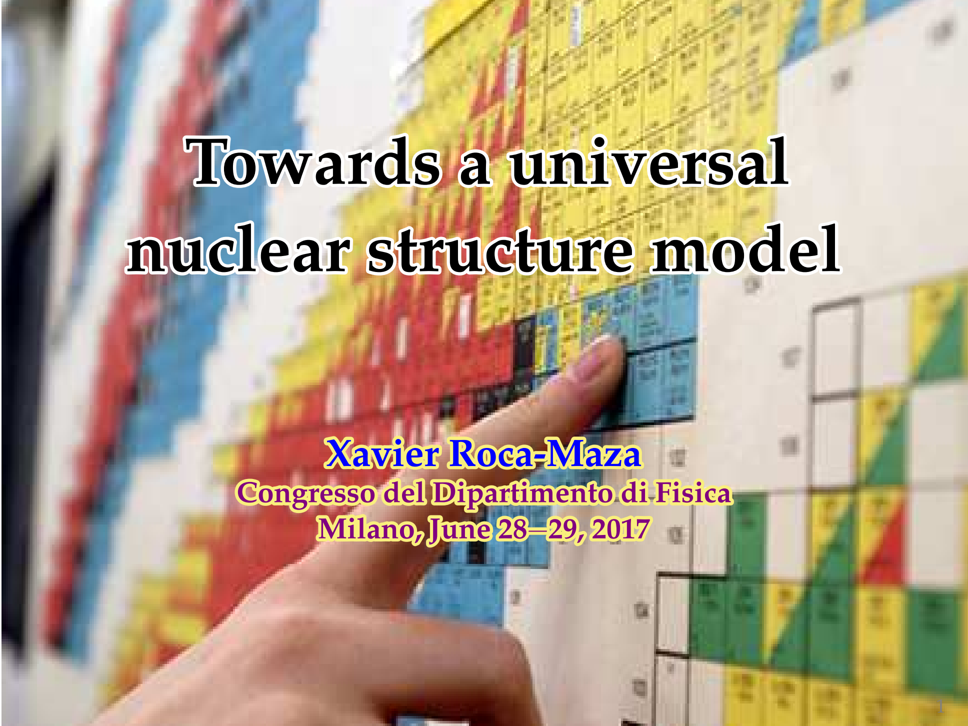

Towards a universal nuclear structure model

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Congresso del Dipartimento di Fisica

Milano, June 28–29, 2017

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Brief presentation of the group:

The group interests focus on theoretical nuclear structure studies: that is, on the study of i) the strong interaction that effectively acts between nucleons in the medium; and ii) suitable many-body techniques to understand the very diverse nuclear phenomenology.

- P. F. Bortignon (PO)

- G. Colò (PO)

- X. Roca-Maza (RTD)

- E. Vigezzi (Dir. Ric.)

Some open lines of research in which the group is involved:

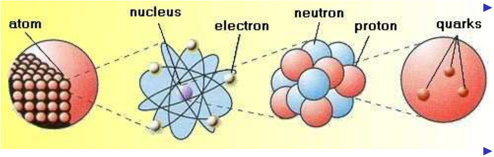
- **Density functional theory:** successful approach to the study of the nuclear matter EoS or of the mass, size and collective excitations in nuclei
- **Nuclear field theory:** successful approach to the study of fragmentation of sp states, resonance widths or half-lives
- **Parity violating and conserving e-N scattering:** characterize the electric and weak charge distribution in nuclei
- **Superfluidity in nuclei:** driven by the pairing interaction
- **Nucleon transfer reactions:** probe spectroscopic properties and superfluidity
- **Astrophysical appliactions:** neutron stars, electron capture, $\beta-$decay, Gamow-Teller resonances

Today...

I will concentrate on some of the basic complexities of the nuclear many-body problem and briefly explain the way along our group is working

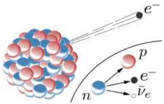
Motivation: The Nuclear Many-Body Problem

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions** (**neutrons and protons**).
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **3 of the 4 fundamental forces in nature** are contributing to the nuclear phenomena (**as a whole driven by the strong interaction**).

