

# Dipole Polarizability, parity violating asymmetry and the neutron skin thickness

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# INTRODUCTION

# The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems:** **spin, isospin, pairing, deformation, ...**
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different nuclear interactions in the medium** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

# Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

**Relativistic mean-field models**, based on Lagrangians where effective mesons carry the interaction:

$$\begin{aligned}\mathcal{L}_{\text{int}} &= \bar{\Psi}\Gamma_{\sigma}(\bar{\Psi}, \Psi)\Psi\Phi_{\sigma} & + \bar{\Psi}\Gamma_{\delta}(\bar{\Psi}, \Psi)\boldsymbol{\tau}\Psi\Phi_{\delta} \\ &- \bar{\Psi}\Gamma_{\omega}(\bar{\Psi}, \Psi)\gamma_{\mu}\Psi A^{(\omega)\mu} & - \bar{\Psi}\Gamma_{\rho}(\bar{\Psi}, \Psi)\gamma_{\mu}\boldsymbol{\tau}\Psi A^{(\rho)\mu} \\ &- e\bar{\Psi}\hat{Q}\gamma_{\mu}\Psi A^{(\gamma)\mu}\end{aligned}$$

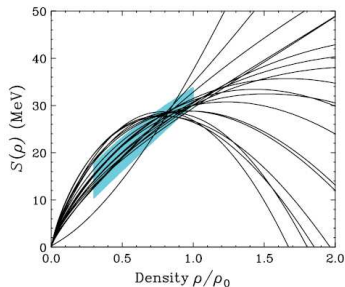
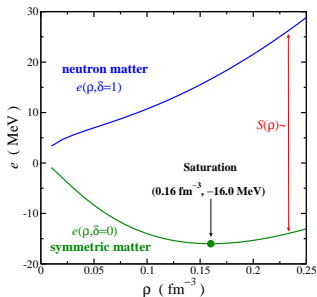
**Non-relativistic mean-field models**, based on Hamiltonians where effective interactions are proposed and tested:

$$V_{\text{Nucl}}^{\text{e}} = V_{\text{attractive}}^{\text{long-range}} + V_{\text{repulsive}}^{\text{short-range}} + V_{\text{SO}} + V_{\text{pair}}$$

Fitted **parameters contain** (important) **correlations beyond the mean-field**

Nuclear energy functionals are **phenomenological** → **not directly connected to any NN (or NNN) interaction**

# The Nuclear Equation of State: Infinite System

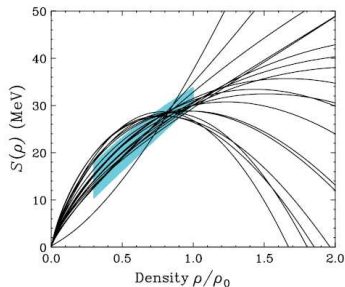
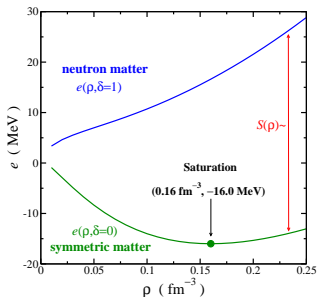


$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

Nuclear  
Matter

$$\left[ \beta = \frac{\rho_n - \rho_p}{\rho} \right]$$

# The Nuclear Equation of State: Infinite System



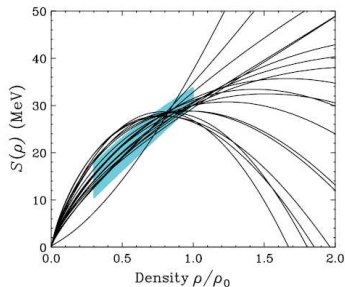
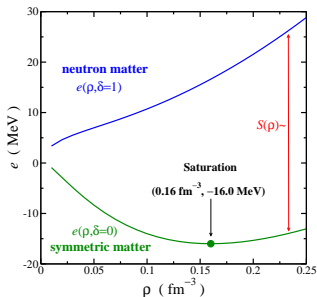
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

