Dipole Polarizability, parity violating asymmetry and the neutron skin thickness

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INTRODUCTION

The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **di erent nuclear interactions in the medium** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on e ective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where e ective mesons carry the interaction:

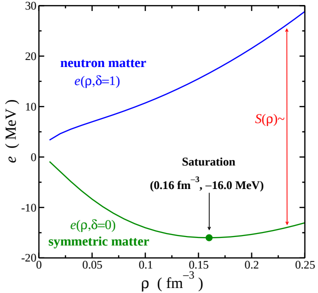
$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\Gamma_{\sigma}(\bar{\Psi},\Psi)\Psi\Phi_{\sigma} \quad +\bar{\Psi}\Gamma_{\delta}(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_{\delta} \\ & -\bar{\Psi}\Gamma_{\omega}(\bar{\Psi},\Psi)\gamma_{\mu}\Psi A^{(\omega)\mu} \quad -\bar{\Psi}\Gamma_{\rho}(\bar{\Psi},\Psi)\gamma_{\mu}\tau\Psi A^{(\rho)\mu} \\ & -e\bar{\Psi}\hat{Q}\gamma_{\mu}\Psi A^{(\gamma)\mu}\end{aligned}$$

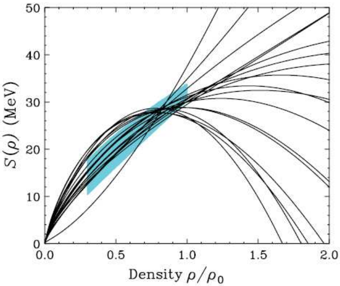
Non-relativistic mean-field models, based on Hamiltonians where e ective interactions are proposed and tested:

$$V^{e}_{\mathrm{Nucl}} = V^{\mathrm{long-range}}_{\mathrm{attractive}} + V^{\mathrm{short-range}}_{\mathrm{repulsive}} + V_{\mathrm{SO}} + V_{\mathrm{pair}}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** → **not directly connected to any NN** (or NNN) **interaction**

The Nuclear Equation of State: Infinite System



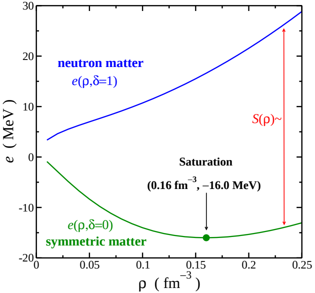


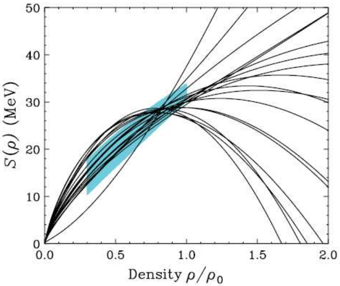
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

Nuclear
Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System





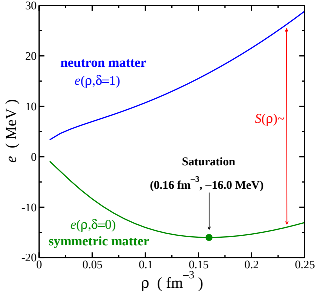
$$\boxed{\frac{E}{A}(\rho, \beta)} = \boxed{\frac{E}{A}(\rho, \beta = 0)} + \boxed{S(\rho)\beta^2} + \mathcal{O}(\beta^4)$$

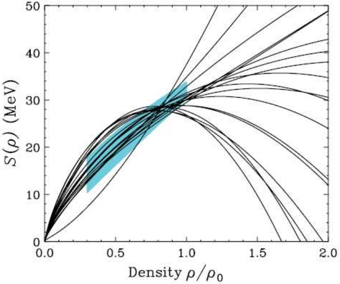
Nuclear Matter

Symmetric Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System





$$\boxed{\frac{E}{A}(\rho,\beta)} = \boxed{\frac{E}{A}(\rho,\beta=0)} + \boxed{S(\rho)\beta^2} + \mathcal{O}(\beta^4)$$

