

**The Isobaric Analog State, the
Antianalog Giant Dipole Resonance
and the neutron skin thickness**

**Xavier Roca-Maza
Università degli Studi di Milano and INFN**

**Faculty of Science, University of Zagreb, Croatia
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INTRODUCTION

Nuclear Energy Density Functionals (EDFs):

Based on effective interactions solved at the HF level, EDFs are successful in the description of ground and excited state properties such as m , $\langle r^2 \rangle^{1/2}$ or GR along the nuclear chart

Main types of EDFs:

Relativistic mean-field models (RMF), based on Lagrangians where effective mesons carry the interaction:

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \bar{\Psi} \Gamma_{\sigma} (\bar{\Psi}, \Psi) \Psi \Phi_{\sigma} & + \bar{\Psi} \Gamma_{\delta} (\bar{\Psi}, \Psi) \tau \Psi \Phi_{\delta} \\ & - \bar{\Psi} \Gamma_{\omega} (\bar{\Psi}, \Psi) \gamma_{\mu} \Psi A^{(\omega)\mu} & - \bar{\Psi} \Gamma_{\rho} (\bar{\Psi}, \Psi) \gamma_{\mu} \tau \Psi A^{(\rho)\mu} \end{aligned}$$

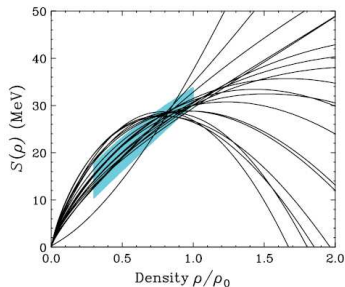
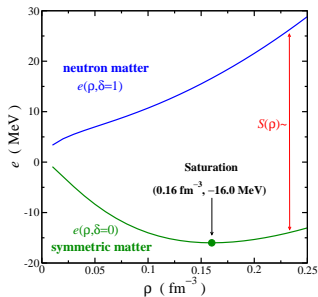
Non-relativistic mean-field models (NRMF), based on Hamiltonians where effective interactions are proposed and tested:

$$V_{\text{Nucl}}^{\text{eff}} = V_{\text{attractive}}^{\text{long-range}} + V_{\text{repulsive}}^{\text{short-range}} + V_{\text{SO}} + \dots$$

-EDFs are **phenomenological** → **not directly connected to any NN (or NNN) interaction** in the vacuum

-EDFs derived from a **Mean-Field** → we expect bulk properties more accurate as heavier is the nucleus

The Nuclear Equation of State: Infinite System

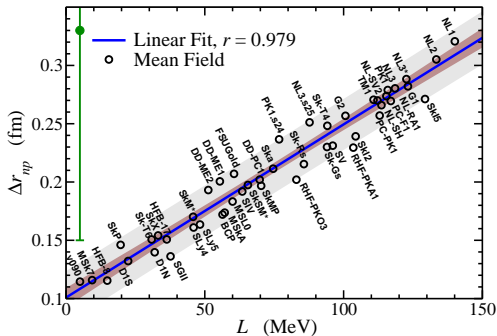


$$e(\rho, \beta) = e(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \quad \text{where } \beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

-Isovector properties not well determined in current **EDFs**

But how we can better constraint the isovector channel from observables? (Example)

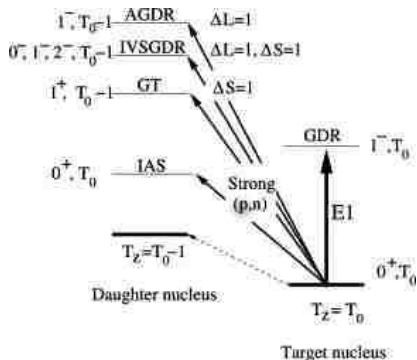
Neutron skin thickness → is one of the most paradigmatic example of an **isovector sensitive observable**.



$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \sim \frac{1}{12} \frac{IR}{J} L \quad \text{where} \quad L \equiv 3\rho_0 \left. \frac{\partial S}{\partial \rho} \right|_{\rho_0} = 3\rho_0 p_0^{\text{neut}}$$

The correlation is physically meaningful

IAS and AGDR within a simple model



AGDR corresponds to $\Delta J^\pi = 1^-$, $\Delta L = 1$, and $\Delta S = 0$ excitation, and represents the anti-analog giant dipole resonance because it is the $T_0 - 1$ component of the charge-exchange of the IVGDR.