Nuclear  
Matter

Symmetric  
Matter

$$\left[ \beta = \frac{\rho_n - \rho_p}{\rho} \right]$$

# The Nuclear Equation of State: Infinite System



$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

Nuclear  
Matter

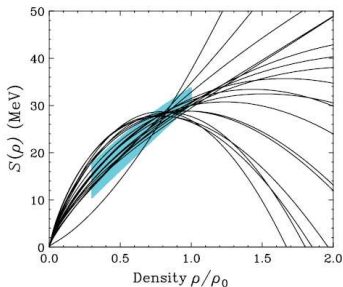
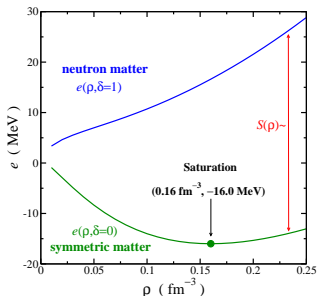
Symmetric  
Matter

Symmetry energy

$$\left[ \beta = \frac{\rho_n - \rho_p}{\rho} \right]$$



# The Nuclear Equation of State: Infinite System

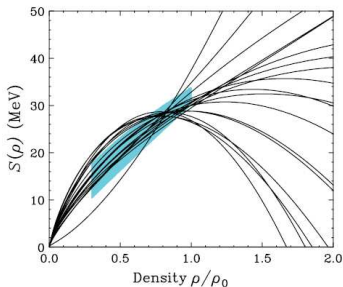
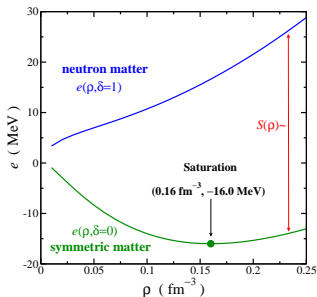


$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho, \beta = 0) + \beta^2 \left( J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}(x^3) \right)$$

$$\left[ \beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0} \right]$$

# The Nuclear Equation of State: Infinite System



$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + \beta^2 \left( \boxed{J} + \boxed{L} x + \frac{1}{2} \boxed{K_{\text{sym}}} x^2 + \mathcal{O}(x^3) \right)$$

$$S(\rho_0) = J$$

$$\left. \frac{d}{d\rho} S(\rho) \right|_{\rho_0} = \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2}$$

$$\left. \frac{d^2}{d\rho^2} S(\rho) \right|_{\rho_0} = \frac{K_{\text{sym}}}{9\rho_0^2}$$

$$\left[ \beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0} \right]$$

# Isvector properties in nuclei

In the past (and also in the present), **neutron properties** in stable medium and heavy nuclei have been mainly measured by using **strongly interacting probes**.



**Limited knowledge of isovector properties**

**At present,**

- the use of **rare ion beams** has opened the possibility of measuring properties of **exotic nuclei**  $\Rightarrow$  **more info**
- parity violating elastic electron scattering** (PVES), a **model independent technique**, has allowed to estimate the **weak form factor at low  $q$**  of a stable heavy nucleus like  $^{208}\text{Pb}$



**Promising perspectives** for the near future

# STATISTIC UNCERTAINTIES IN EDFs

## Covariance analysis: $\chi^2$ test

Observables  $\mathcal{O}$  used to calibrate the parameters  $\mathbf{p}$

$$\chi^2(\mathbf{p}) = \sum_{i=1}^m \left( \frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the  $\chi^2$  can be approximated by an hyper-parabola around the minimum  $\mathbf{p}_0$ ,

$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{i,j} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2(p_j - p_{0j})$$

where  $\mathcal{M} \equiv \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2$  (curvature m.) and  $\mathcal{E} \equiv \mathcal{M}^{-1}$  (error m.).

errors between predicted observables  $\mathcal{A}$

$$\Delta \mathcal{A} = \sqrt{\sum_i^n \partial_{p_i} \mathcal{A} \mathcal{E}_{ii} \partial_{p_i} \mathcal{A}}$$

correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}}$$

where,  $C_{AB} = \overline{(A(\mathbf{p}) - \bar{A})(B(\mathbf{p}) - \bar{B})} \approx \sum_{i,j} \partial_{p_i} \mathcal{A} \mathcal{E}_{ij} \partial_{p_j} \mathcal{B}$