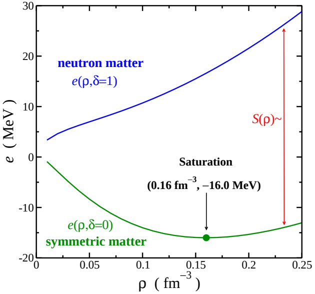
Nuclear Matter

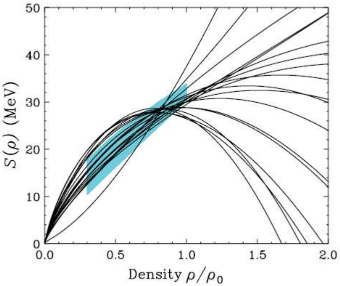
Symmetric Matter

Symmetry energy

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



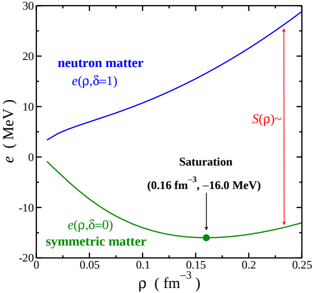


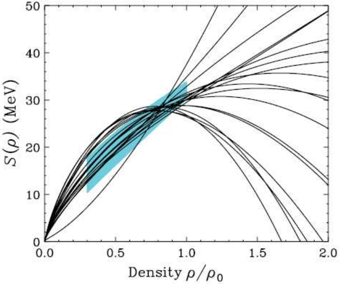
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3)\right)$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho};\quad x = \frac{\rho - \rho_0}{3\rho_0}\right]$$

The Nuclear Equation of State: Infinite System





$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + L x + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3)\right)$$

$$S(\rho_0) = J \qquad \left.\frac{d}{d\rho}S(\rho)\right|_{\rho_0} = \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2} \qquad \left.\frac{d^2}{d\rho^2}S(\rho)\right|_{\rho_0} = \frac{K_{sym}}{9\rho_0^2}$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho-\rho_0}{3\rho_0}\right]$$

Isovector properties in nuclei

- **In the past** (and also in the present), **neutron properties** in stable medium and heavy nuclei have been mainly measured by using **strongly interacting probes**.

$$\Downarrow$$

Limited knowledge of isovector properties

- **At present**,
  - the use of **rare ion beams** has opened the possibility of measuring properties of **exotic nuclei** $\Rightarrow$ **more info**
  - **parity violating elastic electron scattering** (PVES), a **model independent technique**, has allowed to estimate the **weak form factor at low** q of a stable heavy nucleus like ^{208}Pb

$$\Downarrow$$

Promising perspectives for the near future

STATISTIC UNCERTAINTIES IN EDFs

Covariance analysis: χ^2 test

- Observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

- Assuming that the χ^2 can be approximated by an hyper-parabola around the minimum $\mathbf{p}_0$,

$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j})$$

where $\mathcal{M} \equiv \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2$ (curvature m.) and $\mathcal{E} \equiv \mathcal{M}^{-1}$ (error m.).

- errors between predicted observables $\mathcal{A}$

$$\Delta \mathcal{A} = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A}$$

- correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}}$$

where, $C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$

Example: Fitting protocol of SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

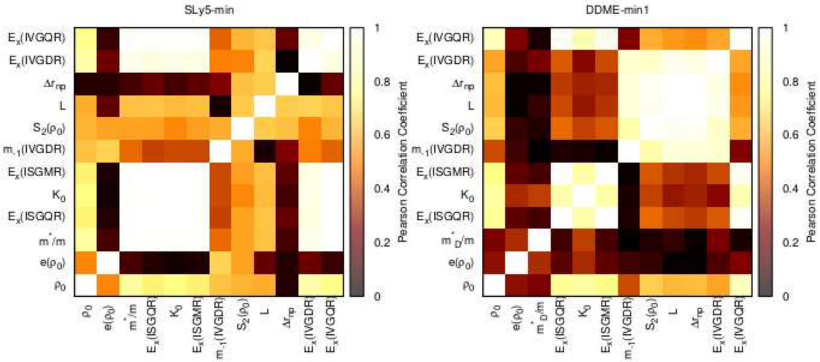
SLy5-min:

