

# New Skyrme energy density functional for a better description of charge-exchange resonances

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## Table of contents:

- ▶ Spin and Isospin excitations in Nuclei
- ▶ Some general comments on EDFs
- ▶ Motivation and present situation: example GTR
- ▶ Propose a new fitting protocol: Example with a Skyrme interaction

## Spin and Isospin excitations in Nuclei

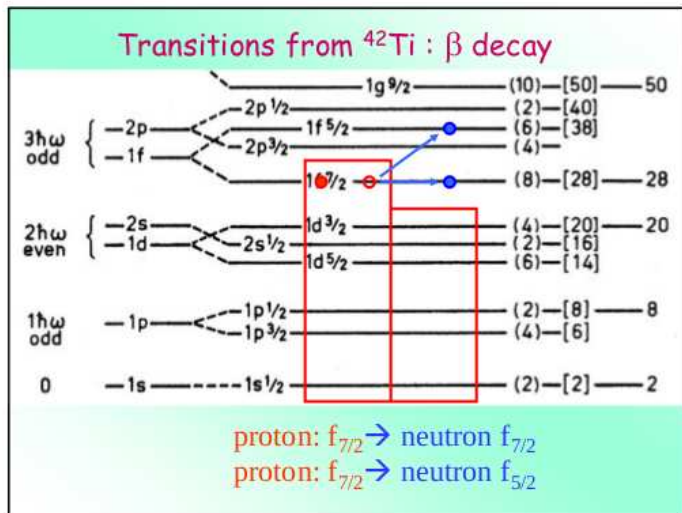
- ▶ **Nucleons** are fermions charac. by their spin and isospin
- ▶ **Nucleons** with spin (isospin) may **change their state** in **phase**: spin-scalar  $S=0$  modes (isospin-scalar  $T=0$  modes); or **out of phase**: spin-vector  $S=1$  modes (isospin-vector  $T=1$  modes)
- ▶ They can be **excited by strong probes** (charge-exchange reactions) and they can **decay via the weak interaction** (axial-vector current couples to the spin and induces  $\beta$ -decay processes)

One of the most important nuclear excitation modes is the

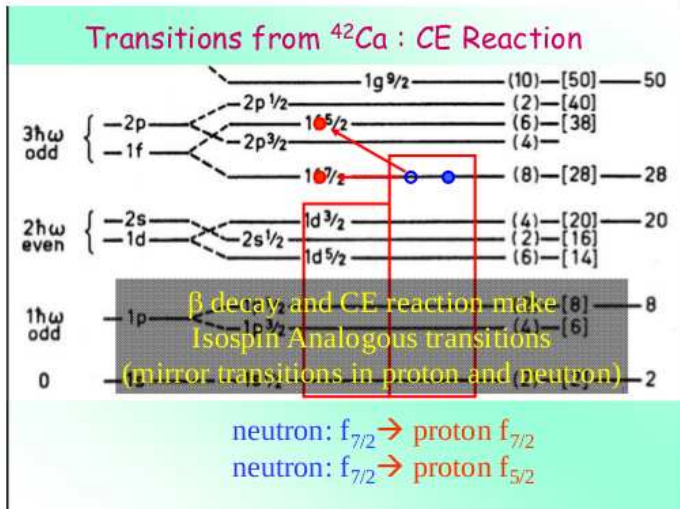
- ▶ **Gamow Teller Resonance** which is a pure **spin-isospin mode** (i.e., from a theoretical picture, it is excited by an operator  $\hat{O} \sim \sigma\tau$ )

**Spin-isospin modes** of excitation (such as the **GTR**) give **direct information** on the spin-isospin channel of the **effective interaction**

## Example: $\beta$ -decay transition



# Example: Gamow Teller transition



## Therefore,

- ▶ allowed **GT** transitions mainly determine  **$\beta$ -decay half-lives**
- ▶ **GT** transitions determine **weak interaction rates** essential role in the **core-collapse dynamics** of massive stars leading to supernova explosion
- ▶ In neutron-rich environment, **neutrino-induced nucleosynthesis** may take place via **GT** processes
- ▶ **GT** matrix elements are necessary for the study of **double- $\beta$ -decay**
- ▶ may be useful in the **calibration of detectors** used to measure neutrinos that reach the Earth