## Example: Fitting protocol of SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

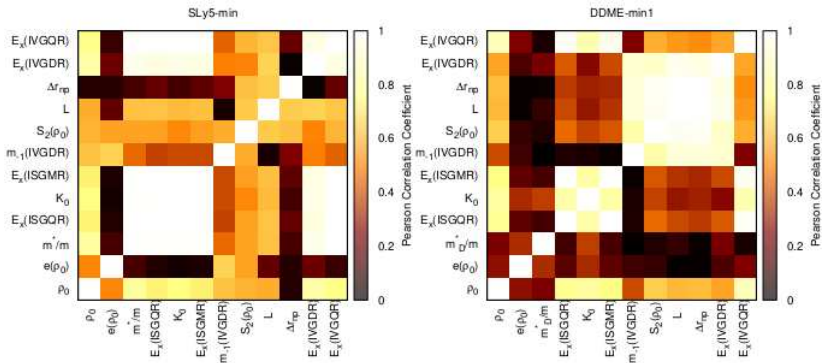
### SLy5-min:

- Binding energies of  $^{40,48}\text{Ca}$ ,  $^{56}\text{Ni}$ ,  $^{130,132}\text{Sn}$  and  $^{208}\text{Pb}$  with a fixed adopted error of 2 MeV
- the charge radius of  $^{40,48}\text{Ca}$ ,  $^{56}\text{Ni}$  and  $^{208}\text{Pb}$  with a fixed adopted error of 0.02 fm
- the neutron matter Equation of State calculated by Wiringa *et al.* (1988) for densities between 0.07 and 0.40  $\text{fm}^{-3}$  with an adopted error of 10%
- the saturation energy ( $e(\rho_0) = -16.0 \pm 0.2 \text{ MeV}$ ) and density ( $\rho_0 = 0.160 \pm 0.005 \text{ fm}^{-3}$ ) of symmetric nuclear matter.

### DD-ME-min1:

- binding energies, charge radii, dipole polarizabilities and surface thicknesses of 17 even-even spherical nuclei,  $^{16}\text{O}$ ,  $^{40,48}\text{Ca}$ ,  $^{56,58}\text{Ni}$ ,  $^{88}\text{Sr}$ ,  $^{90}\text{Zr}$ ,  $^{100,112,120,124,132}\text{Sn}$ ,  $^{136}\text{Xe}$ ,  $^{144}\text{Sm}$  and  $^{202,208,214}\text{Pb}$ . The assumed errors of these observables are 0.2%, 0.5%, 0.5%, and 1.5%, respectively.

# Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)



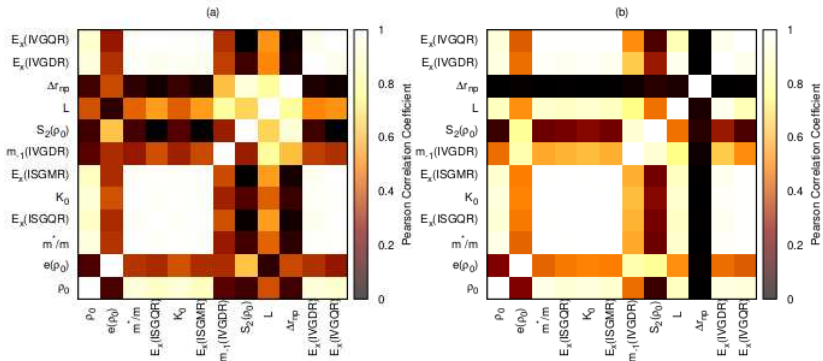
*J. Phys. G: Nucl. Part. Phys.* 42 034033 (2015).

The **neutron skin** is **strongly correlated** with **J** and **L** in both models but **NOT** with  $\alpha_D$ . **(I will discuss on that latter)**

# Covariance analysis: modifying the $\chi^2$

→ **SLy5-a:**  $\chi^2$  as in SLy5-min except for the neutron EoS (relaxed the required accuracy, increasing associated error).

→ **SLy5-b:**  $\chi^2$  as in SLy5-min except the neutron EoS (not employed) and used instead a tight constraint on the  $\Delta r_{np}$  in  $^{208}\text{Pb}$



*J. Phys. G: Nucl. Part. Phys.* 42 034033 (2015).