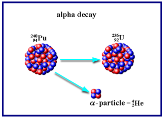


- Nuclei: self-bound system by the strong interaction [In the binding energy $B_{coul} \sim -B_{strong}/(3 \text{ to } 10)$]

- β−decay: weak process

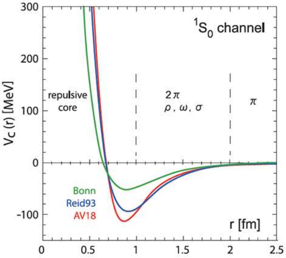


- α−decay: interplay between the strong and electromagnetic interaction



Motivation: The Nuclear Many-Body Problem

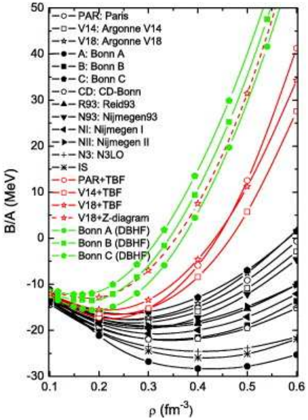
- **Underlying interaction:** the "so called" **residual strong interaction = nuclear force**, **the one acting effectively between nucleons**, has **not been derived yet** from first principles as **QCD is non-perturbative** at the low-energies relevant for the description of nuclei.



The nuclear force in practice: effective potential fitted to nucleon-nucleon scattering data in the vacuum. 3 Body force are needed. 4 Body?

Motivation: The Nuclear Many-Body Problem

- **State-of-the-art many-body** calculations based on **these potentials** are **not conclusive** yet:



- Which parametrization of the residual strong interaction should we use?
- Which many-body technique is the most suitable?
- Exp. $B/A(0.16\ \mathrm{fm}^{-3}) = -16$ MeV

So, two important points to remember:

Working with the *exact* **Hamiltonian and most general wave function** is **not possible** at the moment

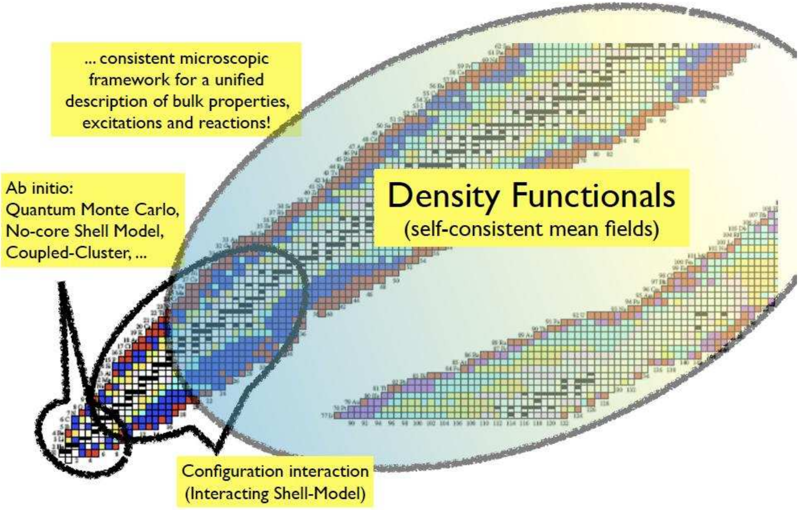
Best attempts so far show too large **discrepancies** and **can** be applied **only to nuclei up to mass number 10-20 approx.**

Motivation: So what to do?

- **Effective interactions** solved at the **Hartree-Fock or Mean-Field** but **fitted to experimental data in many-body system** have been shown to be **successful** in the description of bulk properties **of all nuclei** (**masses, nuclear radii, deformations, Giant Resonances...**)
- These **effective models** can be understood as an **approximate realization of a nuclear energy density functional** $\mathsf{E}[\rho]$.
- **Density Functional Theory rooted on the Hohenberg-Kohn theorem** $\Rightarrow$ **exact functional exists**.