- Binding energies of ^{40,48}Ca, ^{56}Ni, ^{130,132}Sn and ^{208}Pb with a fixed adopted error of 2 MeV
- the charge radius of ^{40,48}Ca, ^{56}Ni and ^{208}Pb with a fixed adopted error of 0.02 fm
- the neutron matter Equation of State calculated by Wiringa *et al.* (1988) for densities between 0.07 and 0.40 fm^{-3} with an adopted error of 10%
- the saturation energy ($e(\rho_0) = -16.0 \pm 0.2$ MeV) and density ($\rho_0 = 0.160 \pm 0.005$ fm^{-3}) of symmetric nuclear matter.

DD-ME-min1:

- binding energies, charge radii, di raction radii and surface thicknesses of 17 even-even spherical nuclei, ^{16}O, ^{40,48}Ca, ^{56,58}Ni, ^{88}Sr, ^{90}Zr, ^{100,112,120,124,132}Sn, ^{136}Xe, ^{144}Sm and ^{202,208,214}Pb. The assumed errors of these observables are 0.2%, 0.5%, 0.5%, and 1.5%, respectively.

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

The **neutron skin** is **strongly correlated** with **J** and **L** in both models but **NOT** with α_D. **(I will discuss on that latter)**

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

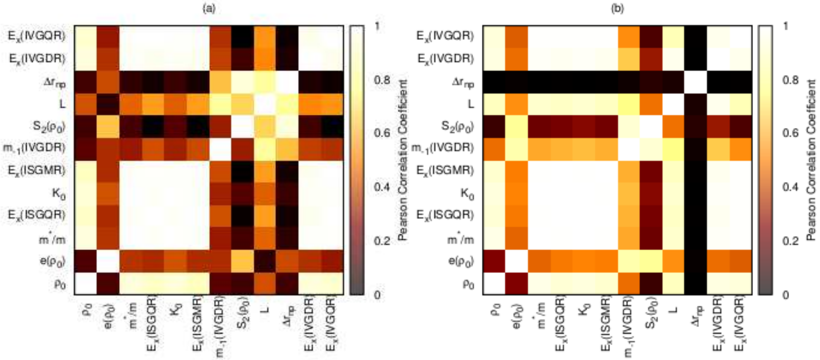
	SLy5-min			DDME-min1			
A	A_0		$\sigma(A_0)$	A_0		$\sigma(A_0)$	units
SNM							
ρ_0	0.162	$\pm$	0.002	0.150	$\pm$	0.001	fm^{-3}
$e(\rho_0)$	-16.02	$\pm$	0.06	-16.18	$\pm$	0.03	MeV
m^*/m	0.698	$\pm$	0.070	0.573	$\pm$	0.008	
J	32.60	$\pm$	0.71	33.0	$\pm$	1.7	MeV
K_0	230.5	$\pm$	9.0	261	$\pm$	23	MeV
L	47.5	$\pm$	4.5	55	$\pm$	16	MeV
^{208}Pb							
E_x^{ISGMR}	14.00	$\pm$	0.36	13.87	$\pm$	0.49	MeV
E_x^{ISGQR}	12.58	$\pm$	0.62	12.01	$\pm$	1.76	MeV
Δr_{np}	0.1655	$\pm$	0.0069	0.20	$\pm$	0.03	fm
E_x^{IVGDR}	13.9	$\pm$	1.8	14.64	$\pm$	0.38	MeV
m_{-1}^{IVGDR}	4.85	$\pm$	0.11	5.18	$\pm$	0.28	$MeV^{-1}\ fm^2$
E_x^{IVGQR}	21.6	$\pm$	2.6	25.19	$\pm$	2.05	MeV

Statistical uncertainties depend on the fitting protocol, that is on the **data (or pseudo-data) and associated errors used for the fits: Let us see an example...**

Covariance analysis: modifying the χ^2

→ **SLy5-a:** χ^2 as in SLy5-min except for the neutron EoS (relaxed the required accuracy   increasing associated error).