When a **constraint** on a property is **relaxed**, **correlations** of other observables with such a property should become **larger** → **SLy5-a:  $\alpha_D$  is now better correlated with  $\Delta r_{np}$**

When a **constraint** on a property is **enhanced**—artificially or by an accurate experimental measurement—**correlations** of other observables with such a property should become **small** → **SLy5-b:  $\Delta r_{np}$  is not correlated with any other observable**



# DIPOLE POLARIZABILITY

# Polarizability, Strength distribution and its moments

The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s.,  $|0\rangle$ , to an excited state,  $|\nu\rangle$ , due to the **action of an external isovector oscillating field** (dipolar in our case) of the form  $(F e^{i\omega t} + F^\dagger e^{-i\omega t})$ :

$$F_{JM} = \sum_i^A r^J Y_{JM}(\hat{r}) \tau_z(i) \quad (\Delta L = 1 \rightarrow \text{Dipole})$$

is proportional to the **static polarizability** for small oscillations

$$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_\nu |\langle \nu | F | 0 \rangle|^2 / E \quad \text{where } m_{-1} \text{ is}$$

**the inverse energy weighted moment** of the **strength function**, defined as,  $S(E) = \sum_\nu |\langle \nu | F | 0 \rangle|^2 \delta(E - E_\nu)$

# Isvector Giant Dipole Resonance:



## Dipole polarizability: a macroscopic approach

**electric polarizability measures tendency of the nuclear charge distribution to be distorted** ( $\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$ )

The dielectric theorem establishes that the  $m_{-1}$  moment can be computed from the expectation value of the Hamiltonian in the constrained ground state  $\mathcal{H}' = \mathcal{H} + \lambda \mathcal{D}$ .

Adopting the Droplet Model:

$$m_{-1} \approx \frac{A \langle r^2 \rangle^{1/2}}{48J} \left( 1 + \frac{15}{4} \frac{J}{Q} A^{-1/3} \right)$$

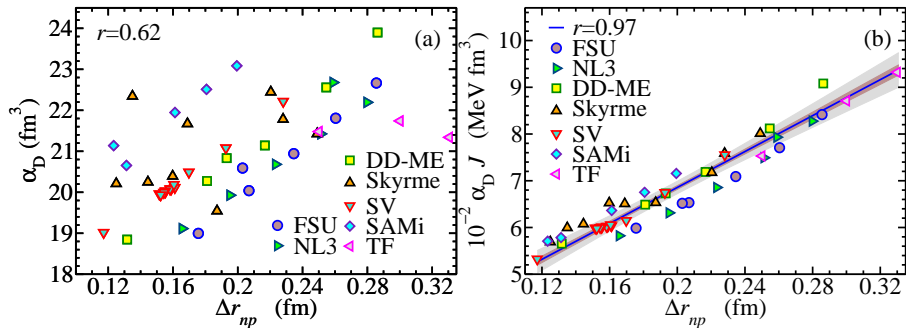
within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A \langle r^2 \rangle}{12J} \left[ 1 + \frac{5}{2} \frac{\Delta r_{np} + \sqrt{\frac{3}{5}} \frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2 \rangle^{1/2} (I - I_C)} \right]$$

# Isvector Giant Dipole Resonance in $^{208}\text{Pb}$ :



## Dipole polarizability: microscopic results HF RPA



X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

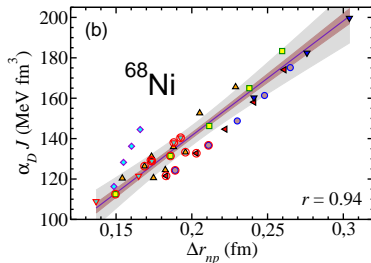
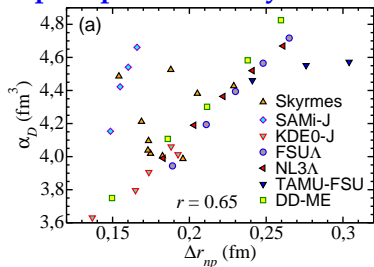
Experimental dipole polarizability  $\alpha_D = 20.1 \pm 0.6 \text{ fm}^3$ ; A. Tamii *et al.*, PRL 107, 062502 (RCNP) [No quasi-deuteron  $\alpha_D = 19.6 \pm 0.6 \text{ fm}^3$ ].