# Some comments on the nuclear many-body problem:

- ▶ **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - ▶ **different predictions** are found **depending** on the **approach**
  - ▶ **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- ▶ Based on effective interactions, **Nuclear Energy Density Functionals** are **successful (but still not perfect)** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

# Nuclear Energy Density Functionals:

## Main types of successful EDFs derived from the mean-field approximation

- ▶ **Relativistic HF models**, based on Lagrangians where effective (heavy) mesons carry the interaction.

$$\begin{aligned}\mathcal{L}_{\text{int}} &= \bar{\Psi}\Gamma_{\sigma}(\bar{\Psi}, \Psi)\Psi\Phi_{\sigma} && + \bar{\Psi}\Gamma_{\delta}(\bar{\Psi}, \Psi)\boldsymbol{\tau}\Psi\Phi_{\delta} \\ &- \bar{\Psi}\Gamma_{\omega}(\bar{\Psi}, \Psi)\gamma_{\mu}\Psi\mathcal{A}^{(\omega)\mu} && - \bar{\Psi}\Gamma_{\rho}(\bar{\Psi}, \Psi)\gamma_{\mu}\boldsymbol{\tau}\Psi\mathcal{A}^{(\rho)\mu} \\ &- e\bar{\Psi}\hat{Q}\gamma_{\mu}\Psi\mathcal{A}^{(\gamma)\mu}\end{aligned}$$

- ▶ **Non-relativistic HF models**, based on Hamiltonians where effective interactions are proposed and tested:

$$V_{\text{Nucl}}^{\text{eff}} = V_{\text{attractive}}^{\text{long-range}} + V_{\text{repulsive}}^{\text{short-range}} + V_{\text{SO}}$$

- ▶ Fitted **parameters contain** (important) **correlations beyond the mean-field**
- ▶ Nuclear energy functionals are **phenomenological** → **not directly connected to any NN (or NNN) interaction**



# Drawbacks on current EDFs ???

On the one side,

- ▶ **we expect** that the **H(F)+RPA** method based on nuclear effective interactions of the **Skyrme, Gogny or Relativistic** (can be understood as an **approximate realization of an EDF**)  $\Rightarrow$  **reasonable description of g.s. energy and density of the system** (<one-body operators>)

On the other side,

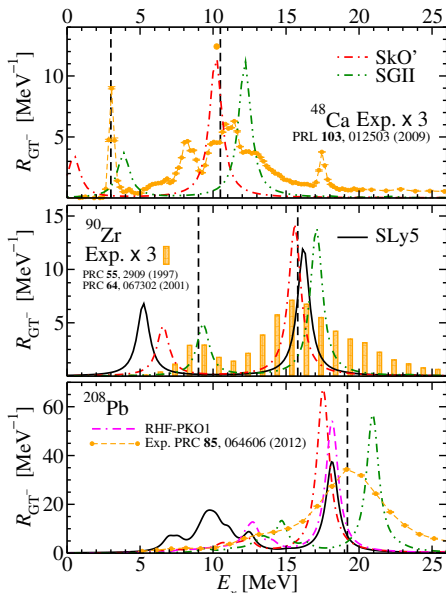
- ▶ there are still some **open problems** ... but we will concentrate here on how to

**improve the spin-isospin properties of our EDF**

# Motivation: Gamow Teller Resonance

The  $E_x$  is not properly described in H(F)+RPA

- ▶ **SGII**<sup>a</sup>: earliest attempt to give a quantitative description of the GTR
- ▶ **SkO'**<sup>b</sup>: accurate in ground state finite nuclear properties and improves the GTR
- ▶ **PKO1**<sup>c</sup>: relativistic HF, reasonable GTR still not perfect
- ▶ Relativistic H<sup>d</sup>: residual interaction modified *ad-hoc*



