Dipole Polarizability and the neutron skin thickness

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INTRODUCTION

Nuclear Energy Density Functionals (EDFs):

Based on effective interactions solved at the HF level, **EDFs** are **successful** in the description of **ground and excited state** properties such as m, $\langle r^2 \rangle^{1/2}$ **or GR** along the **nuclear chart**

Main types of EDFs:

Relativistic mean-field models (RMF), based on Lagrangians where effective mesons carry the interaction:

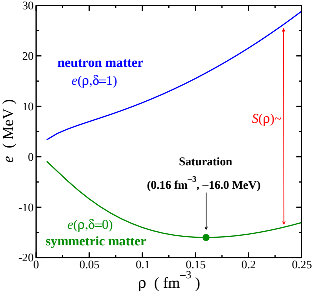
$$\begin{aligned}\mathcal{L}_{int} \quad &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma \quad &&+\bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ &-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} \quad &&-\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\boldsymbol{\tau}\Psi A^{(\rho)\mu}\end{aligned}$$

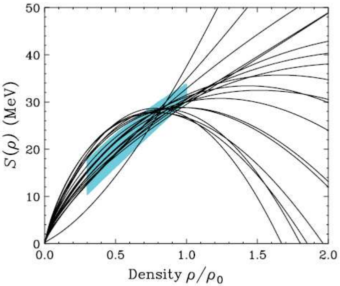
Non-relativistic mean-field models (NRMF), based on Hamiltonians where ef f. interactions are proposed and tested:

$$V_{\rm Nucl}^{\rm eff} = V_{\rm attractive}^{\rm long-range} + V_{\rm repulsive}^{\rm short-range} + V_{\rm SO} + ...$$

-Fitted **param. contain** (important) **correlations beyond MF**
-EDFs are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction** in the vacuum

The Nuclear Equation of State: Infinite System



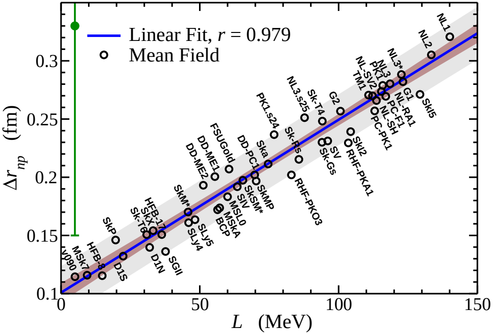


$$e(\rho, \beta) = e(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \quad \text{where} \;\; \beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

-**Isovector properties** not well determined in current **EDFs**
-**Rare Ion Beam Facilites:** systematic study of properties in exotic nuclei (large neutron to proton asymmetry) → more **sensitive to the isovector channel** of the effective interaction (**promising prespectives**)

But how we can better constraint the isovector channel from observables? (Example)

Neutron skin thickness → is one of the most paradigmatic example of an **isovector sensitive observable**.



$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \sim \frac{1}{12}\frac{IR}{J} L \quad \text{where} \quad L \equiv 3\rho_0 \frac{\partial S}{\partial \rho}\Big|_{\rho_0} = 3\rho_0 p_0^{neut}$$

The correlation is physically meaningful

THE DIPOLE POLARIZABILITY

(It is an isovector sensitive observable?)

Dipole polarizability: definition

From a macroscopic perspective

The electric **polarizability** measures the tendency of the nuclear **charge distribution to be distorted**

$$\alpha_D \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

From a microscopic perspective

The electric **polarizability** is proportional to the **inverse energy weighted sum rule (IEWSR)** of the electric dipole response in nuclei

$$\alpha_D = \frac{8\pi}{9} e^2 \sum \frac{B(E1)}{E}$$

or

$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_{\text{ph. abs.}}(E)}{E^2} dE$$

In more detail (from theory) ...

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form $(Fe^{iwt} + F^\dagger e^{-iwt})$:

$$F_{JM} = \sum_i^A r^J Y_{JM}(\hat{r})\tau_z(i) \; (\Delta L = 1 \rightarrow \mathrm{Dipole})$$

- is proportional to the **static polarizability** for small oscillations

$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_\nu |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**

STATISTIC UNCERTAINTIES AND CORRELATIONS IN EDFs

Example on the dipole polarizability

Covariance analysis: χ^2 test

- Observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$ (e.g. of an EDF)

$$\chi^2(\mathbf{p}) = \frac{1}{m - n_p - 1} \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

- Assuming that the χ^2 can be approximated by an hyper-parabola around the minimum $\mathbf{p}_0$,

$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j})$$

where $\mathcal{M} \equiv \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2$ (curvature m.) and $\mathcal{E} \equiv \mathcal{M}^{-1}$ (error m.).