→ **SLy5-b:** χ^2 as in SLy5-min except the neutron EoS (not employed) and used instead a tight constraint on the Δr_{np} in ^{208}Pb



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

- When a **constraint** on a property is **relaxed**, **correlations** of other observables with such a property should become **larger** → **SLy5-a: α_D is now better correlated with Δr_{np}**
- When a **constraint** on a property is **enhanced** —artificially or by an accurate experimental measurement— **correlations** of other observables with such a property should become **small** → **SLy5-b: Δr_{np} is not correlated with any other observable**

DIPOLE POLARIZABILITY

Polarizability, Strength distribution and its moments

The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

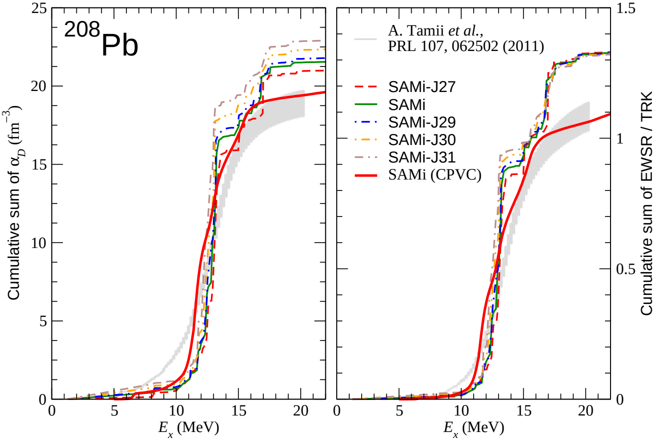
$$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \; (\Delta L = 1 \rightarrow \text{Dipole})$$

is proportional to the **static polarizability** for small oscillations

$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_{\nu} |\langle\nu|F|0\rangle|^2 \delta(E - E_{\nu})$

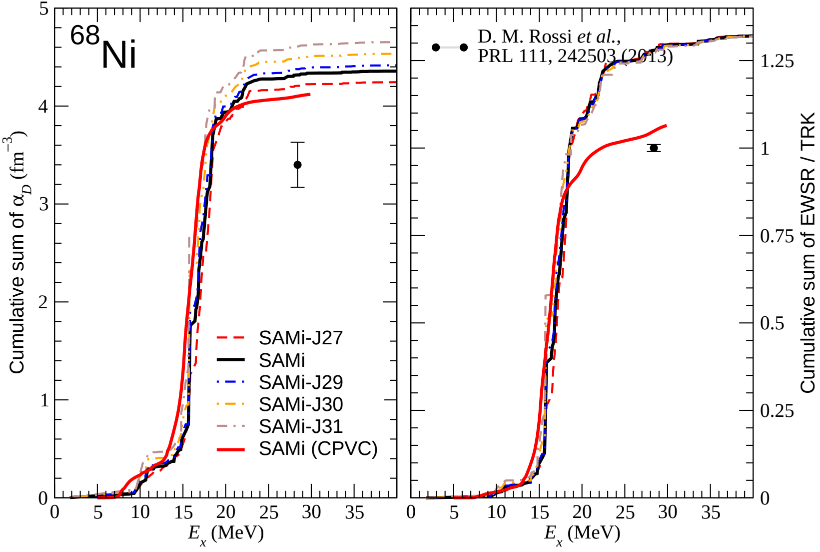
Let us compare some experiment with theory within the measured energy range...

