8
8,5
9
9,5
10
10,5


(c)
48Ca
r = 0.84
αD J (MeV fm3)
Δrnp (fm)
100
90
80
70
60
0,12
0,16
0,2
0,24
(d)
90Zr
r = 0.92
αD J (MeV fm3)
Δrnp (fm)
250
200
150
0,04
0,06
0,08
0,1
0,12