IAS and AGDR within a simple model

- Charge-exchange excitations of nuclei having a neutron excess
⇒ Tamm-Dancoff \approx Random Phase approximation
- Assuming a separable interaction of the type $\kappa \hat{O}^\dagger \hat{O}$ and one unperturbed configuration (ε) exhausting the whole unperturbed strength m_0 , the dispersion relation:

$$\left. \frac{m_0}{\varepsilon - E} - \frac{m_0}{\varepsilon + E} \right|_{\text{RPA}} \approx \left. \frac{m_0}{\varepsilon - E} \right|_{\text{TDA}} \approx -\frac{1}{\kappa}$$

and therefore,

$$E \approx \varepsilon + \kappa m_0 \approx \varepsilon + \kappa \frac{m_1}{\varepsilon}$$

where m_0 and m_1 are the energy and non-energy weighted sum rules, respectively

IAS

- IAS excitation operator is $\sum_i \tau_{-}(i)$ and the unperturbed energy $\varepsilon \approx -U_{\text{Lane}} + \Delta E_C$ (where $U_{\text{Lane}} = V_{\text{sym}} \frac{N-Z}{2A}$)

$$E_{\text{IAS}} \approx -U_{\text{Lane}} + \Delta E_C + \kappa_{\text{IAS}} m_0 \approx -U_{\text{Lane}} + \Delta E_C + \kappa_{\text{IAS}} 2(N-Z)$$

- The nuclear Hamiltonian commutes with isospin operator if Coulomb is neglected. Thus, one may expect that U_{Lane} cancels $2(N-Z)\kappa_{\text{IAS}}$ to a large extent

$$E_{\text{IAS}} \approx \Delta E_C \approx 2 \left(\frac{3}{5} \right)^{3/2} \frac{e^2 Z}{\langle r_{\text{ch}}^2 \rangle^{1/2}}$$

- This estimates $E_{\text{IAS}}(^{208}\text{Pb}) \approx 20\text{MeV}$ with a typical dispersion

$$\delta E_{\text{IAS}} \approx E_{\text{IAS}} \frac{\delta \langle r_{\text{ch}}^2 \rangle^{1/2}}{\langle r_{\text{ch}}^2 \rangle^{1/2}} \lesssim 100 \text{ keV}$$

assuming EDFs $\delta \langle r_{\text{ch}}^2 \rangle^{1/2} \lesssim 0.03 \text{ fm}$.

- **Experimental value $18.8 \pm 0.02 \text{ MeV}$ and a width of $\sim 200 \text{ keV}$**

AGDR

- AGDR excitation operator is $\sum_i r_i Y_{10} \tau_-(i)$ and the unperturbed energy $\varepsilon \approx \varepsilon_0 - U_{\text{Lane}} + \Delta E_C$

$$E_{\text{AGDR}} \approx \varepsilon_0 - U_{\text{Lane}} + \Delta E_C + \kappa_{\text{AGDR}} m_0$$

where $m_0(\tau_-) - m_0(\tau_+) \approx m_0(\tau_-) = \frac{N \langle r_n^2 \rangle^{1/2} - Z \langle r_p^2 \rangle^{1/2}}{2\pi}$

- $\kappa_{\text{AGDR}} = \kappa_{\text{IVGDR}}$ due to isospin invariance of the strong interaction and that the latter can be estimated by a simple HO model, including the effects of m^* on the major shell gap $\varepsilon_0 \approx 41A^{-1/3} \sqrt{m/m^*}$ and relating V_{sym} with DM parameters

$$E_{\text{AGDR}} - E_{\text{IAS}} \propto a \frac{N - Z}{A} - b \frac{\Delta r_{np}}{\langle r^2 \rangle^{1/2}}$$

where a and b represent a combination of HO and DM parameters.