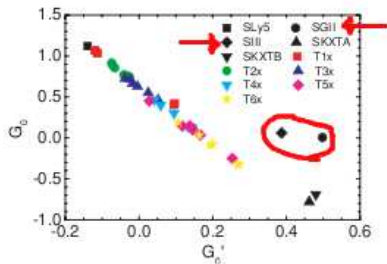
<sup>a</sup> PLB 106, 379 (1981), <sup>b</sup> PRC 60, 014316 (1999), <sup>c</sup> PRL 101, 122502 (2008), <sup>d</sup> PRC 69, 054303

# Motivation: which gs properties are important for describing the $E_x^{\text{GTR}}$ ?

The study<sup>a</sup> of the GTR and the spin-isospin Landau-Migdal parameter  $G'_0$  using several Skyrme sets,

- ▶ concluded that  $G'_0$  is not the only important quantity in determining the excitation energy of the GTR
- ▶ spin-orbit splittings also influences the GTR

- ▶ Empirical indications<sup>b</sup> suggest that  $G'_0 > G_0 > 0$
- ▶ Not a very common feature within available Skyrme forces<sup>c</sup>



<sup>a</sup>M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C **65**, 054322 (2002); <sup>b</sup>T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005); T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999), <sup>c</sup>Li-Gang Cao, G. Colo, and H. Sagawa, Phys. Rev. C **81**, 044302 (2010)

# Landau-Migdal vs Skyrme parameters

- ▶ The p-h interaction is derived as the second functional derivative of the total energy with respect to density at the Fermi surface. Matrix elements of the form:

$$\langle \vec{k}_1 \vec{k}_2 | V | \vec{k}_1 \vec{k}_2 \rangle = \sum_{\alpha=1, \tau, \sigma, \tau \cdot \sigma} \frac{\delta \mathcal{H}}{\delta \rho_\alpha \delta \rho_\alpha} = N_0^{-1} (F + F' \tau_1 \tau_2 + G \sigma_1 \sigma_2 + G' \tau_1 \tau_2 \sigma_1 \sigma_2)$$

\*  $N_0 = 2k_F m^* / \hbar^2 \pi^2$  is the density of states per energy at the Fermi surface

\* Parameters are functions of  $\vec{k}_1$  and  $\vec{k}_2$

\*  $\vec{k}_1$  and  $\vec{k}_2$  are taken at the Fermi surface (Landau parameters are only functions of the angle between them and the Fermi momentum  $\Rightarrow F = \sum_l F_l P_l(\cos \theta)$ ).

- ▶ One can do the same functional derivative with Skyrme and compare both expressions to find the relation between the Landau-Migdal and the Skyrme parameters (here just two of them):

$$G_0 N_0 = -\frac{1}{4} t_0 + \frac{1}{2} t_0 x_0 - \frac{1}{8} t_1 k_F^2 + \frac{1}{4} t_1 x_1 k_F^2 + \frac{1}{8} t_2 k_F^2 + \frac{1}{4} t_2 x_2 k_F^2 - \frac{1}{24} t_3 \rho^\alpha + \frac{1}{12} t_3 x_3 \rho^\alpha$$

$$G'_0 N_0 = -\frac{1}{4} t_0 - \frac{1}{8} t_1 k_F^2 + \frac{1}{8} t_2 k_F^2 - \frac{1}{24} t_3 \rho^\alpha$$

# Empirical constraints on $G_0$ and $G'_0$

- ▶ **Gamow-Teller Resonance** using RPA based on the Woods-Saxon potential have been studied and the **Landau-Migdal parameters estimated by comparing experiment** with theoretical calculations in Refs. [T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005) and T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999)].
  - ▶ Landau-Migdal parameter  $G'_0$  **dominates the excitation energy in the GTR**
- ▶ In our fit, **we do not use the obtained values as pseudodata** because **our theoretical framework is different and the results are associated to different  $m^*$**  (our sp energies are based on HF calculations instead of a Wood-Saxon potential).
- ▶ **We use** the empirical result in which a **hierarchy** between spin and spin-isospin parameters is suggested:

$$G'_0 > G_0 > 0$$

# Why spin-orbit splittings are important?

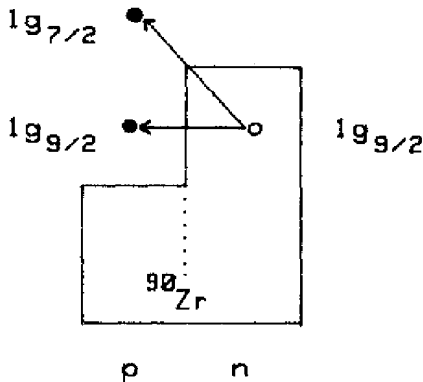
$$E_x^1 \approx$$

$$\epsilon_{\pi 1g_{7/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^1$$

$$E_x^2 \approx$$

$$\epsilon_{\pi 1g_{9/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^2$$

$$\Delta E_x \approx \Delta \epsilon_{\pi 1g} + \Delta \epsilon_{ph}$$



Schematic picture of single-particle transitions involved in the Gamow Teller Resonance of  $^{90}\text{Zr}$ . Transitions excited by  $\sigma\tau_-$  operator.

# Skyrme Model

## Hamiltonian<sup>a</sup>

Includes **central tensor terms ( $J^2$  terms)** due to the coupling of tensor and spin and gradients terms and **two spin-orbit parameters** (same as SkO and some SkI forces)

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}} + \mathcal{H}_{\text{SO}} + \mathcal{H}_{\text{sg}} + \mathcal{H}_{\text{Coul}}$$

$$\mathcal{K} = \hbar^2 \tau / 2m$$

$$\mathcal{H}_0 = (1/4)t_0[(2 + x_0)\rho^2 - (2x_0 + 1)(\rho_n^2 + \rho_p^2)]$$

$$\mathcal{H}_3 = (1/24)t_3\rho^\alpha[(2 + x_3)\rho^2 - (2x_3 + 1)(\rho_n^2 + \rho_p^2)]$$

$$\mathcal{H}_{\text{eff}} = (1/8)[t_1(2 + x_1) + t_2(2 + x_2)]\tau\rho \\ + (1/8)[t_2(2x_2 + 1) - t_1(2x_1 + 1)](\tau_n\rho_n + \tau_p\rho_p)$$

$$\mathcal{H}_{\text{fin}} = (1/32)[3t_1(2 + x_1) - t_2(2 + x_2)](\nabla\rho)^2 \\ - (1/32)[3t_1(2x_1 + 1) + t_2(2x_2 + 1)][(\nabla\rho_n)^2 + (\nabla\rho_p)^2]$$

$$\mathcal{H}_{\text{SO}} = (1/2)W_0\mathbf{J} \cdot \nabla\rho + (1/2)W'_0(\mathbf{J} \cdot \mathbf{n} \nabla\rho_n + \mathbf{J}_p \cdot \nabla\rho_p)$$

$$\mathcal{H}_{\text{sg}} = -(1/16)(t_1x_1 + t_2x_2)\mathbf{J}^2 + (1/16)(t_1 - t_2)(\mathbf{J}_n^2 + \mathbf{J}_p^2)$$

<sup>a</sup>E. Chabanat et al., Nucl. Phys. A **635**, 231 (1998); E. Chabanat *et al.*, *ibid.* **643**, 441 (1998)

# Fitting Protocol

$\chi^2$  definition: 
$$\chi^2 = \frac{1}{N_{\text{data}}} \sum_i N_{\text{data}} \frac{(\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{data}})^2}{(\Delta \mathcal{O}_i^{\text{data}})^2}$$

**Landau-Migdal parameters** in infinite nuclear matter  $G_0$  and  $G'_0$  fixed to **0.15** and **0.35**, respectively, at  $\rho_0$ .

**Table:** Data and *pseudo*-data  $\mathcal{O}_i$ , adopted errors for the fit  $\Delta \mathcal{O}_i$  and selected finite nuclei and EoS.