Example 1: Cumulative sum of α_D and EWSR



The correction in the RPA predictions for the α_D^{RPA} due to the fact that not all possible correlations have been included in the model (α_D^{corr}) that lead to a fair reproduction of the experimental resonance width can be estimated to be $\frac{\alpha_D^{RPA} - \alpha_D^{corr}}{\alpha_D^{RPA}} \sim \frac{\Gamma^2}{4E_x^2}$ **This correction might not be negligible**

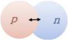
Example 2: Cumulative sum of α_D and EWSR



Theory versus experiment

- **RPA** strength concentrated at a ~ single peak: **EWSR and α_D "saturate" earlier in** E
- **PVC** strength follow **reasonable** trends with E, thought **no** calculations exist for the Δr_{np} **and refitting** of the interaction would be needed at the **PVC** level.
- **RPA** has been proben to be successful in describing experimental E_x and sum rules
- **Experimental data** on the α_D analysed via RPA need to include the **full dipole response** with low and high-energy parts:
  - **Data** within a given energy range, better if it is **extrapolated**: low energy part is more important than high energy part
  - **Data** in the high energy range, should be taken carefully due to quasi-deuteron excitations contaminations (small but sizeble correction to α_D)

Isovector Giant Dipole Resonance:



Dipole polarizability: a macroscopic approach

electric polarizability measures tendency of the nuclear charge distribution to be distorted ($\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$)

The dielectric theorem establishes that the m_{-1} moment can be computed from the expectation value of the Hamiltonian in the constrained ground state $\mathcal{H}' = \mathcal{H} + \lambda \mathcal{D}$.

Adopting the Droplet Model:

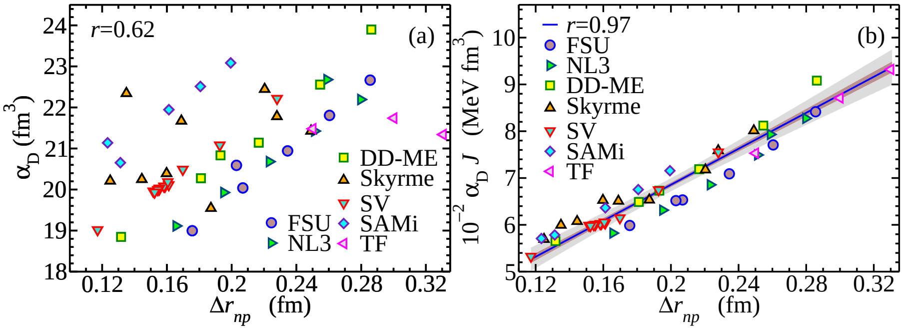
$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Isovector Giant Dipole Resonance in ^{208}Pb:

Dipole polarizability: microscopic results HF RPA



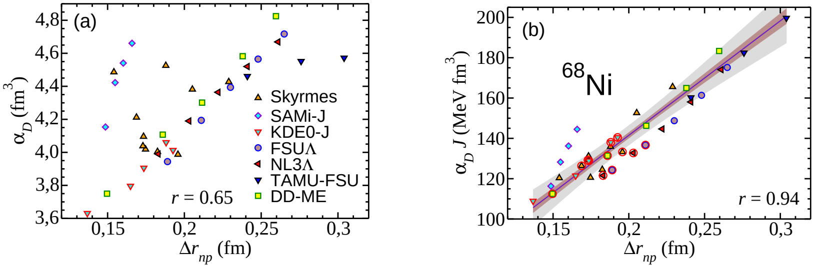
X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

Experimental dipole polarizability $\alpha_D = 20.1 \pm 0.6$ fm^3; A. Tamii *et al.*, PRL 107, 062502 (RCNP) [No quasi-deuteron $\alpha_D = 19.6 \pm 0.6$ fm^3].