AGDR

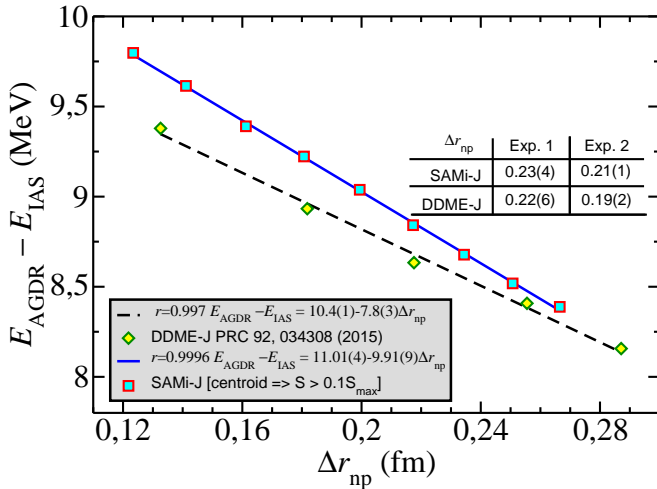
- Alternatively, one may try to connect $E_{AGDR} - E_{IAS}$ with E_{IVGDR} employing the HO model:

$$E_{AGDR} - E_{IAS} \approx \frac{\varepsilon_0}{\Delta E_C} (E_{IVGDR} - \varepsilon) \frac{m_0^{AGDR}}{m_0^{IVGDR}}$$

- The physics encoded in the energy difference $E_{AGDR} - E_{IAS}$ reflects that of the IVGDR (expected because of isospin invariance).
- This equations perfectly reproduces the trends of (SAMi-J), although the absolute values are shifted by $\sim 10\%$
- This finding gives us confidence in using these simple arguments to interpret the microscopic results.

EDF results on $E(\text{AGDR}) - E(\text{IAS})$

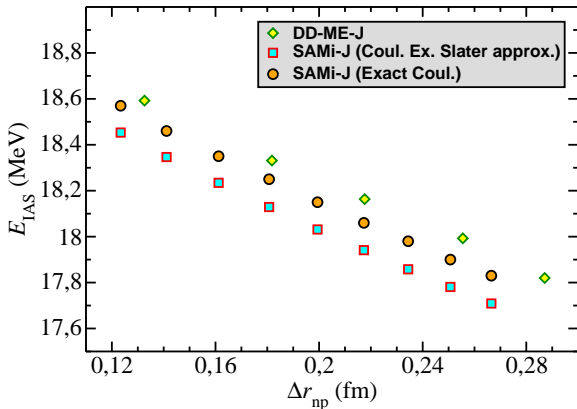
EDF results on $E(\text{AGDR})-E(\text{IAS})$ for ^{208}Pb



Centroid calculated for an energy range large as the experimental one but shifted to symmetrically cover the theoretical peak.

IAS and the residual interaction in the isovector channel

E_{IAS} in ^{208}Pb



$-\delta E_{IAS} \approx 800 \text{ keV}$

-Exact Coulomb shift E_{IAS} by 100 keV

-Non included charge symmetry breaking ($V_{nn} \neq V_{pp}$) and charge independence breaking ($V_{nn/pp} \neq V_{np}$) effects shift E_{IAS} by about 150 keV (PRC 47, 1360 (R)).

E(IAS): a simple case ^{48}Ca

Results neglecting the Coulomb interaction $\Delta\varepsilon \equiv \varepsilon_p(1f_{7/2}) - \varepsilon_n(1f_{7/2})$

Force	E_x	$S(E_x)$	$\Delta\varepsilon$	$V_{p,h}$	$\Delta\varepsilon + V_{p,h}$	$\langle V_C \rangle$
SAMi-J27	0.017	7.99	-6.08	7.06	0.98	7.08
SAMi-J28	0.010	7.99	-6.32	7.05	0.72	7.09
SAMi-J29	0.062	7.99	-6.40	6.92	0.51	7.10
SAMi-J30	0.039	7.99	-6.40	6.81	0.40	7.11
SAMi-J31	0.036	7.99	-6.33	6.73	0.39	7.12
SAMi-J32	0.049	7.99	-6.22	6.66	0.44	7.13
SAMi-J33	0.075	7.99	-6.07	6.60	0.53	7.14
SAMi-J34	0.011	7.99	-5.92	6.56	0.64	7.15
SAMi-J35	0.015	7.99	-5.77	6.53	0.75	7.16

$E_{IAS} \approx 0$ MeV since $[\mathcal{H} - V_C, \tau] = 0$, $\langle V_C \rangle$ is almost constant, E_{IAS} show no trend with the interaction

Results including the Coulomb interaction $\Delta\varepsilon \equiv \varepsilon_p(1f_{7/2}) - \varepsilon_n(1f_{7/2})$