$\mathcal{O}_i$	$\Delta \mathcal{O}_i$	
B	1.00 MeV	$^{40,48}\text{Ca}$ , $^{90}\text{Zr}$ , $^{132}\text{Sn}$ and $^{208}\text{Pb}$
$r_c$	0.01 fm	$^{40,48}\text{Ca}$ , $^{90}\text{Zr}$ and $^{208}\text{Pb}$
$\Delta E_{\text{SO}}$	$0.04 \times \mathcal{O}_i$	$\pi 1g$ in $^{90}\text{Zr}$ and $\pi 2f$ in $^{208}\text{Pb}$
$e_n(\rho)$	$0.20 \times \mathcal{O}_i$	R. B. Wiringa <i>et al.</i> , PRC 38, 1010 (1988)



# Skyrme Aizu Milano interaction: SAMi

## Parameter set and nuclear matter properties:

Table: SAMi parameter set and saturation properties with the estimated standard deviations inside parenthesis

	value( $\sigma$ )			value( $\sigma$ )	
$t_0$	-1877.75(75)	MeV fm <sup>3</sup>	$\rho_\infty$	0.159(1)	fm <sup>-3</sup>
$t_1$	475.6(1.4)	MeV fm <sup>5</sup>	$e_\infty$	-15.93(9)	MeV
$t_2$	-85.2(1.0)	MeV fm <sup>5</sup>	$m_{IS}^*$	0.6752(3)	
$t_3$	10219.6(7.6)	MeV fm <sup>3+3<math>\alpha</math></sup>	$m_{IV}^*$	0.664(13)	
$x_0$	0.320(16)		J	28(1)	MeV
$x_1$	-0.532(70)		L	44(7)	MeV
$x_2$	-0.014(15)		$K_\infty$	245(1)	MeV
$x_3$	0.688(30)		$G_0$	0.15	(fixed)
$W_0$	137(11)		$G'_0$	0.35	(fixed)
$W'_0$	42(22)				
$\alpha$	0.25614(37)				

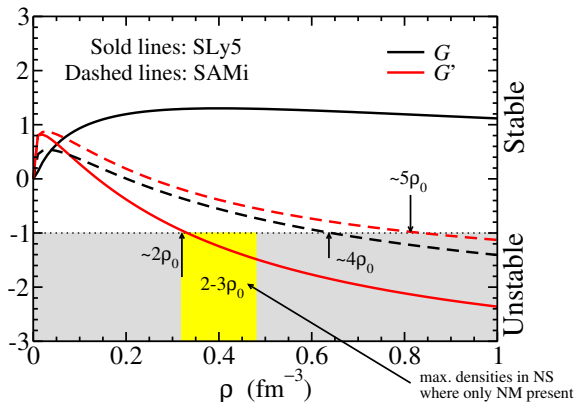
# SAMi: spin and spin-isospin instabilities in NM

## Qualitative test

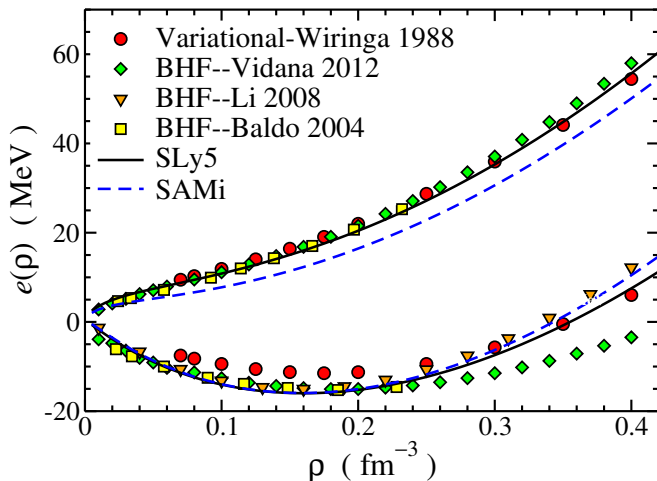
Imposing that spin and isospin dof at the Fermi surface are stable under generalized deformations [S.-O. Bäckman *et al.*, Nucl. Phys. A **321**, 10 (1979)]