The **advantage** of these approximate $\mathsf{E}[\rho]$ is that they are quite versatile: **nowadays our unique tool to self-consistently access the ground state and some excited state properties of ALL atomic nuclei**

Applicability of nuclear E[ρ] as compared to other methods



Nuclear mean-field models (or EDFs)

$$\langle \Psi | \mathcal{H} | \Psi \rangle \approx \langle \Phi | \mathcal{H}_{\text{eff}} | \Phi \rangle = E[\rho]$$

where Φ is a Slater determinant and ρ is a one-body density matrix $\Rightarrow$ we expect reliable description for the expectation value of one-body operators.

Main types of models:

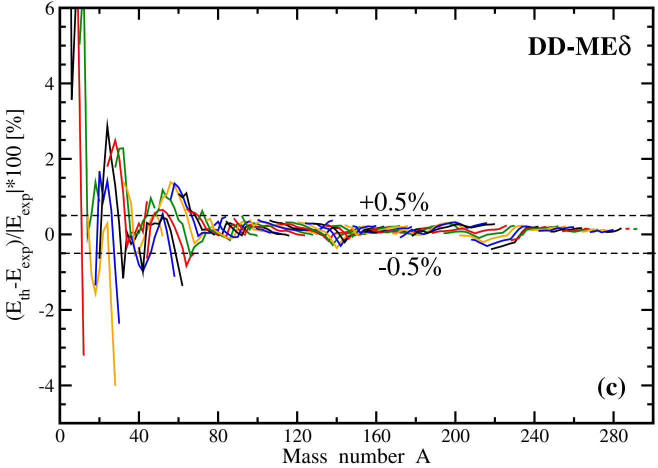
- **Relativistic** based on Lagrangians where effective mesons carry the interaction (π, σ, ω...).
- **Non-relativistic** based on effective Hamiltonians (Yukawa, Gaussian or zero-range two body forces)

$\Rightarrow$ **Both give similar results**

$\Rightarrow$ **phenomenological** models $\rightarrow$ **difficult to connect to the residual strong interaction**

Examples: Binding energies

Relativistic model by Milano and Barcelona groups

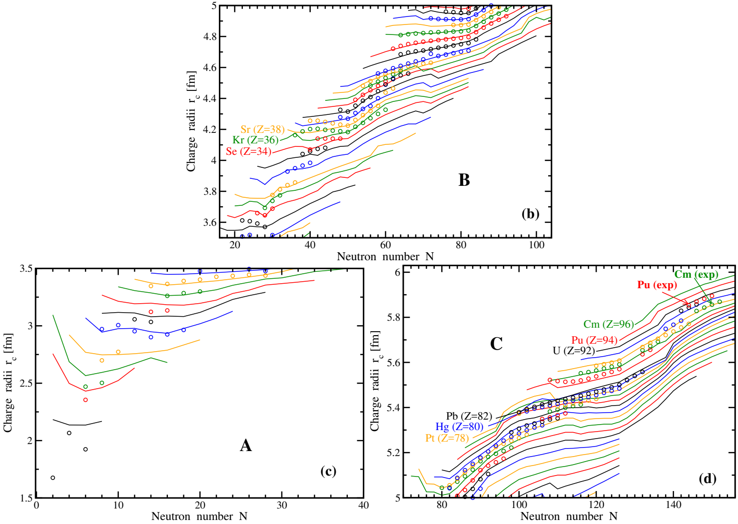


[PHYSICAL REVIEW C 89, 054320 (2014)]

Remarkable accuracy on thousands of measured binding energies

Examples: Charge radii

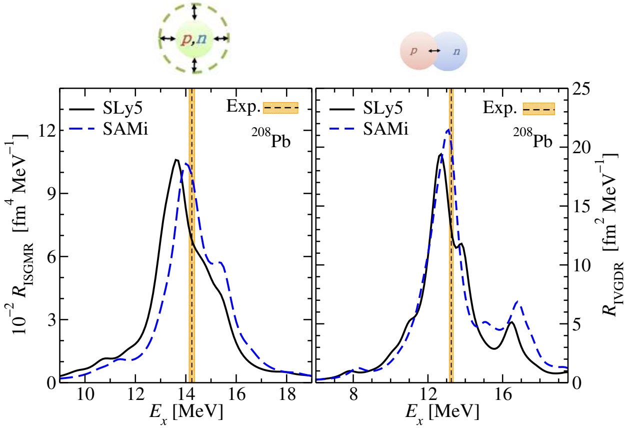
Theory-lines / Experiment - circles



[PHYSICAL REVIEW C 89, 054320 (2014)]