- errors between predicted observables $\mathcal{A}$

$$\Delta \mathcal{A} = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A}$$

- correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}}$$

where, $C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$

Example: Fitting protocol of SLy5-min (NON-Rel) and DD-ME-min1 (Rel)

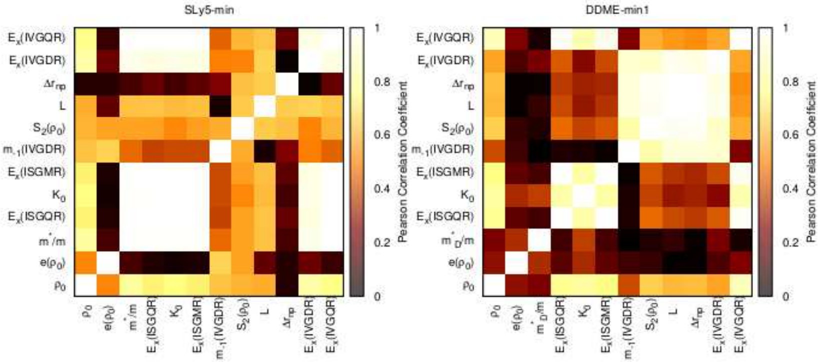
SLy5-min:

- **Binding energies** of ^{40,48}Ca, ^{56}Ni, ^{130,132}Sn and ^{208}Pb with a fixed adopted error of 2 MeV
- the **charge radius** of ^{40,48}Ca, ^{56}Ni and ^{208}Pb with a fixed adopted error of 0.02 fm
- the **neutron matter** Equation of State calculated by Wiringa *et al.* (1988) for densities between 0.07 and 0.40 fm^{-3} with an adopted error of 10%
- the **saturation energy** ($e(\rho_0) = -16.0 \pm 0.2$ MeV) and **density** ($\rho_0 = 0.160 \pm 0.005$ fm^{-3}) of **symmetric nuclear matter**.

DD-ME-min1:

- **binding energies, charge radii, diffraction radii and surface thicknesses** of 17 even-even spherical nuclei, ^{16}O, ^{40,48}Ca, ^{56,58}Ni, ^{88}Sr, ^{90}Zr, ^{100,112,120,124,132}Sn, ^{136}Xe, ^{144}Sm and ^{202,208,214}Pb. The assumed errors of these observables are 0.2%, 0.5%, 0.5%, and 1.5%, respectively.

Covariance analysis: SLy5-min (NON-Rel) and DD-ME-min1 (Rel)



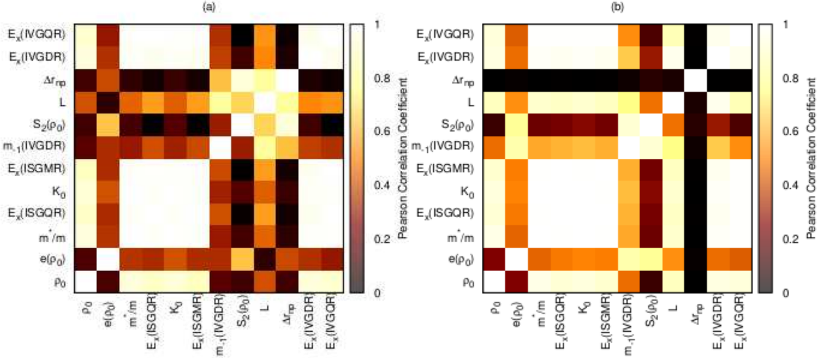
J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

The **neutron skin** is **strongly correlated** with **L** in both models but **NOT** with α_D. **(I will come back on that latter)**

Covariance analysis: modifying the χ^2

→ **SLy5-a:** χ^2 as in SLy5-min except for the neutron EoS (relaxed the required accuracy increasing associated error).

→ **SLy5-b:** χ^2 as in SLy5-min except the neutron EoS (not employed) and used instead a tight constraint on the Δr_{np} in ^{208}Pb



J. Phys. G: Nucl. Part. Phys. 42 034033 (2015).

- When a **constraint** on a property is **relaxed**, **correlations** of other observables with such a property should become **larger** → **SLy5-a: α_D is now better correlated with Δr_{np}**
- When a **constraint** on a property is **enhanced** —artificially or by an accurate experimental measurement— **correlations** of other observables with such a property should become **small** → **SLy5-b: Δr_{np} is not correlated with any other observable**

SISTEMATIC UNCERTAINTIES AND CORRELATIONS IN EDFs

Example on the dipole polarizability

Dipole polarizability: macroscopic approach

> The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda \mathcal{D}$.