$$1 + G_0 > 0$$

$$1 + G'_0 > 0$$

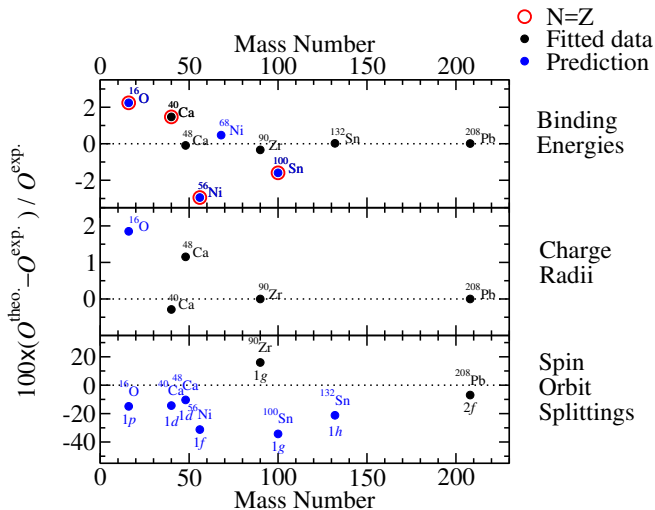


# Equation of State: SAMi vs *ab-initio* calculations



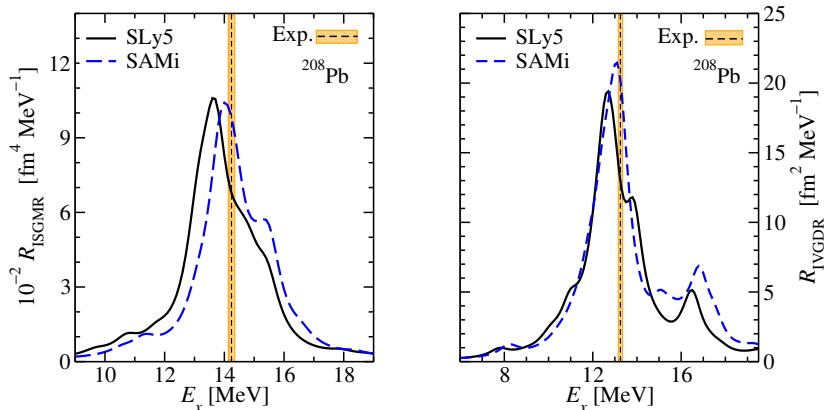
**Figure:** Neutron and symmetric matter EoS as predicted by the HF SAMi (dashed line) and SLy5 (solid line) interactions and by the benchmark microscopic calculations of R. B. Wiringa *et al.*, PRC **38**, 1010 (1988) (circles). State-of-the-art BHF calculations are shown by diamonds I. Vidaña, private communication, triangles Z. H. Li *et al.*, Phys. Rev. C **77**, 034316 (2008) and squares M. Baldo *et al.*, Nucl. Phys. A **736**, 241 (2004).

# Finite Nuclei: spherical double-magic nuclei



**Figure:** Finite nuclei properties as predicted by the HF SAMi (black circles) and some predictions (blue circles) for spherical double-magic nuclei. Experimental data taken from Refs. G. Audi *et al.*, NPA **729**, 337 (2003), I. Angeli, ADNDT **87**, 185 (2004), M. Zalewski *et al.*, PRC **77**, 024316 (2008)