$\alpha_D J$ is linearly correlated with Δr_{np} and no α_D alone within EDFs

Isovector Giant Dipole Resonance in ^{68}Ni:

Dipole polarizability: microscopic results HF RPA

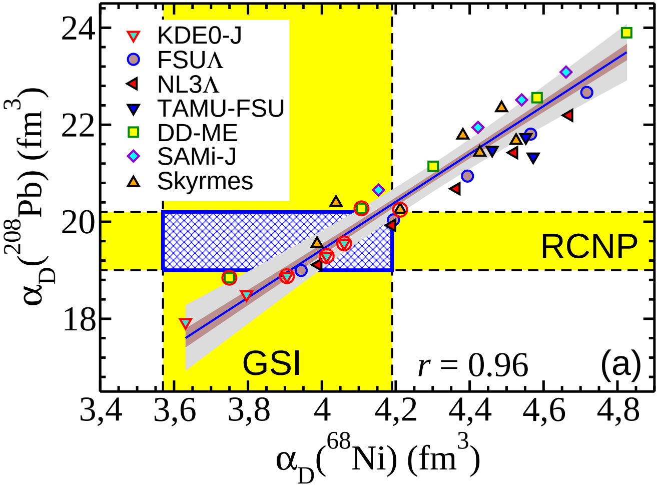


X. Roca-Maza et al. PRC 92

Experimental dipole polarizability $\alpha_D = 3.40 \pm 0.23$ fm^3 D. M. Rossi *et al.*, PRL 111, 242503 (GSI). $\alpha_D = 3.88 \pm 0.31$ fm^3 "full" response D. M. Rossi, T. Aumann, and K. Boretzky.

^{208}Pb vs ^{68}Ni:

Dipole polarizability: microscopic results HF RPA



Just as an indication: $\alpha_D(A = 208)/\alpha_D(A = 68) \sim (208/68)^{5/3}$;
Cercled models predict $\Delta r_{np}(^{208}Pb) = 0.125 - 0.207$ fm and $\Delta r_{np}(^{68}Ni) = 0.146 - 0.211$ fm; J = 30 − 35 MeV; L = 30 − 65 MeV.

PARITY VIOLATING ASYMMETRY

Some basics ...

- **Electrons** interact by exchanging a γ or a Z_0 boson.
- While **protons** couple basically to γ, **neutrons** do it to Z_0.
- Electron motion governed by the Dirac equation:

$$[\vec{\alpha} \cdot \vec{p} + \beta m_e + V(r)]\psi = E\psi$$

where $V(r) = V_C(r) + \gamma^5 V_W(r)$

- Dirac equation for helicity states ($m_e \approx 0$)

$$[\vec{\alpha} \cdot \vec{p} + (V_C(r) \pm V_W(r))]\psi_\pm = E\psi_\pm$$

- **Ultra-relativistic electrons, depending on their helicity,** will interact with the nucleus seeing a slightly di erent potential "αZ" $\pm$ "G_F".

Refs: Phys. Rev. C **57** 3430 (1998); Phys. Rev. C **63**, 025501 (2001); Phys. Rev. C **78**, 044332 (2008); Phys. Rev. C **82**, 054314 (2010); Phys. Rev. Lett. **106** 252501 (2011)

Some basics ...

- The **interference** between the DCS of electrons with + and − helicity states,

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega}$$

- **Ultra-relativistic electrons** moving under the e ect of $V_\pm$ where **Coulomb distortions** are important ⇒ solution of the Dirac equation via the Distorted Wave Born Approximation (**DWBA**).

- Input for the calculation of $V_\pm$ are the ρ_n **and** ρ_p **(main uncertainty in** ρ_n**)** and **nucleon form factors** for the e-m and the weak neutral current.

Qualitative considerations ...

Within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)}\right]$$

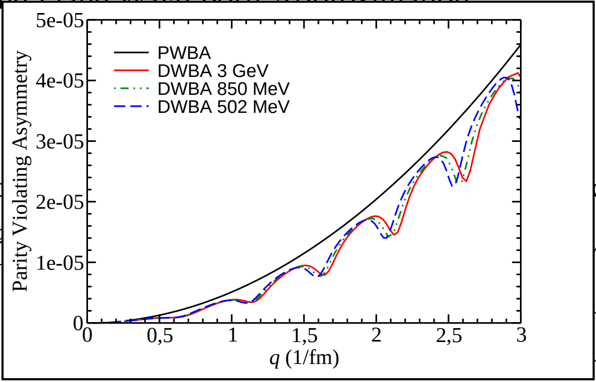
... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$\begin{aligned} -\frac{q^2}{6}\left(\langle r_n^2\rangle - \langle r_p^2\rangle\right) &= -\frac{q^2}{6}\left[\Delta r_{np}(\langle r_n^2\rangle^{1/2} + \langle r_p^2\rangle^{1/2})\right] \\ &= -\frac{q^2}{6}\left(2\langle r_p^2\rangle^{1/2}\Delta r_{np} + \Delta r_{np}^2\right) \end{aligned}$$