Adopting the Droplet Model ($m_{-1} \propto \alpha_D$):

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

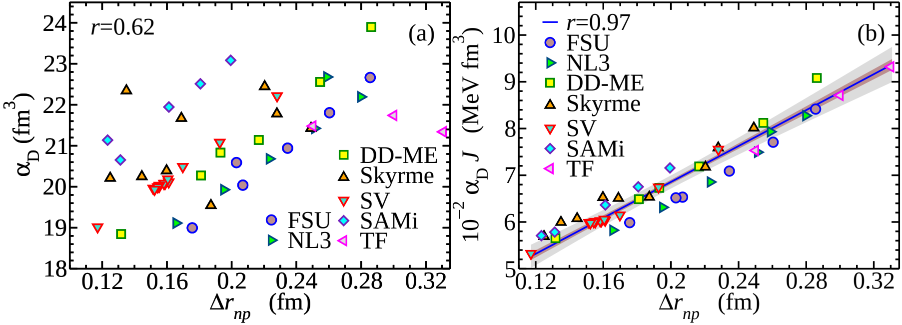
within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Is this correlation appearing also in EDFs?

Isovector Giant Dipole Resonance in ^{208}Pb:

Dipole polarizability: microscopic results HF RPA

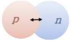


X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

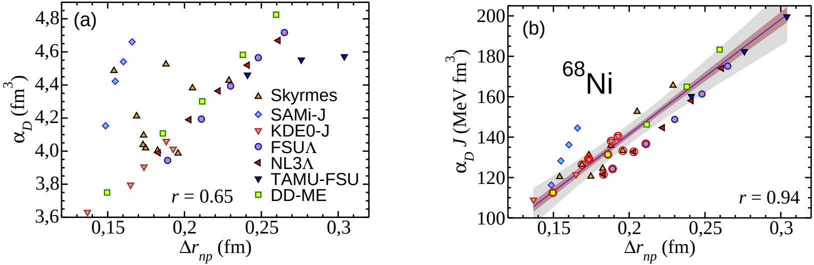
Experimental dipole polarizability $\alpha_D = 20.1 \pm 0.6$ fm^3; A. Tamii *et al.*, PRL 107, 062502 (RCNP) [No quasi-deuteron $\alpha_D = 19.6 \pm 0.6$ fm^3].

$\alpha_D J$ is linearly correlated with Δr_{np} and no α_D alone within EDFs

Isovector Giant Dipole Resonance in ^{68}Ni:



Dipole polarizability: microscopic results HF RPA



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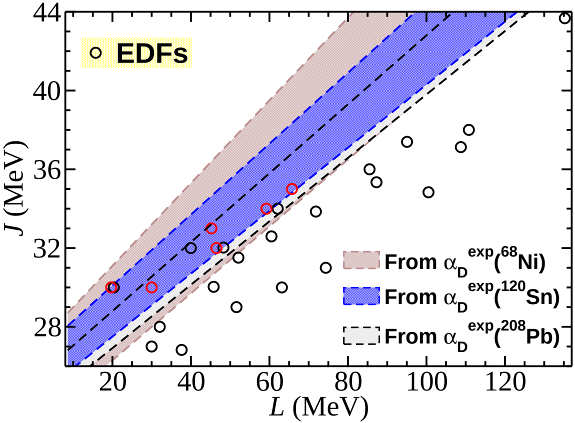
Experimental dipole polarizability $\alpha_D = 3.40 \pm 0.23$ fm^3 D. M. Rossi *et al.*, PRL 111, 242503 (GSI). $\alpha_D = 3.88 \pm 0.31$ fm^3 “full” response D. M. Rossi, T. Aumann, and K. Boretzky.

Constraints of this analysis on the $J - L$ plane



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Experimental dependence of $J - L$ does not exactly follows the trend of theoretical models used to analyze the data!

This might be an indication of the limitation of current (employed) models and the need to imporve them.