# Giant Monopole and Dipole Resonances in $^{208}\text{Pb}$



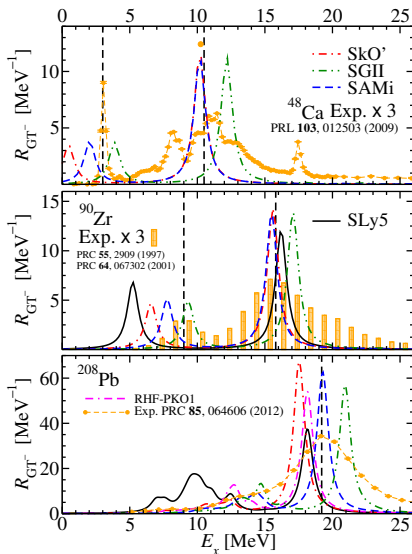
**Figure:** Strength function at the relevant excitation energies in  $^{208}\text{Pb}$  as predicted by SLy5 and the SAMi interaction for GMR and GDR. A Lorentzian smearing parameter equal to 1 MeV is used. Experimental data for the centroid energies are also shown:  $E_c (\text{GMR}) = 14.24 \pm 0.11$  MeV [D. H. Youngblood, et al., Phys. Rev. Lett. **82**, 691 (1999)] and  $E_c (\text{GDR}) = 13.25 \pm 0.10$  MeV [N. Ryezayeva et al., Phys. Rev. Lett. **89**, 272502 (2002)].

# Gamow Teller Resonance in $^{48}\text{Ca}$ , $^{90}\text{Zr}$ and $^{208}\text{Pb}$

Operator:

$$\sum_{i=1}^A \sigma(i) \tau_{\pm}(i)$$

**Figure:** Gamow Teller strength distributions in  $^{48}\text{Ca}$  (upper panel),  $^{90}\text{Zr}$  (middle panel) and  $^{208}\text{Pb}$  (lower panel) as measured in the experiment [T. Wakasa *et al.*, Phys. Rev. C **55**, 2909 (1997), K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009), A. Krasznaborkay *et al.*, Phys. Rev. C **64**, 067302 (2001), H. Akimune *et al.*, Phys. Rev. C **52**, 604 (1995) and T. Wakasa *et al.*, Phys. Rev. C **85**, 064606 (2012)] and predicted by SLy5, SkO', SGII and SAMi forces.



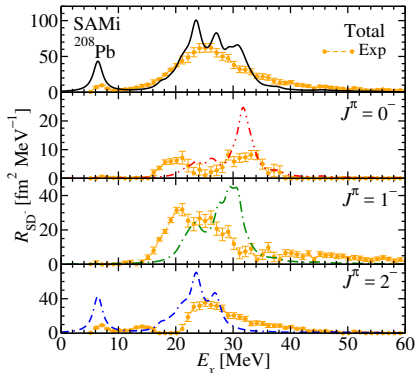
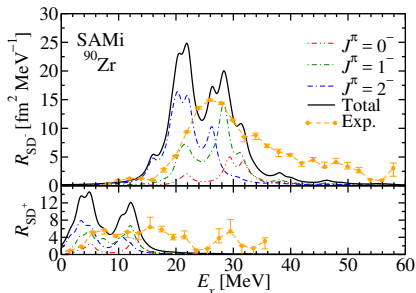
# Spin Dipole Resonances in $^{90}\text{Zr}$ and $^{208}\text{Pb}$

Operator:

$$\sum_{i=1}^A \sum_M \tau_{\pm}(i) r_i^L [Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i)]_{JM}$$

Sum Rule:

$$\int [R_{SD^-}(E) - R_{SD^+}(E)] dE = \frac{2}{4\pi} (N \langle r_n^2 \rangle - Z \langle r_p^2 \rangle)$$



T. Wakasa *et al.*, Phys. Rev. C **85**, 064606 (2012)

K. Yako *et al.*, Phys. Rev. C **74**, 051303(R) (2006)

## Conclusions:

- ▶ we have **successfully determined a new Skyrme** energy density functional which **accounts** for the most relevant quantities in order to improve the description of **charge-exchange nuclear resonances**:
  - ▶ the **hierarchy** and **positive values** of the spin and spin-isospin Landau-Migdal parameters  $G_0$  and  $G'_0$
  - ▶ the **proton spin-orbit splittings** of different **high angular momenta** single-particle levels
- ▶ the **GTR** in  $^{48}\text{Ca}$  and the **GTR, IAR, and SDR** in  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$  are predicted with **good accuracy by SAMi**
- ▶ **SAMi does not deteriorate** the description of other **nuclear observables**
- ▶ **applicability in nuclear physics and astrophysics**