variation of A_{pv} at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

Qualitative considerations ...

Within the Plane Wave Born Approximation



... which o... oximately,

$-\frac{q^2}{6}$ ... $^2)]$

variation of $A_{p\nu}$ at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

Qualitative considerations ...

Within the Plane Wave Born Approximation,

$$A_{p\nu} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)}\right]$$

... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$\begin{aligned} -\frac{q^2}{6}\left(\langle r_n^2\rangle - \langle r_p^2\rangle\right) &= -\frac{q^2}{6}\left[\Delta r_{np}(\langle r_n^2\rangle^{1/2} + \langle r_p^2\rangle^{1/2})\right] \\ &= -\frac{q^2}{6}\left(2\langle r_p^2\rangle^{1/2}\Delta r_{np} + \Delta r_{np}^2\right) \end{aligned}$$

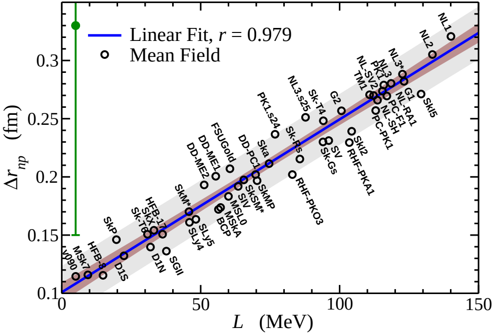
variation of $A_{p\nu}$ at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

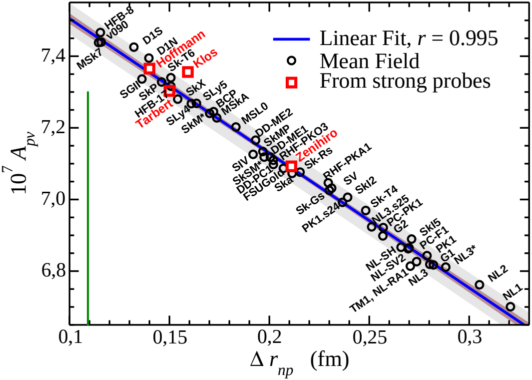
So, let us check DWBA results...

^{208}Pb: direct correlations

$\delta A_{pv} \sim$ **1%;**
$\delta \Delta r_{np} \sim 0.02$ **fm;**
$\delta L \sim$ **10 MeV (systematic)**

X. Roca-Maza, *et al.*, PRL **106** 252501 (2011)





EDF correlations allows to determine Δr_{np} and L without direct assumpt. on ρ, JLab and Mainz forthcoming experiments

Di erent experiments on proton elastic scattering, antirpotonic atoms and pion-photoproduction agrees with the correlation

CONCLUSIONS

Conclusions:

- A precise and **model-independent** determination of Δr_{np} in ^{208}Pb via PVES experiments **probes** the **symmetry energy**.
- We demonstrate a close **linear correlation** between $A_{p\nu}$ and Δr_{np} within the same framework in which the Δr_{np} is correlated with L (expected to be better as heavier the nucleus).
- Other **experiments** fairly **agree** with the **correlation** between $A_{p\nu}$ and Δr_{np} in ^{208}Pb.
- EDFs (RPA) show a linear correlation between $\alpha_D J$ and Δr_{np}
- $A_{p\nu}$ and α_D are complementary **observables** that may set **tight constraints** on the **density dependence of the symmetry energy around saturation density, if precisely and or systematically measured.**

Collaborators:

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