Examples: Giant Monopole and Dipole Resonances

Non-relativistic model by Milano and Aizu (Japan) groups

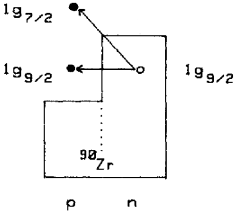


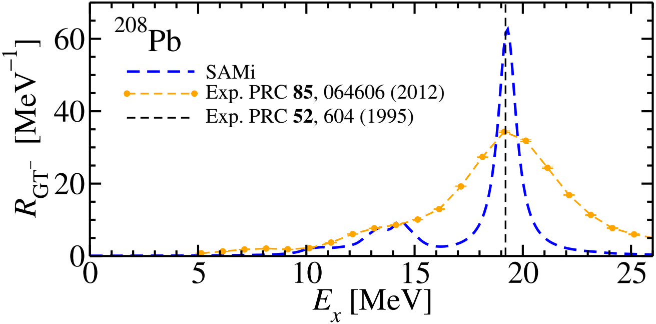
R = nuclear response function (in dipole resonance is related with the probability of a photon absorption by the nucleus or σ_γ)

Good description excitation energy and integrated R but not the width of the resonance.

Examples: Gamow Teller Resonance

Gamow-Teller Resonance driven by strong nuclear force (analogous transitions to $\beta-$decay).





[Phys.Rev. C86 (2012) 031306]

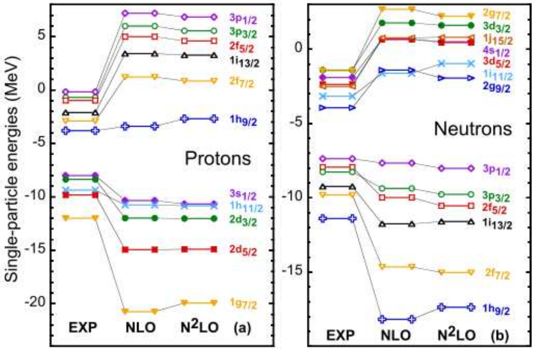
Mean-field approach: overall good description of ground state and excited state properties in nuclei

It is beyond the MF approach: accurate description of the fragmentation of the single-particle and collective states

Examples: Observables beyond the MF approach

Single particle (sp) states:

Density of the system is well described within the MF approach (E[ρ]) while sp are not satisfactorily reproduced.



[J. Phys. G. 44 (2017) 045106]

Examples: Observables beyond the MF approach

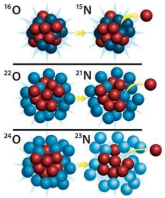
Spectroscopic factor S is associated to n, j, l:

Removal probability for valence protons

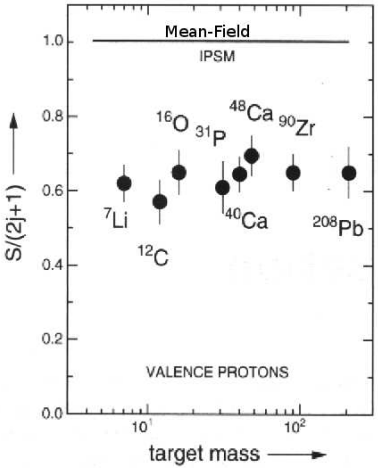
*** From theory:** For a bound $A-1$ final state

$$S_{\text{theo}} = \int d\vec{p} |\langle \Phi_{A-1} | a_{\vec{p}} | \Phi_A \rangle|^2$$

*** Transfer reaction:** $A \to A-1$



$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{exp}} = S_{\text{exp}} \left.\frac{d\sigma}{d\Omega}\right|_{\text{theo}}$$