Force	E_x	$S(E_x)$	$\Delta\varepsilon$	$V_{p,h}$	$\Delta\varepsilon + V_{p,h}$
SAMi-J27	6.74	7.97	0.72	6.96	7.68
SAMi-J28	6.67	7.97	0.45	6.94	7.39
SAMi-J29	6.60	7.97	0.34	6.79	7.13
SAMi-J30	6.54	7.97	0.31	6.66	6.97
SAMi-J31	6.49	7.98	0.34	6.55	6.90
SAMi-J32	6.44	7.98	0.43	6.47	6.90
SAMi-J33	6.41	7.98	0.55	6.40	6.95
SAMi-J34	6.38	7.98	0.68	6.34	7.02
SAMi-J35	6.37	7.98	0.82	6.29	7.11

$\delta E_{IAS} \approx 400$ keV, E_{IAS} show a trend with the interaction, $V_{p,h}$ decreases with increasing $J \sim 600$ keV and $\Delta\varepsilon$ show some not well definite trend.

E(IAS): a simple case ^{90}Zr

Results neglecting the Coulomb interaction $\Delta\varepsilon \equiv \varepsilon_p(1g_{9/2}) - \varepsilon_n(1g_{9/2})$

Force	E_x	$S(E_x)$	$\Delta\varepsilon$	V_{ph}	$\Delta\varepsilon + V_{ph}$	$\langle V_C \rangle$
SAMi-J27	0.008	9.99	-4.61	5.39	0.78	11.78
SAMi-J28	0.005	10.00	-4.86	5.44	0.58	11.79
SAMi-J29	0.003	10.00	-4.99	5.41	0.42	11.80
SAMi-J30	0.002	10.00	-5.05	5.40	0.34	11.80
SAMi-J31	0.002	9.99	-5.06	5.39	0.33	11.81
SAMi-J32	0.002	9.99	-5.03	5.41	0.37	11.81
SAMi-J33	0.003	9.99	-4.97	5.42	0.45	11.82
SAMi-J34	0.004	9.99	-4.91	5.45	0.54	11.83
SAMi-J35	0.006	9.99	-4.84	5.48	0.63	11.84

$E_{IAS} \approx 0$ MeV since $[\mathcal{H} - V_C, \tau] = 0$, $\langle V_C \rangle$ is almost constant, E_{IAS} show no trend with the interaction

Results including the Coulomb interaction $\Delta\varepsilon \equiv \varepsilon_p(1g_{9/2}) - \varepsilon_n(1g_{9/2})$

Force	E_x	$S(E_x)$	$\Delta\varepsilon$	V_{ph}	$\Delta\varepsilon + V_{ph}$
SAMi-J27	11.42	9.94	6.87	5.29	12.17
SAMi-J28	11.33	9.94	6.58	5.32	11.90
SAMi-J29	11.22	9.94	6.39	5.26	11.65
SAMi-J30	11.12	9.94	6.26	5.21	11.48
SAMi-J31	11.03	9.94	6.19	5.18	11.38
SAMi-J32	10.95	9.94	6.16	5.17	11.34
SAMi-J33	10.89	9.94	6.17	5.16	11.34
SAMi-J34	10.84	9.94	6.20	5.16	11.37
SAMi-J35	10.80	9.95	6.24	5.17	11.42

$\delta E_{IAS} \approx 600$ keV, E_{IAS} show a trend with the interaction, V_{ph} is almost constant with increasing J and $\Delta\varepsilon$ decreases with increasing J ~ 800 keV.

Conclusions

- ▶ The $E_{\text{AGDR}} - E_{\text{IAS}}$ is correlated with the neutron skin thickness
- ▶ The $E_{\text{AGDR}} - E_{\text{IAS}}$ should be sensitive to the same physics of the E_{IVGDR}
- ▶ Analysis using DDME predict $\Delta r_{\text{np}} = 0.16 - 0.28$ fm which would imply $J = 32 - 38$ MeV and $L = 45 - 110$ MeV
- ▶ Analysis using SAMi predict $\Delta r_{\text{np}} = 0.19 - 0.27$ fm which would imply $J = 31 - 35$ MeV and $L = 50 - 110$ MeV
- ▶ The E_{IAS} decreases with increasing J by about 800 keV in ^{208}Pb which is larger than the experimental error and width.
- ▶ Such a trend depends exclusively on the isovector channel of the effective interaction.
- ▶ No dependency of the trend shown by E_{IAS} has been found with Coulomb (r_{ch}), CSB, CIB, or spin-orbit effects.