$\alpha_D J$  is linearly correlated with  $\Delta r_{np}$  and no  $\alpha_D$  alone within EDFs

# Isvector Giant Dipole Resonance in $^{68}\text{Ni}$ :

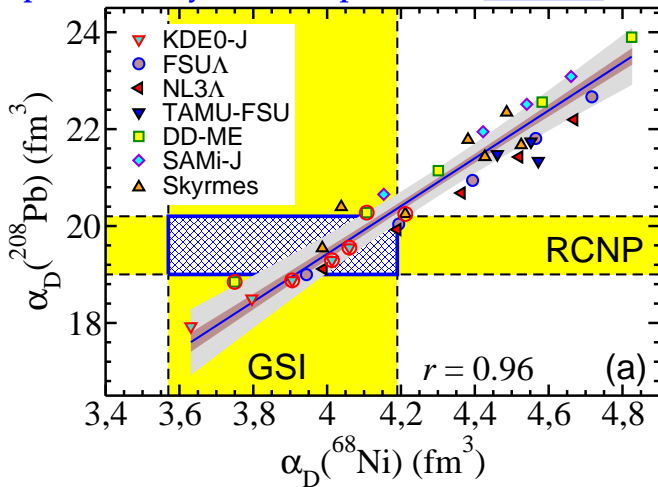


## Dipole polarizability: microscopic results HF RPA



X. Roca-Maza *et al.* PRC 92

Experimental dipole polarizability  $\alpha_D = 3.40 \pm 0.23 \text{ fm}^3$  D. M. Rossi *et al.*, PRL 111, 242503 (GSI).  $\alpha_D = 3.88 \pm 0.31 \text{ fm}^3$  “full” response D. M. Rossi, T. Aumann, and K. Boretzky.

Dipole polarizability: microscopic results HF RPA

**Just as an indication:**  $\alpha_D(A = 208)/\alpha_D(A = 68) \sim (208/68)^{5/3}$ ;  
Circled models predict  $\Delta r_{np}(^{208}\text{Pb}) = 0.125 - 0.207$  fm and  
 $\Delta r_{np}(^{68}\text{Ni}) = 0.146 - 0.211$  fm;  $J = 30 - 35$  MeV;  $L = 30 - 65$  MeV.

# PARITY VIOLATING ASYMMETRY

## Some basics ...

- **Electrons** interact by exchanging a  $\gamma$  or a  $Z_0$  boson.
- While **protons** couple basically to  $\gamma$ , **neutrons** do it to  $Z_0$ .
- Electron motion governed by the Dirac equation:
$$[\vec{\alpha} \cdot \vec{p} + \beta m_e + V(r)]\psi = E\psi$$
where  $V(r) = V_C(r) + \gamma^5 V_W(r)$
- Dirac equation for helicity states ( $m_e \approx 0$ )
$$[\vec{\alpha} \cdot \vec{p} + (V_C(r) \pm V_W(r))]\psi_{\pm} = E\psi_{\pm}$$
- **Ultra-relativistic electrons, depending on their helicity,** will interact with the nucleus seeing a slightly different potential " $\alpha Z$ "  $\pm$  " $G_F$ ".

**Refs:** Phys. Rev. C **57** 3430 (1998); Phys. Rev. C **63**, 025501 (2001); Phys. Rev. C **78**, 044332 (2008); Phys. Rev. C **82**, 054314 (2010); Phys. Rev. Lett. **106** 252501 (2011)



## Some basics ...

The **interference** between the DCS of electrons with + and - helicity states,

$$A_{pv} = \frac{d\sigma_{+}/d\Omega - d\sigma_{-}/d\Omega}{d\sigma_{+}/d\Omega + d\sigma_{-}/d\Omega}$$

**Ultra-relativistic electrons** moving under the effect of  $V_{\pm}$  where **Coulomb distortions** are important  $\Rightarrow$  solution of the Dirac equation via the Distorted Wave Born Approximation (**DWBA**).

Input for the calculation of  $V_{\pm}$  are the  $\rho_n$  **and**  $\rho_p$  (**main uncertainty in  $\rho_n$** ) and **nucleon form factors** for the e-m and the weak neutral current.