[Nuclear Physics A 553 (1993) 297-308]

Possible solution to these problems: PVC

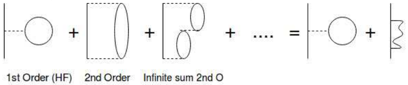
There is NO explicit interplay of collective motion (e.g. giant resonances) and single-particle motion in the mean-field approach

Solution: take into account their interplay

For example,

- in **spherical nuclei**: mainly sp+surface vibrations $\rightarrow$ **Particle Vibration Coupling model (PVC)**
- while in **deformed nuclei**: mainly sp+rotations

PVC model in a diagramtic way (e.g. Σ or sp potential):



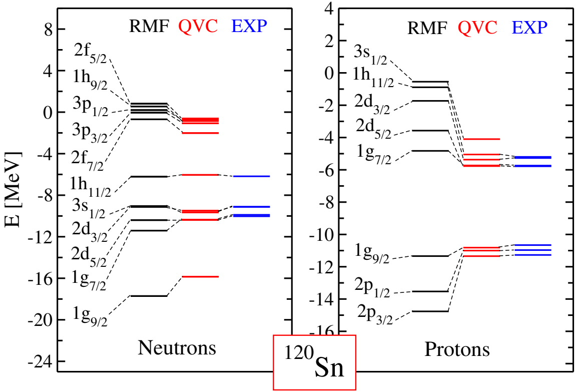
$\Rightarrow$ "Selection of most relevant diagrams"

$\Rightarrow$ same $V_{\rm eff}$ at all vertices (fitted at Mean-Field level)

Some selected examples

Examples: Observables beyond the MF approach

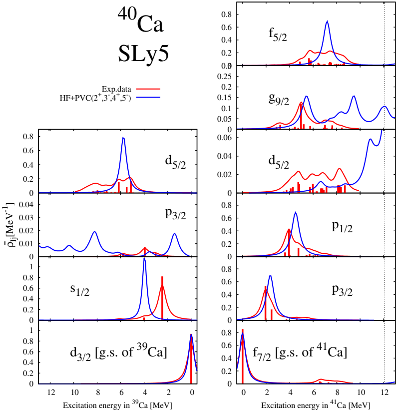
Single particle states:



[Phys. Rev. C 85, 021303(R)]

Examples: Observables beyond the MF approach

Total single particle strength associated to n, j, l: transfer reaction

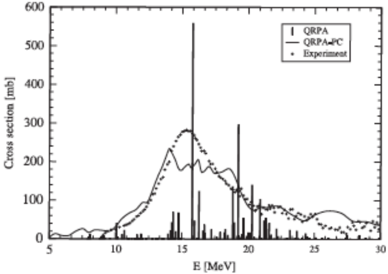


[Phys. Rev. C 86, 034318]

(Remember spectroscopic factor)

Collective strength

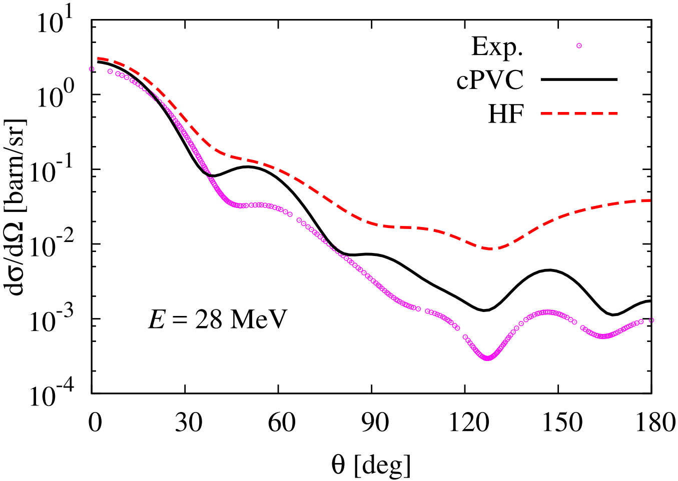
photoabsorption cross section in ^{120}Sn



Examples: Observables beyond the MF approach

Optical potential model based on PVC.