**Thank you for your  
attention!**

# **Extra Material**

# Motivation:

## Gamow Teller Resonance II: quenching of the strength

- ▶ **Experimentally**, the **GTR** exhausts **60–70%** of the **Ikeda sum rule**:  $\int [R_{GT^-}(E) - R_{GT^+}(E)] dE = 3(N - Z)$
- ▶ To **explain** the problem, two possibilities that go beyond RPA correlations have been drawn:
  - ▶ the effects of the second-order configuration mixing: **2p-2h correlations**
  - ▶ within the quark model, a **n(p)** can become a **p(n)** or a  $\Delta^+(\Delta^{++})$  under the action of the  $GT^-$  operator and since there is **no Pauli blocking for  $\Delta$ -h excitations**  $\Rightarrow$  it may **contribute to the GTR**.
- ▶ The **experimental analysis of  $^{90}\text{Zr}$**   $\Rightarrow$  **quenching** (2/3) has to be **mainly attributed to 2p-2h** coupling and not to  $\Delta$ -isobar effects much smaller [T. Wakasa *et. al.*, Phys. Rev. C 55, 2909 (1997)].
- ▶  $E_x$  **GTR in nuclei** mainly in the region of several **tens of MeV** and the  $\Delta$ -h states are hundreds of MeV above the GT  $\Rightarrow$  **hard to excite the  $\Delta$**  in the nuclear medium.

## Covariance analysis: $\chi^2$ test

Observables  $\mathcal{O}$  are used to calibrate the parameters  $\mathbf{p}$  of a given model. The optimum parametrization  $\mathbf{p}_0$  is determined by a least-squares fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{i=1}^m \left( \frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the  $\chi^2$  is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned} \chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{i,j}^n (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2(p_j - p_{0j}) \\ &\equiv \sum_{i,j}^n (p_i - p_{0i}) \mathcal{M}_{ij} (p_j - p_{0j}) \end{aligned}$$

where  $\mathcal{M}$  is the curvature matrix.

## Covariance analysis: $\chi^2$ test

$\mathcal{M}$  provides us access to estimate the errors between predicted observables ( $A(\mathbf{p})$ ),

$$\Delta A = \sqrt{\sum_i^n \partial_{p_i} A \varepsilon_{ii} \partial_{p_i} A} \quad (1)$$

$\varepsilon = \mathcal{M}^{-1}$  and the correlations between predicted observables,

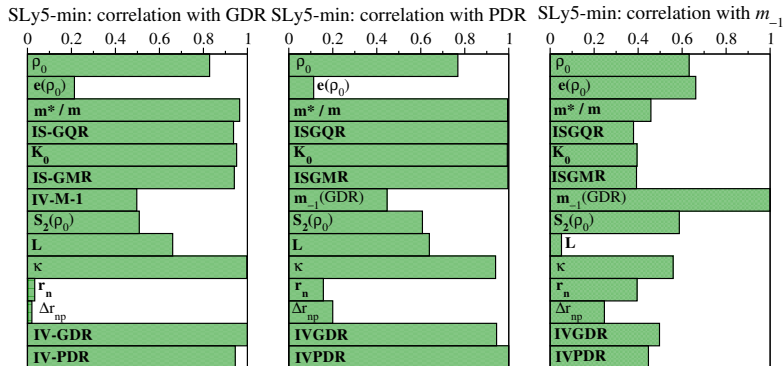
$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA}C_{BB}}} \quad (2)$$

where,

$$C_{AB} = \overline{(A(\mathbf{p}) - \bar{A})(B(\mathbf{p}) - \bar{B})} \approx \sum_{ij}^n \partial_{p_i} A \varepsilon_{ij} \partial_{p_j} B$$



# Covariance analysis: SLy5-min as an example



**Figure:** Pearson product-moment correlation coefficient for the IVGDR (left panel), IVPDR (middle panel) and  $m_{-1}$  (IVGDR) (right panel) with all other studied properties as predicted by the covariance analysis of SLy5.