## Qualitative considerations ...

Within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}} \left[ 4 \sin^2 \theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)} \right]$$

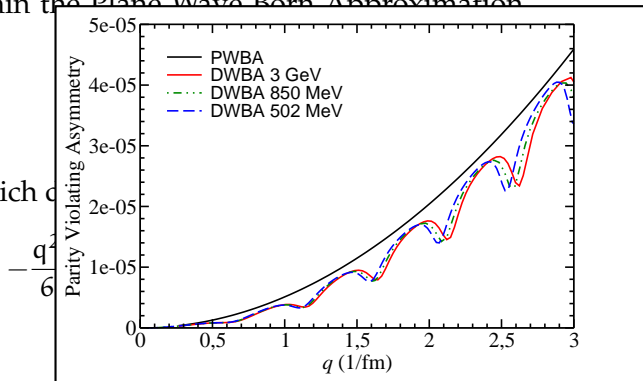
... which depends on  $F_n(q) - F_p(q)$ . For  $q \rightarrow 0$ , it is approximately,

$$\begin{aligned} -\frac{q^2}{6} (\langle r_n^2 \rangle - \langle r_p^2 \rangle) &= -\frac{q^2}{6} \left[ \Delta r_{np} (\langle r_n^2 \rangle^{1/2} + \langle r_p^2 \rangle^{1/2}) \right] \\ &= -\frac{q^2}{6} \left( 2\langle r_p^2 \rangle^{1/2} \Delta r_{np} + \Delta r_{np}^2 \right) \end{aligned}$$

variation of  $A_{pv}$  at a fixed  $q$  dominated by the variation of  $\Delta r_{np}$ .  $F_p(q)$  well fixed by experiment

# Qualitative considerations ...

Within the Plane Wave Born Approximation



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variation of  $A_{pv}$  at a fixed  $q$  dominated by the variation of  $\Delta r_{np}$ .  $F_p(q)$  well fixed by experiment

**So, let us check DWBA results...**

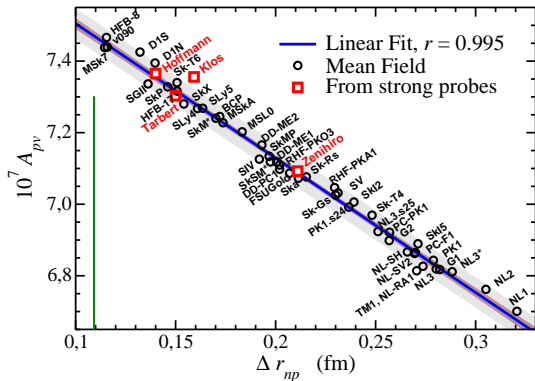
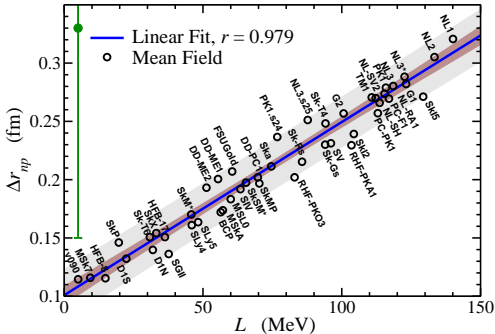
# $^{208}\text{Pb}$ : direct correlations

$\delta A_{pv} \sim 1\%$ ;

$\delta \Delta r_{np} \sim 0.02 \text{ fm}$ ;

$\delta L \sim 10 \text{ MeV (systematic)}$

X. Roca-Maza, *et al.*, PRL **106** 252501 (2011)



EDF correlations allows to determine  $\Delta r_{np}$  and  $L$  without direct assumpt. on  $\rho$ , JLab and Mainz forthcoming experiments

Different experiments on proton elastic scattering, antiprotonic atoms and pion-photoproduction agrees with the correlation

# CONCLUSIONS

## Conclusions:

- A precise and **model-independent** determination of  $\Delta r_{np}$  in  $^{208}\text{Pb}$  via PVES experiments **probes** the **symmetry energy**.
- We demonstrate a close **linear correlation** between  $A_{pv}$  and  $\Delta r_{np}$  within the same framework in which the  $\Delta r_{np}$  is correlated with  $L$  (expected to be better as heavier the nucleus).
- Other **experiments** fairly **agree** with the **correlation** between  $A_{pv}$  and  $\Delta r_{np}$  in  $^{208}\text{Pb}$ .
- EDFs (RPA) show a linear correlation between  $\alpha_D J$  and  $\Delta r_{np}$
- $A_{pv}$  and  $\alpha_D$  are complementary **observables** that may set **tight constraints** on the **density dependence of the symmetry energy around saturation density, if precisely and or systematically measured.**



# Collaborators:

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**P.-G. Reinhard**<sup>9</sup>  
**Michal Warda**<sup>10</sup>  
**Mario Centelles, Xavier**  
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