Neutron elastic cross section by ^{16}O at 28 MeV



[Phys. Rev. C 86, 041603(R) (2012)]

PVC: How to renormalize the effective interacrion?

- **PVC includes many-body correlations not explicitely included at the Mean-Field level.**
- While V_{eff} **fitted at the Mean-Field** to experimental data.
- This implies **double counting** since parameters contain correlations beyond Mean-Field

> Solution: **refit the interaction, that is, renormalize the theory.**

- **Divergences** may also appear in beyond Mean-Field calculations if **zero-range** V_{eff} **are used**.
- **The group is now studiyng different strategies for the renormalization of** V_{eff}. Essentially:
  - In a simplified case (2nd order term is kept, no summation up to infinity). **Cutoff and dimensional renormalization** have been performed
  - In the full PVC approach by appliyng the **subtraction method**

Conclusions

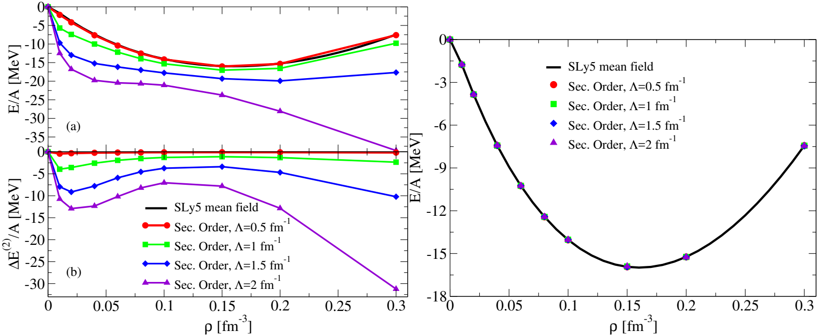
- **Effective interactions** solved at **Hartree-Fock or Mean-Field** level have been shown to be **successful** in the description of all nuclei (**masses, nuclear sizes, deformations, Giant Resonances...**
- These **effective models** can be understood as an **approximate realization of a nuclear energy density functional** $E[\rho] \Rightarrow$ **exact functional exist**.
- An accurate description of **the fragmentation of single-particle and collective states** is reached **beyond the MF approach**
- **Particle Vibration model improve the description of these observables**, although renormalization of the interaction needs to be investigated.

Thank you!

Simplified PVC: Cutoff renormalization on EoS

$$E_{\text{potential}} = \langle 0|V|0\rangle + \sum_{\nu\neq 0} \frac{|\langle \nu|V|0\rangle|^2}{E_0 - E_\nu}$$

where $|0\rangle$ is the GS and $|\nu\rangle$ an excited state [Phys. Rev. C 94, 034311 (2016)]



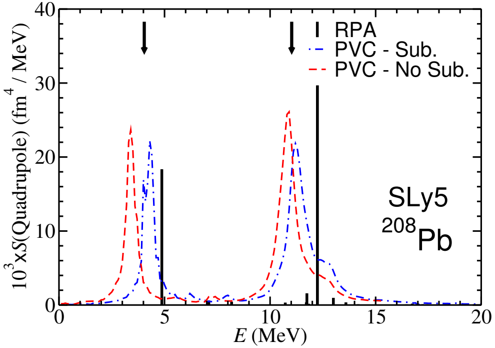
Note: since we do not know the TRUE EoS we chose one calculation of EoS as a benchmark

Full PVC: Subtraction method

Subtraction method is based on a simple idea: **modify the theory** so that the expectation value of any one-body operator (idealy accurate at the MF) **do not change with respect the MF prediction**.

* **Its realization is simple**
("$\Sigma_{sub}(E) = \Sigma_{Nosub}(E) - \Sigma_{HF}$" induced eff. interaction)
* Unfortunatelly **corrects only approximately** some of the studied observables (such as different moments of the response function)

* **One (different) subtraction recipe should be applied for each observable that needs to be renormalized**.



[Phys. Rev. C 94, 034311 (2016)]