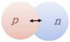
CONCLUSIONS

Conclusions:

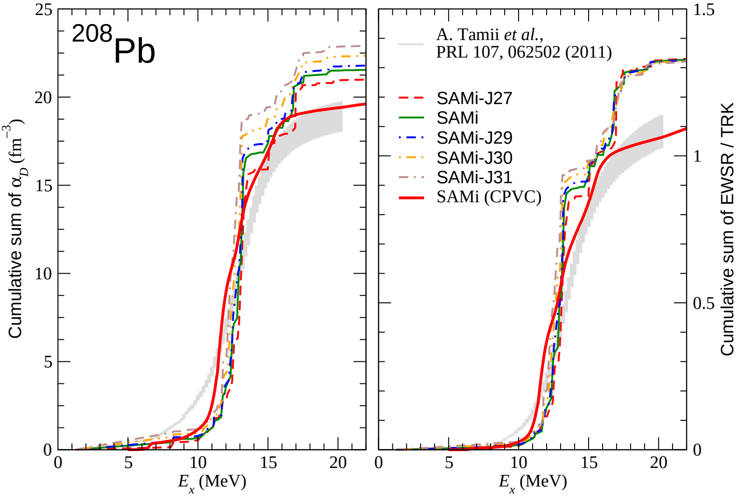
- We have **studied** theoretically **how sensitive is the isovector** channel of the interaction to a **measurement** of the **dipole polarizability** in a heavy nucleus such as ^{208}Pb.
- we have proposed a **physically meaningful correlation** between the polarizability and the properties of the effective interaction: $\alpha_D J$ vs Δr_{np} and not α_D alone.
- Our **results** for ^{208}Pb can be **extended to other nuclei** such as the exotic ^{68}Ni.
- Within our approach, we have **derived three bands in the** $J - L$ **plane** consistent with the recent measurements of the polarizability in ^{68}Ni, ^{120}Sn and ^{208}Pb
- **one EDF consistent** with all the three **bands** is **not consistent with one of the experiments** on the polarizability.
- **one EDF consistent** with all the three **experimental results** do **not overlap all three bands** (although it is close).
- The **slope shown by the derived bands** in the $J - L$ is **not strictly followed by the models** used for the analysis

EXTRA MATERIAL

^{208}Pb:

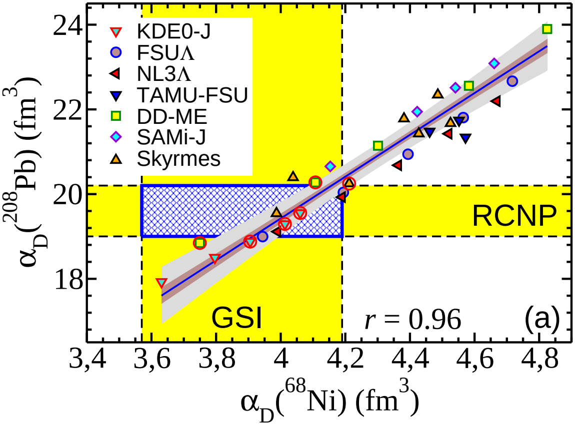


Dipole polarizability: microscopic results



^{208}Pb vs ^{68}Ni:

Dipole polarizability: microscopic results HF RPA



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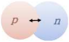
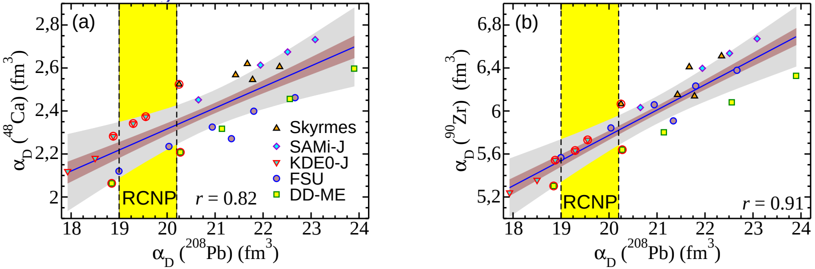
Just as an indication: $\alpha_D(A = 208)/\alpha_D(A = 68) \sim (208/68)^{5/3}$;
Cercled models predict $\Delta r_{np}(^{208}Pb) = 0.125 - 0.207$ fm and $\Delta r_{np}(^{68}Ni) = 0.146 - 0.211$ fm; J = 30 − 35 MeV; L = 30 − 65 MeV.

Can we use this information to predict the polarizability in other nuclei?





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Nucleus	Δr_{np} (fm)	α_D (fm^3)
^{48}Ca	0.15−0.18 (0.16 ± 0.01)	2.06−2.52 (2.30 ± 0.14)
^{90}Zr	0.058−0.077 (0.067 ± 0.008)	5.30−6.06 (5.65 ± 0.23)

Table: Estimates for the neutron skin thickness and electric dipole polarizability of ^{48}Ca and ^{90}Zr from models that predict α_{exp} in ^{68}Ni, ^{132}Sn and ^{208}Pb.